Novel Collider Signals for Unparticle

Kingman Cheung SUSY07, Aug 2007

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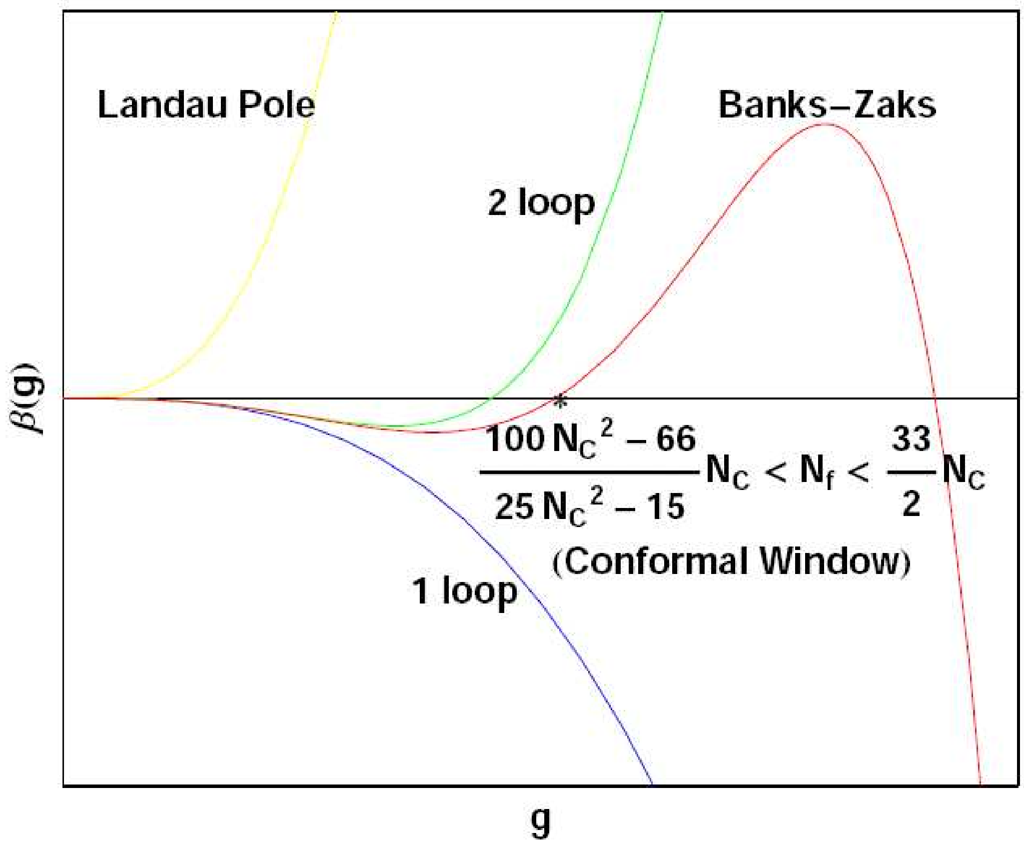
Outline

- Motivations of unparticle
- Derivation of unparticle propagator
- Real emissions of unparticle and constraints
- Virtual exchange: 4-fermion interactions and constraints

Motivations for Unparticle

(Georgi hep-ph/0703260 PRL)

Banls-Zaks (1982) – Perturbative IR fixed point



Scale Invariant Sector

- Suppose there exists an exact scale-invariant sector at a high energy scale $M_{\mathcal{U}}$ with a non-trivial IR fixed point (e.g., Bank-Zaks).
- The underlying conformal dynamics can be strongly interacting or complex. But suppose it decouples at low energy and interacts weakly with the SM, we can EFT approach to study this sector.
- Below $M_{\mathcal{U}}$

$$\frac{1}{M_{\mathcal{U}}^{d_{\rm SM}+d_{\mathcal{BZ}}-4}}\,\mathcal{O}_{\rm SM}\,\mathcal{O}_{\mathcal{BZ}}$$

- Renormalization effects in the scale invariant $\mathcal{BZ}$ sector induce dimensional transmutation at an energy scale $\Lambda_{\mathcal{U}}$. Below $\Lambda_{\mathcal{U}}$

$$C_{\mathcal{O}_{\mathcal{U}}}\,\frac{\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^{d_{\rm SM}+d_{\mathcal{BZ}}-4}}\,\mathcal{O}_{\rm SM}\,\mathcal{O}_{\mathcal{U}}\ ,$$

Phase space for emission of unparticle

Consider a two-point function for a scalar unparticle operator $O_{\mathcal{U}}$

$$\langle 0 | O_{\mathcal{U}}(x) O^\dagger_{\mathcal{U}}(0) | 0 \rangle = \int \frac{d^4 P}{(2\pi)^4} \, e^{-iP\cdot x} \, \rho_{\mathcal{U}}(P^2)$$

where

$$\begin{aligned} \rho_{\mathcal{U}}(P^2) &= \int d^4x \, e^{iP\cdot x} \langle 0 | O_{\mathcal{U}}(x) O^\dagger_{\mathcal{U}}(0) | 0 \rangle \\ &= A_{d_{\mathcal{U}}} \; \theta(P^0) \; \theta(P^2) \; (P^2)^\alpha \end{aligned}$$

where α is an index to be determined based on scale invariance and $A_{d_{\mathcal{U}}}$ is a normalization factor also required to be fixed.

- Under a scale transformation $x \to s\,x$ and $O_{\mathcal{U}}(sx) \to s^{-d_{\mathcal{U}}} O_{\mathcal{U}}(x)$, we have

$$\begin{aligned} A_{d_{\mathcal{U}}} \; \theta(P^0) \; \theta(P^2)(P^2)^\alpha &= \int d^4x s^4 e^{isP\cdot x} \langle 0 | s^{-2d_{\mathcal{U}}} O_{\mathcal{U}}(x) O^\dagger_{\mathcal{U}}(0) | 0 \rangle \\ &= s^{-2(d_{\mathcal{U}}-2)} A_{d_{\mathcal{U}}} \; \theta(sP^0) \; \theta(s^2 P^2) \; (s^2 P^2)^\alpha \end{aligned}$$

  that determines $\alpha = d_{\mathcal{U}} - 2$.

- $A_{d_\mathcal{U}}$ is normalized to interpret as the $d_\mathcal{U}$-body phase space of massless particles (Georgi PRL)

$$A_{d_\mathcal{U}} = \frac{16\pi^2\sqrt{\pi}}{(2\pi)^{2d_\mathcal{U}}} \frac{\Gamma(d_\mathcal{U}+\frac{1}{2})}{\Gamma(d_\mathcal{U}-1)\Gamma(2\,d_\mathcal{U})} \,.$$

$d_\mathcal{U}$ can take on non-integral values.

- The differential cross section for $ij \to \mathcal{U} + \text{particles}$

$$d\sigma(p_1, p_2 \to P_\mathcal{U}, k_1, k_2, \ldots) = \frac{1}{2s}\,|\overline{\mathcal{M}}|^2\, d\Phi$$

$$\begin{aligned} d\Phi \quad &= \quad (2\pi)^4\,\delta^{(4)}\,(p_1+p_2-P_\mathcal{U}-k_1-k_2-\ldots)\prod_i \left[2\pi\,\theta(k_i^0)\,\delta(k_i^2)\,\frac{d^4k_i}{(2\pi)^4}\right] \\ &\times \quad A_{d_\mathcal{U}}\,\theta(P_\mathcal{U}^0)\,\theta(P_\mathcal{U}^2)\,\left(P_\mathcal{U}^2\right)^{d_\mathcal{U}-2}\,\frac{d^4P_\mathcal{U}}{(2\pi)^4} \end{aligned}$$

- In the limit $d_\mathcal{U} \to 1$ from above

$$\lim_{d_\mathcal{U}\to 1^+} A_{d_\mathcal{U}}\,(P_\mathcal{U}^2)^{d_\mathcal{U}-2}\,\theta(P_\mathcal{U}^0)\,\theta(P_\mathcal{U}^2) = 2\pi\theta(P_\mathcal{U}^0)\,\delta(P_\mathcal{U}^2)\;,$$

so that the phase-space factor associated with the unparticle behaves just a single massless particle.

Conjecture to Extra Dimensions

(KC, Keung, Yuan; Stephanov)

In flat large extra dimensions with periodic boundary conditions all the momenta k_i are quantized to integral multiples of $2\pi/R$.

For large R, the summation over the KK modes turns into an integral and the density of states is introduced as

$$\sum_{\vec{k}} \longrightarrow \int \left(\frac{R}{2\pi}\right)^n d^n k = \int \frac{R^n (m^2)^{\frac{n}{2}-1} dm^2}{(4\pi)^{\frac{n}{2}} \Gamma(\frac{n}{2})}$$

Identifying the power of m^2 in the density of states with the power of P^2 in the spectral density of the unparticle, we obtain

$$(m^2)^{n/2-1} = (P_{\mathcal{U}}^2)^{d_{\mathcal{U}}-2} \quad \Longrightarrow \quad d_{\mathcal{U}} = \frac{n}{2} + 1 \; .$$

We may also interpret $A_{d_{\mathcal{U}}}$ as

$$A_{d_{\mathcal{U}}} = \frac{R^{2(d_{\mathcal{U}}-1)}}{(4\pi)^{d_{\mathcal{U}}-1} \Gamma(d_{\mathcal{U}}-1)}$$

Virtual propagator of unparticle

The Feynman propagator $i\Delta_F(P^2)$ of the unparticle is given by the spectral formula

$$\begin{aligned}\Delta_F(P^2) &= \frac{1}{2\pi}\int_0^\infty \frac{R(M^2)dM^2}{P^2-M^2+i\epsilon} \\ &= \frac{1}{2\pi}\mathcal{P}\int_0^\infty \frac{R(M^2)dM^2}{P^2-M^2} - i\frac{1}{2}R(P^2)\theta(P^2)\end{aligned}$$

where $R(M^2) = A_{d_\mathcal{U}}(M^2)^{d_\mathcal{U}-2}$.

The scale-invariant form of $\Delta_F(P^2) = Z_{d_\mathcal{U}}(-P^2)^{d_\mathcal{U}-2}$. The complex function $(-P^2)^{d_\mathcal{U}-2}$ has a branch cut in the positive P^2:

$$(-P^2)^{d_\mathcal{U}-2} = \begin{cases} |P^2|^{d_\mathcal{U}-2} & \text{if } P^2 \text{ is negative and real,} \\ |P^2|^{d_\mathcal{U}-2}e^{-id_\mathcal{U}\pi} & \text{for positive } P^2 \text{ with an infinitesimal } i0^+. \end{cases}$$

The factor $Z_{d_\mathcal{U}}$ is determined by the dispersion relation:

$$\Im\mathrm{m}\Delta_F(P^2) = -Z_{d_\mathcal{U}} \sin(d_\mathcal{U}\pi)(P^2)^{d_\mathcal{U}-2} = -\frac{1}{2} A_{d_\mathcal{U}} \, (P^2)^{d_\mathcal{U}-2} \, .$$

$$i\Delta_F(P^2) = i\frac{A_{d_\mathcal{U}}}{2\sin(d_\mathcal{U}\pi)}(-P^2)^{d_\mathcal{U}-2} \, ,$$

Other spin-1 and spin-2 propagators:

$$\begin{aligned} \left[i\Delta_F(P^2)\right]_{\mu\nu} &= i\frac{A_{d_\mathcal{U}}}{2\sin(d_\mathcal{U}\pi)} \, (-P^2)^{d_\mathcal{U}-2} \, \pi_{\mu\nu}(P) \, , \\ \left[i\Delta_F(P^2)\right]_{\mu\nu,\rho\sigma} &= i\frac{A_{d_\mathcal{U}}}{2\sin(d_\mathcal{U}\pi)} \, (-P^2)^{d_\mathcal{U}-2} \, T_{\mu\nu,\rho\sigma}(P) \, . \end{aligned}$$

where

$$\begin{aligned} \pi^{\mu\nu}(P) &= -g^{\mu\nu} + \frac{P^\mu P^\nu}{P^2} \\ T^{\mu\nu,\rho\sigma}(P) &= \frac{1}{2}\left[\pi^{\mu\rho}(P)\,\pi^{\nu\sigma}(P) + \pi^{\mu\sigma}(P)\,\pi^{\nu\rho}(P) - \frac{2}{3}\,\pi^{\mu\nu}(P)\,\pi^{\rho\sigma}(P)\right] \end{aligned}$$

Unparticle operators

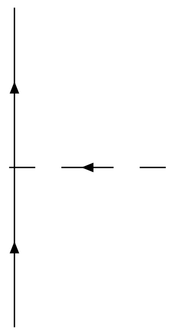
Spin $-$ 0 $$\lambda_0 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} f O_{\mathcal{U}} \ , \quad \lambda_0 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} i \gamma^5 f O_{\mathcal{U}} \ ,$$

$$\lambda_0 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{f} \gamma^\mu f (\partial_\mu O_{\mathcal{U}}) \ , \lambda_0 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} G_{\alpha\beta} G^{\alpha\beta} O_{\mathcal{U}} \ ,$$

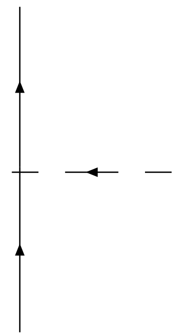
Spin $-$ 1 $$\lambda_1 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} \gamma_\mu f \, O_{\mathcal{U}}^\mu \ , \quad \lambda_1 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} \gamma_\mu \gamma_5 f \, O_{\mathcal{U}}^\mu \ ,$$

Spin $-$ 2 $$-\frac{1}{4} \lambda_2 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{\psi} \, i \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \psi \, O_{\mathcal{U}}^{\mu\nu} \ , \lambda_2 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} G_{\mu\alpha} G_\nu^{\ \alpha} O_{\mathcal{U}}^{\mu\nu}$$

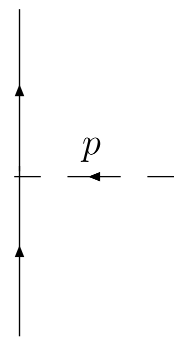
Three-point Vertices



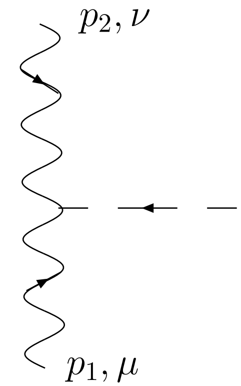
$i\frac{\lambda_0}{\Lambda_U^{d_U-1}}$



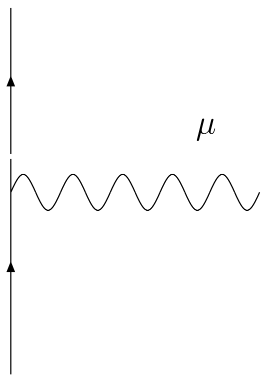
$-\frac{\lambda_0}{\Lambda_U^{d_U-1}}\gamma^5$



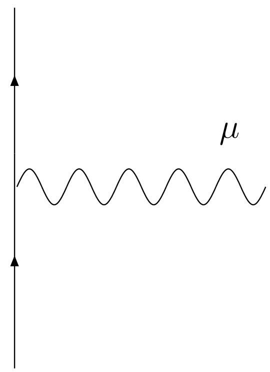
$\frac{\lambda_0}{\Lambda_U^{d_U}}\ \not{p}$



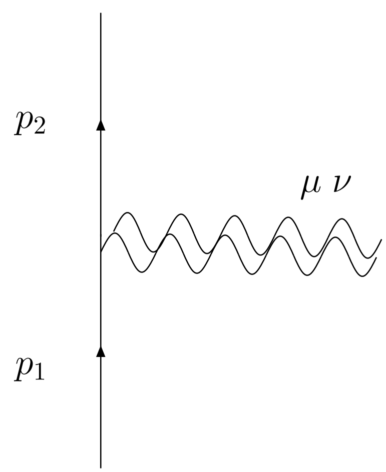
$4i\frac{\lambda_0}{\Lambda_U^{d_U}}(-p_1 \cdot p_2 g^{\mu\nu} + p_1^\nu p_2^\mu)$



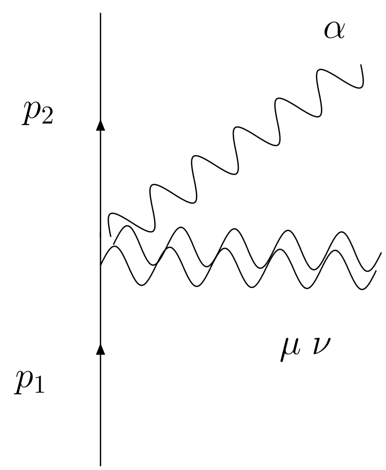
$i\frac{\lambda_1}{\Lambda_U^{d_U-1}}\gamma^\mu$



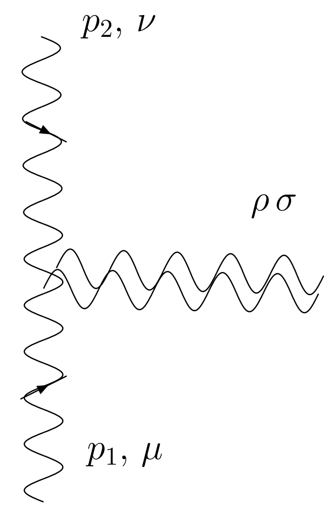
$i\frac{\lambda_1}{\Lambda_U^{d_U-1}}\gamma^\mu\gamma^5$



$-\frac{i}{4}\frac{\lambda_2}{\Lambda_U^{d_U}}[\gamma^\mu(p_1^\nu + p_2^\nu) + \gamma^\nu(p_1^\mu + p_2^\mu)]$

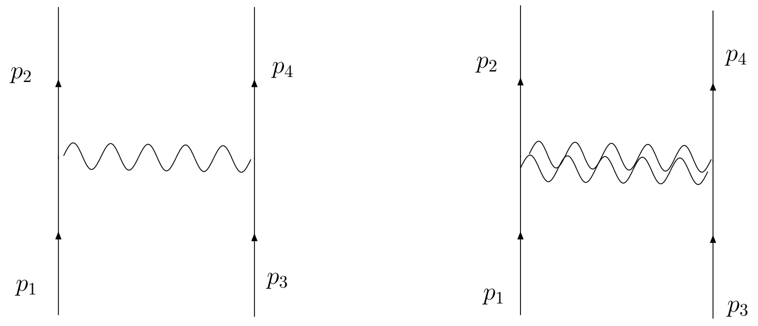


$\frac{i}{2}\frac{e\lambda_2}{\Lambda_U^{d_U}}[\gamma^\mu g^{\nu\alpha} + \gamma^\nu g^{\mu\alpha}]Q_f$



$i\frac{\lambda_2}{\Lambda_U^{d_U}}[K^{\mu\nu\rho\sigma} + K^{\mu\nu\sigma\rho}]$

Effective four-fermion interactions



(a) (b)

Virtual exchange of spin-1 unparticle $O^\mu_{\mathcal{U}}$ between two fermionic currents results in 4-fermion interaction

$$\mathcal{M}_1^{4f} = \lambda_1^2 \, Z_{d_{\mathcal{U}}} \, \frac{1}{\Lambda^2_{\mathcal{U}}} \left(- \frac{P^2_{\mathcal{U}}}{\Lambda^2_{\mathcal{U}}} \right)^{d_{\mathcal{U}}-2} \, (\bar{f}_2 \gamma_\mu P_\alpha f_1) \, (\bar{f}_4 \gamma^\mu P_\beta f_3)$$

The exchange of spin-2 unparticle gives the following 4-fermion interaction

$$\begin{aligned}\mathcal{M}_2^{4f} \quad &= \quad -\frac{1}{8} \lambda_2'^2 \, Z_{d_{\mathcal{U}}} \, \frac{1}{\Lambda^4_{\mathcal{U}}} \left(- \frac{(p_1-p_2)^2}{\Lambda^2_{\mathcal{U}}} \right)^{d_{\mathcal{U}}-2} \left(\bar{f}_2 \gamma^\mu f_1 \right) \, \left(\bar{f}_4 \gamma^\nu f_3 \right) \\ &\times \quad [(p_1+p_2)\cdot(p_3+p_4) g_{\mu\nu} + (p_1+p_2)_\nu (p_3+p_4)_\mu]\end{aligned}$$

Phenomenological Applications

Real Emissions: missing energy signals

- Single photon, single Z production at e^+e^- colliders
- Mono-jet production at hadronic colliders
- Fermion-pair plus missing energy at Z decay

Virtual exchange:

- Drell-Yan
- Fermion-pair production at e^+e^- colliders
- Diphoton production

Real Emissions

Single Z and single γ production at e^+e^- Machine

The differential cross section for $f(p)\,\bar{f}(p') \to Z(k)\,\mathcal{U}(P_{\mathcal{U}})$

$$d\sigma = \frac{1}{2s}\,|\overline{\mathcal{M}}|^2\,\frac{\sqrt{E_Z^2 - M_Z^2}\,A_{d_{\mathcal{U}}}}{16\pi^3\Lambda_{\mathcal{U}}^2}\left(\frac{P_{\mathcal{U}}^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-2}\,\theta(P_{\mathcal{U}}^0)\theta(P_{\mathcal{U}}^2)dE_Z d\Omega_Z \ ,$$

$$P_{\mathcal{U}}^2 = s + M_Z^2 - 2\sqrt{s}\,E_Z \ ,$$

We do not actually measured $P_{\mathcal{U}}^2$, but we can use the recoil mass spectrum of the Z.

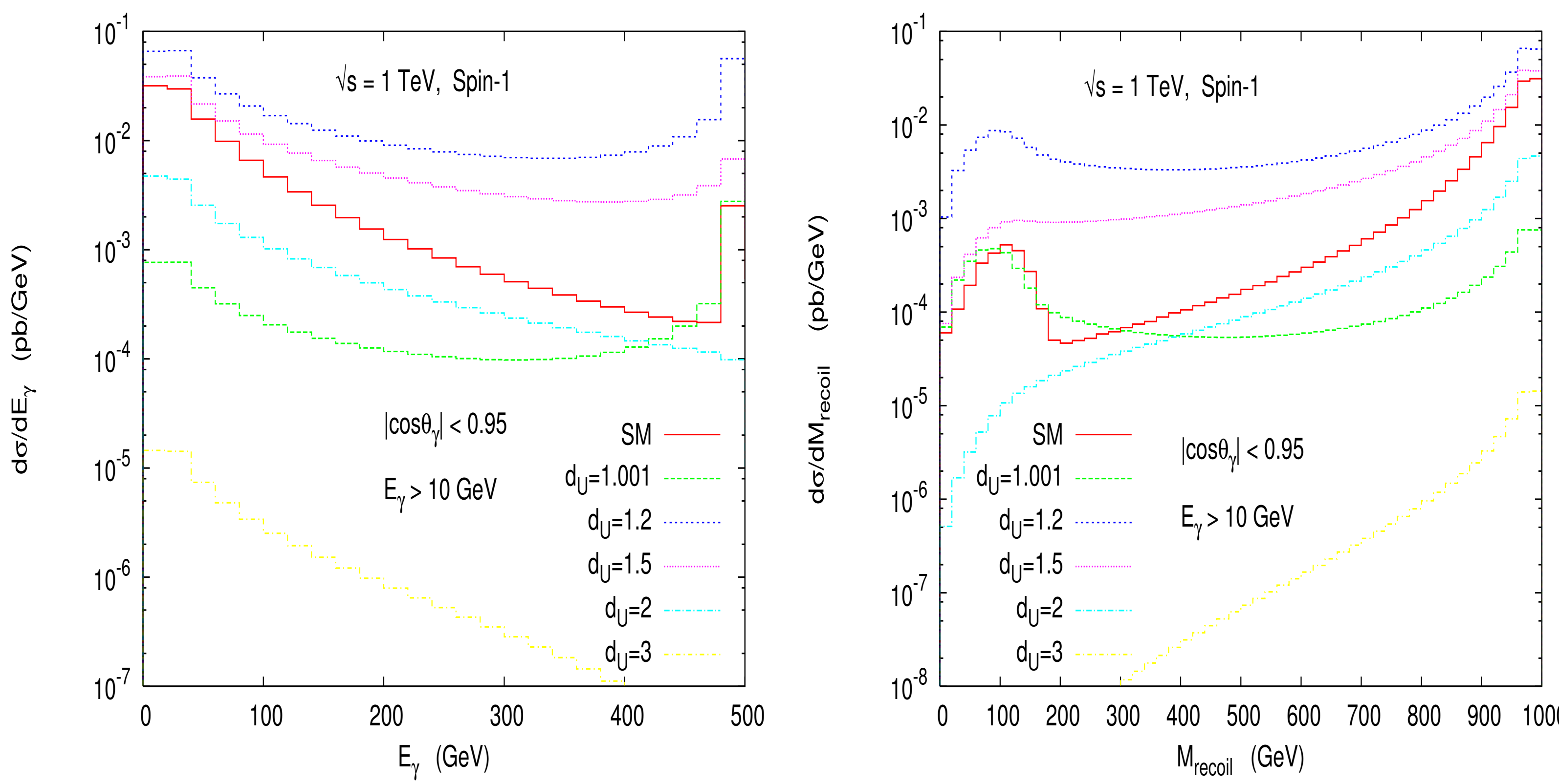
Spin 1 unparticle:

$$|\overline{\mathcal{M}}|^2 = \frac{2}{N_c}\lambda_1^2\frac{e^2({g_L^f}^2 + {g_R^f}^2)}{\sin^2\theta_{\rm w}\cos^2\theta_{\rm w}}\,g\left(t/M_Z^2, u/M_Z^2, P_{\mathcal{U}}^2/M_Z^2\right)$$

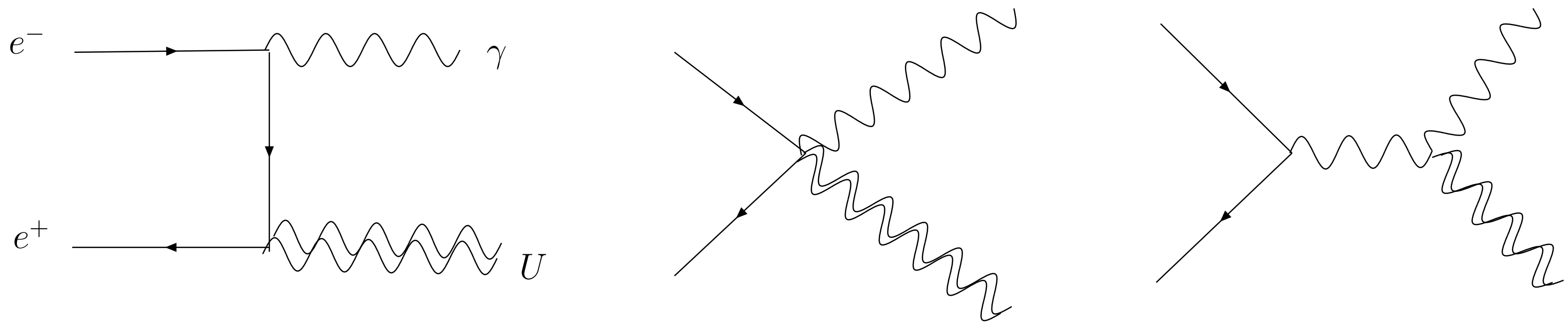
By putting $M_Z \to 0$ we also obtain $e^-e^+ \to \gamma\mathcal{U}$:

$$d\sigma = \frac{1}{2s}\,|\overline{\mathcal{M}}|^2\,\frac{E_\gamma A_{d_{\mathcal{U}}}}{16\pi^3\Lambda_{\mathcal{U}}^2}\left(\frac{P_{\mathcal{U}}^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-2}\,dE_\gamma d\Omega$$

$$|\overline{\mathcal{M}}|^2 = \frac{2}{N_c}\lambda_1^2 e^2 Q_f^2\,\frac{u^2 + t^2 + 2sP_{\mathcal{U}}^2}{ut} \ .$$

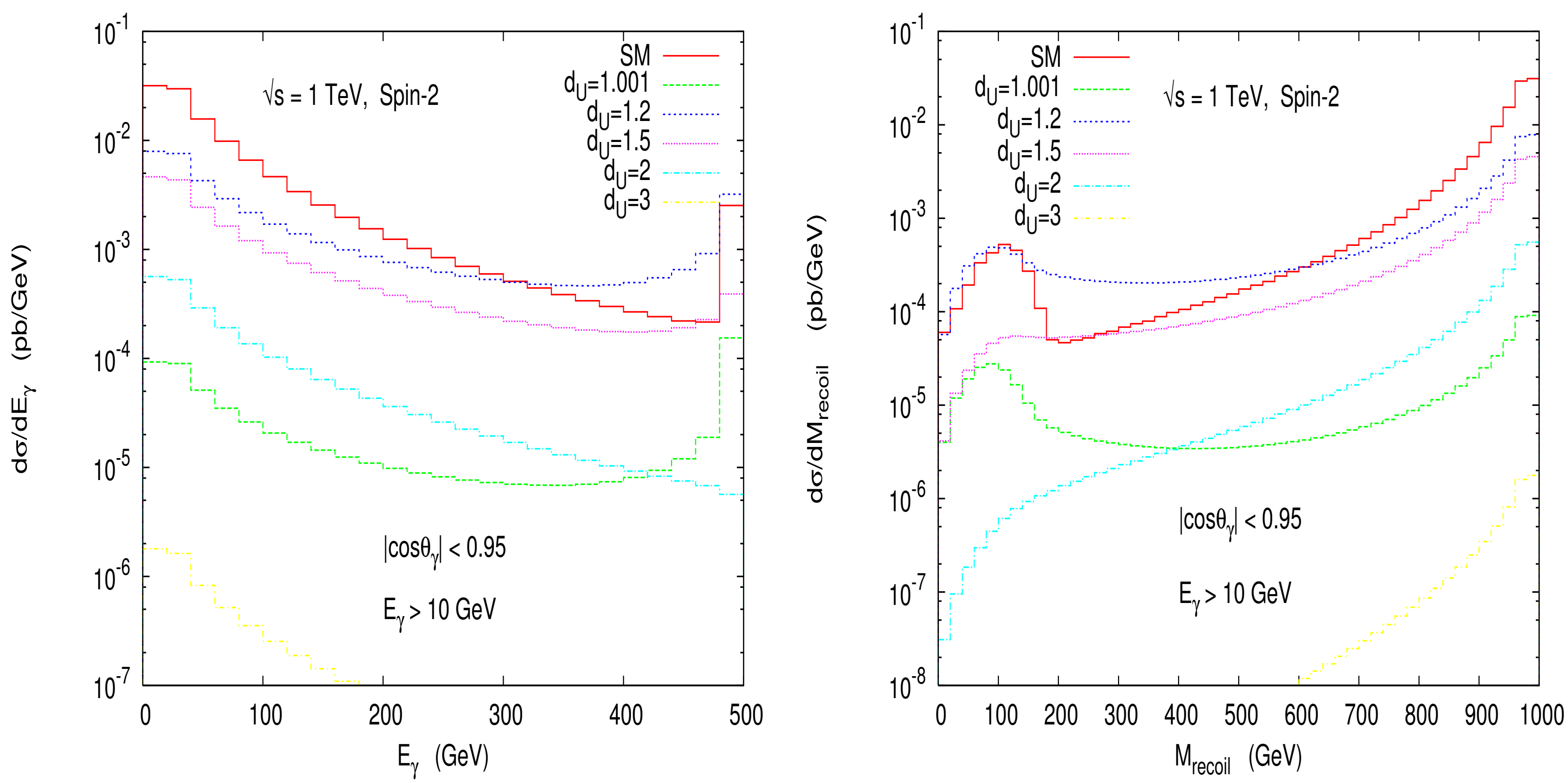

√s = 1 TeV, Spin-1
|cosθ_γ| < 0.95
E_γ > 10 GeV
SM
d_U=1.001
d_U=1.2
d_U=1.5
d_U=2
d_U=3
dσ/dE_γ (pb/GeV)
E_γ (GeV)
dσ/dM_recoil (pb/GeV)
M_recoil (GeV)

Spin 2 unparticle:

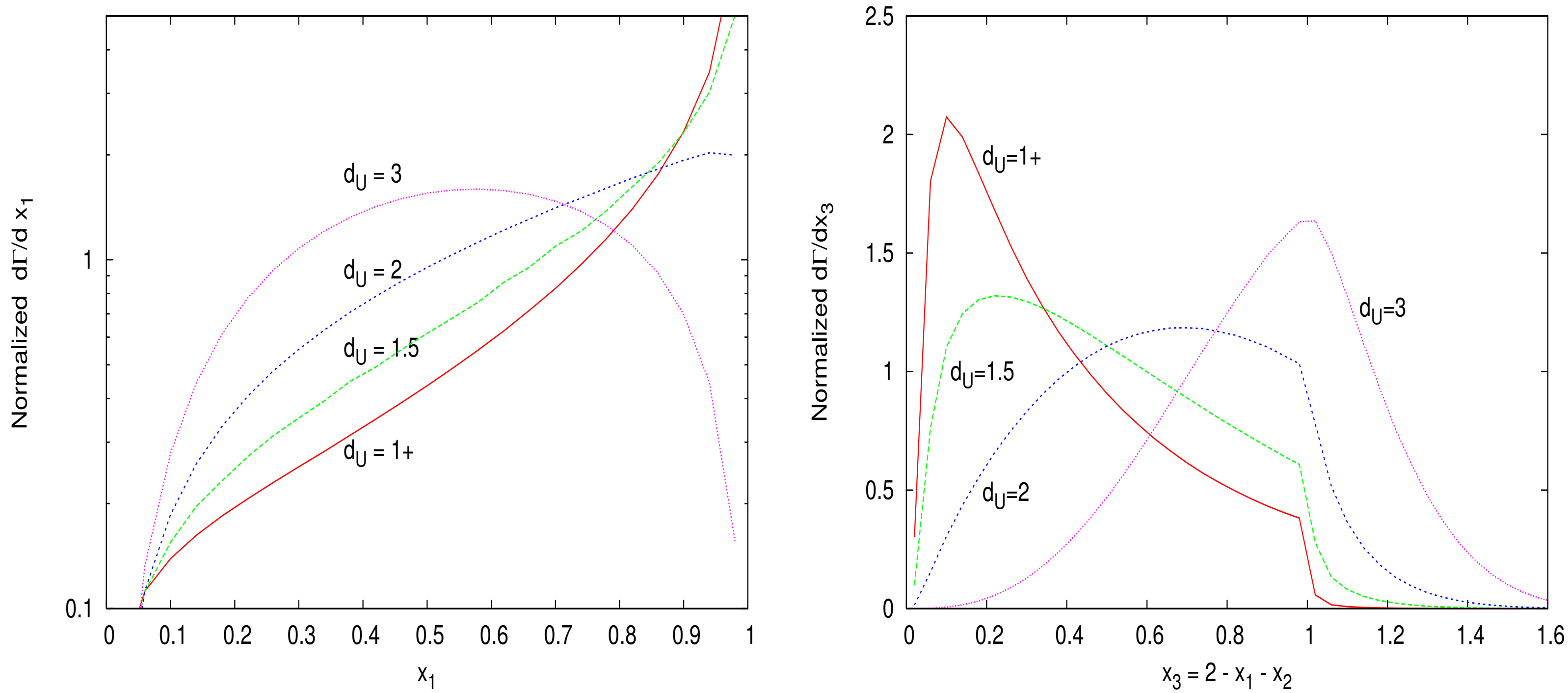


The amplitude square for $f\bar{f} \to \gamma\mathcal{U}$ (equal couplings for the two operators)

$$|\overline{\mathcal{M}}|^2 = \frac{1}{4N_c}\frac{\lambda_2^2}{\Lambda_{\mathcal{U}}^2} e^2 Q_f^2 \frac{1}{stu}\left(2\,s\,P_{\mathcal{U}}^2 + t^2 + u^2\right)\left(s\,P_{\mathcal{U}}^2 + 4\,t\,u\right) \ .$$


√s = 1 TeV, Spin-2
SM
d_U=1.001
d_U=1.2
d_U=1.5
d_U=2
d_U=3
|cosθ$_\gamma$| < 0.95
E_γ > 10 GeV
dσ/dE_γ (pb/GeV)
E_γ (GeV)
dσ/dM_{recoil} (pb/GeV)
M_{recoil} (GeV)

$Z \to f\bar{f}\mathcal{U}$



Monojet production at the LHC

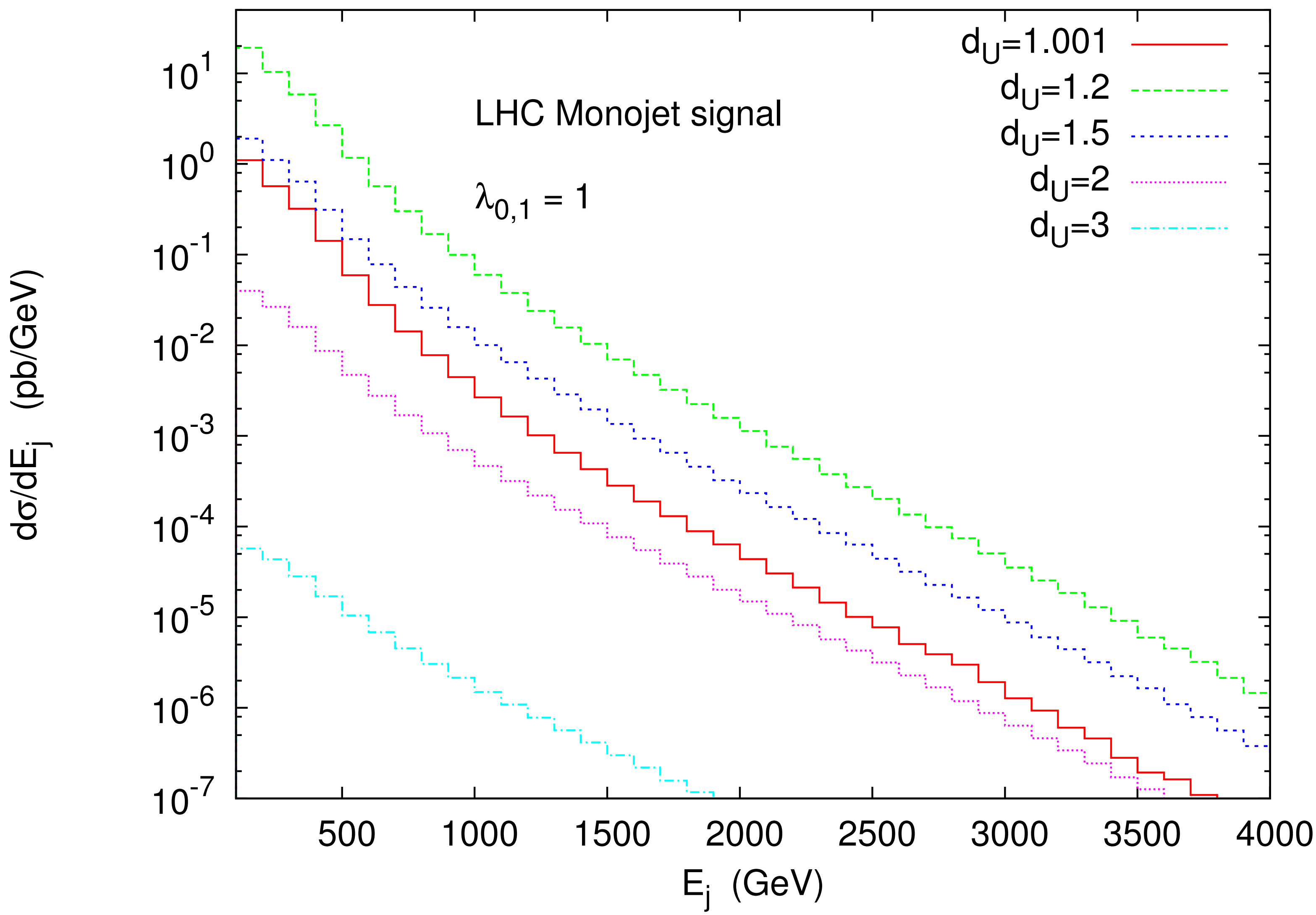
Contributing subprocesses:

$$gg \to g\mathcal{U}\ ,\ \ q\bar{q} \to g\mathcal{U}\ ,\ \ qg \to q\mathcal{U}\ ,\ \ \bar{q}g\ \to\ \bar{q}\mathcal{U}$$

Here we consider the operator $O^\mu_{\mathcal{U}}$ and $O_{\mathcal{U}}$ for gluon fusion. Note that the recoil mass relation becomes:

$$P^2_{\mathcal{U}} = \hat{s} - 2\sqrt{\hat{s}}E_j$$

But $\hat{s} = x_1 x_2 s$ is the center-of-mass energy squared of the partons. It is an unknown so that $P^2_{\mathcal{U}}$ cannot be reconstructed. Information is lost in the E_j spectrum.



LHC Monojet signal
$\lambda_{0,1} = 1$
d_U=1.001
d_U=1.2
d_U=1.5
d_U=2
d_U=3
$d\sigma/dE_j$ (pb/GeV)
E_j (GeV)
500
1000
1500
2000
2500
3000
3500
4000
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}

Experimental Constraint from LEP2

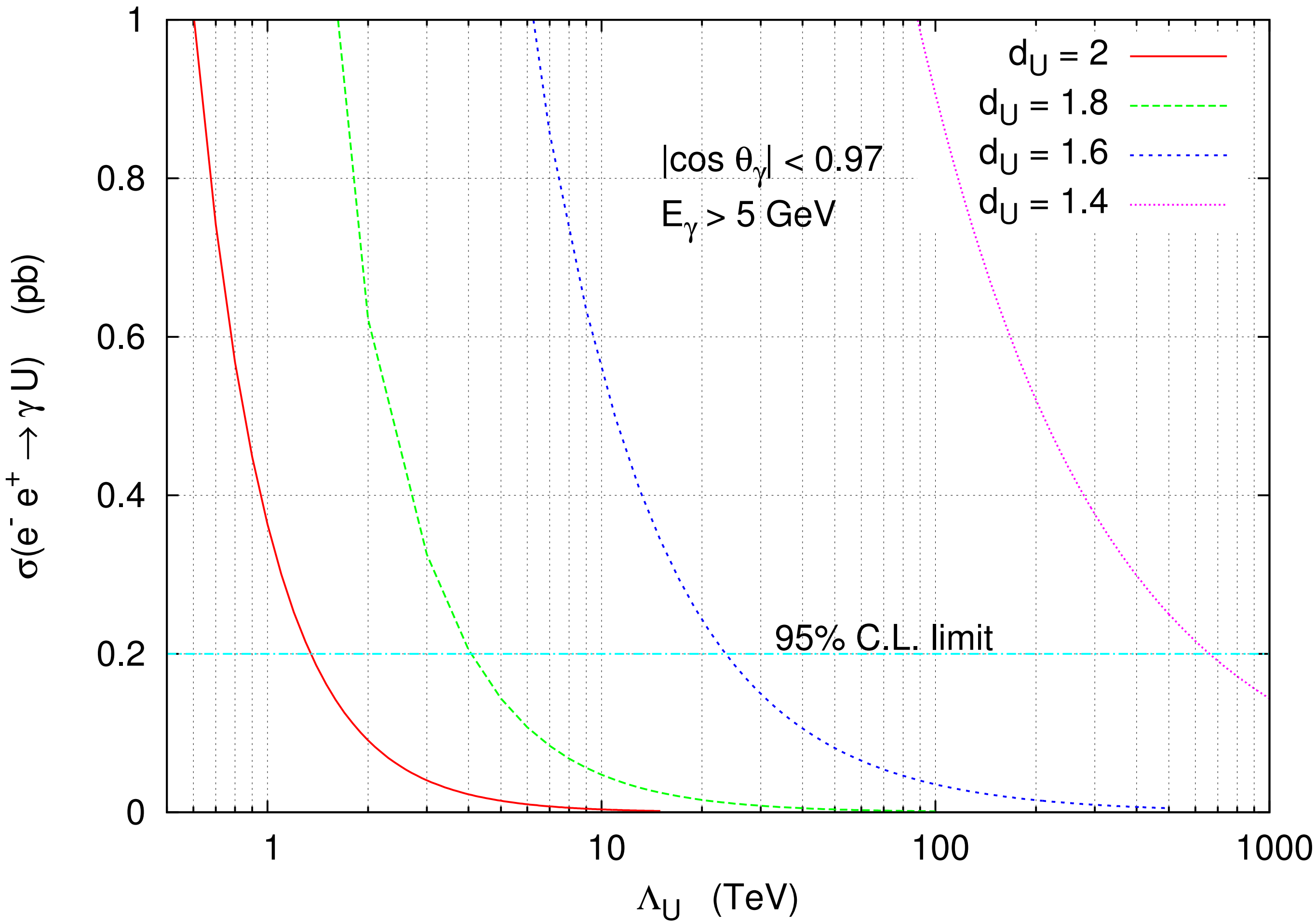
LEP Coll. have measured single-photon plus missing energy in the context of ADD, GMSB, and other models that can produce a single γ plus $\not\!\!E_T$ in the final state.

We use the strongest from L3 at $\sqrt{s} = 207$ GeV:

$$\sigma^{95} \simeq 0.2 \text{ pb under } \quad E_\gamma > 5\,\text{GeV}, \quad |\cos\theta_\gamma| < 0.97$$

Limits on $\Lambda_{\mathcal{U}}$ from single-photon production

$d_{\mathcal{U}}$	$\Lambda_{\mathcal{U}}$ (TeV)
2.0	1.35
1.8	4
1.6	23
1.4	660



L3 95% C.L. upper limit on $\sigma(e^- e^+ \to \gamma + X) \simeq 0.2$ pb under the cuts: $E_\gamma > 5$ GeV and $|\cos\theta_\gamma| < 0.97$ at $\sqrt{s} = 207$ GeV

Virtual Exchange

Drell-Yan: spin 1 unparticle exchange

Spin-1 unparticle exchange will give rise to 4-fermion interactions:

$$\mathcal{M}_1^{ef} = \lambda_1^2 \, Z_{d_\mathcal{U}} \, \frac{1}{\Lambda_\mathcal{U}^2} \left(- \frac{P_\mathcal{U}^2}{\Lambda_\mathcal{U}^2} \right)^{d_\mathcal{U} - 2} \sum_{\alpha,\beta = L,R} \eta_{\alpha\beta} (\bar{e} \gamma_\mu P_\alpha e) \, (\bar{f} \gamma^\mu P_\beta f)$$

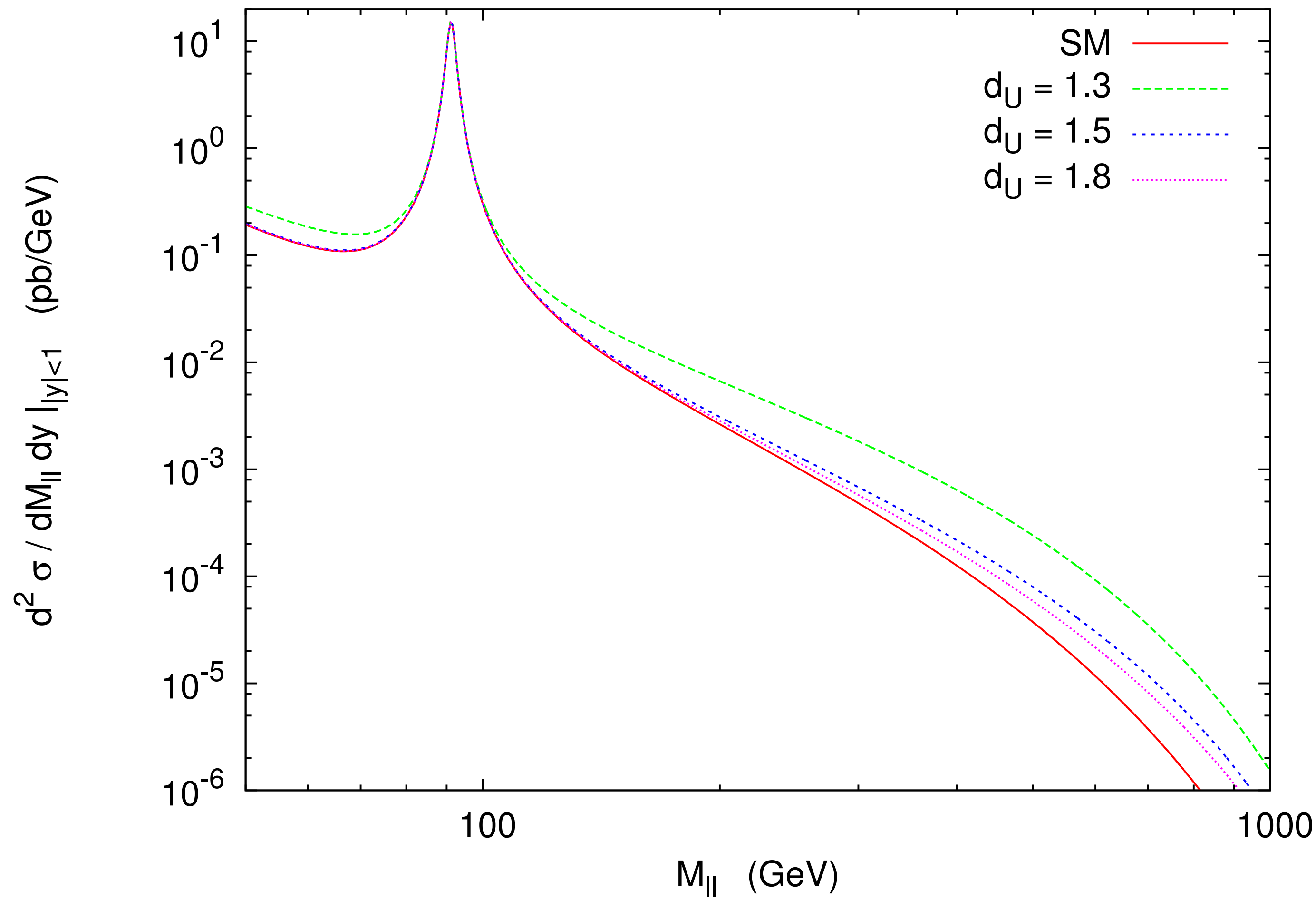
We can have LL, RR, LR, RL interactions. It can generate parity violation, interference effects with the SM amplitudes.

In DY we consider the simplest case $LL = RR = LR = RL$. The hadronic production is

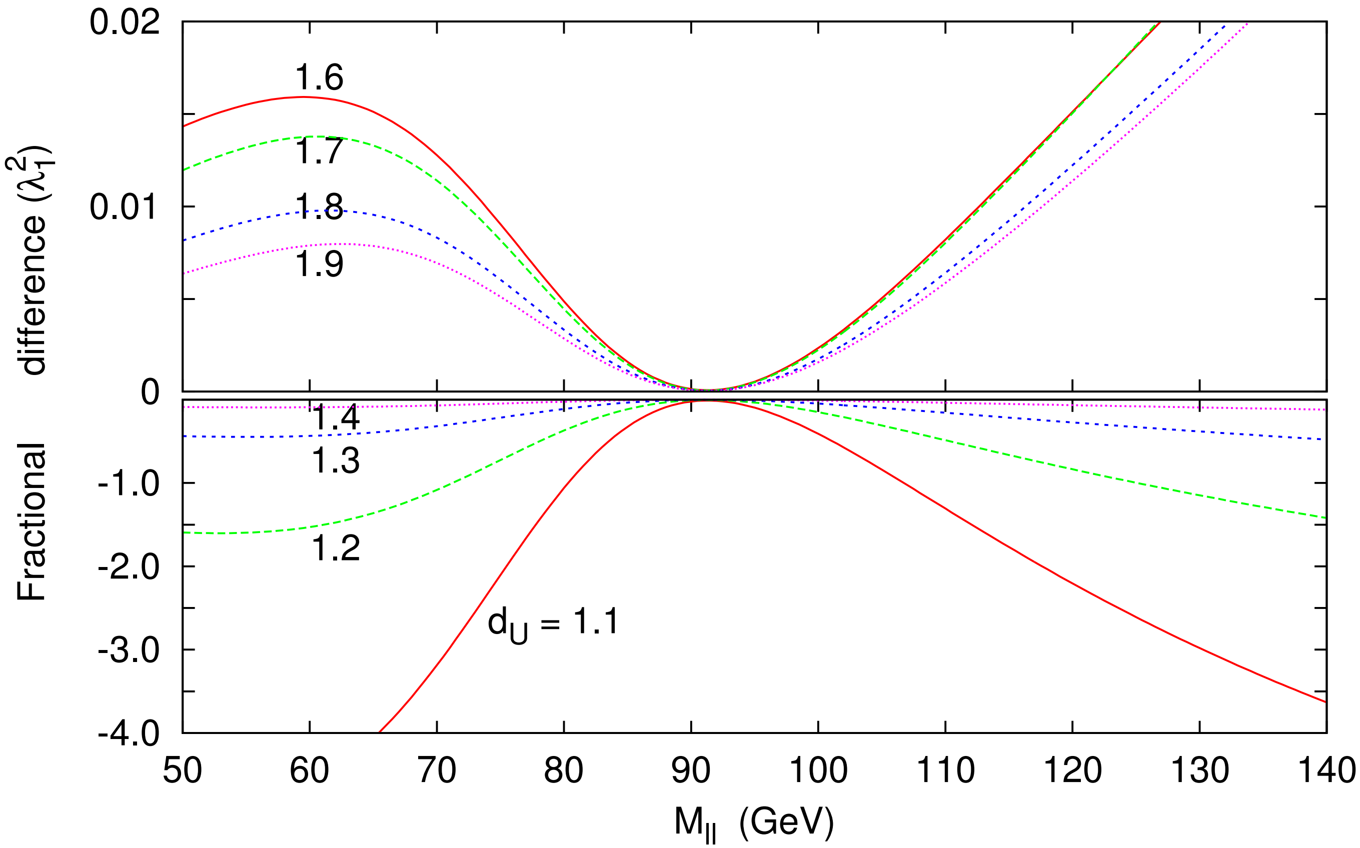
$$\frac{d^2\sigma}{dM_{\ell\ell}\, dy} = K \frac{M_{\ell\ell}^3}{72\pi s} \sum_q f_q(x_1) f_{\bar{q}}(x_2) \; \times \left(|M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2 + |M_{RR}|^2 \right) ,$$

where

$$M_{\alpha\beta} = \; \lambda_1^2 Z_{d_\mathcal{U}} \frac{1}{\Lambda_\mathcal{U}^2} \left(- \frac{\hat{s}}{\Lambda_\mathcal{U}^2} \right)^{d_\mathcal{U} - 2} + \frac{e^2 Q_l Q_q}{\hat{s}} + \frac{e^2 g_\alpha^l g_\beta^q}{\sin^2\theta_{\rm w} \cos^2\theta_{\rm w}} \, \frac{1}{\hat{s} - M_Z^2 + i M_Z \Gamma_Z} \; .$$



Tevatron: Enhancement at the high invariant mass



Interference of $\mathcal{U}$ with γ, Z propagators

Drell-Yan: spin 2 unparticle exchange

The amplitude:

$$i\mathcal{M}_{\mathcal{U}} = -\frac{i}{8}\lambda_2^2\, Z_{d_{\mathcal{U}}}\,\frac{1}{\Lambda_{\mathcal{U}}^4}\left(-\frac{\hat{s}}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-2}\Big[(p_1-p_2)\cdot(p_3-p_4)\,\bar{v}(p_2)\gamma^\mu u(p_1)\,\bar{u}(p_3)\gamma_\mu v(p_4)$$

$$+\,\bar{v}(p_2)\,(\not{p}_3-\not{p}_4)\,u(p_1)\,\bar{u}(p_3)\,(\not{p}_1-\not{p}_2)\,v(p_4)\Big]$$

It interferes with the SM γ, Z amplitudes.

$$\sum|\mathcal{M}|^2 = \Big\{4\hat{u}^2\left(|M_{LL}^{\rm sm}|^2+|M_{RR}^{\rm sm}|^2\right)+4\hat{t}^2\left(|M_{LR}^{\rm sm}|^2+|M_{RL}^{\rm sm}|^2\right)$$

$$+8|A|^2\left(\hat{t}^4+\hat{u}^4-6\hat{t}^3\hat{u}-6\hat{t}\hat{u}^3+18\hat{t}^2\hat{u}^2\right)+16\frac{e^2Q_eQ_q}{\hat{s}}\,\Re{\rm e}(A)\,(\hat{u}-\hat{t})^3$$

$$+16\frac{e^2}{\sin^2\theta_{\rm w}\cos^2\theta_{\rm w}}\Re{\rm e}\left(\frac{A^*}{\hat{s}-M_Z^2+iM_Z\Gamma_Z}\right)\Big[g_a^e g_a^q\left(\hat{t}^3-3\hat{t}^2\hat{u}-3\hat{t}\hat{u}^2+\hat{u}^3\right)$$

$$+ \, g_v^e g_v^q (\hat{u} - \hat{t})^3 \Big] \Big\} \, ,$$

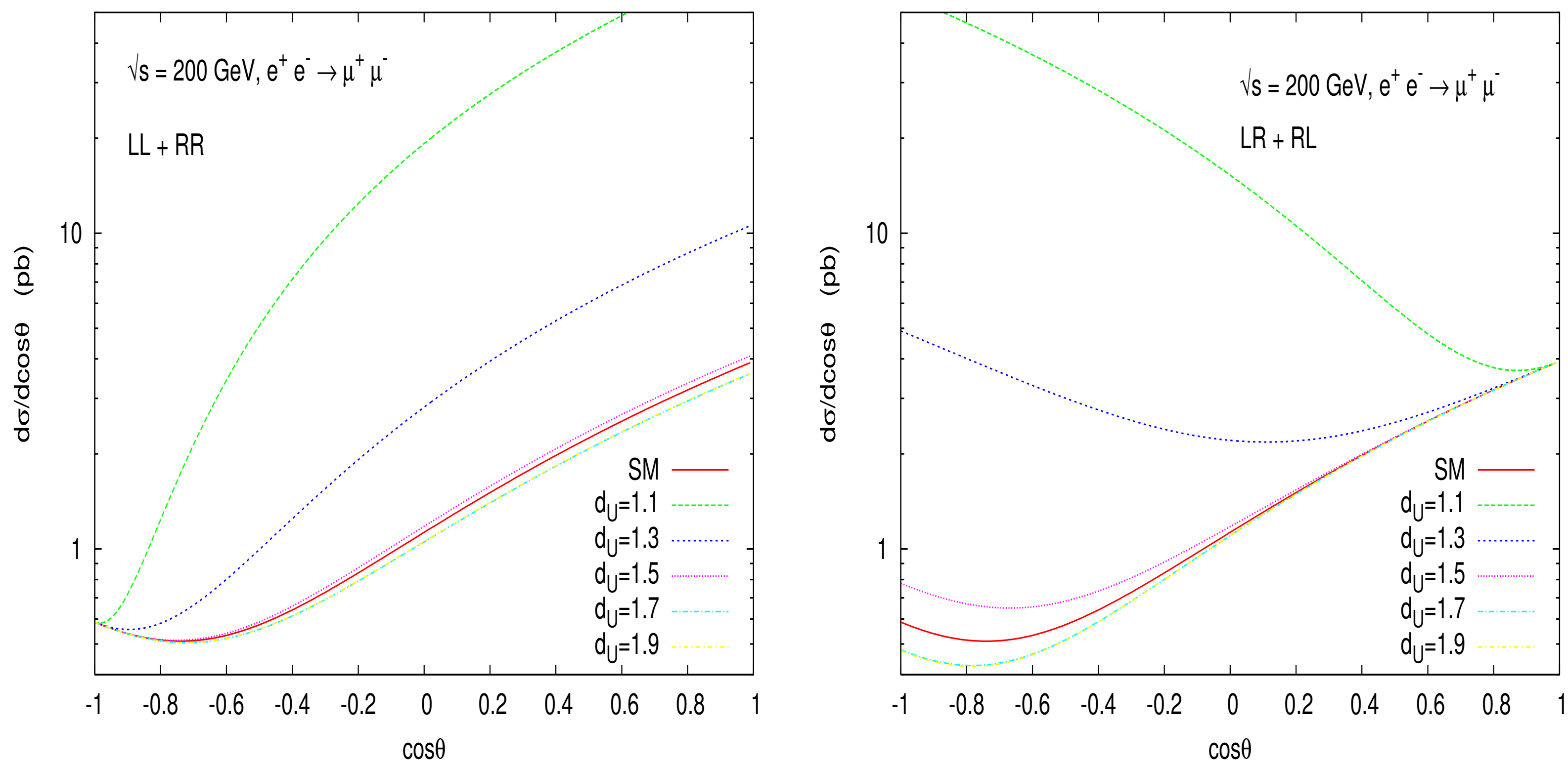
- Angular distribution can see the spin-2 structure,
- The total cross section has accidental cancellation in the interference terms.

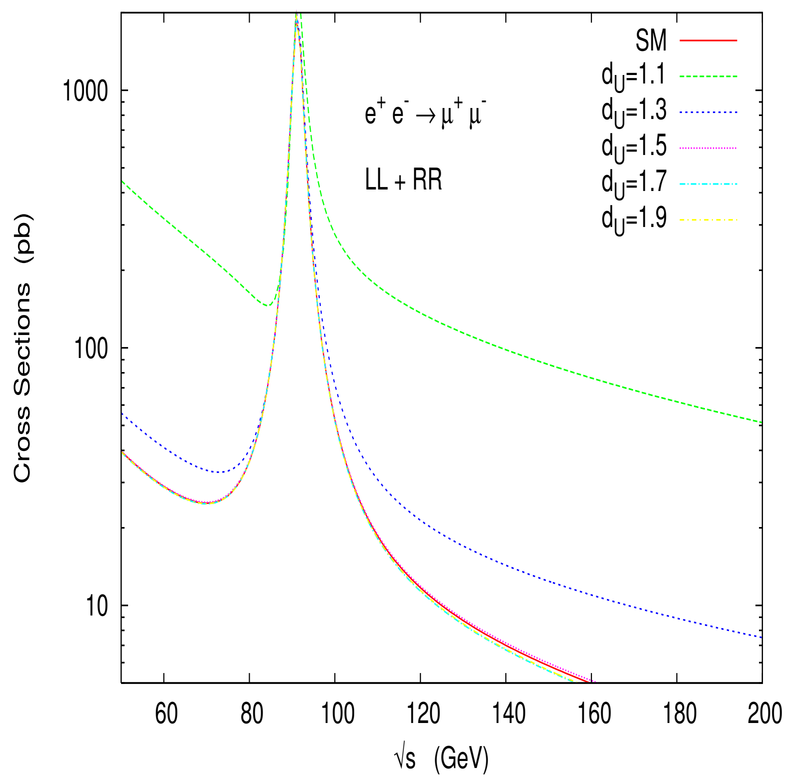
Fermion pair production at e^+e^- colliders: spin 1 unparticle

The formulas are similar to DY case, but experimental uncertainties are much smaller. The differential cross section including the spin-1 unparticle exchange is

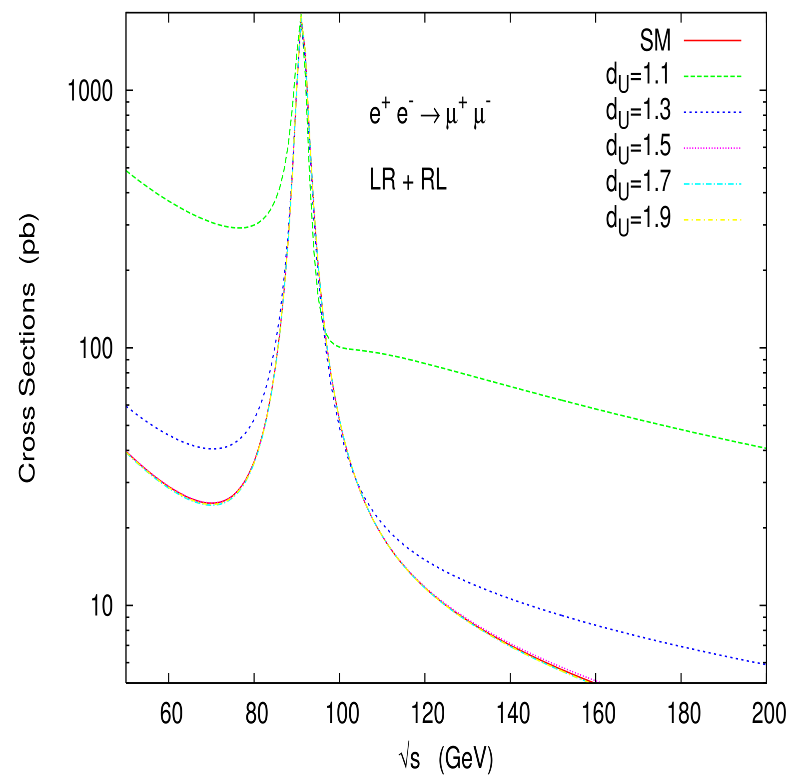
$$\frac{d\sigma(e^-e^+ \to f\bar{f})}{d\cos\theta} = \frac{N_c s}{128\pi}\Big[(1+\cos\theta)^2(|M_{LL}|^2 + |M_{RR}|^2) + (1-\cos\theta)^2(|M_{LR}|^2 + |M_{RL}|^2)\Big]$$

- It is easy to see LL, RR enhance $+\cos\theta$ region
- While LR, RL enhance $-\cos\theta$ region.
- FB asymmetry can help distinguish the new interactions.
- The total cross sections around the Z pole show interesting interference pattern.


√s = 200 GeV, e+ e- → μ+ μ-
LL + RR
dσ/dcosθ (pb)
10
1
SM
dU=1.1
dU=1.3
dU=1.5
dU=1.7
dU=1.9
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
cosθ
√s = 200 GeV, e+ e- → μ+ μ-
LR + RL
dσ/dcosθ (pb)
10
1
SM
dU=1.1
dU=1.3
dU=1.5
dU=1.7
dU=1.9
-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
cosθ

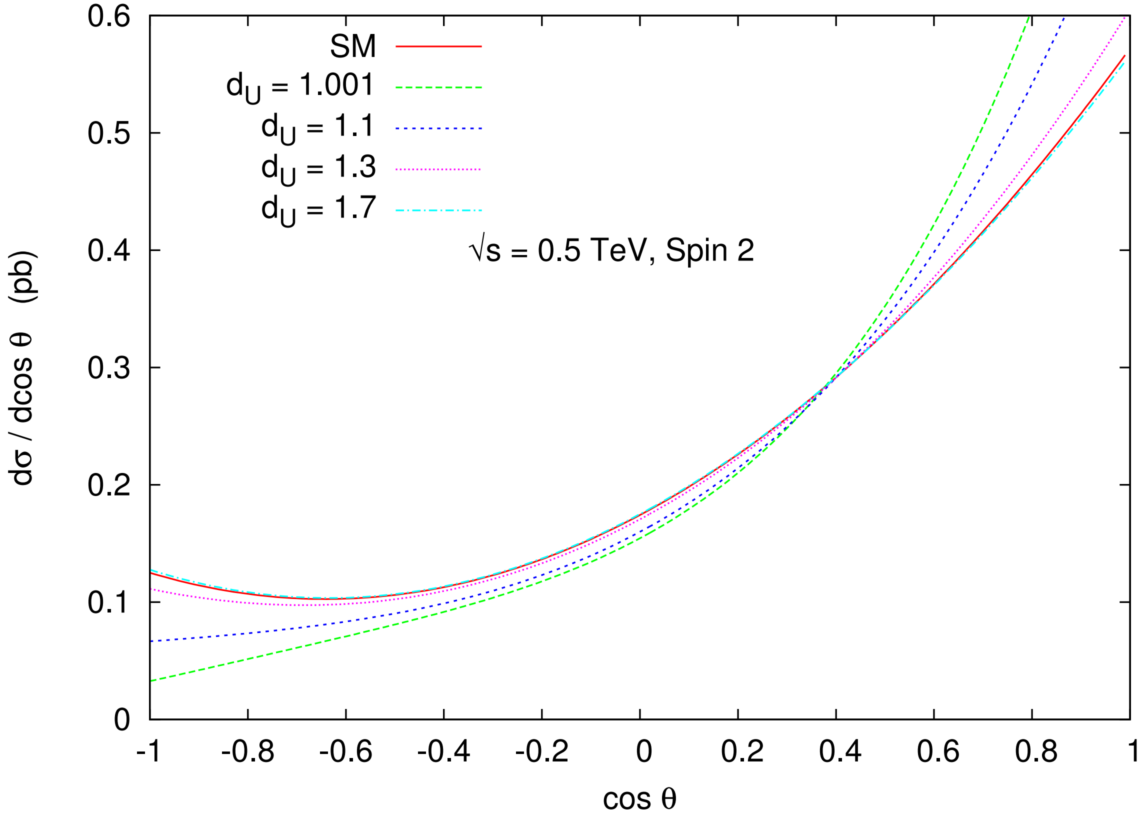


Cross Sections (pb)
1000
100
10
60
80
100
120
140
160
180
200
√s (GeV)
$e^+ e^- \to \mu^+ \mu^-$
LL + RR
SM
d_U=1.1
d_U=1.3
d_U=1.5
d_U=1.7
d_U=1.9



Cross Sections (pb)
1000
100
10
60
80
100
120
140
160
180
200
√s (GeV)
$e^+ e^- \to \mu^+ \mu^-$
LR + RL
SM
d_U=1.1
d_U=1.3
d_U=1.5
d_U=1.7
d_U=1.9

Fermion pair production at e^+e^- colliders: spin 2 unparticle



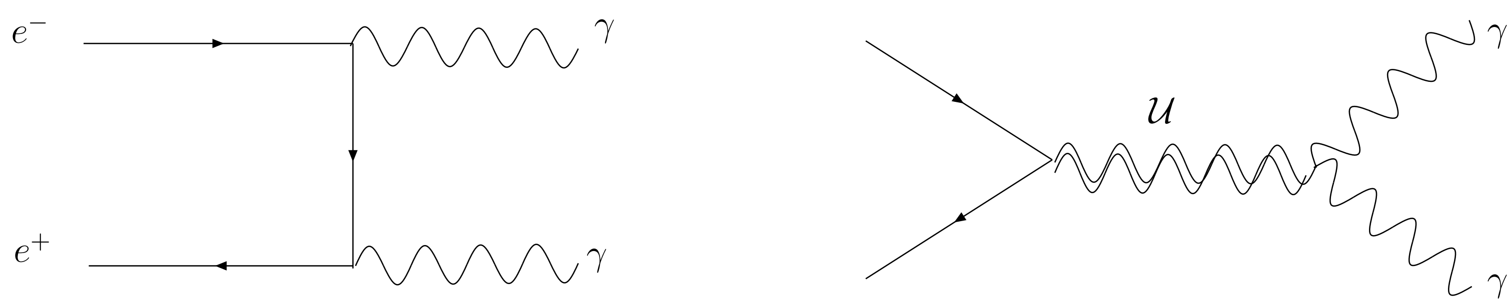
Diphoton production at e^+e^- colliders: spin 2 unparticle

The amplitude for $f(p_1)\,\bar{f}(p_2) \to \gamma(k_1)\,\gamma(k_2)$ due to the s-channel unparticle exchange is given by

$$i\mathcal{M}_{\mathcal{U}} = -\frac{i}{4}\frac{\lambda_2^2 Z_{d_{\mathcal{U}}}}{\Lambda_{\mathcal{U}}^4}\left(\frac{-s}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-2} \bar{v}(p_2)\left[\gamma_\rho(p_1-p_2)_\sigma + \gamma_\sigma(p_1-p_2)_\rho\right]u(p_1)\epsilon_\mu(k_1)\epsilon_\nu(k_2)$$

$$\times\Big[g^{\mu\nu}\left(k_1^\rho k_2^\sigma + k_2^\rho k_1^\sigma\right) + k_1\cdot k_2\left(g^{\rho\mu}g^{\sigma\nu} + g^{\sigma\mu}g^{\rho\nu}\right)$$

$$-k_1^\nu\left(k_2^\rho g^{\sigma\mu} + k_2^\sigma g^{\rho\mu}\right) - k_2^\mu\left(k_1^\rho g^{\sigma\nu} + k_1^\sigma g^{\rho\nu}\right)\Big]$$



The spin- and color-averaged amplitude squared is given by

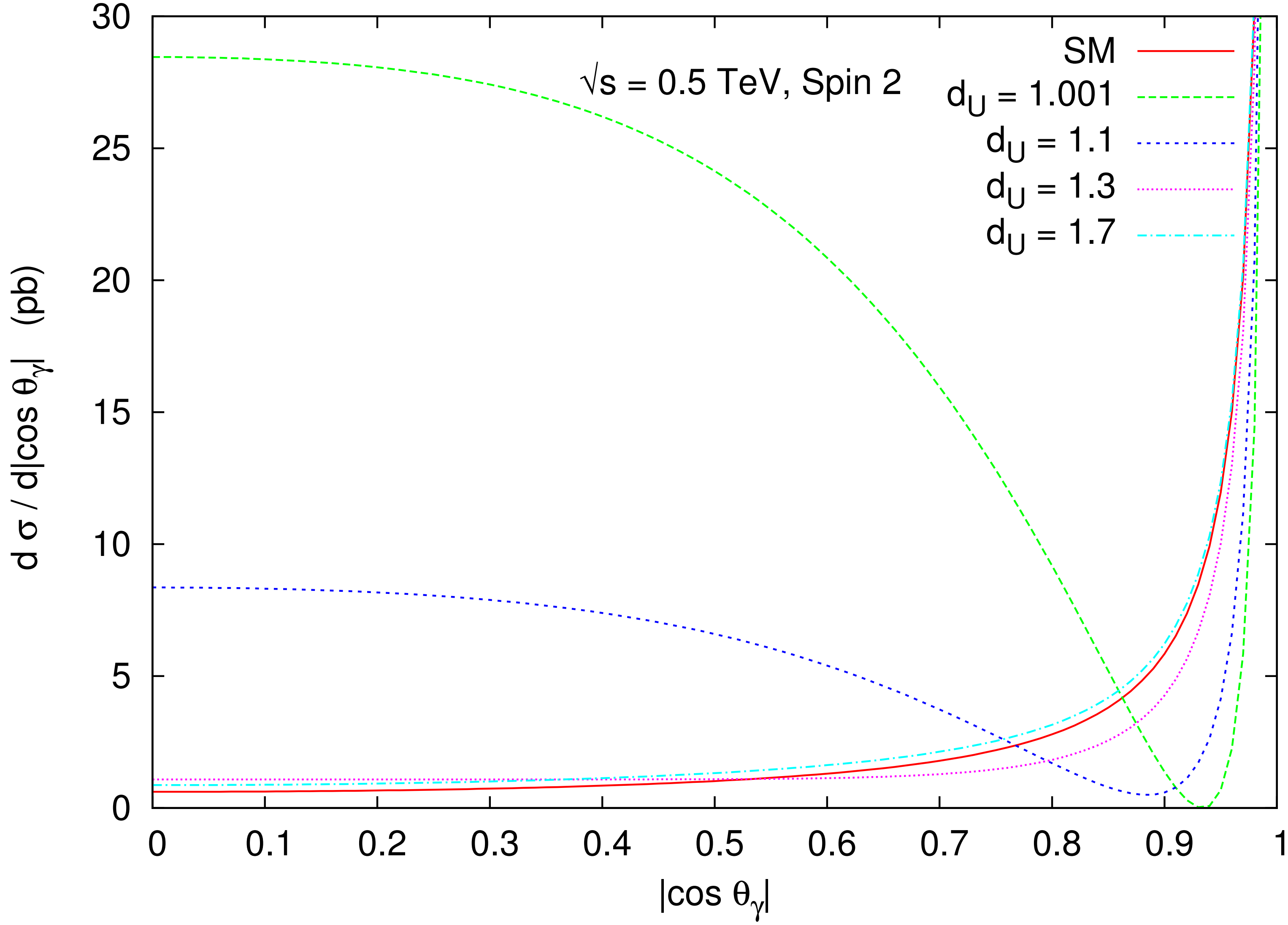
$$|\overline{\mathcal{M}}|^2 = \frac{1}{4}\frac{1}{N_c}\left\{8e^4 Q_f^4\left(\frac{u}{t}+\frac{t}{u}\right)+32ut(u^2+t^2)|A'|^2+32e^2Q_f^2(u^2+t^2)\Re\mathrm{e}(A')\right\}$$

The differential cross section is given by

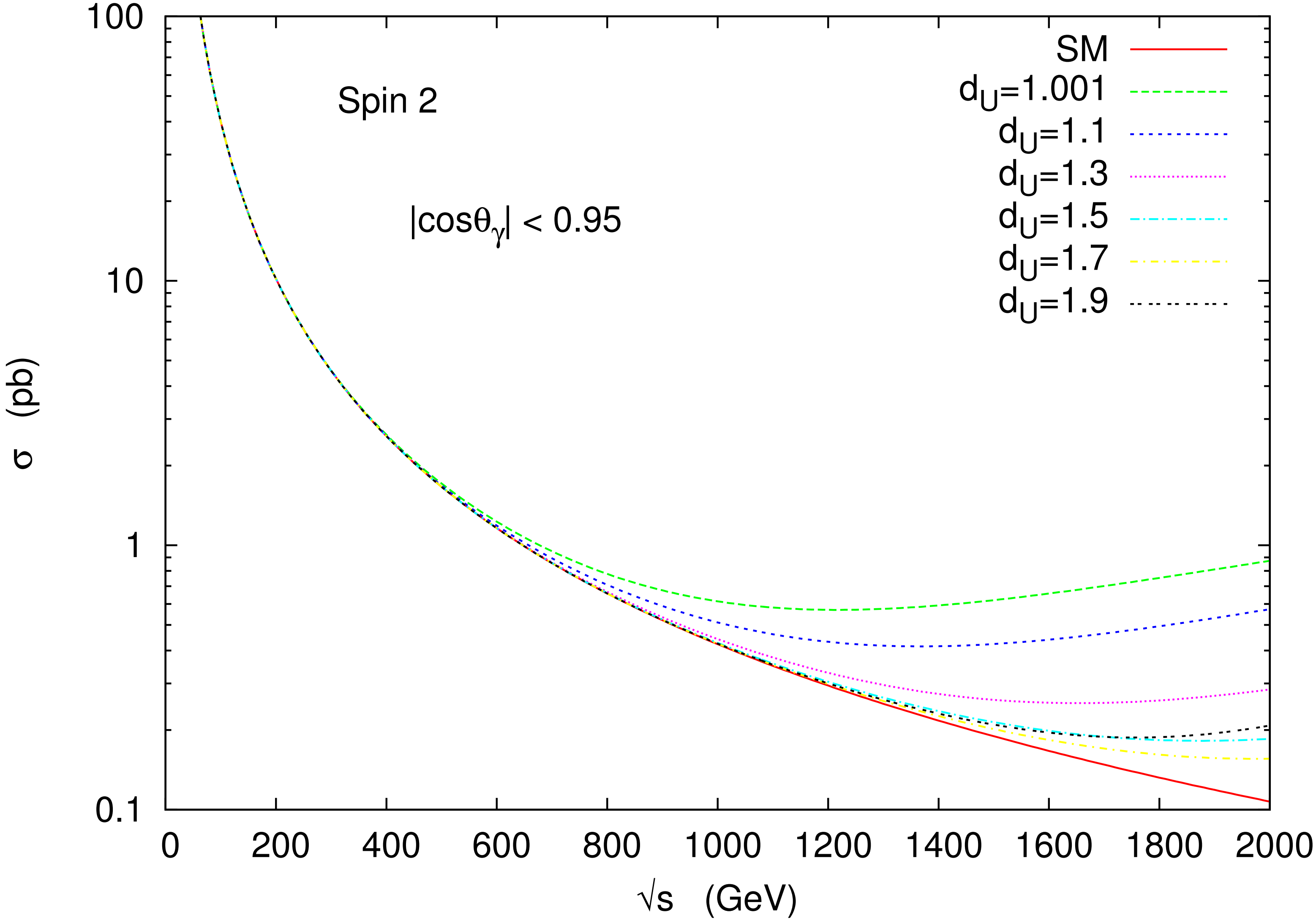
$$\frac{d\sigma}{d|\cos\theta_\gamma|}(f\bar{f}\to\gamma\gamma)=\frac{1}{32\pi s}\,|\overline{\mathcal{M}}|^2\,,$$

Integrated cross section as

$$\sigma(f\bar{f}\to\gamma\gamma)\Big|_{0\le|\cos\theta_\gamma|<z} = \frac{1}{32\pi s}\frac{1}{4N_c}\left\{8e^4Q_f^4\left[-2z-2\log\left|\frac{1-z}{1+z}\right|\right]\right.$$
$$\left.+32s^4\left(\frac{z}{8}-\frac{z^5}{40}\right)|A'|^2+32e^2Q_f^2s^2\left(\frac{z}{2}+\frac{z^3}{6}\right)\Re\mathrm{e}(A')\right\}$$



√s = 0.5 TeV, Spin 2
SM
d_U = 1.001
d_U = 1.1
d_U = 1.3
d_U = 1.7
d σ / d|cos θ_γ| (pb)
|cos θ_γ|
0
5
10
15
20
25
30
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Spin 2
|cosθγ| < 0.95
SM
dU=1.001
dU=1.1
dU=1.3
dU=1.5
dU=1.7
dU=1.9
σ (pb)
√s (GeV)
100
10
1
0.1
0
200
400
600
800
1000
1200
1400
1600
1800
2000

Experimental constraints on unparticle scale $\Lambda_{\mathcal{U}}$

We can compare the 4-fermion interaction of $\mathcal{U}$ exchange with the conventional 4-fermion contact interactions

$$\mathcal{L}_{4f} = \frac{4\pi}{\Lambda^2} \sum_{\alpha,\beta=L,R} \eta_{\alpha\beta} (\bar{e}\gamma_\mu P_\alpha e)\,(\bar{f}\gamma^\mu P_\beta f)\ ,$$

which results in the following equality:

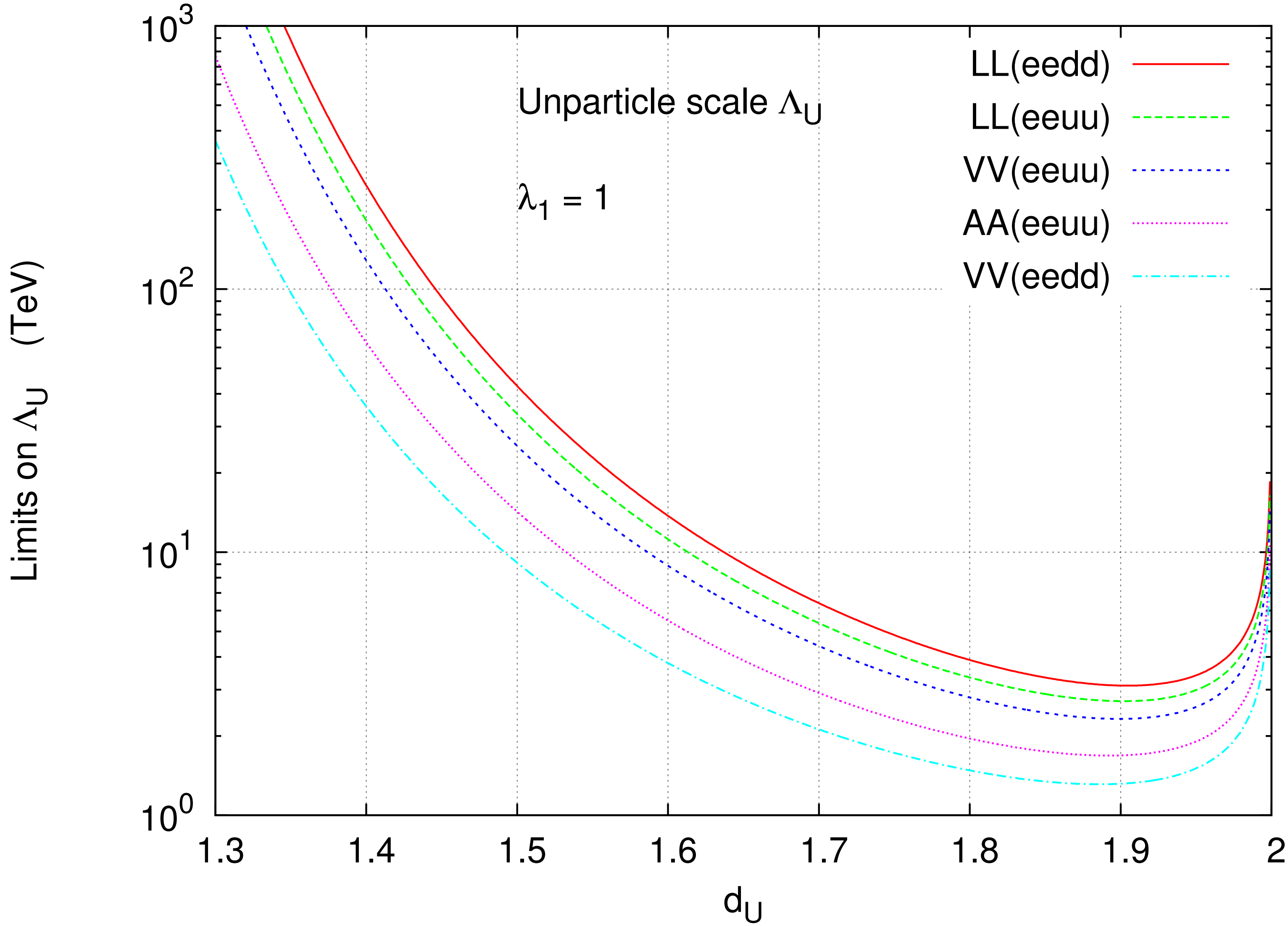
$$\lambda_1^2\, Z_{d_{\mathcal{U}}}\, \frac{1}{\Lambda_{\mathcal{U}}^2} \left(-\frac{P_{\mathcal{U}}^2}{\Lambda_{\mathcal{U}}^2} \right)^{d_{\mathcal{U}}-2} = \frac{4\pi}{(\Lambda^{95})^2}$$

We use the global fits on fermion-pair production at LEP, Drell-Yan production at the Tevatron, deep-inelastic scattering at HERA, and a number of low-energy parity-violating experiments:

$$\Lambda^{95}_{LL}(eeuu) \simeq 23\,\mathrm{TeV}, \quad \Lambda^{95}_{LL}(eedd) \simeq 26\,\mathrm{TeV}$$

Parity conserving:

$$\Lambda^{95}_{VV}(eeuu) \simeq 20\,\mathrm{TeV}, \quad \Lambda^{95}_{VV}(eedd) \simeq 12\,\mathrm{TeV}, \quad \Lambda^{95}_{AA}(eedd) \simeq \Lambda^{95}_{VV}(eeuu) = 15\,\mathrm{TeV}$$


Unparticle scale Λ_U
$\lambda_1 = 1$
LL(eedd)
LL(eeuu)
VV(eeuu)
AA(eeuu)
VV(eedd)
Limits on Λ_U (TeV)
d_U
10^0
10^1
10^2
10^3
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

Conclusions

- Conjecture to large extra dimension model. Unparticle is a generalization of extra dimensions

$$d_{\mathcal{U}} = \frac{n}{2} + 1$$

- Real emission of unparticle yields missing energy, like a fractional number of massless particles.
- Unparticle exchange gives rise to 4-fermion interaction, which can interfere with the SM amplitudes. Diphoton, Dilepton, Diboson, Dijet production are useful probes.
- There are many channels or processes that are sensitive to unparticle, but not yet explored.