Leptogenesis in $SO(10)$ models with a left-right symmetric seesaw mechanism

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Abstract. We study leptogenesis in supersymmetric $SO(10)$ models with a left-right symmetric seesaw mechanism, including flavour effects and the contribution of the next-to-lightest right-handed neutrino. Assuming $M_D = M_u$ and hierarchical light neutrino masses, we find that successful leptogenesis is possible for 4 out of the 8 right-handed neutrino mass spectra that are compatible with the observed neutrino data. An accurate description of charged fermion masses appears to be an important ingredient in the analysis.

PACS. 12.10.Dm Unified theories and models of strong and electroweak interactions – 14.60.St Non-standard-model neutrinos, right-handed neutrinos, etc.

1 Introduction

Testing the seesaw mechanism [1] is almost certainly an hopeless goal, except for specific low-energy realizations. The main reasons we have to believe in it are its elegance and the fact that it fits so nicely into $SO(10)$ unification. This motivates us to investigate its observable implications, such as leptogenesis [2] and, in supersymmetric theories, lepton flavour violation.

So far most studies of leptogenesis have been done in the framework of the type I (heavy right-handed neutrino exchange) seesaw mechanism, or assumed dominance of either the type I or the type II (heavy scalar $SU(2)_L$ triplet exchange) seesaw mechanism. It is interesting, though, to investigate whether the generic situation where both contributions are comparable in size can lead to qualitatively different results. A further motivation to do so comes from the well-known fact that successful leptogenesis is difficult to achieve in $SO(10)$ models with a type I seesaw mechanism, which generally[1] present a very hierarchical right-handed neutrino mass spectrum, with M_1 lying below the Davidson-Ibarra bound [3].

In this talk, we present results on leptogenesis in $SO(10)$ models with a left-right symmetric seesaw mechanism. Details can be found in Refs. [4,5] (for related work, see Refs. [6,7]).

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[1] This might not be the case in models where the relation $M_D = M_u$ receives large corrections from Yukawa couplings involving a $\overline{\mathbf{126}}$ or $\mathbf{120}$ Higgs representation, or from non-renormalizable interactions.

2 Right-handed neutrino spectra in the left-right symmetric seesaw mechanism

2.1 The left-right symmetric seesaw mechanism

In left-right symmetric extensions of the Standard Model, the light neutrino mass matrix is often given by the following formula [8]:

$$M_\nu \ = \ f v_L - \frac{v^2}{v_R} Y_\nu^T f^{-1} Y_\nu \ . \tag{1}$$

In Eq. (1), v_R is the scale of $B-L$ breaking, v is the electroweak scale, and $v_L \sim v^2 v_R / M^2_{\Delta_L}$ is the vev of the heavy $SU(2)_L$ triplet. A discrete left-right symmetry ensures that a single symmetric matrix f determines both the couplings of the $SU(2)_L$ triplet to lepton doublets, to which the type II contribution (first term) is proportional, and the right-handed neutrino mass matrix $M_R = f v_R$, which enters the type I contribution (second term). The discrete symmetry also constrains the Dirac coupling matrix Y_ν to be symmetric.

In order to study leptogenesis, the knowledge of the masses and couplings of the right-handed neutrinos and of the $SU(2)_L$ triplet is needed. Therefore, in a theory which predicts the Dirac matrix Y_ν, one must solve Eq. (1) for the f_{ij} couplings, assuming a given pattern of the light neutrino masses and mixings. In Ref. [9], it was shown that this "reconstruction" problem has exactly 2^n solutions for n families, and explicit expressions for the f_{ij}'s were provided up to $n = 3$. Here we use the alternative reconstruction procedure proposed in Ref. [4].

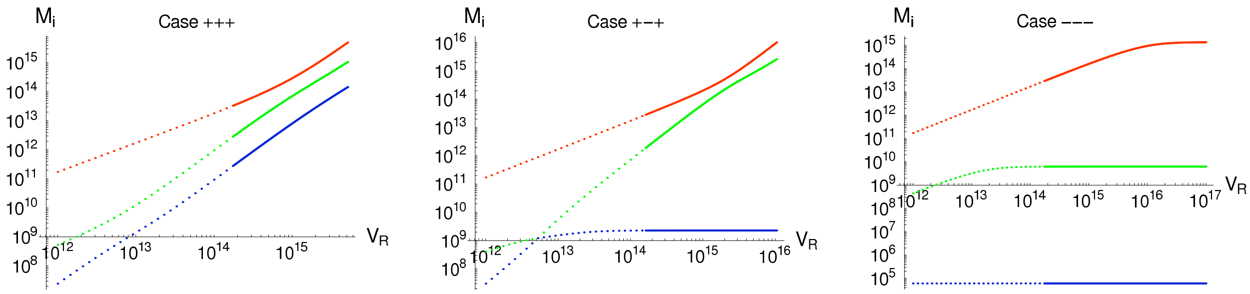

Fig. 1. Right-handed neutrino masses as a function of v_R (in GeV) for solutions $(+,+,+)$ *(left)*, $(+,-,+)$ *(middle)* and $(-,-,-)$ *(right panel)*. Inputs: hierarchical light neutrino masses with $m_1 = 10^{-3}$ eV, $\sin^2\theta_{13} = 0.009$, $\beta/\alpha = 0.1$, no CP violation beyond the CKM phase. The range of variation of v_R is restricted from above by the requirement that $f_3 \leq 1$. Dotted lines indicate a fine-tuning greater than 10% in the $(3,3)$ entry of the light neutrino mass matrix.

2.2 Reconstruction procedure

In order to solve Eq. (1), we first rewrite it as

$$Z \;=\; \alpha X - \beta X^{-1}\,, \tag{2}$$

with $\alpha \equiv v_L$, $\beta \equiv v^2/v_R$ and

$$Z \;\equiv\; N_\nu^{-1} M_\nu (N_\nu^{-1})^T\,, \quad X \;\equiv\; N_\nu^{-1} f (N_\nu^{-1})^T\,, \tag{3}$$

where N_ν is a matrix such that $Y_\nu = N_\nu N_\nu^T$, and Y_ν is assumed to be invertible. Being complex and symmetric, Z can be diagonalized by a complex orthogonal matrix if its eigenvalues (i.e. the roots of the characteristic polynomial $\det(Z - z\mathbf{1}) = 0$) are all distinct:

$$Z \;=\; O_Z \mathrm{Diag}\,(z_1, z_2, z_3) O_Z^T\,, \qquad O_Z O_Z^T \;=\; \mathbf{1}\,. \tag{4}$$

Then, upon an O_Z transformation, Eq. (2) reduces to 3 independent quadratic equations for the eigenvalues of X, $z_i = \alpha x_i - \beta x_i^{-1}$. For a given choice of (x_1, x_2, x_3), the solution of Eq. (1) is given by:

$$f \;=\; N_\nu\, O_Z\, \mathrm{Diag}\,(x_1, x_2, x_3)\, O_Z^T\, N_\nu^T\,. \tag{5}$$

The right-handed neutrino masses $M_i = f_i v_R$ are obtained upon diagonalizing f by a unitary matrix U_f, and the couplings of the right-handed neutrino mass eigenstates are given by $Y \equiv U_f^\dagger Y_\nu$.

Since each equation $z_i = \alpha x_i - \beta x_i^{-1}$ has two solutions x_i^- and x_i^+, there are 8 different solutions for the matrix f, which we label in the following way: $(+,+,+)$ refers to the solution (x_1^+, x_2^+, x_3^+), $(+,+,-)$ to the solution (x_1^+, x_2^+, x_3^-), and so on. It is convenient to define x_i^- and x_i^+ such that, in the $4\alpha\beta \ll |z_i|^2$ limit:

$$x_i^- \;\simeq\; -\frac{\beta}{z_i}\,, \qquad x_i^+ \;\simeq\; \frac{z_i}{\alpha}\,. \tag{6}$$

With this definition, the large v_R limit $(4\alpha\beta \ll |z_1|^2)$ of solutions $(-,-,-)$ and $(+,+,+)$ corresponds to the "pure" type I and type II cases, respectively:

$$f^{(-,-,-)} \stackrel{4\alpha\beta \ll |z_1|^2}{\longrightarrow} -\frac{v^2}{v_R}\, Y_\nu M_\nu^{-1} Y_\nu\,, \tag{7}$$

$$f^{(+,+,+)} \stackrel{4\alpha\beta \ll |z_1|^2}{\longrightarrow} \frac{M_\nu}{v_L}\,. \tag{8}$$

The remaining 6 solutions correspond to mixed cases where the light neutrino mass matrix receives significant contributions from both types of seesaw mechanisms. In the opposite, small v_R limit ($|z_3|^2 \ll 4\alpha\beta$), one has $x_i^\pm \;\simeq\; \pm\,\mathrm{sign}(\mathrm{Re}(z_i))\sqrt{\beta/\alpha}$, which indicates a partial cancellation between the type I and type II contributions to light neutrino masses.

2.3 Application to $SO(10)$ models

Let us now apply the reconstruction procedure to supersymmetric $SO(10)$ models with two $\mathbf{10}$s, a $\mathbf{54}$ and a $\overline{\mathbf{126}}$ representations in the Higgs sector. The two $\mathbf{10}$s generate the charged fermion masses, leading to the well-known relations:

$$M_u \;=\; M_D\;(\equiv Y_\nu v_u)\,, \qquad M_d \;=\; M_e\,. \tag{9}$$

The $\mathbf{54}$ and the $\overline{\mathbf{126}}$ contain the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ representations needed for the left-right symmetric seesaw mechanism. In particular, the $SU(2)_L$ triplet as well as the $SU(2)_R$ triplet whose vev v_R breaks $B-L$ are components of the $\overline{\mathbf{126}}$. The equality $f_L = f_R$ and the symmetry of Y_ν are ensured by $SO(10)$ gauge symmetry.

Then, for a given choice of the light neutrino mass parameters and of the high energy phases contained in M_u, the matrix Z is known[2] and f can be reconstructed as a function of the $B-L$ breaking scale v_R and of β/α. Perturbativity of the f_{ij} couplings constrains $\beta/\alpha \leq \mathcal{O}(1)$ and restricts the range of v_R from above. In Fig. 1, we show the right-handed neutrino mass spectrum of three representative solutions as a function of v_R for a hierarchical light neutrino mass spectrum. The 4 solutions with $x_3 = x_3^-$ are characterized by a constant value of the lightest right-handed neutrino mass, $M_1 \approx 6\times 10^4$ GeV; the 2 solutions with $x_3 = x_3^+$ and $x_2 = x_2^-$ by $M_1 \approx 2\times 10^9$ GeV; and the 2 solutions with $x_3 = x_3^+$ and $x_2 = x_2^+$ by a rising M_1.

[2] The implicit additional inputs are $\tan\beta$ (we choose $\tan\beta = 10$) and the values of the up quark masses and of the CKM matrix at the seesaw scale.

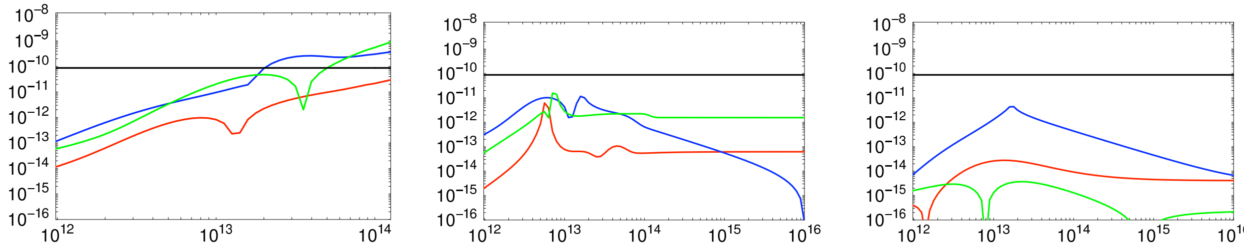

Fig. 2. Y_B as a function of v_R (in GeV) for solutions $(+,+,+)$ *(left)*, $(+,-,+)$ *(middle)* and $(-,-,-)$ *(right panel)*. Inputs: hierarchical light neutrino mass spectrum with $m_1 = 10^{-3}$ eV, $\sin^2\theta_{13} = 0.009$ and $\delta_{PMNS} = 0$; $\beta/\alpha = 0.1$; three different choices for the Majorana and high-energy phases *(blue: $\Phi_2^u = \pi/4$; green: $\Phi_2^\nu = \pi/4$; red: no CP violation beyond the CKM phase)*; vanishing initial abundance for N_1 and N_2.

3 Implications for leptogenesis

Since $M_{\Delta_L} \sim (\beta/\alpha)\, v_R$ and $M_1 \ll v_R$ in all solutions, one can safely assume that the $SU(2)_L$ triplet is heavier than the lightest right-handed neutrino. Then the dominant contribution to leptogenesis comes from out-of-equilibrium decays of N_1 (in some cases to be discussed below, the next-to-lightest neutrino N_2 will also be relevant). The CP asymmetry in N_1 decays, $\epsilon_{N_1} \equiv \left[\Gamma(N_1 \to lH) - \Gamma(N_1 \to \bar{l}H^\star)\right] / \left[\Gamma(N_1 \to lH) +\Gamma(N_1 \to \bar{l}H^\star)\right]$, receives two contributions: the standard type I contribution $\epsilon^I_{N_1}$ [2,10], and an additional contribution $\epsilon^{II}_{N_1}$ from a vertex diagram containing a virtual triplet [11,12]:

$$\epsilon^I_{N_1} \; = \; \frac{1}{8\pi} \sum_k \frac{\mathrm{Im}\left[(YY^\dagger)_{1k}\right]^2}{(YY^\dagger)_{11}}\, f(x_k)\ , \qquad (10)$$

$$\epsilon^{II}_{N_1} \; = \; \frac{3}{8\pi} \sum_{k,l} \frac{\mathrm{Im}\left[Y_{1k} Y_{1l} f^\star_{kl} v^\star_L\right]}{(YY^\dagger)_{11}}\, \frac{M_1}{v_u^2}\, g(x_\Delta)\ , \qquad (11)$$

where $f(x) = -\sqrt{x}\,[\,2/(x-1) + \ln(1+1/x)]$, $g(x) = x\ln(1+1/x)$, $x_k \equiv M_k^2/M_1^2$, $x_\Delta = M^2_{\Delta_L}/M_1^2$, and $Y \equiv U_f^\dagger Y_\nu$. The final baryon asymmetry is given by:

$$Y_B \; \equiv \; \frac{n_B}{s} \; = \; -1.48 \times 10^{-3}\, \eta\, \epsilon_{N_1}\ , \qquad (12)$$

where η is an efficiency factor to be determined by integrating the Boltzmann equations. For leptogenesis to be successful, Eq. (12) should reproduce the observed baryon-to-entropy ratio $Y_B^{obs.} = (8.7 \pm 0.3) \times 10^{-11}$ [13].

One can anticipate the behaviour of the different solutions from the observation of the mass spectra in Fig. 1 [4]. Indeed, successful leptogenesis requires $|\epsilon_{N_1}| \geq \mathcal{O}(10^{-7})$, while for $M_1 \ll M_2, M_{\Delta_L}$ Eqs. (10) and (11) yield the upper bound [12]:

$$|\epsilon_{N_1}| \; \leq \; 2 \times 10^{-7} \left(\frac{M_1}{10^9\,\mathrm{GeV}}\right) \left(\frac{m_{max}}{0.05\,\mathrm{eV}}\right). \qquad (13)$$

Thus, the 4 solutions with $x_3 = x_3^-$ will fail to generate the observed baryon asymmetry from N_1 decays, a conclusion that generalizes a well-known fact in the type I case. However, N_2 decays could do the job if they generated a large asymmetry in a lepton flavour that is only mildly washed out by N_1 decays and inverse decays [14]. The 2 solutions with $x_3 = x_3^+$ and $x_2 = x_2^+$ have a rising M_1 and should be able to reproduce the observed asymmetry, as in the pure type II case. Finally, the situation is less conclusive for the 2 solutions with $x_3 = x_3^+$ and $x_2 = x_2^-$, for which flavour effects and the contribution of N_2 could be decisive.

It is clear from the above discussion that a careful study of leptogenesis requires the inclusion of the next-to-lightest right-handed neutrino and of flavour effects [15]. As is well known in the type I case, flavour effects can significantly affect the final baryon asymmetry when there is a hierarchy between the washout parameters for different lepton flavours [16]. We performed such an analysis in Ref. [5], and present our results here. Fig. 2 shows the final baryon asymmetry Y_B as a function of v_R for solutions $(+,+,+)$, $(+,-,+)$ and $(-,-,-)$. Not surprisingly, the $(+,+,+)$ solution leads to successful leptogenesis; however there is a tension with the upper bound on the reheating temperature from gravitino overproduction [17] above $v_R \approx 3 \times 10^{13}$ GeV, where $M_1 > 10^{10}$ GeV. By contrast, the solutions $(+,-,+)$ and $(-,-,-)$ fail to reproduce the observed baryon asymmetry[3] . In the $(-,-,-)$ case, flavour effects prevent an exponential washout of the $B-L$ asymmetry generated in N_2 decays (N_1 decays alone would give $Y_B \sim (10^{-17} - 10^{-15})$), but this is not sufficient for "N_2 leptogenesis" to work.

However, this is not the whole story, since the above results were obtained assuming the $SO(10)$ mass relation $M_d = M_e$, which is in gross conflict with experimental data. Corrections to this formula, e.g. from non-renormalizable operators of the form $\mathbf{16_i 16_j 10_d 45}$, will modify the reconstructed f_{ij}'s by introducting a mismatch U_m between the bases of charged lepton and down quark mass eigenstates. Fig. 3 shows how the final baryon asymmetry is modified when the effect of U_m is taken into account. We can see that several choices for U_m (the measured charged lepton and down quark mass eigenstates do not fix all parameters in U_m) lead to successful leptogenesis in the $(+,-,+)$ case, but not in the $(-,-,-)$ case. There is some tension between successful leptogenesis and gravitino

[3] In Ref. [6], a different conclusion has been obtained for the solution $(+,-,+)$ in the case of an inverted light neutrino mass hierarchy.

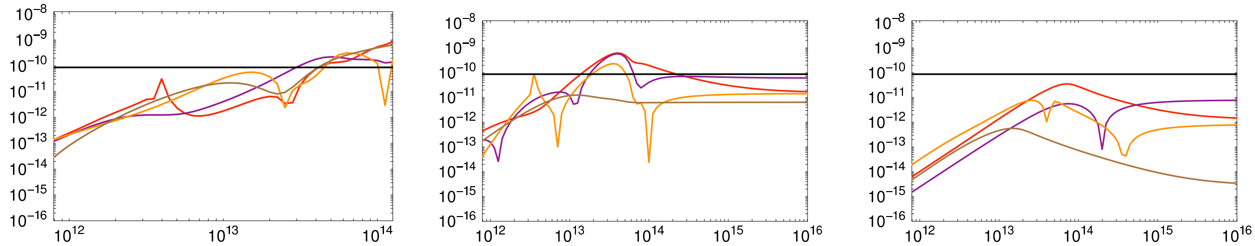

Fig. 3. Same as Fig. 2, but with corrections to the relation $M_d = M_e$ from the non-renormalizable operators $\mathbf{16_i 16_j 10_d 45}$, keeping the relation $M_D = M_u$. Four different choices of the matrix U_m and of the CP-violating phases.

overproduction in the $(+,-,+)$ solution, but, exactly as in the $(+,+,+)$ solution, the observed asymmetry is generated over a significant portion of the parameter space with $M_1 < 10^{10}$ GeV.

4 Conclusions

We have studied leptogenesis in supersymmetric $SO(10)$ models with a left-right symmetric seesaw mechanism, including flavour effects and the contribution of the next-to-lightest right-handed neutrino. Assuming the relation $M_D = M_u$ and a hierarchical light neutrino mass spectrum, we found that the "type II-like" solutions $(+,+,+)$ and $(-,+,+)$, as well as the solutions $(+,-,+)$ and $(-,-,+)$, can lead to successful leptogenesis. An accurate description of charged fermion masses was a crucial ingredient in the analysis. By contrast, the solution $(-,-,-)$ fails to generate the observed baryon asymmetry from N_2 decays, and a similar conclusion holds for the 3 other solutions with $x_3 = x_3^-$ if one requires $M_1 < 10^{10}$ GeV.

Some comments about the generality of our results are in order: (i) Although the above results were obtained for $M_D = M_u$, the same qualitative behaviour of the 8 solutions is expected for a more generic hierarchical Dirac matrix. Of course, whether leptogenesis is successful or not in a given solution can only be decided on a model-by-model basis; (ii) At the quantitative level, different input parameters (other than the various phases and U_m) can significantly affect the results presented in Figs. 1 to 3. This is most notably the case of the light neutrino mass parameters: θ_{13}, m_1 and the type of the mass hierarchy (see Ref. [5] for details). Also, corrections to the relation $M_D = M_u$ could have a significant impact, since e.g. both M_1 in the $(+,-,+)$ solution and M_2 in the $(-,-,-)$ solution are proportional to $m_c^2(M_{GUT})/m_3$.

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