Constraints on the very early universe from thermal WIMP Dark Matter

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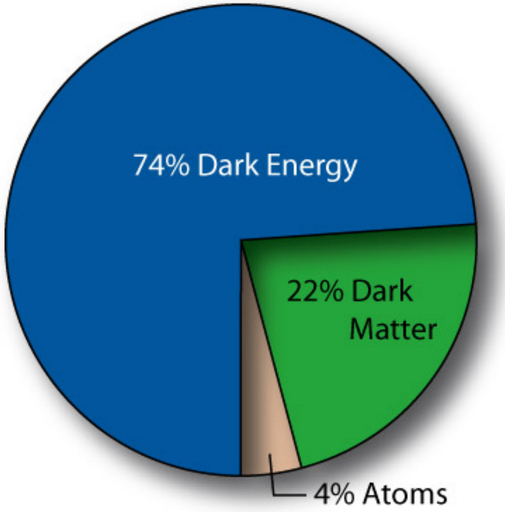
Refs:

- PRD73 (2006) 123502 [hep-ph/0603165]
- arXiv:0704.1590 [hep-ph]

1. Motivation



[http://map.gsfc.nasa.gov]

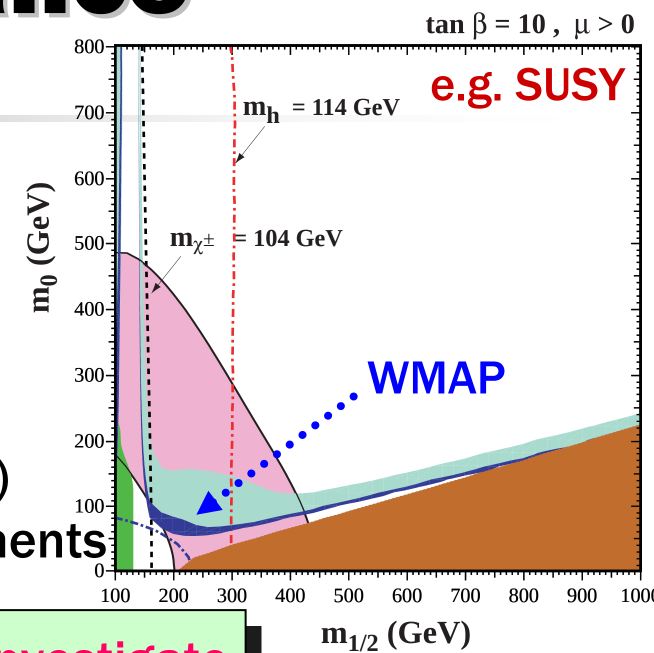
- Observations of
  - cosmic microwave background
  - structure of the universe
  - etc.

Non-baryonic cold dark matter (CDM): $0.8 < \Omega_{\rm CDM} h^2 < 0.12$ (95% CL)

- **Neutral, stable (long-lived) weakly interacting massive particles (WIMPs) χ are good candidates for CDM**
  - Neutralino (LSP); 1st KK mode of the B boson (LKP); etc.

When WIMPs were in full thermal eq., the relic abundance naturally falls around the observed CDM abundacne: $\Omega_{\chi,{\rm standard}} h^2 \sim 0.1$

Investigation of early universe using CDM abundance



[From Ellis et al., PLB565 (2003) 176]

- The relic abundance of thermal WIMPs is determined by the Boltzmann equation:

$$\dot{n}_\chi + 3 H n_\chi = -\langle \sigma_{\rm eff} v \rangle (n_\chi^2 - n_{\chi,{\rm eq}}^2)$$

(and the reheat temperature: T_R)

- The (effective) cross section $\sigma_{\rm eff}$ can be (hopefully) determined from collider and DM detection experiments

We can test the standard CDM scenario and investigate the conditions of very early universe: $T_R, H, \cdots$

- Standard scenario:
  - χ was in chemical eq. $\Omega_\chi h^2$ is independent of T_R
  - $H = \frac{\pi T^2}{M_{\rm Pl}} \sqrt{\frac{g_*}{90}}$ (g_*: Rel. dof)
- Non-standard scenarios:
  - Low reheat temperature
  - Entropy production
  - Modified Hubble parameter
  - Non-thermal production

[Scherrer et al., PRD(1985); Salati,PLB(2003); Fernengo et al., PRD(2003); Chung et al., PRD (1999); ...]

Outline

This work

- We provide an approximate analytic treatment that is applicable to low-reheat-temperature scenarios
- Based on the assumption of CDM = thermal WIMP
  - we derive the lower bound on the maximal temperature of RD epoch
  - we constrain possible modifications of the Hubble parameter

c.f. Cosmic p^+, γ ⇒ Bounds on pre-BBN expansion

[Schelke, Catena, Fornengo, Masiero, Pietroni PRD74 (2006); Donato, Fornengo, Schelke, JCAP0703 (2007)]

1. Motivation
2. Standard calculation of WIMP relic abundance
3. Low-temperature scenario
4. Constrains on the very early universe from WIMP dark matter
5. Summary

2. Standard calculation of the WIMP relic abundance

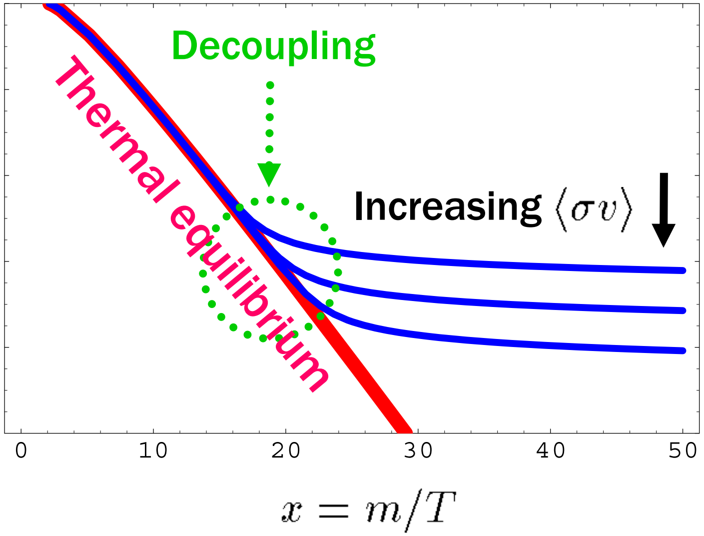
[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- Conventional assumptions for χ:
  - $\chi = \bar{\chi}$, single production of χ is forbidden
  - Thermal equilibrium was maintained
- For adiabatic expansion the Boltzmann eq. is

$$\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\langle\sigma v\rangle s}{Hx}(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2),$$

$$Y_{\chi(,\mathrm{eq})} = \frac{n_{\chi(,\mathrm{eq})}}{s}, \, x = \frac{m_\chi}{T}$$

Co-moving number density



- During the RD epoch, χ and decoupled when they were non-relativistic:

$$\langle\sigma v\rangle = a + 6b/x + \mathcal{O}(1/x^2), \quad n_{\chi,\mathrm{eq}} = g_\chi \, (m_\chi T/2\pi)^{3/2} \mathrm{e}^{-m_\chi/T}$$

$$\boxed{\Omega_{\chi,\mathrm{standard}} h^2 \simeq 0.1 \times \left(\frac{a + 3b/x_F}{10^{-9}\ \mathrm{GeV}^{-2}}\right)^{-1} \left(\frac{x_F}{22}\right) \left(\frac{g_*}{90}\right)^{-1/2} \sim \Omega_{\mathrm{CDM}} h^2}$$

3. Low-temperature scenario

- T_R : Reheat temperature

  The initial abundance is assumed to be negligible: $Y_\chi(x_0) = 0\,, \quad x_0 = \frac{m_\chi}{T_R}$

- Zeroth order approximation:

$T_R < T_F$ ⇒ χ annihilation is negligible:

$$\frac{dY_0}{dx} = 0.028\ g_\chi^2 g_*^{-3/2} m_\chi M_{\rm Pl} {\rm e}^{-2x} x \left(a + \frac{6b}{x}\right)$$

⇒ The solution is proportional to the cross section:

At late times,

$$\boxed{Y_0(x \gg x_0) \simeq 0.014\ g_\chi^2 g_*^{-3/2} m_\chi M_{\rm Pl} {\rm e}^{-2x_0} x_0 \left(a + \frac{6b}{x_0}\right)}$$

This solution should be smoothly connected to the standard result

First order approximation

- Add a correction term describing annihilation to Y_0 : $Y_1 = Y_0 + \delta \ (\delta < 0)$
- As long as $|\delta| \ll Y_0$, the evolution equation for δ is

$$\frac{d\delta}{dx} = -1.3\,\sqrt{g_*}m_\chi M_{\rm PL}\left(a+\frac{6b}{x}\right)\frac{Y_0(x)^2}{x^2}$$

➡ The solution is proportional to σ^3

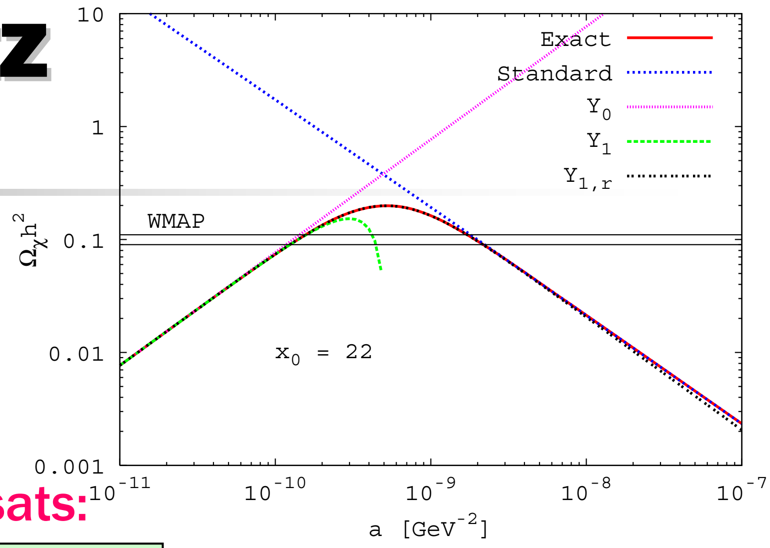
At late times,

$$\boxed{\delta(x \gg x_0) \simeq -2.5\times 10^{-4}\ g_\chi^4 g_*^{-5/2} m^3 M_{\rm Pl}^3 {\rm e}^{-4x_0} x_0 \left(a+\frac{3b}{x_0}\right)\left(a+\frac{6b}{x_0}\right)^2}$$

- $|\delta|$ soon dominates over Y_0 for not very small cross section

➡ Y_1 fails to track the exact solution

Re-summed ansatz



- It is noticed that $Y_0 \propto \sigma > 0, \ \delta \propto \sigma^3 < 0$

  For large cross section,

  $Y_\chi(x \to \infty)$ should be $\propto 1/\langle \sigma v \rangle$

- This observation suggests the re-summed ansats:

$$\boxed{Y(x) = Y_0 + \delta = Y_0\left(1 + \frac{\delta}{Y_0}\right) \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}}$$

- For $|\delta| \gg Y_0$, $\boxed{Y_{1,r}(x) \simeq -\frac{Y_0^2}{\delta} \propto \frac{1}{\sigma}}$

$x_0 \to x_F$ ⇨ Standard formula

At late times, $Y_{1,r}(x \to \infty) = \dfrac{x_0}{1.3\ \sqrt{g_*} m_\chi M_{\mathrm{Pl}}(a + 3b/x_0)}$

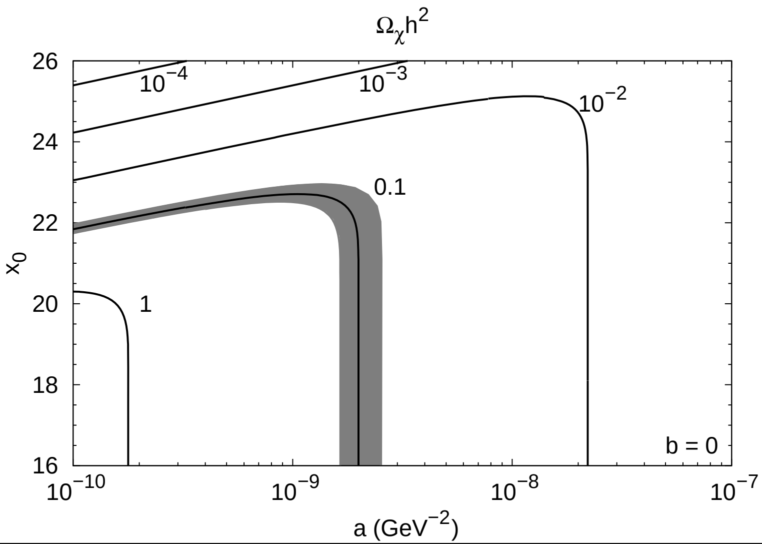
- In the case where χ production is negligible but the initial abundance is sizable, $Y_{1,r}$ is exact

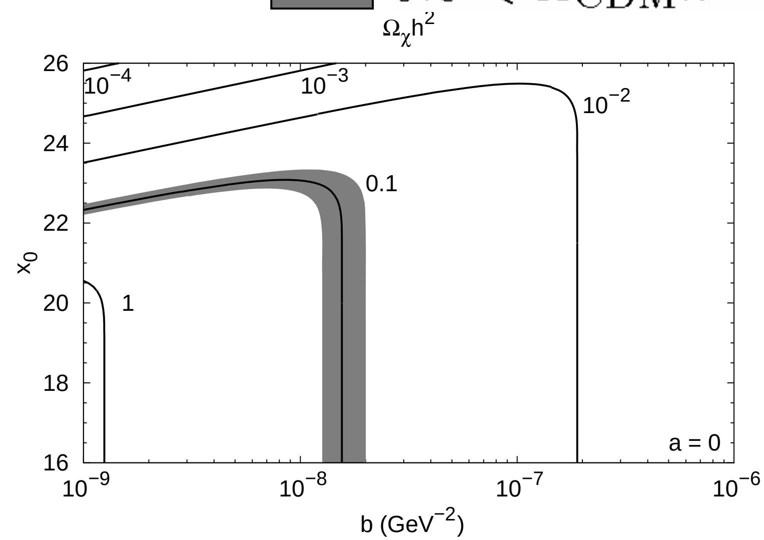
4. Constrains on the very early universe from WIMP DM

- **Out-of-equilibrium case:** $\sigma \nearrow \Rightarrow \Omega h^2 \nearrow$; $T_0 = m_\chi / x_0 \nearrow \Rightarrow \Omega h^2 \nearrow$

  Equilibrium case: $\sigma \nearrow \Rightarrow \Omega h^2 \searrow$; Ωh^2 Independent of T_0

- **Thermal relic abundance in the RD universe:** $0.8 < \Omega_{\rm CDM} h^2 < 0.12$





Assumption of $\Omega_{\rm CDM} h^2 = \Omega_{\chi,\rm thermal} h^2$,

$\Rightarrow$ Lower bound on the maximal temperature: $T_R > m_\chi / 23$

Modified expansion rate

- Various cosmological models predict a non-standard early expansion
  [e.g. Scherrer et al.,PRD(1985); Salati,PLB(2003); Fernengo et al.,PRD(2003); Chung et al., PRD (1999); ...]

  ⇨ Predicted WIMP relic abundances are also changed

- When WIMPs were in full thermal equilibrium, in terms of the modification parameter $A(x) = H_{\rm st}(x)/H(x)$ the relic abundance is

$$\Omega_\chi h^2 = 0.1 \left(\frac{I(x_F)}{8.5 \times 10^{-10}\ \mathrm{GeV}^{-2}} \right)^{-1}$$

$$I(x_F) = \int_{x_F}^{\infty} dx \frac{\sqrt{g_*}\langle \sigma v \rangle A(x)}{x^2}, \quad x_F = \ln \left[\sqrt{\frac{45}{\pi^5}} \xi m_\chi M_{\rm Pl} g_\chi \frac{\langle \sigma v \rangle A(x)}{\sqrt{x g_*}} \right] \Bigg|_{x = x_F}$$

If $A(x) = 1$, $x_F = x_{F,\rm st}$ and we recover the standard formula

This formula is capable of predicting the final relic density correctly

Constrains on modifications of the Hubble parameter

- In terms of $z \equiv T/m_\chi = 1/x$

we need to know $A(z)$ only for $z_{\rm BBN} = 10^{-5} - 10^{-4} \le z \le z_F \sim 1/20 \ll \mathcal{O}(1)$

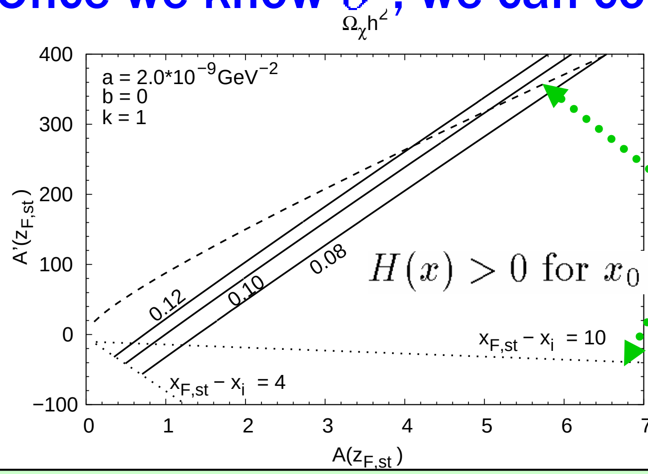
$\Rightarrow$ This suggests a parametrization of $A(z)$ in powers of $(z - z_{F,{\rm st}})$:

$$A(z) = A(z_{F,{\rm st}}) + (z - z_{F,{\rm st}})A'(z_{F,{\rm st}}) + \frac{1}{2}(z - z_{F,{\rm st}})^2 A''(z_{F,{\rm st}})$$

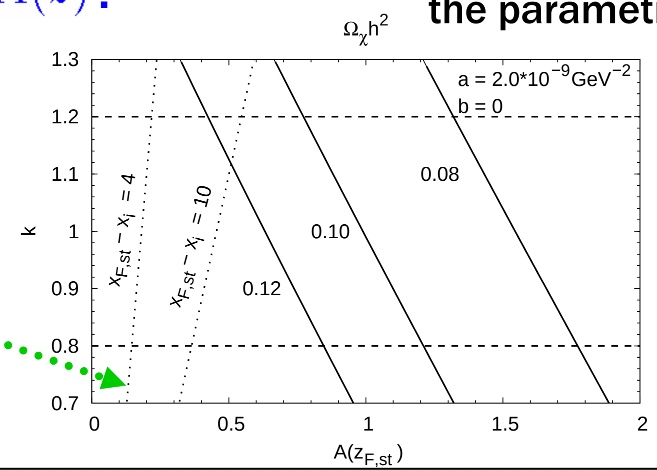
subject to the BBN limit: $0.8 \le k \equiv A(z \to z_{\rm BBN}) \le 1.2$

x_i :Maximal temperature where the parametrization is valid

- Once we know σ, we can constrain $A(z)$:



$H(x) > 0$ for $x_0 < x < \infty$



$\Omega_\chi h^2$ depends on all $H(T_{\rm BBN} < T < T_F)$ $\Rightarrow$ Larger allowed region for $H(T_F)$

5. Summary

- Using the CDM relic density we can examine very early universe around $T \sim m_\chi/20 \sim \mathcal{O}(10)\ \mathrm{GeV}$ (well before BBN $T_{\mathrm{BBN}} \sim \mathcal{O}(1)\ \mathrm{MeV}$)

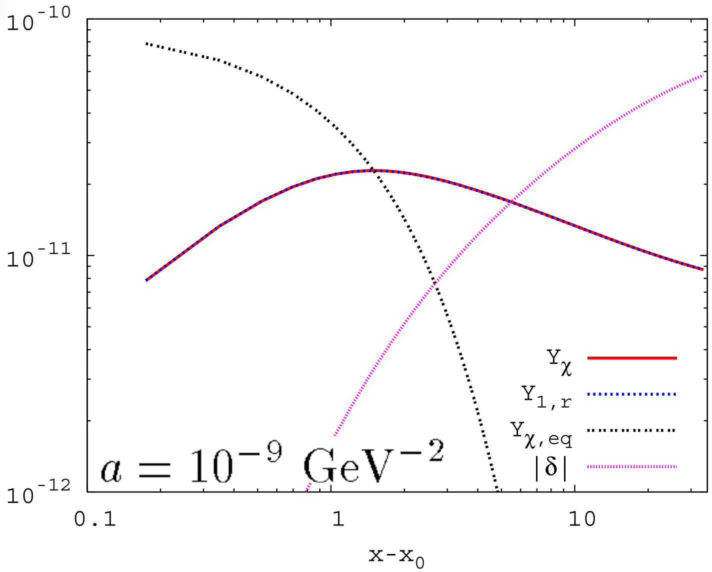
- The relic density of thermal WIMPs depends on the reheat temperature T_R and on the Hubble parameter $H(T_{\mathrm{BBN}} < T < T_F)$

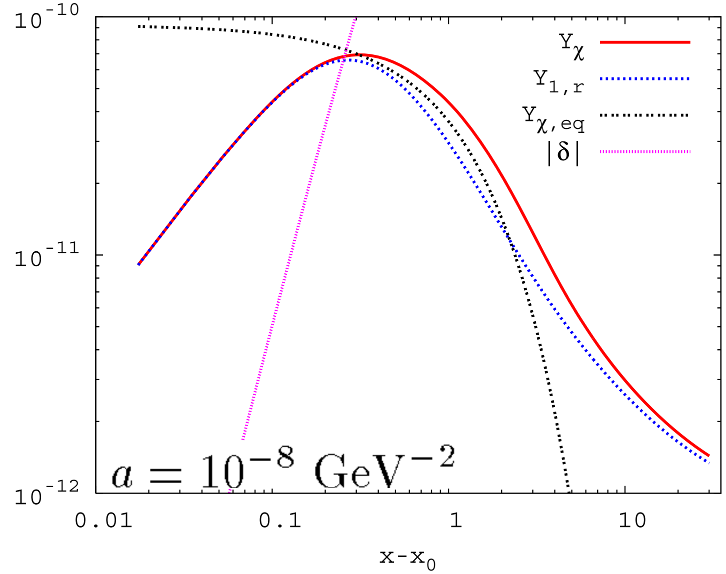
- By applying $\Omega_{\mathrm{CDM}} h^2 = \Omega_{\chi,\mathrm{thermal}} h^2$, we found the lower bound on the maximal temperature: $T_R > m_\chi/23$

- The sensitivity of $\Omega_{\chi,\mathrm{thermal}} h^2$ on $H(T_F)$ is weak because $\Omega_{\chi,\mathrm{thermal}} h^2$ depends on all $H(T_{\mathrm{BBN}} < T < T_F)$

Backup slides

Evolution of solutions

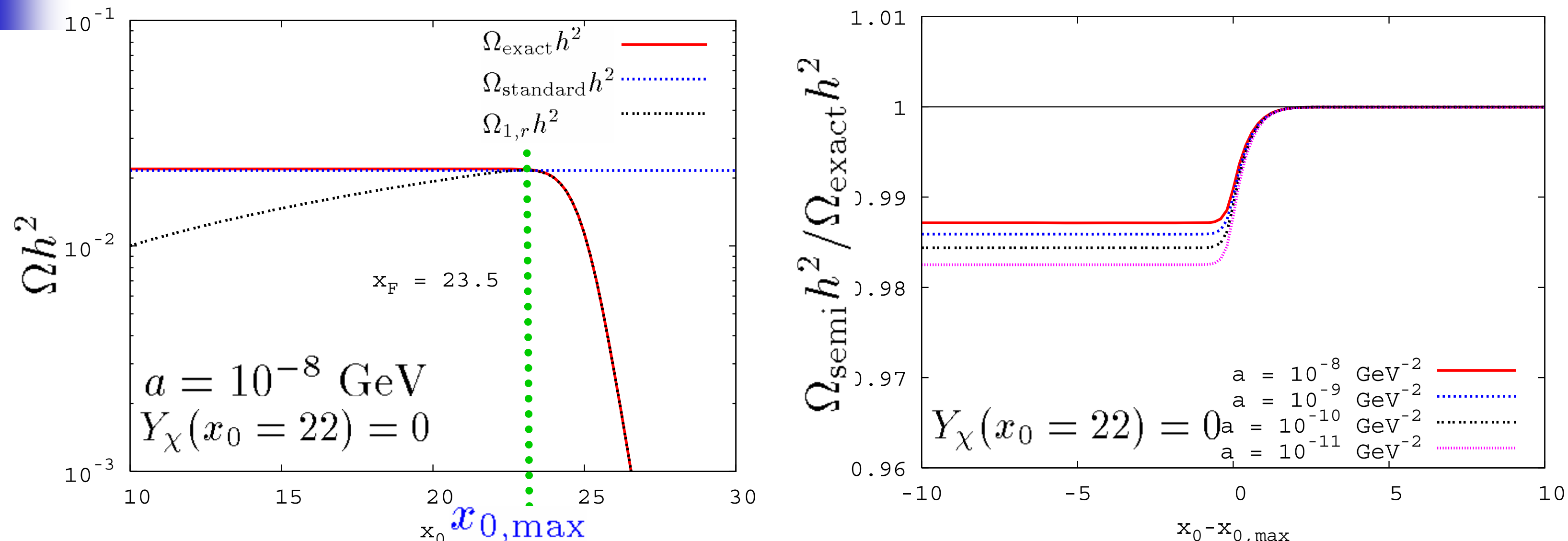




Y_χ : Exact result, $Y_{1,r}$: Re-summed ansatz, $b=0,\ Y_\chi(x_0=22)=0$

- The re-summed ansatz $Y_{1,r}$ describes the full temperature dependence of the abundance when equilibrium is not reached
- For larger cross section the deviation becomes sizable for $x - x_0 \sim 1$, but the deviation becomes smaller for $x \gg x_0$

Semi-analytic solution



- $Y_{1,r}(x_0, x \to \infty)$ $(\propto \Omega_{1,r} h^2)$ has a maximum (left)
- New semi-analytic solution can be constructed: $\Omega_{\rm semi} h^2$ (right)

For $x_0 > x_{0,\max}$, use $Y_{1,r}(x_0)$; for $x_0 < x_{0,\max}$, use $Y_{1,r}(x_{0,\max})$

The semi-analytic solution $\Omega_{\rm semi} h^2$ reproduces the correct final relic density $\Omega_{\rm exact} h^2$ to an accuracy of a few percent