Local SU(5) Unification from the Heterotic String

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1 Introduction
2 The Model
3 Anomaly Cancellation
4 Local GUT
5 Outlook
Introduction

- **GUT:** Attractive features:
  - $SU(3) \times SU(2) \times U(1) \subset SU(5), SO_{10} \ldots$, gauge couplings unify
  - Unification matter into larger multiplets

- Drawbacks in 4d GUTS
  - Large Higgs representations required
  - Doublet–triplet–splitting
  - Yukawa couplings do not unify

- Drawbacks can be addressed in higher-dimensional orbifold GUTs

- Nice possibility: Heterotic String:
  - $E_8 \times E_8$ gauge symmetry included
  - Simple orbifold compactifications with realistic four-dimensional matter content and gauge group possible
  - UV completion
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[Kobayashi, Raby, Zhang; Buchmüller, Hamaguchi, Lebedev, Ratz; Kim, Kim, Kyae; Förste, Nilles, Vaudrevange, Wingerter, Ramos-Sanchez, \ldots]
Heterotic Orbifold Compactification

- Choose a torus with discrete isometry ("twist") with fixed points
- Mod out by this isometry, fixed points become singularities
- Fixing boundary conditions at fixed points requires embedding the twist into gauge group and choosing Wilson lines
- Gauge symmetry reduced at fixed points (but rank usually preserved)
- Twisted sectors: States localised at fixed points
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The Model: Geometry

- Torus: $G_2 \times SU(3) \times SO(4)$ root lattice, $\mathbb{Z}_{6-II} = \mathbb{Z}_3 \times \mathbb{Z}_2$ twist:

[Kobayashi, Raby, Zhang]

- Obtain effective 6D Theory on $T^2/\mathbb{Z}_2$ orbifold
- Internal zero modes and twisted states show up as bulk states, twisted states are localised at orbifold fixed points

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- String Scale

- Obtain effective 6D Theory on $T^2/\mathbb{Z}_2$ orbifold
  
  - Internal zero modes and $\mathbb{Z}_3$ twisted states show up as bulk states, $\mathbb{Z}_2$ twisted states are localised at orbifold fixed points

[C. Lüdeling (ITP, Universität Heidelberg)]

[SUSY ‘07, July 26, 2007]
The Model: Effective $T^2/\mathbb{Z}_2$ Orbifold
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SU(5)   SU(2) $\times$ SU(4)

SU(6)

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SU(5) \to SU(2) \times SU(4)

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SU(6)

$35 = 9 \times (6 + \bar{6})$

20
The Model: Effective $T^2/\mathbb{Z}_2$ Orbifold
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SU(5) $\rightarrow$ $\bar{5} + 10$

SU(2) $\times$ SU(4) $\rightarrow$ $4 \times 2$

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SU(6) $\rightarrow$ $35$, $9 \times (6 + \bar{6})$, $20$

2 MSSM generations $\rightarrow$ exotics
Anomalies

- Orbifold have bulk and brane anomalies
  - Anomaly cancellation by Green–Schwarz mechanism requires factorisation of anomaly polynomials, $l_8 = X_4 Y_4$ and $l_6^f = X_4^f Y_2$
  - $\mathcal{O}(500)$ conditions, but guaranteed by string theory (and modular invariance conditions on twist vectors and Wilson lines): Check of spectrum
  - Anomalous U(1)’s induce localised FI terms

\[
\begin{align*}
\xi_0 &= 148 \left( \frac{g M_P^2}{384 \pi^2} \right) \delta^{(2)}(z - z_0) \\
\xi_1 &= 80 \left( \frac{g M_P^2}{384 \pi^2} \right) \delta^{(2)}(z - z_1)
\end{align*}
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- These lead to localisation of bulk fields, break the U(1) and need to be cancelled to obtain SUSY vacuum
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[Lee, Nilles, Zucker]
Local SU(5) GUT

- Local GUT: At fixed points, boundary conditions break bulk gauge group to smaller groups,

\[
\text{in our case: } \quad \text{SU}(6) \rightarrow \left\{ \begin{array}{c}
\text{SU(5)} \\
\text{SU(2)} \times \text{SU(4)}
\end{array} \right.
\]

- In zero mode spectrum, only the intersection of local groups survives, which is \( G_{\text{SM}} = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \)
- Localised fields come in complete multiplets of local GUT group
- Due to other branes, bulk fields form split multiplets
- Due to higher symmetry, decoupling of exotics much more transparent than in four-dimensional limit
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On branes, SUSY is broken to $\mathcal{N} = 1$

Bulk Matter:
- Hypermultiplets, split as $H = (H_L, H_R)$ into chiral multiplet
- Bulk vector multiplets split as $V = (A, \phi)$ into vector and chiral multiplets

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Several pairs of $\mathbf{5} + \bar{\mathbf{5}}$ and most exotics decoupled easily

Remaining $\mathbf{5}$'s and $\bar{\mathbf{5}}$'s:

<table>
<thead>
<tr>
<th>Bulk:</th>
<th>$\mathbf{5}$</th>
<th>$\mathbf{5}_1$</th>
<th>$\bar{\mathbf{5}}_c^0$</th>
<th>$\bar{\mathbf{5}}$</th>
<th>$\bar{\mathbf{5}}_1$</th>
<th>$\bar{\mathbf{5}}_2$</th>
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<tbody>
<tr>
<td>Zero modes:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SU(3) × SU(2)</td>
<td>(1, 2)</td>
<td>(1, 2)</td>
<td>(3, 1)</td>
<td>(1, 2)</td>
<td>(1, 2)</td>
<td>(3, 1)</td>
<td>(3, 1)</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>U(1)$_{B-L}$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{2}{3}$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{1}{3}$</td>
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</tr>
<tr>
<td>MSSM content</td>
<td>$H_u$</td>
<td></td>
<td></td>
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<td></td>
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- $2 \times (\bar{\mathbf{5}} + \mathbf{10})$ generations on the branes
- $2 \times (\mathbf{5} + \bar{\mathbf{10}})$ generations in the bulk
- $\mathbf{5} + \bar{\mathbf{5}}$ Higgses in the bulk
Split Multiplets

- Bulk generations:
  \[
  \tilde{5}_{(3)} = (\bar{3}, 1) + (1, 2) \quad \quad 10_{(3)} = (3, 2) + (\bar{3}, 1) + (1, 1)
  \]
  \[
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One generation remains, avoiding SU(5) mass relations

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Orbifold projection solves doublet–triplet–splitting
Yukawa Couplings

\[ W = C_{(ij)}^{(u)} 5_u 10_{(i)} 10_{(j)} + C_{(ij)}^{(d)} 5_d \bar{5}_{(i)} 10_{(j)} \]

\[ C_{(ij)}^{(u)} = \begin{pmatrix} a_1 & 0 & a_2 & a_3 \\ 0 & a_1 & a_2 & a_3 \\ a_2 & a_2 & 0 & g \\ a_3 & a_3 & g & a_4 \end{pmatrix}, \quad C_{ij}^{(d)} = \begin{pmatrix} 0 & 0 & b_1 & b_2 \\ 0 & 0 & b_1 & b_2 \\ b_3 & b_3 & b_4 & 0 \\ b_5 & b_5 & b_6 & b_5^2 \end{pmatrix} \]
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\[ a_1 = \langle Y^c_0 \bar{Y}^c_0 S_1 S_3 \rangle, \quad a_2 = \langle (\bar{Y}^c_0 S_1)^2 S_5 \rangle, \quad a_3 = \langle Y^c_0 \bar{Y}^c_0 S_1 S_3 S_5 \rangle, \]
\[ a_4 = \langle Y^c_0 \bar{Y}^c_0 S_1 S_3 (S_5)^2 \rangle, \]
\[ b_1 = \langle Y_0 \bar{Y}_1 (S_5)^3 (S_7)^2 \rangle, \quad b_2 = \langle X^c_1 \bar{Y}^c_2 U^c_1 S_7 \rangle, \quad b_3 = \langle X^c_1 \bar{Y}_1 S_3 (S_5 S_7)^2 \rangle, \]
\[ b_4 = \langle (X^c_1)^2 \bar{Y}_1 U^c_1 S_4 S_7 \rangle, \quad b_5 = \langle S_5 \rangle, \quad b_6 = \langle (X^c_1)^2 Y_1 S_1 S_7 \rangle \]
Yukawa Couplings

\[ W = C_{(ij)}^{(u)} 5_u 10(i) 10(j) + C_{(ij)}^{(d)} 5_d \bar{5}(i) 10(j) \]

\[ C_{(ij)}^{(u)} = \begin{pmatrix} a_1 & 0 & a_2 & a_3 \\ 0 & a_1 & a_2 & a_3 \\ a_2 & a_2 & 0 & g \\ a_3 & a_3 & g & a_4 \end{pmatrix}, \quad C_{ij}^{(d)} = \begin{pmatrix} 0 & 0 & b_1 & b_2 \\ 0 & 0 & b_1 & b_2 \\ b_3 & b_3 & b_4 & 0 \\ b_5 & b_5 & b_6 & b_5^2 \end{pmatrix} \]

\[ W = Y_{ij}^u h_u u_i^c q_j + Y_{ij}^d h_d d_i^c q_j + Y_{ij}^l h_d l_i e_j^c \]

\[ Y_{ij}^u = \begin{pmatrix} a_1 & 0 & a_3 \\ 0 & a_1 & a_3 \\ a_2 & a_2 & g \end{pmatrix}, \quad Y_{ij}^d = \begin{pmatrix} 0 & 0 & b_2 \\ 0 & 0 & b_2 \\ b_5 & b_5 & b_7 \end{pmatrix}, \quad Y_{ij}^l = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_1 \\ b_3 & b_3 & b_4 \end{pmatrix} \]
Outlook

• Constructed local 6D GUT from the heterotic string
  • Doublet–triplet splitting achieved easily, SU(5) mass relations avoided due to split bulk multiplets
  • More symmetry in 6D → simple decoupling of unwanted states
  • Supersymmetric vacuum: four-dimensional $D$-term vanishes

Open Questions:
  • Phenomenology needs to be improved (CKM mixing, $R$-parity)
  • Stabilisation of moduli, in particular, size of two-dimensional torus
  • Profiles of bulk fields due to localised FI terms
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