Curvature and isocurvature perturbations in two-field inflation

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based on arXiv:0704.0212

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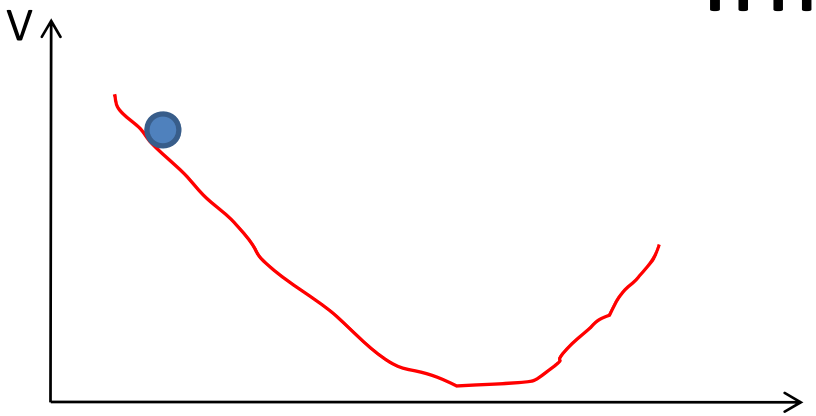
Inflation



$$\ddot{\sigma} + 3H\dot{\sigma} + V_\sigma = 0$$

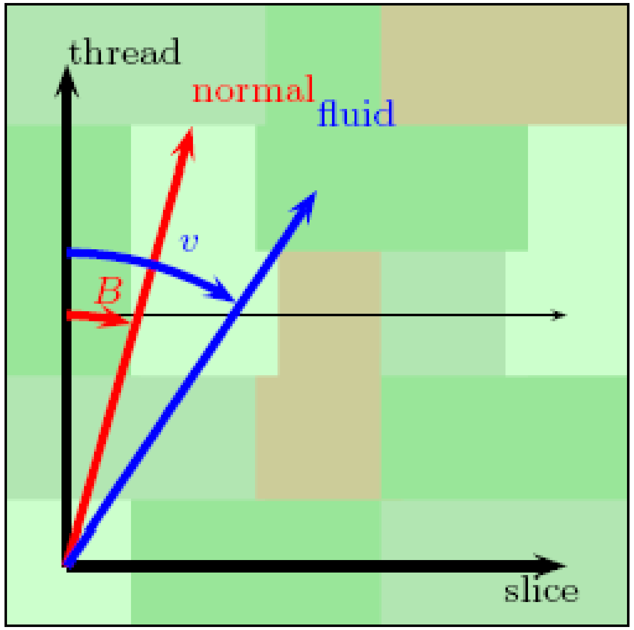
$$H \equiv \frac{\dot{a}}{a} = \frac{1}{3M_{Pl}^2}\rho$$

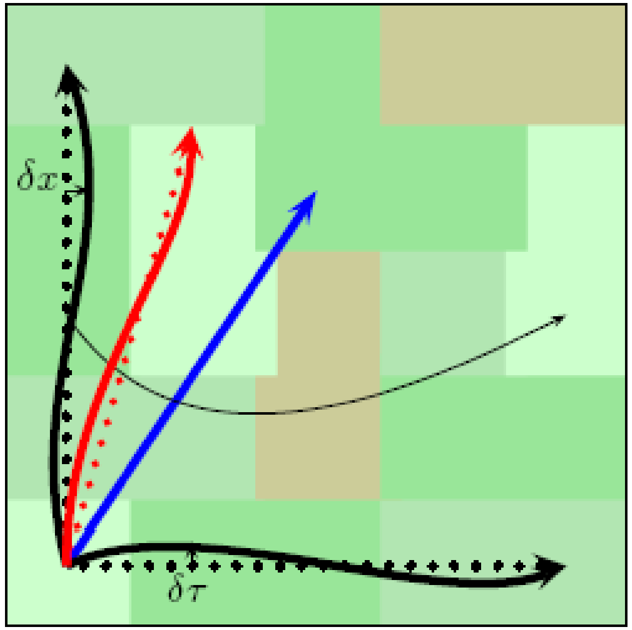
$$\varepsilon = \frac{\dot{\sigma}^2}{2H^2 M_{Pl}^2} \qquad \eta_{\sigma\sigma} = \frac{V_{\sigma\sigma}}{3H^2}$$

small = slow-roll

- solves the flatness problem
- solves the horizon problem
- dilutes any preexisting relics
- generates primordial density perturbations through quantum fluctuations of the coupled inflaton-gravity system

Curvature perturbations

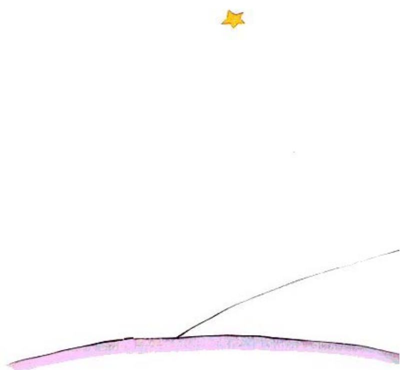


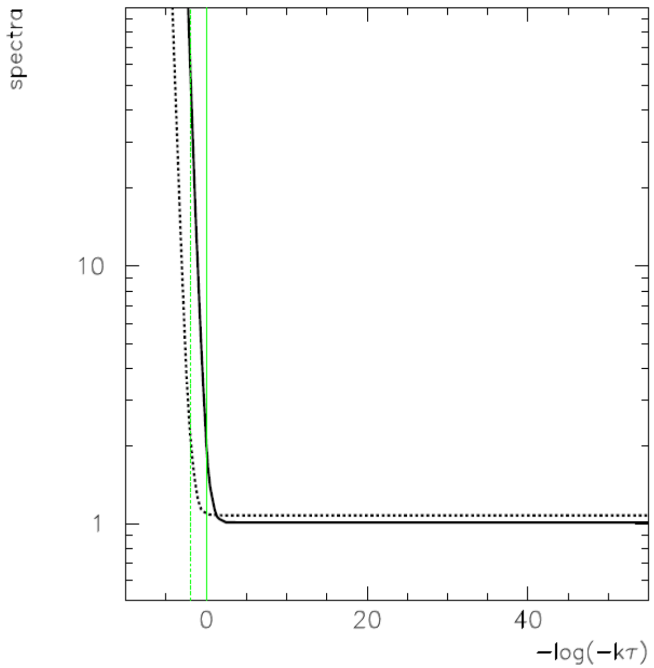


Q_σ gauge invariant variable describing perturbations

$$R = \frac{H}{\dot{\phi}} Q_\sigma$$ 3-dim curvature of the comoving hypersurfaces

Evolution of perturbations



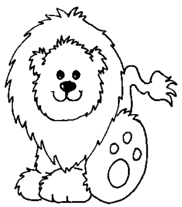


$$\langle Q_{\sigma}{}^{*}_{\mathbf{k}} Q_{\sigma \mathbf{k}'} \rangle = \frac{2\pi^2}{k^3} \mathcal{P}_{Q_\sigma}(k) \delta(\mathbf{k} - \mathbf{k}')$$

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H_*^2}{2\pi \dot{\sigma}_*} \right)^2$$

$$n_s = \underbrace{1 + \mathrm{d} \ln \mathcal{P}_{\mathcal{R}} / \mathrm{d} \ln k}_{1 - 6\epsilon_* + 2\eta}$$

Road map



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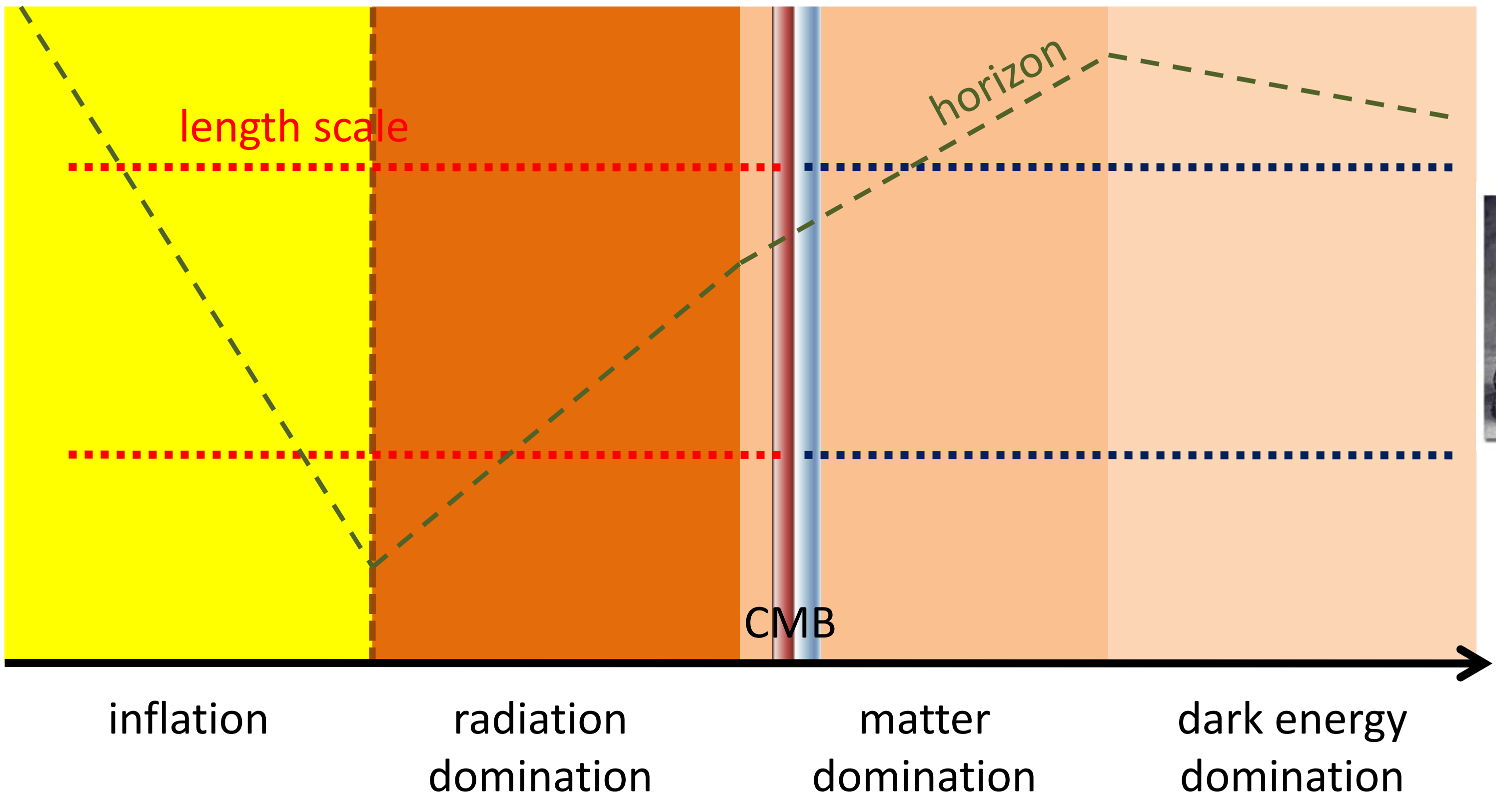
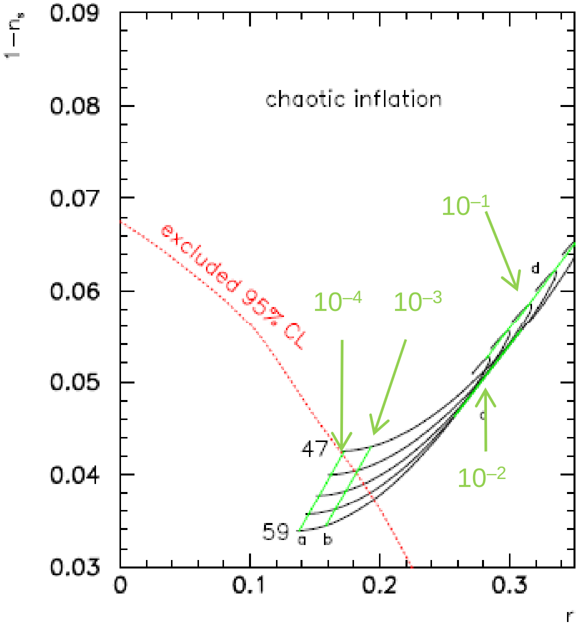
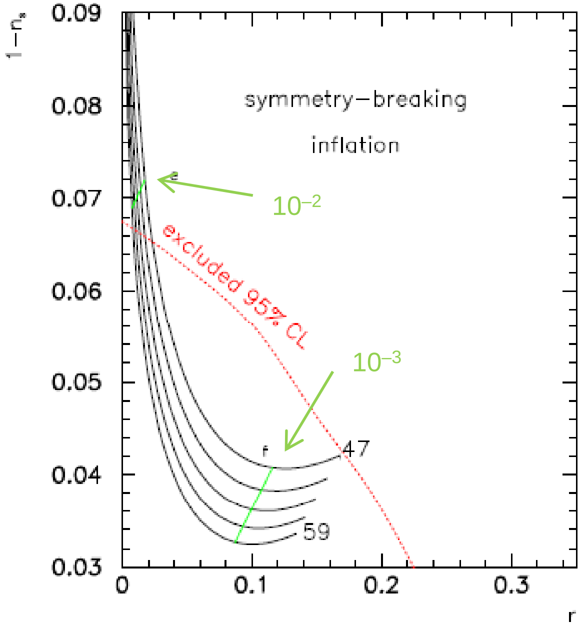


Observations and inflation

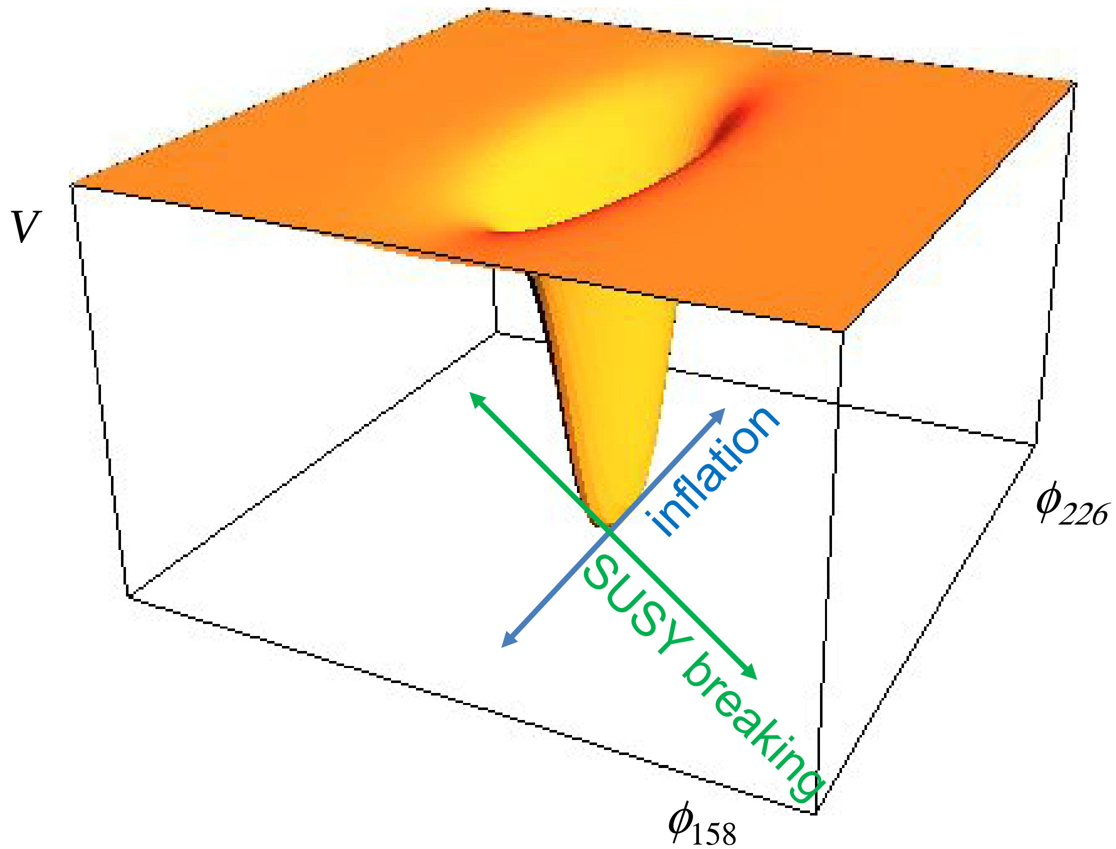
Ellis, Lalak, Pokorski, kt, 2006





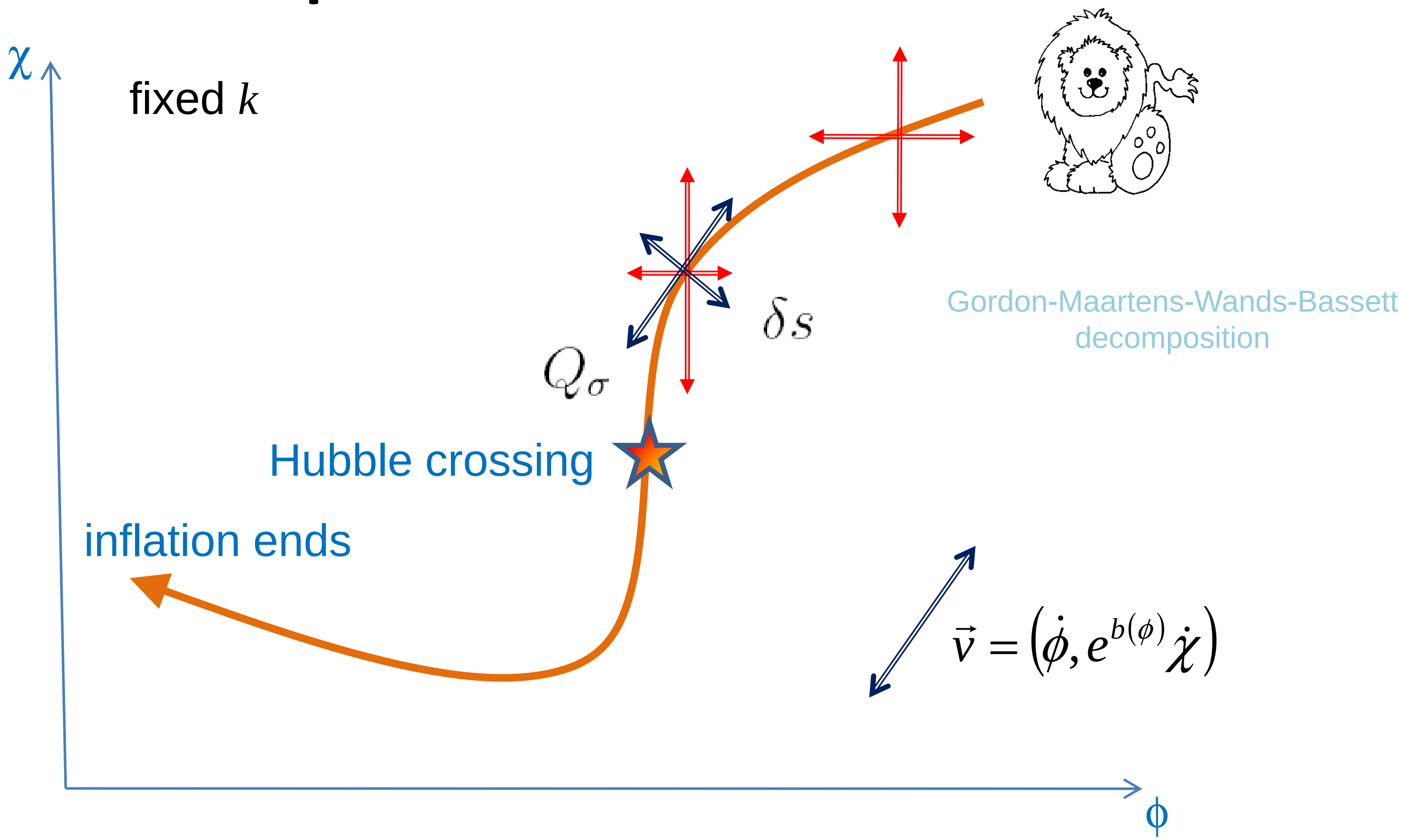
$$V(\phi) = \frac{1}{2}m^2\phi^2\left(1 + \frac{1}{2}\kappa_c^2\phi^2\right) \qquad V(\phi) = \lambda m_P^4\left(1 - \kappa_s^2\frac{\phi^2}{m_P^2}\right)^2$$

Motivation for multi-field scenarios



$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{e^{2b(\phi)}}{2} (\partial_\mu \chi)(\partial^\mu \chi) - V(\phi, \chi) \right]$$

Curvature and isocurvature perturbations



$$\mathcal{R} \equiv \frac{H}{\dot{\sigma}} Q_\sigma$$ curvature perturbation

$$\mathcal{S} \equiv \frac{H}{\dot{\sigma}} \delta s$$ isocurvature perturbation

Equations

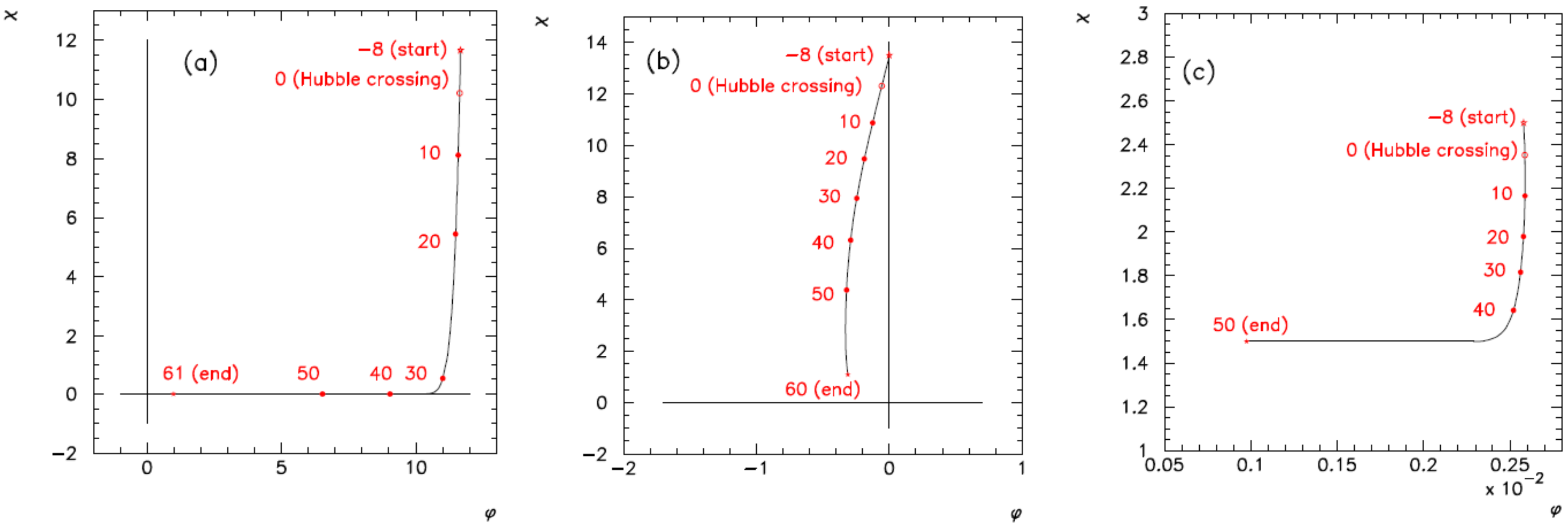
$$\ddot{Q}_\sigma + 3H\dot{Q}_\sigma + \left(\frac{k^2}{a^2} + C_{\sigma\sigma}\right) Q_\sigma + \frac{2V_s}{\dot{\sigma}}\dot{\delta s} + C_{\sigma s}\,\delta s = 0$$

$$\ddot{\delta s} + 3H\dot{\delta s} + \left(\frac{k^2}{a^2} + C_{ss}\right)\delta s - \frac{2V_s}{\dot{\sigma}}\dot{Q}_\sigma + C_{s\sigma}Q_\sigma = 0\,,$$

$$\begin{aligned}
C_{\sigma\sigma} &= V_{\sigma\sigma} - \left(\frac{V_s}{\dot{\sigma}}\right)^2 + 2\frac{\dot{\sigma}V_\sigma}{M_P^2 H} + \frac{3\dot{\sigma}^2}{M_P^2} - \frac{\dot{\sigma}^4}{2M_P^4H^2} - b_\phi\left(s_\theta^2 c_\theta V_\sigma + (c_\theta^2+1)s_\theta V_s\right)\\
C_{\sigma s} &= 6H\frac{V_s}{\dot{\sigma}} + \frac{2V_\sigma V_s}{\dot{\sigma}^2} + 2V_{\sigma s} + \frac{\dot{\sigma}V_s}{M_P^2 H} + 2b_\phi(s_\theta^3 V_\sigma - c_\theta^3 V_s)\\
C_{ss} &= V_{ss} - \left(\frac{V_s}{\dot{\sigma}}\right)^2 + b_\phi(1+s_\theta^2)c_\theta V_\sigma + b_\phi c_\theta^2 s_\theta V_s - \dot{\sigma}^2(b_{\phi\phi} + b_\phi^2)\\
C_{s\sigma} &= -6H\frac{V_s}{\dot{\sigma}} - \frac{2V_\sigma V_s}{\dot{\sigma}^2} + \frac{\dot{\sigma}V_s}{M_P^2 H}
\end{aligned}$$

$$\cos\theta \equiv \frac{\dot{\phi}}{\dot{\sigma}}, \qquad \sin\theta \equiv \frac{\dot{\chi}\,e^b}{\dot{\sigma}} \qquad \text{with} \qquad \dot{\sigma} \equiv \sqrt{\dot{\phi}^2 + e^{2b}\dot{\chi}^2}$$

Examples



$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$

$$7m_\phi = m_\chi$$

$$b = 0$$

$$m_\phi = m_\chi$$

$$b(\phi) = -\phi/M_P$$

double inflation

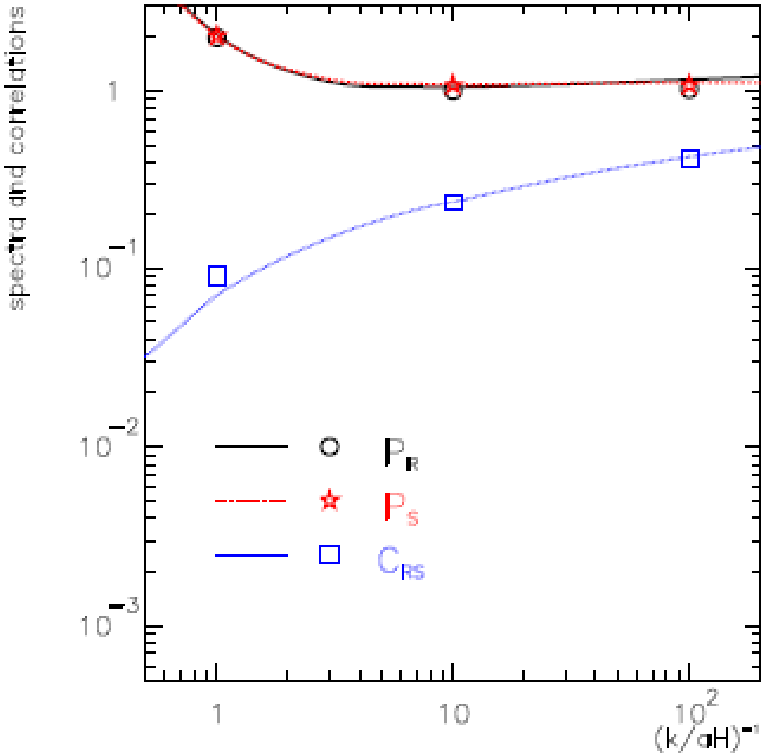
$$V(\phi,\chi) = V_0 + V_1\sqrt{\psi(\phi)}e^{-2\beta_1\psi(\phi)} +$$
$$+ V_2\psi(\phi)e^{-\beta_1\psi(\phi)}\cos(\beta_2\chi)$$

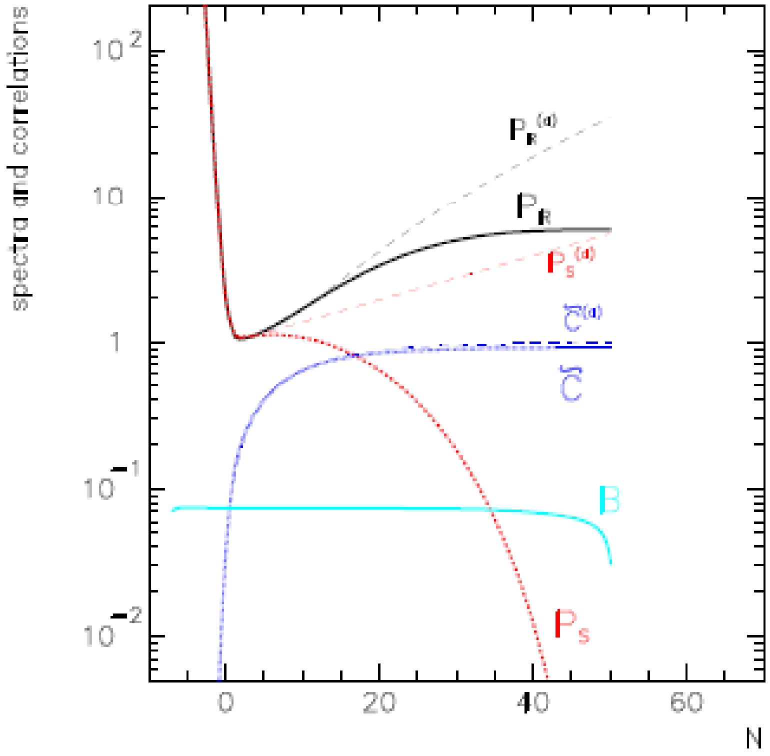
$$b(\phi) = b_0 - \frac{1}{3}\ln\left(\frac{\phi}{M_P}\right)$$

roulette inflation

Bond et al., 2006

Examples





roulette
inflation

Examples

n_s	$1-6\epsilon_* + 2\eta_{\sigma\sigma *}$	single-field result	full result
double inflation (canonical)	0.929	0.982	0.967
double inflation (non-canonical)	0.953	0.968	0.934
roulette inflation	1.017	1.019	0.932

$$\mathcal{P}_{\mathcal{R}}^{\rm sf}(k) \simeq \frac{H^4}{4\pi^2\dot{\sigma}^2} = \frac{H^4}{8\pi^2\mathcal{L}_{\rm kin}}$$

Conclusions

good motivation to look for the implications
of next-to-minimal models of inflation

richer dynamics; possible generation of isocurvature perturbations
(after reheating)

cautionary tale: predictions may differ from the single-field case

We have a tool
(for studying the evolution of the multi-inflaton systems)
and won’t hesitate to use it!