A_4 and its double-covering T' as flavor symmetries

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Observations

- Masses of the charged fermions are strongly hierarchical

$$m_u : m_c : m_t \approx \lambda^8 : \lambda^4 : 1 \,, \quad m_d : m_s : m_b \approx \lambda^4 : \lambda^2 : 1 \,,$$

$$m_e : m_\mu : m_\tau \approx \lambda^5 : \lambda^2 : 1 \quad \text{where } \lambda \approx \theta_C \approx 0.22$$

- Mass hierarchy in the ν sector is milder, ordering till now unknown.
- Mixing parameters:
  small mixings for quarks, large mixings for leptons.
- for lepton mixing special structures are allowed:
  - "tri-bimaximal" (TB): ($1\,\sigma$)
    $\sin^2(\theta_{12}^{TB}) = \frac{1}{3} \,, \quad \sin^2(\theta_{23}^{TB}) = \frac{1}{2} \,, \quad \sin^2(\theta_{13}^{TB}) = 0.$
  - "μ-τ" symmetric (MTS):
    $\sin^2(\theta_{23}^{MTS}) = \frac{1}{2} \,, \quad \sin^2(\theta_{13}^{MTS}) = 0.$

$\Rightarrow$ All these issues need a theoretical description: Flavor symmetry G_F!

good candidates are A_4 and T'

Basics of the model

- the flavor symmetry $G_F = A_4$ or $G_F = T'$ is spontaneously broken at high energies
- low energy effective theory: MSSM
- breaking of G_F is induced by VEVs of flavon fields which a singlets under the SM gauge groups
- (MS)SM fermions transform under G_F
- the MSSM Higgs doublets $h_{u,d}$ are singlets under G_F

A_4 Model (Altarelli/Feruglio (2005))

- Lepton generations transform according to A_4 reps.:

$$l_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \sim 3$$

$$e^c \sim 1\,, \quad \mu^c \sim 1''\,, \quad \tau^c \sim 1'$$

$$h_{u,d} \sim 1$$

- results :
  - TB mixing in the lepton sector at leading order, corrections at level $\lambda^2 \approx 0.04$
  - but no explanation of the quark sector
    idea that quarks transform like charged leptons leads to $V_{CKM} = \mathbb{1}$ at leading order, but corrections are only of order $\lambda^2 \approx 0.04$ instead of $\lambda = \theta_C$ (other models: Ma (2002); He et al. (2006); Bazzocchi et al. (2007))

Group Theory of A_4

- The group A_4 is the symmetry group of a regular tetrahedron, group of even permutations of four objects
- Order of the group: 12
- Four irreducible representations: 1, $1'$, $1''$ and 3
- Generator relations for generators S and T:

$$S^2 = \mathbb{1}\ , \quad T^3 = \mathbb{1}\ , \quad (ST)^3 = \mathbb{1}\ .$$

rep.	S	T
1	1	1
$1'$	1	ω
$1''$	1	ω^2
3	$\frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$

$(\omega = e^{\frac{2\pi i}{3}})$

T' **Model** (Feruglio et al.(2007))

- Quark generations transform according to T' reps.:

$$D_q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, D_u^c = \begin{pmatrix} u^c \\ c^c \end{pmatrix}, D_d^c = \begin{pmatrix} d^c \\ s^c \end{pmatrix} \sim 2''$$

$$q_3, t^c, b^c \sim 1$$

together with the leptons in A_4-like reps.:

$$l_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \sim 3$$

$$e^c \sim 1\,, \quad \mu^c \sim 1''\,, \quad \tau^c \sim 1'$$

- results :
  - TB mixing in the lepton sector at leading order, corrections are still at level $\lambda^2 \approx 0.04$
  - $m_{u,d,s,c} \ll m_{b,t}$, all data can be accommodated at subleading order in the quark sector, esp. θ_C

Group Theory of T'

- The group T' is the double covering of the group A_4. (Compare to $SU(2)$ and $SO(3)$)
- Order of the group: 24
- Irred. reps: $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{1}''$, $\mathbf{3}$ and $\mathbf{2}$, $\mathbf{2}'$, $\mathbf{2}''$
- Generator relations for generators S and T:

$$S^2 = \mathbb{R}\,, \quad T^3 = \mathbb{1}\,, \quad (ST)^3 = \mathbb{1}\,, \quad \mathbb{R}^2 = \mathbb{1}.$$

rep.	S	T
$\mathbf{2}$	A_1	ωA_2
$\mathbf{2}'$	A_1	$\omega^2 A_2$
$\mathbf{2}''$	A_1	A_2

$$A_1 = -\frac{1}{\sqrt{3}} \begin{pmatrix} i & \sqrt{2}e^{i\pi/12} \\ -\sqrt{2}e^{-i\pi/12} & -i \end{pmatrix},$$

$$A_2 = \begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}$$

Particle Content of the T' Model

	LEPTONS				*QUARKS*						*FLAVONS*				
Field	l	e^c	μ^c	τ^c	D_q	D_u^c	D_d^c	q_3	t^c	b^c	φ_T	φ_S	$\xi, \tilde{\xi}$	η	ξ''
G_F	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1''}$	$\mathbf{1'}$	$\mathbf{2''}$	$\mathbf{2''}$	$\mathbf{2''}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2'}$	$\mathbf{1''}$
Z_3	ω	ω^2	ω^2	ω^2	ω	ω^2	ω^2	ω	ω^2	ω^2	1	ω	ω	1	1
$U(1)$	0	$2n$	n	0	0	n	0	0	0	0	0	0	0	0	0

additionally needed:

- Z_3 symmetry to separate charged fermion and neutrino sector:

$$\{\varphi_T, \eta, \xi''\} \rightarrow m_l, m_u, m_d \quad \text{and} \quad \left\{\varphi_S, \xi, \tilde{\xi}\right\} \rightarrow m_\nu$$

- $U(1)$ for hierarchy $m_s \ll m_c$, $m_e \ll m_\mu \ll m_\tau$ (field θ: $Q(\theta) = -1$)

Superpotential

$$w = w_l + w_q + w_d$$

for leptons :

$$w_l = y_e e^c(\varphi_T l)h_d/\Lambda\ \left(\frac{\theta}{\Lambda}\right)^{2n} + y_\mu \mu^c(\varphi_T l)' h_d/\Lambda\ \left(\frac{\theta}{\Lambda}\right)^{n} + y_\tau \tau^c(\varphi_T l)'' h_d/\Lambda$$
$$+(x_a \xi + \tilde{x}_a \tilde{\xi})(ll)h_u h_u/\Lambda^2 + x_b(\varphi_S ll)h_u h_u/\Lambda^2$$

for quarks :

$$w_q = y_t\,(t^c q_3)\,h_u + y_b\,(b^c q_3)\,h_d + y_1(\varphi_T D^c_u D_q)h_u/\Lambda\ \left(\frac{\theta}{\Lambda}\right)^{n} +$$
$$y_5(\varphi_T D^c_d D_q)h_d/\Lambda + y_2\ \xi''(D^c_u D_q)h_u/\Lambda\ \left(\frac{\theta}{\Lambda}\right)^{n} + y_6\ \xi''(D^c_d D_q)h_d/\Lambda$$
$$+\{y_3\ t^c(\eta D_q) + y_4\ (D^c_u \eta)q_3\ \left(\frac{\theta}{\Lambda}\right)^{n}\}h_u/\Lambda + \{y_7\ b^c(\eta D_q) + y_8\ (D^c_d \eta)q_3\}h_d/\Lambda$$

Higgs superpotential w_d: VEV structure:

$$G_S \quad : \quad \langle\varphi_S\rangle = (v_S, v_S, v_S)\ ,\ \ \langle\xi\rangle = u\ ,\ \ \langle\tilde{\xi}\rangle = 0\ ,$$
$$G_T \quad : \quad \langle\varphi_T\rangle = (v_T, 0, 0)\ ,\ \ \langle\eta\rangle = (v_1, 0)\ ,\ \ \langle\xi''\rangle = 0\ .$$

Leading order

- charged leptons: $m_l = \frac{v_T}{\sqrt{2}\Lambda} v_d \, \text{diag}\left(y_e \left(\frac{\langle\theta\rangle}{\Lambda}\right)^{2n}, y_\mu \left(\frac{\langle\theta\rangle}{\Lambda}\right)^{n}, y_\tau\right)$
- neutrinos: $m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a+2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a-b/3 \\ -b/3 & a-b/3 & 2b/3 \end{pmatrix} \Rightarrow$ TB mixing!

  with masses: $\frac{v_u^2}{\Lambda}$ diag$(a+b, a, -a+b)$ and $a = x_a \frac{u}{\Lambda}$, $b = x_b \frac{v_S}{\Lambda}$
- up-type and down-type quarks:

$$m_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_1 v_{\mathsf{T}}/\Lambda \left(\frac{\langle\theta\rangle}{\Lambda}\right)^n & y_4 v_1/\Lambda \left(\frac{\langle\theta\rangle}{\Lambda}\right)^n \\ 0 & y_3 v_1/\Lambda & y_t \end{pmatrix} v_u \;, \quad m_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_5 v_{\mathsf{T}}/\Lambda & y_8 v_1/\Lambda \\ 0 & y_7 v_1/\Lambda & y_b \end{pmatrix} v_d$$

$\Rightarrow m_b$ and m_t large, $\frac{m_c}{m_t} \sim \mathcal{O}(\lambda^4)$, $\frac{m_s}{m_b} \sim \mathcal{O}(\lambda^2)$, $|V_{cb}| \sim \mathcal{O}(\lambda^2)$

Higgs superpotential (I)

$$\begin{aligned} w_d &= M\,(\varphi_T^0\,\varphi_T) + g\,(\varphi_T^0\,\varphi_T\,\varphi_T) + g_1\,(\varphi_S^0\,\varphi_S\,\varphi_S) + g_2\,\tilde{\xi}\,(\varphi_S^0\,\varphi_S) \\ &+ g_3\,\xi^0\,(\varphi_S\,\varphi_S) + g_4\,\xi^0\,\xi^2 + g_5\,\xi^0\,\xi\,\tilde{\xi} + g_6\,\xi^0\,\tilde{\xi}^2 \\ &+ M_\eta\,(\eta\,\eta^0) + M_\xi\,\xi''\,\xi'^0 + g_7\,\xi''\,(\varphi_T^0\,\varphi_T)' + g_8\,(\varphi_T^0\,\eta\,\eta) \\ &+ g_9\,(\varphi_T\,\eta\,\eta^0) + g_{10}\,\xi'^0\,(\varphi_T\,\varphi_T)'' \end{aligned}$$

- further fields needed: “driving fields” $\varphi_T^0 \sim (3,1)$, $\varphi_S^0 \sim (3,\omega)$, $\xi^0 \sim (1,\omega)$, $\eta^0 \sim (2'',1)$, $\xi'^0 \sim (1',1)$ under (T', Z_3)
- introduce $U(1)_R$ to construct Higgs potential:

$$Q(\text{matter}) = +1\;,\quad Q(\text{Higgs}) = 0\;,\quad Q(\text{driving field}) = +2$$

$\rightarrow w_d$ linear in driving fields
(Yukawa couplings are $U(1)_R$ invariant)

Higgs superpotential (II)

$$
\begin{aligned}
\frac{\partial w}{\partial \varphi^S_{01}} &= g_2 \tilde{\xi} \varphi_{S\,1} + \frac{2 g_1}{3} (\varphi_{S\,1}^2 - \varphi_{S\,2} \varphi_{S\,3}) = 0 \\
\frac{\partial w}{\partial \varphi^S_{02}} &= g_2 \tilde{\xi} \varphi_{S\,3} + \frac{2 g_1}{3} (\varphi_{S\,2}^2 - \varphi_{S\,1} \varphi_{S\,3}) = 0 \\
\frac{\partial w}{\partial \varphi^S_{03}} &= g_2 \tilde{\xi} \varphi_{S\,2} + \frac{2 g_1}{3} (\varphi_{S\,3}^2 - \varphi_{S\,1} \varphi_{S\,2}) = 0 \\
\frac{\partial w}{\partial \xi_0} &= g_4 \xi^2 + g_5 \xi \tilde{\xi} + g_6 \tilde{\xi}^2 + g_3 (\varphi_{S\,1}^2 + 2 \varphi_{S\,2} \varphi_{S\,3}) = 0 \\
\frac{\partial\, w}{\partial\, \varphi^0_{T\,1}} &= M\, \varphi_{T\,1} + \frac{2\, g}{3}\, (\varphi^2_{T\,1} - \varphi_{T\,2}\, \varphi_{T\,3}) + g_7\, \xi'\,'\, \varphi_{T\,2} + i\, g_8\, \eta_1^2 = 0 \\
\frac{\partial\, w}{\partial\, \varphi^0_{T\,2}} &= M\, \varphi_{T\,3} + \frac{2\, g}{3}\, (\varphi^2_{T\,2} - \varphi_{T\,1}\, \varphi_{T\,3}) + g_7\, \xi'\,'\, \varphi_{T\,1} + (1 - i)\, g_8\, \eta_1\, \eta_2 = 0 \\
\frac{\partial\, w}{\partial\, \varphi^0_{T\,3}} &= M\, \varphi_{T\,2} + \frac{2\, g}{3}\, (\varphi^2_{T\,3} - \varphi_{T\,1}\, \varphi_{T\,2}) + g_7\, \xi'\,'\, \varphi_{T\,3} + g_8\, \eta_2^2 = 0 \\
\frac{\partial\, w}{\partial\, \eta^0_1} &= -M_\eta\, \eta_2 + g_9\, ((1 - i)\, \eta_1\, \varphi_{T\,3} - \eta_2\, \varphi_{T\,1}) = 0 \\
\frac{\partial\, w}{\partial\, \eta^0_2} &= M_\eta\, \eta_1 - g_9\, ((1 + i)\, \eta_2\, \varphi_{T\,2} + \eta_1\, \varphi_{T\,1}) = 0 \\
\frac{\partial\, w}{\partial\, \xi'\,^0} &= M_\xi\, \xi'\,' + g_{10}\, (\varphi^2_{T\,2} + 2\, \varphi_{T\,1}\, \varphi_{T\,3}) = 0
\end{aligned}
$$

Higgs superpotential (III)

- separate set of eqs. for $\left\{\varphi_{S\,i}, \xi, \tilde{\xi}\right\}$ and $\{\varphi_{T\,i}, \eta_i, \xi''\}$
- distinct number of possible solutions
- `key feature`:

  all VEV structures preserve non-trivial subgroups of T'

  1. Z_4: $\langle\tilde{\xi}\rangle = 0\,,\ \langle\xi\rangle = u\,,\ \langle\varphi_S\rangle = (v_S, v_S, v_S)\,,\ \ v_S^2 = -\frac{g_4}{3g_3}u^2 \qquad (G_S)$

     $\langle\xi''\rangle = -\frac{M}{g_7}\,,\ \langle\eta\rangle = (0,0)\,,\ \langle\varphi_T\rangle = (v_T, v_T, v_T)\,,\ v_T^2 = \frac{M\,M_\xi}{3\,g_7\,g_{10}}$
  2. Z_3: $\langle\xi''\rangle = 0\,,\ \langle\eta\rangle = (v_1, 0)\,,\ \langle\varphi_T\rangle = (v_T, 0, 0)\,, v_1 \neq 0\,, \qquad (G_T)$

     $v_T = \frac{M_\eta}{g_9}$
  3. Z_6: $\langle\xi''\rangle = 0\,,\ \langle\eta\rangle = (0,0)\,,\ \langle\varphi_T\rangle = (v_T, 0, 0)\,,\ \ v_T = -\frac{3M}{2g}$
- choose the desired minimum by constraining the sign of the soft masses $m^2_{\varphi_T}$, $m^2_{\varphi_S}$, m^2_ξ, m^2_η, ...

Subgroups of T'

subgroups are $D_2' \simeq Q$ and the abelian groups Z_6, Z_4, Z_3 and Z_2

T'	D_2'	Z_6	Z_4	Z_3	Z_2
1	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$1'$	$\mathbf{1}$		$\mathbf{1}$		$\mathbf{1}$
$1''$	$\mathbf{1}$		$\mathbf{1}$		$\mathbf{1}$
2					
$2'$				$\mathbf{1}\ +...$	
$2''$				$\mathbf{1}\ +...$	
3		$\mathbf{1}\ +...$	$\mathbf{1}\ +...$	$\mathbf{1}\ +...$	$3\ \mathbf{1}$

$\mathbf{1}$ is total singlet of the subgroup

for comparison:

A_4	D_2	Z_3	Z_2
1	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$1'$	$\mathbf{1}$		$\mathbf{1}$
$1''$	$\mathbf{1}$		$\mathbf{1}$
3		$\mathbf{1}\ +...$	$\mathbf{1}\ +...$

Leading order

- charged leptons: $m_l = \frac{v_T}{\sqrt{2}\Lambda} v_d \,\text{diag}(y_e \left(\frac{\langle\theta\rangle}{\Lambda}\right)^{2n}, y_\mu \left(\frac{\langle\theta\rangle}{\Lambda}\right)^{n}, y_\tau)$
- neutrinos: $m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a+2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a-b/3 \\ -b/3 & a-b/3 & 2b/3 \end{pmatrix} \Rightarrow$ TB mixing!

  with masses: $\frac{v_u^2}{\Lambda}$ diag$(a+b, a, -a+b)$ and $a = x_a \frac{u}{\Lambda}$, $b = x_b \frac{v_S}{\Lambda}$
- up-type and down-type quarks:

$$m_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_1 v_\mathsf{T}/\Lambda \left(\frac{\langle\theta\rangle}{\Lambda}\right)^n & y_4 v_1/\Lambda \left(\frac{\langle\theta\rangle}{\Lambda}\right)^n \\ 0 & y_3 v_1/\Lambda & y_t \end{pmatrix} v_u \;, \quad m_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_5 v_\mathsf{T}/\Lambda & y_8 v_1/\Lambda \\ 0 & y_7 v_1/\Lambda & y_b \end{pmatrix} v_d$$

$\Rightarrow m_b$ and m_t large, $\frac{m_c}{m_t} \sim \mathcal{O}(\lambda^4)$, $\frac{m_s}{m_b} \sim \mathcal{O}(\lambda^2)$, $|V_{cb}| \sim \mathcal{O}(\lambda^2)$

Subleading order (I)

a.) take all terms up to $\mathcal{O}(\frac{1}{\Lambda^2})$ for charged fermions and all terms up to $\mathcal{O}(\frac{1}{\Lambda^3})$ for neutrinos, e.g.
$(f^c\, l\, \varphi_T\, \varphi_T)\, \frac{h_d}{\Lambda^2}$, $(f^c\, l\, \eta\, \eta)\, \frac{h_d}{\Lambda^2}$ and $(\varphi_T\, \varphi_S)'\, (l\, l)''\, \frac{h_u^2}{\Lambda^3}$, $\xi''\, \xi\, (l\, l)'\, \frac{h_u^2}{\Lambda^3}$

b.) take all terms up to $\mathcal{O}(\frac{1}{\Lambda})$ in Higgs sector, e.g.
$\frac{1}{\Lambda}(\varphi_T^0\, \varphi_T)\, (\varphi_T\, \varphi_T)$ and $\frac{1}{\Lambda}\, (\varphi_T^0\, \varphi_T)''\, \xi''\, \xi''$, $\frac{1}{\Lambda}\, (\varphi_T\, \eta)\, (\varphi_T^0\, \eta)$

$\rightarrow$ induce shifts of the VEVs v into $v + \delta v$: $\frac{\delta v}{\Lambda} \approx \left(\frac{v}{\Lambda}\right)^2 \approx \lambda^4$

<u>results</u>:

- important corrections to quarks are due to $\mathrm{b.)}$
- contributions from $\mathrm{a.)}$ and $\mathrm{b.)}$ correct TB mixing
- corrections should be $\lesssim \lambda^2$ for TB mixing and at the same time reproduce θ_C in the quark sector (checked $\surd$)

Subleading order (II)

- orders of mass matrix elements:

$$m_u = \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_u \;, \quad m_d = \begin{pmatrix} \lambda^6 & \lambda^3 & \lambda^4 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_d \;.$$

- two predictions:

$$\sqrt{\frac{m_d}{m_s}} = |V_{us}| + \mathcal{O}(\lambda^2)$$

and $\sqrt{\frac{m_d}{m_s}} = \left|\frac{V_{td}}{V_{ts}}\right| + \mathcal{O}(\lambda^2)$ (due to $|V_{ub}| \sim \mathcal{O}(\lambda^4)$)

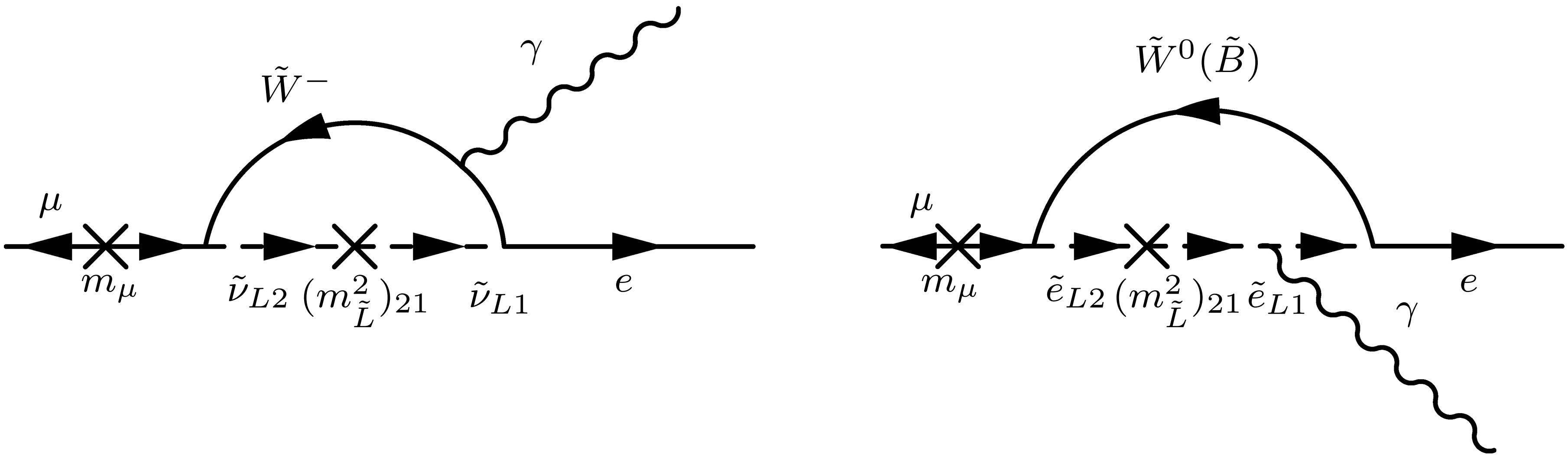
- furthermore:

$$\frac{m_u}{m_c} \sim \mathcal{O}(\lambda^4) \;, \quad \frac{m_d}{m_s} \sim \mathcal{O}(\lambda^2) \;, \quad |V_{ud}| \approx |V_{cs}| \approx 1 + \mathcal{O}(\lambda^2) \;, \quad |V_{tb}| \approx 1 \;,$$

$$|V_{us}| \approx |V_{cd}| \sim \mathcal{O}(\lambda) \;, \quad |V_{cb}| \approx |V_{ts}| \sim \mathcal{O}(\lambda^2) \;, \quad |V_{td}| \sim \mathcal{O}(\lambda^3) \;.$$

LFVs -prediction of $\mu \to e\gamma$ (I)

$\mu \to e\gamma$ is mediated by sfermion-neutralino/chargino loops in the MSSM



In the MSSM with arbitrary soft terms the branching ratio for $\mu \to e\gamma$ exceeds the experimental bound

$$\text{Br}(\mu \to e\gamma) \lesssim 1.2 \times 10^{-11} \quad (\text{MEGA})$$

This bound will be further reduced by the MEG experiment:

$$\text{Br}(\mu \to e\gamma) \lesssim \mathcal{O}(10^{-13})$$

LFVs - prediction of $\mu \to e\gamma$ (II)

Look at the soft masses for the sleptons: $(m^2_{\tilde{L}})_{ij}\, \tilde{L}_i^\dagger\, \tilde{L}_j$ and $(m^2_{\tilde{e}})_{ij}\, \tilde{e}_i^{c\,\star}\, \tilde{e}_j^c$. In general they are arbitrary, but with the use of the flavor symmetry $G_F = A_4$ or $G_F = T'$ they can be constrained.

Since $l_i \sim 3$,

$$m^2_{\tilde{L}} \propto \mathbb{1}$$

and since $e_i^c \sim 1 + 1'' + 1'$,

$$m^2_{\tilde{e}} \propto \begin{pmatrix} \beta_e & 0 & 0 \\ 0 & \beta_\mu & 0 \\ 0 & 0 & \beta_\tau \end{pmatrix}$$

at lowest order, i.e. without insertions of flavon fields. These insertions generate off-diagonal terms in the slepton mass matrices as well as the lepton mass matrices itself. Therefore $\mu \to e\gamma$ will be mediated, but all operators are now controlled by a symmetry.

Conclusion & Outlook

- T' model is a working low energy model which can explain lepton mixing (TB mixing) and properties of the quark sector at the same time
- key feature is the VEV alignment and the preservation of non-trivial subgroups of T' in the charged fermion as well as the neutrino sector
- the model has further phenomenological imprints: LFVs, FCNCs and possibly also inflation

- still missing:
  - group theory reason for success of A_4/ T'/ $Z_3 \rtimes Z_7$
  - simpler mechanism for VEV alignment
  - GUT model with $SO(10) \times G_F$

Thank you.