Constraints on the very early universe from thermal WIMP Dark Matter

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Refs:
1. Motivation

- Observations of
  - cosmic microwave background
  - structure of the universe
  - etc.

Non-baryonic cold dark matter (CDM): \(0.8 < \Omega_{\text{CDM}} h^2 < 0.12\) (95% CL)

- Neutral, stable (long-lived) weakly interacting massive particles (WIMPs) \(\chi\) are good candidates for CDM
  - Neutralino (LSP); 1\(^{\text{st}}\) KK mode of the B boson (LKP); etc.

When WIMPs were in full thermal eq., the relic abundance naturally falls around the observed CDM abundance: \(\Omega_{\chi,\text{standard}} h^2 \sim 0.1\)
We can test the standard CDM scenario and investigate the conditions of very early universe: $T_R, H, \cdots$

- **Standard scenario:**
  - $\chi$ was in chemical eq.
  - $\Omega_\chi h^2$ is independent of $T_R$
  - $H = \frac{\pi T^2}{M_{Pl}} \sqrt{\frac{g_*}{90}}$ ($g_*$: Rel. dof)

- **Non-standard scenarios:**
  - Low reheat temperature
  - Entropy production
  - Modified Hubble parameter
  - Non-thermal production

[Scherrer et al., PRD(1985); Salati, PLB(2003); Fernengo et al., PRD(2003); Chung et al., PRD (1999); ...]

[From Ellis et al., PLB565 (2003) 176]
We provide an approximate analytic treatment that is applicable to low-reheat-temperature scenarios.

Based on the assumption of CDM = thermal WIMP:
- we derive the lower bound on the maximal temperature of RD epoch
- we constrain possible modifications of the Hubble parameter

Outline

1. Motivation
2. Standard calculation of WIMP relic abundance
3. Low-temperature scenario
4. Constrains on the very early universe from WIMP dark matter
5. Summary

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[Schelke, Catena, Fornengo, Masiero, Pietroni PRD74 (2006);
Donato, Fornengo, Schelke, JCAP0703 (2007)]
2. Standard calculation of the WIMP relic abundance

[Scherrer, Turner, PRD33(1986); Griest, Seckel, PRD43(1991); ...]

- Conventional assumptions for $\chi$:
  - $\chi = \bar{\chi}$, single production of $\chi$ is forbidden
  - Thermal equilibrium was maintained

- For adiabatic expansion the Boltzmann eq. is
  \[
  \frac{dY_{\chi}}{dx} = -\frac{\langle \sigma v \rangle s}{Hx} (Y_{\chi}^2 - Y_{\chi,eq}^2),
  \]

  \[
  Y_{\chi,(eq)} = \frac{n_{\chi,(eq)}}{s}, \quad x = \frac{m_{\chi}}{T}
  \]

- During the RD epoch, $\chi$ and decoupled when they were non-relativistic:
  \[
  \langle \sigma v \rangle = a + 6b/x + \mathcal{O}(1/x^2), \quad n_{\chi,eq} = g_{\chi} \left( m_{\chi} T / 2\pi \right)^{3/2} e^{-m_{\chi}/T}
  \]

\[\Omega_{\chi,\text{standard}} h^2 \simeq 0.1 \times \left( \frac{a + 3b/x_F}{10^{-9} \text{ GeV}^{-2}} \right)^{-1} \left( \frac{x_F}{22} \right) \left( \frac{g_*}{90} \right)^{-1/2} \sim \Omega_{\text{CDM}} h^2\]
3. Low-temperature scenario

- $T_R$: Reheat temperature

The initial abundance is assumed to be negligible: $Y_\chi(x_0) = 0$, $x_0 = \frac{m_\chi}{T_R}$

- Zeroth order approximation:

$T_R < T_F \quad \chi$ annihilation is negligible:

$$\frac{dY_0}{dx} = 0.028 \ g_\chi^2 g_*^{-3/2} m_\chi M_{Pl} e^{-2x} x \left( a + \frac{6b}{x} \right)$$

The solution is proportional to the cross section:

At late times,

$$Y_0(x \gg x_0) \approx 0.014 \ g_\chi^2 g_*^{-3/2} m_\chi M_{Pl} e^{-2x_0} x_0 \left( a + \frac{6b}{x_0} \right)$$

This solution should be smoothly connected to the standard result.
First order approximation

- Add a correction term describing annihilation to $Y_C$: $Y_1 = Y_0 + \delta$ ($\delta < 0$)

- As long as $|\delta| \ll Y_0$, the evolution equation for $\delta$ is

$$\frac{d\delta}{dx} = -1.3 \sqrt{g_*} m_x M_{PL} \left( a + \frac{6b}{x} \right) \frac{Y_0(x)^2}{x^2}$$

The solution is proportional to $\sigma^3$

At late times,

$$\delta(x \gg x_0) \simeq -2.5 \times 10^{-4} g_X g_*^{-5/2} m^3 M_{PL}^3 e^{-4x_0} x_0 \left( a + \frac{3b}{x_0} \right) \left( a + \frac{6b}{x_0} \right)^2$$

- $|\delta|$ soon dominates over $Y_C$ for not very small cross section

$Y_1$ fails to track the exact solution
It is noticed that $Y_0 \propto \sigma > 0$, $\delta \propto \sigma^3 < 0$.

For large cross section, $Y_\chi(x \to \infty)$ should be $\propto 1/\langle \sigma v \rangle$

This observation suggests the re-summed ansatz:

$$Y(x) = Y_0 + \delta = Y_0 \left(1 + \frac{\delta}{Y_0}\right) \approx \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}$$

For $|\delta| \gg Y_0$,

$$Y_{1,r}(x) \approx - \frac{Y_0^2}{\delta} \propto \frac{1}{\sigma}$$

At late times, $Y_{1,r}(x \to \infty) = \frac{1.3 \sqrt{g_* m_\chi} M_{Pl}(a + 3b/x_0)}{x_0}$

In the case where $\chi$ production is negligible but the initial abundance is sizable, $Y_{1,r}$ is exact.
4. Constrains on the very early universe from WIMP DM

- Out-of-equilibrium case: $\sigma \xrightarrow{\Omega h^2} ; \quad T_0 = m_\chi / x_0 \xrightarrow{\Omega h^2} $
- Equilibrium case: $\sigma \xrightarrow{\Omega h^2} ; \quad \Omega h^2 \text{ Independent of } T_0$
- Thermal relic abundance in the RD universe:

Assumption of $\Omega_{\text{CDM}} h^2 = \Omega_{\chi,\text{thermal}} h^2$,

Lower bound on the maximal temperature: $T_R > \frac{m_\chi}{23}$
When WIMPs were in full thermal equilibrium, in terms of the modification parameter $A(x) = H_{st}(x)/H(x)$, the relic abundance is modified expansion rate

Various cosmological models predict a non-standard early expansion [e.g. Scherrer et al., PRD(1985); Salati, PLB(2003); Fernengo et al., PRD(2003); Chung et al., PRD (1999); ...] Predicted WIMP relic abundances are also changed

When WIMPs were in full thermal equilibrium, in terms of the modification parameter $A(x) = H_{st}(x)/H(x)$, the relic abundance is

$$\Omega_X h^2 = 0.1 \left( \frac{I(x_F)}{8.5 \times 10^{-10} \text{ GeV}^{-2}} \right)^{-1}$$

$$I(x_F) = \int_{x_F}^{\infty} dx \frac{\sqrt{g}(\sigma v) A(x)}{x^2}, \quad x_F = \ln \left[ \frac{\sqrt{45}}{\pi^5} \xi m_X M_{Pl} g_X \frac{\langle \sigma v \rangle A(x)}{\sqrt{x g_*}} \right]_{x = x_F}$$

If $A(x) = 1$, $x_F = x_{F, st}$ and we recover the standard formula

This formula is capable of predicting the final relic density correctly

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Constrains on modifications of the Hubble parameter

- In terms of $z \equiv T/m_\chi = 1/x$
  we need to know $A(z)$ only for $z_{\text{BBN}} = 10^{-5} - 10^{-4} \leq z \leq z_F \sim 1/20 \ll \mathcal{O}(1)$

  This suggests a parametrization of $A(z)$ in powers of $(z - z_F, st)$:
  $$A(z) = A(z_{F, st}) + (z - z_{F, st})A'(z_{F, st}) + \frac{1}{2}(z - z_{F, st})^2 A''(z_{F, st})$$

  subject to the BBN limit: $0.8 \leq k \equiv A(z \to z_{\text{BBN}}) \leq 1.2$

- Once we know $\sigma_{\Omega h^2}$, we can constrain $A(z)$:

  $$\Omega_{\chi} h^2 \text{ depends on all } H(T_{\text{BBN}} < T < T_F) \text{ Larger allowed region for } H(T_F)$$
5. Summary

- Using the CDM relic density we can examine very early universe around $T \sim m_X/20 \sim \mathcal{O}(10) \text{ GeV}$ (well before BBN $T_{BBN} \sim \mathcal{O}(1) \text{ MeV}$).

- The relic density of thermal WIMPs depends on the reheat temperature $T_R$ and on the Hubble parameter $H(T_{BBN} < T < T_F)$.

- By applying $\Omega_{CDM} h^2 = \Omega_{\chi, \text{thermal}} h^2$, we found the lower bound on the maximal temperature: $T_R > m_X/23$.

- The sensitivity of $\Omega_{\chi, \text{thermal}} h^2$ on $H(T_F)$ is weak because $\Omega_{\chi, \text{thermal}} h^2$ depends on all $H(T_{BBN} < T < T_F)$. 
Backup slides
The re-summed ansatz $Y_{1,r}$ describes the full temperature dependence of the abundance when equilibrium is not reached. For larger cross section the deviation becomes sizable for $x - x_0 \sim 1$, but the deviation becomes smaller for $x \gg x_0$. 

$Y_\chi$: Exact result, $Y_{1,r}$: Re-summed ansatz, $b = 0$, $Y_\chi(x_0 = 22) = 0$
Semi-analytic solution

- $Y_{1,r}(x_0, x \to \infty) \propto \Omega_{1,r} \hbar^2$ has a maximum (left)
- New semi-analytic solution can be constructed: $\Omega_{\text{semi}} \hbar^2$ (right)

For $x_0 > x_{0,\text{max}}$, use $Y_{1,r}(x_0)$; for $x_0 < x_{0,\text{max}}$, use $Y_{1,r}(x_{0,\text{max}})$

The semi-analytic solution $\Omega_{\text{semi}} \hbar^2$ reproduces the correct final relic density $\Omega_{\text{exact}} \hbar^2$ to an accuracy of a few percent.