Lightest neutralino in the MNSSM

Roman Nevzorov
Glasgow University

in collaboration with S.Hesselbach, D.J.Miller, G.Moortgat-Pick and M.Trusov
Outline

Introduction

Neutralino sector in the MNSSM

Upper bound on the mass of the lightest neutralino

Approximate solution for the lightest neutralino mass

Conclusions
Introduction

- Recent observations indicate that 22% – 25% of the energy density of the Universe exists in the form of stable non–baryonic, non–luminos (dark) matter.

- The existence of dark matter is the strongest piece of evidence for physics beyond the SM.

- In the MSSM the lightest SUSY particle (LSP) can play the role of dark matter.

- In most SUSY scenarios the LSP is the lightest neutralino.

- Since neutralinos are heavy weakly interacting particles they can
  - explain the large scale structure of the Universe;
  - provide the correct relic abundance of dark matter.
But MSSM being incorporated in supergravity or GUTs suffers from the $\mu$ problem. Indeed, in SUGRA models

$$W_{SUGRA} = W_0(h_m) + \mu(h_m)(\hat{H}_d\hat{H}_u) + h_t(h_m)(\hat{Q}\hat{H}_2)\hat{U}_R + \ldots ,$$

where $\mu(h_m) \sim M_{Pl}$ or $\mu(h_m) = 0$.

The correct pattern of electroweak symmetry breaking requires

$$\mu(h_m) \sim 100 - 1000 \text{GeV}.$$  

In the NMSSM the superpotential is invariant under $Z_3$ discrete symmetry, i.e.

$$\mu(H_1H_2) \rightarrow \lambda S(H_1H_2) + \frac{\kappa}{3} S^3 .$$

At the EW scale field $S$ acquires VEV inducing an effective $\mu$ term

$$\mu_{eff} = \lambda \langle S \rangle .$$
However VEVs of the Higgs fields break $Z_3$ symmetry resulting in the formation of domain walls which create unacceptably large CMB anisotropies.

Non–renormalizable operators that break $Z_3$ symmetry give rise to quadratically divergent tadpole contributions destabilising the mass hierarchy.


The $Z_2^R$ or $Z_5^R$ symmetries allow to suppress the potentially harmful operators.


High order operators do not affect mass hierarchy but prevent the appearance of domain walls.
Neutralino sector in the MNSSM

The superpotential of the corresponding simplest extension of the MSSM – Minimal Non–minimal Supersymmetric Standard Model (MNSSM) is

\[ W_{\text{MNSSM}} = \lambda \hat{S}(\hat{H}_d \hat{H}_u) + \xi \hat{S} + W_{\text{MSSM}}(\mu = 0). \]

where \( \xi \lesssim (\text{TeV})^2. \)


High order operators which are not forbidden by \( Z_5^R \) symmetry induce linear term \( \xi \hat{S} \) in the superpotential that breaks \( Z_3 \) and Peccei–Quinn symmetries.

The neutralino sector of the MNSSM is formed by the superpartners of the neutral gauge bosons (\( \tilde{W}_3, \tilde{B} \)) and neutral Higgsino fields (\( \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S} \)).
In the field basis \((\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})\) the neutralino mass matrix takes a form

\[
M_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta & 0 \\
0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta & 0 \\
-M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu_{\text{eff}} & -\frac{\lambda v}{\sqrt{2}} s_\beta \\
M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu_{\text{eff}} & 0 & -\frac{\lambda v}{\sqrt{2}} c_\beta \\
0 & 0 & -\frac{\lambda v}{\sqrt{2}} s_\beta & -\frac{\lambda v}{\sqrt{2}} c_\beta & 0
\end{pmatrix},
\]

\(s_W = \sin \theta_W, \quad c_W = \cos \theta_W, \quad s_\beta = \sin \beta, \quad c_\beta = \cos \beta, \quad \mu_{\text{eff}} = \frac{\lambda s}{\sqrt{2}},\)

\(\tan \beta = \frac{v_2}{v_1}, \quad \langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle = \frac{s}{\sqrt{2}}.\)
The spectrum of neutralino is defined by
\[ \lambda, \; \mu_{\text{eff}}, \; \tan \beta, \; M_1, \; M_2. \]

The direct chargino searches set limits on
\[ |M_2|, \; |\mu_{\text{eff}}| > 90 - 100 \text{ GeV}. \]

In SUSY GUT's one gets
\[ M_2 = \frac{\alpha_2(M_Z)}{\alpha_1(M_Z)} M_1 \approx 2 M_1. \]

The requirement of validity of perturbation theory up to the GUT scale constrains the allowed range of \( \lambda \)
\[ \lambda(M_Z) \lesssim 0.7. \]

When \( \lambda \) is small the non-observation of Higgs boson at LEP rules out low values of \( \tan \beta \lesssim 2.5 \).
Upper bound on the mass of $\chi^0_1$

- In order to find theoretical bounds on the neutralino masses $m_{\chi^0_i}$ it is convenient to consider matrix $M_{\tilde{\chi}^0} M^\dagger_{\tilde{\chi}^0}$.

- The eigenvalues of $M_{\tilde{\chi}^0} M^\dagger_{\tilde{\chi}^0}$ are equal to $|m_{\chi^0_i}|^2$.

- In the basis $(\tilde{B}, \tilde{W}_3, -\tilde{H}^0_d s_\beta + \tilde{H}^0_u c_\beta, \tilde{H}^0_d c_\beta + \tilde{H}^0_u s_\beta, \tilde{S})$ the bottom-right $2 \times 2$ block of $M_{\tilde{\chi}^0} M^\dagger_{\tilde{\chi}^0}$ takes a form

$$
\begin{pmatrix}
|\mu_{eff}|^2 + \sigma^2 & \nu^* \mu_{eff} \\
\nu \mu_{eff}^* & |\nu|^2
\end{pmatrix},
$$

$$
\sigma^2 = M_Z^2 \cos^2 2\beta + |\nu|^2 \sin^2 2\beta, \quad \nu = \frac{\lambda v}{\sqrt{2}}.
$$

- Since the minimal eigenvalue of $M_{\tilde{\chi}^0} M^\dagger_{\tilde{\chi}^0}$ is less than its smallest diagonal element, $|m_{\chi^0_1}| \lesssim |\nu|$.
The mass of the lightest neutralino must be also smaller than the minimal eigenvalue of bottom-right $2 \times 2$ submatrix of $M_{\tilde{\chi}^0} M_{\tilde{\chi}^0}^\dagger$, i.e.

$$|m_{\chi_1^0}|^2 \lesssim \frac{1}{2} \left[ |\mu_{eff}|^2 + \sigma^2 + |\nu|^2 - \sqrt{\left( |\mu_{eff}|^2 + \sigma^2 + |\nu|^2 \right)^2 - 4|\nu|^2 \sigma^2} \right].$$

The lightest neutralino mass vanish when $\lambda \to 0$.

The upper bound on $m_{\chi_1^0}$ decreases when $|\mu_{eff}|$ grow and at large $|\mu_{eff}| \gg M_Z$

$$|m_{\chi_1^0}|^2 \lesssim \frac{|\nu|^2 \sigma^2}{\left( |\mu_{eff}|^2 + \sigma^2 + |\nu|^2 \right)}.$$

Taking into account the restrictions on $\mu_{eff}$ and $\lambda(M_Z)$
we find

$$|m_{\chi_1^0}|^2 < 0.8 M_Z^2 \implies m_{\chi_1^0} \lesssim 80 - 85 \text{GeV}.$$
Approximate solution for $m_{\chi_1^0}$

Neutralino masses obey characteristic equation

$$\det(M_{\tilde{\chi}^0} - \kappa I) = \left( M_1 M_2 - (M_1 + M_2)\kappa + \kappa^2 \right) \left( \kappa^3 - (\mu_{eff}^2 + \nu^2)\kappa + \nu^2 \mu_{eff} \sin 2\beta \right) + M_Z^2 \left( \tilde{M} - \kappa \right) \left( \kappa^2 + \mu_{eff} \sin 2\beta \kappa - \nu^2 \right) = 0,$$

where $\tilde{M} = M_1 c_W^2 + M_2 s_W^2$ and $\kappa$ is an eigenvalue of $M_{\tilde{\chi}^0}$.

Because in the MNSSM $|m_{\chi_1^0}|$ is considerably smaller than $|m_{\chi_2^0}|$ one can ignore $\kappa^3$, $\kappa^4$ and $\kappa^5$ terms in the characteristic equation so that it reduces to

$$\kappa^2 - B \kappa + C = 0.$$

Then the approximate solution for $m_{\chi_1^0}$ can be written as

$$|m_{\chi_1^0}| = \text{Min} \left\{ \frac{1}{2} \left| B - \sqrt{B^2 - 4C} \right|, \frac{1}{2} \left| B + \sqrt{B^2 - 4C} \right| \right\}.$$
$B$ and $C$ are given by

\[
B = \frac{M_1 M_2}{M_1 + M_2} + \left( \frac{\nu^2}{\mu_{eff}^2 + \nu^2} - \frac{M_Z^2}{M_1 + M_2} \right) \mu_{eff} \sin 2\beta \\
- \frac{M_Z^2 \nu^2}{(M_1 + M_2)(\mu_{eff}^2 + \nu^2)}
\]

\[
C = \frac{\nu^2}{\mu_{eff}^2 + \nu^2} \left( \frac{M_1 M_2}{M_1 + M_2} \mu_{eff} \sin 2\beta - \frac{\tilde{M}}{M_1 + M_2} M_Z^2 \right).
\]

Lightest neutralino mass for $\lambda = 0.7$, $M_1 = 0.5 M_2$ and $\tan \beta = 10, 3$. 

\[
\begin{array}{cccc}
\mu_{eff} & \mu_{eff} & \mu_{eff} & \mu_{eff} \\
\hline
10 & 20 & 30 & 40 \\
20 & 30 & 40 & 50 \\
30 & 40 & 50 & 60 \\
40 & 50 & 60 & 70
\end{array}
\]

\[
\begin{array}{cccc}
M_2 & M_2 & M_2 & M_2 \\
\hline
-400 & -200 & 0 & 200 \\
-200 & 0 & 200 & 400 \\
0 & 200 & 400 & 600 \\
200 & 400 & 600 & 800
\end{array}
\]
When $|m_{\chi_1^0}|$ is close to its maximum value the lightest neutralino is basically formed by $\tilde{B}$ and $\tilde{S}$.

Lightest neutralino mass for $\lambda = 0.7$, $M_1 = 0.5 M_2$, $M_2 = \mu_{\text{eff}}$
When $|m_{\chi_1^0}|$ is considerably less than $M_Z$ the lightest neutralino is predominantly singlino.

If either $\mu_{eff}$ or $M_{1,2} \gg M_Z$ then

$$|m_{\chi_1^0}| \sim \frac{|\mu_{eff}| \nu^2 \sin 2\beta}{\mu_{eff}^2 + \nu^2}.$$  

The lightest neutralino mass decreases with raising of $\mu_{eff}$ and $\tan \beta$.

Since the correct EW symmetry breaking requires $\mu_{eff} = \text{const}$ when $\lambda \to 0$ the lightest neutralino mass is proportional to $\lambda^2$ at small values of $\lambda$.

At very large $\tan \beta$

$$|m_{\chi_1^0}| \to \frac{\nu^2 M_Z^2}{\mu^2 + \nu^2} \left| \frac{\tilde{M}}{M_1 M_2} \right|.$$  

The lightest neutralino mass reduces when $M_1$ and $M_2$ grow.
Conclusions

We have argued that in contrast with the MSSM the allowed range of the mass of the lightest neutralino in the MNSSM is limited.

In the allowed part of the parameter space the lightest neutralino mass does not exceed $80 - 85 \text{ GeV}$.

We have found the approximate solution for the lightest neutralino mass.

- At large values of $\mu$–term $m_{\chi_1^0}$ is inversely proportional to $\mu_{\text{eff}}$.
- $|m_{\chi_1^0}|$ vanishes in the limit when $\lambda \rightarrow 0$.
- $|m_{\chi_1^0}|$ decreases with raising of $\tan \beta$, $M_1$, and $M_2$.

In the allowed part of the parameter space the lightest neutralino is predominantly singlino that makes rather difficult its observation at future colliders.