Local SU(5) Unification from the Heterotic String

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W. Buchmüller, CL, J. Schmidt, `arXiv:0707.1651`

1. Introduction
2. The Model
3. Anomaly Cancellation
4. Local GUT
5. Outlook

Introduction

- GUT: Attractive features:
  - $SU(3) \times SU(2) \times U(1) \subset SU(5), SO_{10} \ldots$, gauge couplings unify
  - Unification matter into larger multiplets
- Drawbacks in 4d GUTS
  - Large Higgs representations required
  - Doublet–triplet–splitting
  - Yukawa couplings do not unify
- Drawbacks can be addressed in higher-dimensional orbifold GUTs
- Nice possibility: Heterotic String:
  - $E_8 \times E_8$ gauge symmetry included
  - Simple orbifold compactifications with realistic four-dimensional matter content and gauge group possible
  - UV completion

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[Kobayashi, Raby, Zhang; Buchmüller, Hamaguchi, Lebedev, Ratz; Kim, Kim, Kyae; Förste, Nilles, Vaudrevange, Wingerter, Ramos-Sanchez,. . .]

Heterotic Orbifold Compactification

- Choose a torus with discrete isometry ("twist") with fixed points
- Mod out by this isometry, fixed points become singularities
- Fixing boundary conditions at fixed points requires embedding the twist into gauge group and choosing Wilson lines
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- Twisted sectors: States localised at fixed points

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The Model: Geometry

[Buchmüller, Hamaguchi, Lebedev, Ratz]

- Torus: $G_2 \times SU(3) \times SO(4)$ root lattice, $\mathbb{Z}_{6-II} = \mathbb{Z}_3 \times \mathbb{Z}_2$ twist:

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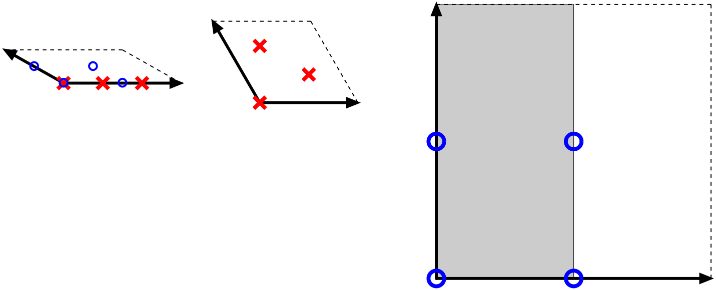

- Obtain effective 6D Theory on $T^2/\mathbb{Z}_2$ orbifold
- Internal zero modes and twisted states show up as bulk states, twisted states are localised at orbifold fixed points

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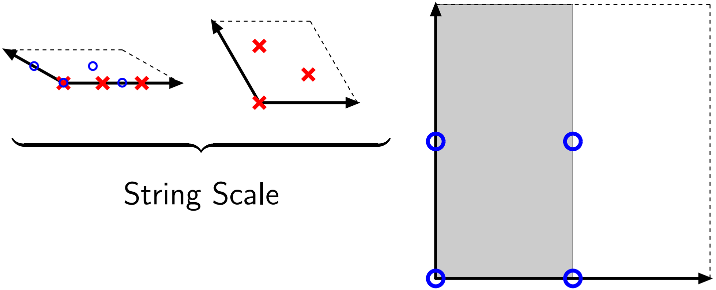

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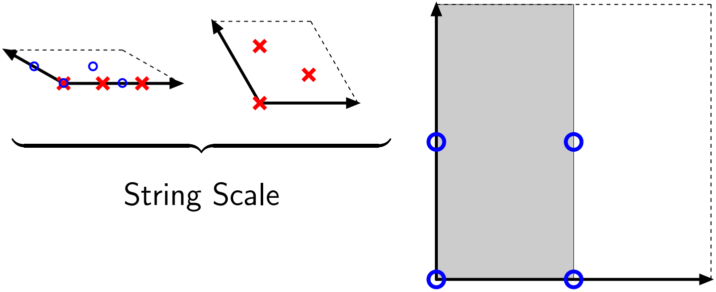

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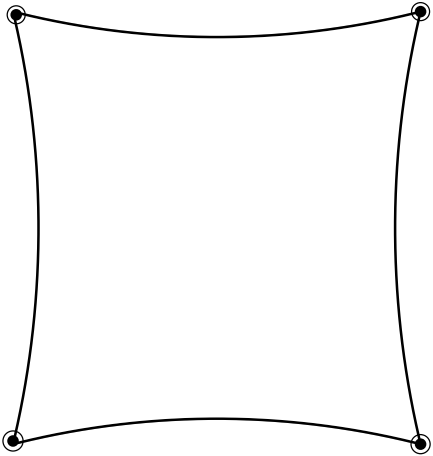

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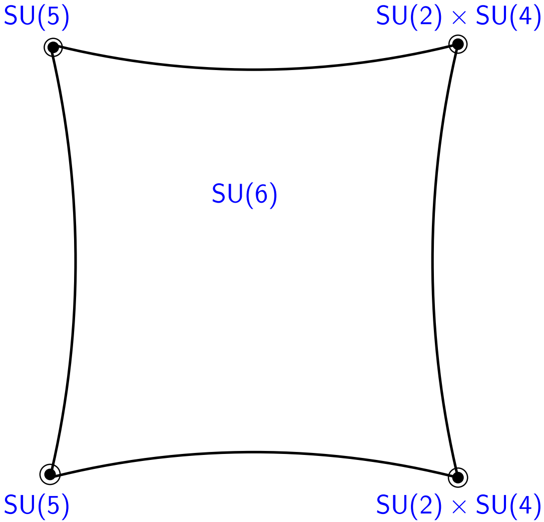

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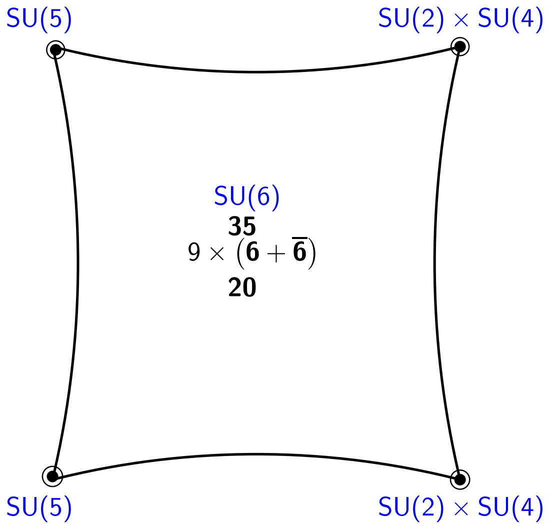

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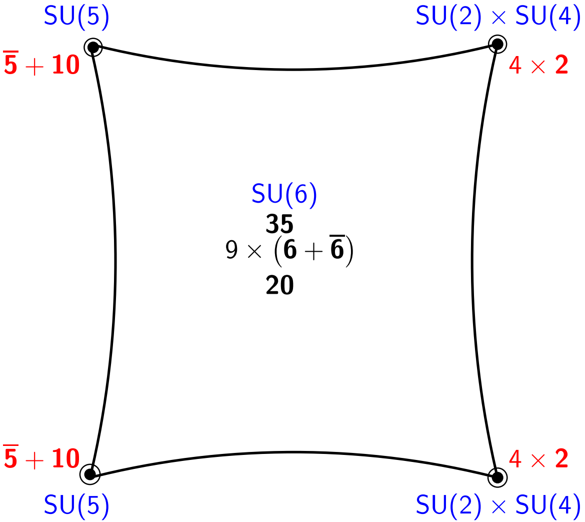

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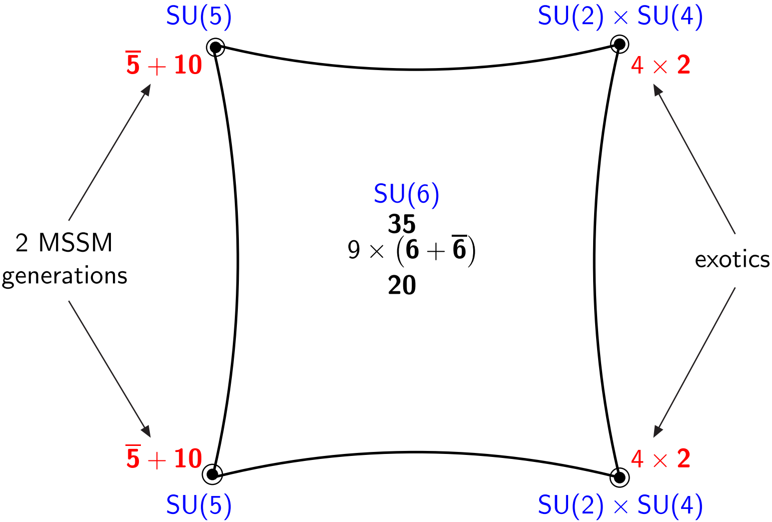

Anomalies

- Orbifold have bulk and brane anomalies
- Anomaly cancellation by Green–Schwarz mechanism requires factorisation of anomaly polynomials, $I_8 = X_4 Y_4$ and $I_6^f = X_4^f Y_2$
- $\mathcal{O}(500)$ conditions, but guaranteed by string theory (and modular invariance conditions on twist vectors and Wilson lines): Check of spectrum
- Anomalous U(1)'s induce localised FI terms

$$\xi_0 = 148 \left(\frac{g M_{\mathrm{P}}^2}{384\pi^2}\right) \delta^{(2)}(z - z_0)$$
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Local SU(5) GUT

- Local GUT: At fixed points, boundary conditions break bulk gauge group to smaller groups,

$$\text{in our case:} \quad \mathrm{SU}(6) \longrightarrow \begin{cases} \mathrm{SU}(5) \\ \mathrm{SU}(2) \times \mathrm{SU}(4) \end{cases}$$

- In zero mode spectrum, only the intersection of local groups survives, which is $G_{SM} = SU(3) \times SU(2) \times U(1)$
- Localised fields come in complete multiplets of local GUT group
- Due to other branes, bulk fields form split multiplets
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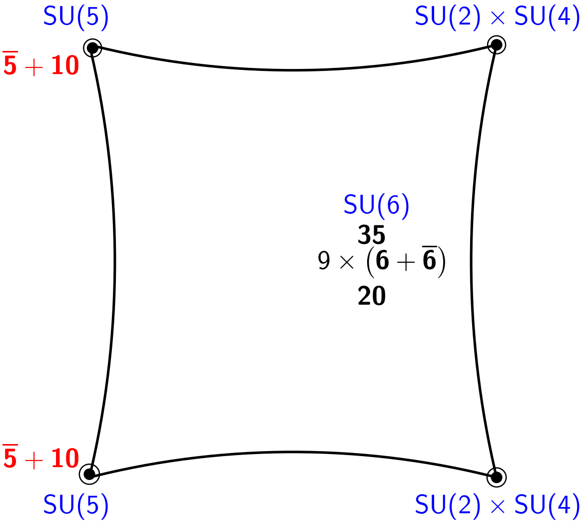
Projection



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- Bulk Matter: Hypermultiplets, split as $H = (H_L, H_R)$ into chiral multiplet
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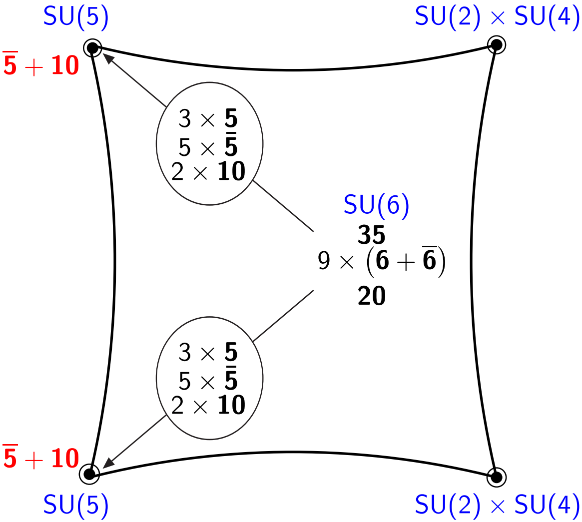

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Decoupling

- Several pairs of $\mathbf{5} + \bar{\mathbf{5}}$ and most exotics decoupled easily
- Remaining $\mathbf{5}$'s and $\bar{\mathbf{5}}$'s:

Bulk: / Zero modes:	$\mathbf{5}$	$\mathbf{5}_1$	$\bar{\mathbf{5}}_0^c$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}_1$	$\bar{\mathbf{5}}_2$	$\mathbf{5}_0^c$	$\mathbf{5}_2^c$
SU(3) × SU(2)	$(1,\mathbf{2})$	$(1,\mathbf{2})$	$(\mathbf{3},1)$	$(1,\mathbf{2})$	$(1,\mathbf{2})$	$(\bar{\mathbf{3}},1)$	$(\bar{\mathbf{3}},1)$	$(1,\mathbf{2})$
$U(1)_{B-L}$	0	0	$-\frac{2}{3}$	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	-1
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$2 \times (\bar{\mathbf{5}} + \mathbf{10})$ generations on the branes
$2 \times (\bar{\mathbf{5}} + \mathbf{10})$ generations in the bulk
$\mathbf{5} + \bar{\mathbf{5}}$ Higgses in the bulk

Split Multiplets

- Bulk generations:

$$\bar{\mathbf{5}}_{(3)} = \left(\bar{\mathbf{3}}, 1\right) + (1, \mathbf{2}) \qquad \mathbf{10}_{(3)} = (\mathbf{3}, \mathbf{2}) + \left(\bar{\mathbf{3}}, 1\right) + (1, 1)$$
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Orbifold projection solves doublet–triplet–splitting

Yukawa Couplings

$$W = C^{(u)}_{(ij)} \mathbf{5}_u \mathbf{10}_{(i)} \mathbf{10}_{(j)} + C^{(d)}_{(ij)} \mathbf{5}_d \bar{\mathbf{5}}_{(i)} \mathbf{10}_{(j)}$$

$$C^{(u)}_{(ij)} = \begin{pmatrix} a_1 & 0 & a_2 & a_3 \\ 0 & a_1 & a_2 & a_3 \\ a_2 & a_2 & 0 & g \\ a_3 & a_3 & g & a_4 \end{pmatrix}, \qquad C^{(d)}_{ij} = \begin{pmatrix} 0 & 0 & b_1 & b_2 \\ 0 & 0 & b_1 & b_2 \\ b_3 & b_3 & b_4 & 0 \\ b_5 & b_5 & b_6 & b_5^2 \end{pmatrix}$$

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$$a_1 = \langle Y_0^c \bar{Y}_0^c S_1 S_3 \rangle, \qquad a_2 = \langle \left(\bar{Y}_0^c S_1 \right)^2 S_5 \rangle, \qquad a_3 = \langle Y_0^c \bar{Y}_0^c S_1 S_3 S_5 \rangle,$$

$$a_4 = \langle Y_0^c \bar{Y}_0^c S_1 S_3 (S_5)^2 \rangle,$$

$$b_1 = \langle Y_0 \bar{Y}_1 (S_5)^3 (S_7)^2 \rangle, \qquad b_2 = \langle X_1^c \bar{Y}_2^c U_1^c S_7 \rangle, \qquad b_3 = \langle X_1^c \bar{Y}_1 S_3 (S_5 S_7)^2 \rangle,$$

$$b_4 = \langle (X_1^c)^2 \bar{Y}_1 U_1^c S_4 S_7 \rangle, \qquad b_5 = \langle S_5 \rangle, \qquad b_6 = \langle (X_1^c)^2 Y_1 S_1 S_7 \rangle$$

Yukawa Couplings

$$W = C^{(u)}_{(ij)} \mathbf{5}_u \mathbf{10}_{(i)} \mathbf{10}_{(j)} + C^{(d)}_{(ij)} \mathbf{5}_d \bar{\mathbf{5}}_{(i)} \mathbf{10}_{(j)}$$

$$C^{(u)}_{(ij)} = \begin{pmatrix} a_1 & 0 & a_2 & a_3 \\ 0 & a_1 & a_2 & a_3 \\ a_2 & a_2 & 0 & g \\ a_3 & a_3 & g & a_4 \end{pmatrix}, \qquad C^{(d)}_{ij} = \begin{pmatrix} 0 & 0 & b_1 & b_2 \\ 0 & 0 & b_1 & b_2 \\ b_3 & b_3 & b_4 & 0 \\ b_5 & b_5 & b_6 & b_5^2 \end{pmatrix}$$

$$W = Y^u_{ij} h_u u^c_i q_j + Y^d_{ij} h_d d^c_i q_j + Y^l_{ij} h_d l_i e^c_j$$

$$Y^u_{ij} = \begin{pmatrix} a_1 & 0 & a_3 \\ 0 & a_1 & a_3 \\ a_2 & a_2 & g \end{pmatrix}, \quad Y^d_{ij} = \begin{pmatrix} 0 & 0 & b_2 \\ 0 & 0 & b_2 \\ b_5 & b_5 & b_7 \end{pmatrix}, \quad Y^l_{ij} = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_1 \\ b_3 & b_3 & b_4 \end{pmatrix}$$

Outlook

- Constructed local 6D GUT from the heterotic string
- Doublet–triplet splitting achieved easily, SU(5) mass relations avoided dur to split bulk multiplets
- More symmetry in 6D $\rightsquigarrow$ simple decoupling of unwanted states
- Supersymmetric vacuum: four-dimensional D-term vanishes
- Open Questions:
  - Phenomenology needs to be improved (CKM mixing, R-parity)
  - Stabilisation of moduli, in particular, size of two-dimensional torus
  - Profiles of bulk fields due to localised FI terms
  - Blowup/resolution of singularities, generalisation to K3 internal space

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