Local SU(5) Unification in 6D from the Heterotic String

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Abstract. We present a six-dimensional $T^2/\mathbb{Z}_2$ orbifold model which arises as an intermediate step in the compactification of the heterotic string to the MSSM. The orbifold contains two pairs of inequivalent fixed points, with unbroken local gauge groups $\mathsf{SU}(5)$ and $\mathsf{SU}(2)\times\mathsf{SU}(4)$, respectively, the intersection of which gives the standard model gauge group. All bulk and brane anomalies are cancelled by the Green–Schwarz mechanism. At each $\mathsf{SU}(5)$ fixed point, there is a localised $\bar{\mathbf{5}}\oplus\mathbf{10}$ standard model generation, while the third generation and the Higgs fields come in split bulk multiplets due to the breaking of $\mathsf{SU}(5)$ at the other fixed point.

PACS. 11.25Mj Compactification and four-dimensional models – 12.10.-g Unified field theories and models

1 Introduction

An attractive route for physics beyond the standard model is provided by the idea of grand unification where the standard model group $G_{\mathrm{SM}}=\mathsf{SU}(3)\times\mathsf{SU}(3)\times\mathsf{U}(1)$ is realised as a subgroup of a larger semisimple group G_{GUT}. This idea is supported by the observed unification of gauge couplings in the (supersymmetric) standard model at the GUT scale $M_{\mathrm{GUT}}\simeq 10^{16}$ GeV. In the simplest case, $G_{\mathrm{GUT}}=\mathsf{SU}(5)$, and one standard model family fits into a $\bar{\mathbf{5}}\oplus\mathbf{10}$ representation of $\mathsf{SU}(5)$. Extending this approach via larger groups such as $\mathsf{SO}(10)$, one arrives at E_8, which is realised in the heterotic string. However, there are drawbacks to this naïve picture of four-dimensional grand unification. For example, large Higgs representations are required to break the GUT group to the standard model. Furthermore, the standard model Higgs doublet comes in a $\mathbf{5}$ of $\mathsf{SU}(5)$, together with a colour triplet which needs to get a mass of the order of the GUT scale to avoid proton decay while the doublet stays light. Phenomenologically, the unification of matter in larger multiplets predicts a unification of Yukawa couplings at the GUT scale which is not observed.

These problems can be addressed in higher-dimensional orbifold GUTs, where symmetry breaking and doublet–triplet–splitting can be achieved via projection conditions. A promising possibility for such GUTs the heterotic string, which includes an $\mathsf{E}_8\times\mathsf{E}_8$ gauge symmetry and allows for comparatively simple compactifications on orbifolds with realistic matter content and gauge groups in the four-dimensional limit. At the same time, it guarantees the absence of anomalies and provides a UV completion for the higher-dimensional effective field theory.

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2 $T^2/\mathbb{Z}_2$Orbifold Model

Our model [1] is based on the compactification of the heterotic string on the orbifold $T^6/\mathbb{Z}_{6-\mathrm{II}}$, where the T^6 is specified by the Lie algebra lattice of $\mathsf{G}_2\times\mathsf{SU}(3)\times\mathsf{SO}(4)$ [2]. This model is known to give the MSSM in the four-dimensional limit[3,4]. We take the limit in which the $\mathsf{G}_2\times\mathsf{SU}(3)$ tori become small (of $\mathcal{O}\Big(M_{\mathrm{string}}^{-1}\Big)$) while the $\mathsf{SO}(4)$ torus stays larger, $\mathcal{O}\Big(M_{\mathrm{string}}^{-1}\Big)$. Hence, we end up with an effective six-dimensional theory on $T^2/\mathbb{Z}_2$. This orbifold has four fixed points, but due to one Wilson line, they come in two inequivalent pairs.

The bulk gauge group is

$$G_{\mathrm{bulk}}=\mathsf{SU}(6)\times\mathsf{U}(1)^3 \times\Big[\mathsf{SU}(3)\times\mathsf{SO}(8)\times\mathsf{U}(1)^2\Big]\,. \tag{1}$$

Brackets denote subgroups of the second E_8. Bulk matter comprises the untwisted sector and the sector twisted by the $\mathbb{Z}_3$ subtwist of $\mathbb{Z}_{6-\mathrm{II}}$, whose fields are localised at fixed points in the G_2 and $\mathsf{SU}(3)$ tori. The untwisted sector contains the supergravity and dilaton multiplet $(G_{MN},B_{MN},\Phi,\Psi_M,\chi)$, the vector multiplets corresponding to G_{bulk} and hypermultiplets transforming as

$$(\mathbf{20};1,1)+(1;1,\mathbf{8})+(1;1,\mathbf{8}_s) +(1;1,\mathbf{8}_c)+4\times(1;1,1) \tag{2}$$

under the non-Abelian group factors. (We suppress $\mathsf{U}(1)$ charges here. The complete list of all states and charges is given in [1].) For the untwisted sector, we note that at each fixed point in the $\mathsf{SU}(3)$ torus, there is a local $\mathsf{SO}(14)\times\mathsf{U}(1)\times[\mathsf{SO}(14)\times\mathsf{U}(1)]$ gauge group (differently embedded into $\mathsf{E}_8\times\mathsf{E}_8$ each time due to a

Table 1. Local gauge groups and their intersection. The standard model is part of $\mathsf{SU}(5)$ at $n_2 = 0$.

n_2	Gauge group
0	$\mathsf{SU}(5) \times \mathsf{U}(1)^4 \times \left[\mathsf{SU}(3) \times \mathsf{SO}(8) \times \mathsf{U}(1)^2\right]$
1	$\mathsf{SU}(4) \times \mathsf{SU}(2) \times \mathsf{U}(1)^4 \times \left[\mathsf{SU}(4)' \times \mathsf{SU}(2)' \times \mathsf{U}(1)^4\right]$
$\cap$	$\mathsf{SU}(3) \times \mathsf{SU}(2) \times \mathsf{U}(1)^5 \times \left[\mathsf{SU}(4)' \times \mathsf{SU}(2)' \times \mathsf{U}(1)^4\right]$

Wilson line), and localised hypermultiplets transforming as $(\mathbf{14}; 1)$ and $(1; \mathbf{14})$. With respect to the six-dimensional gauge group, they split into

$$(\mathbf{14}; 1) = (\mathbf{6}; 1, 1) + (\bar{\mathbf{6}}; 1, 1) + 2 \times (1; 1, 1)\ , \qquad (3a)$$

$$(1; \mathbf{14}) = (1; \mathbf{3}, 1) + (1; \bar{\mathbf{3}}, 1) + \left(1; 1, \hat{\mathbf{8}}\right)\ . \qquad (3b)$$

Here $\hat{\mathbf{8}}$ refers to $\mathbf{8}$, $\mathbf{8}_s$ and $\mathbf{8}_c$, at the three fixed points, respectively. Furthermore, at each fixed point there are two non-Abelian singlet oscillator hypermultiplets. Note that the $\mathbb{Z}_3$ twisted sector comes in three copies due to the three fixed points in the G_2 torus.

The fixed points in the $\mathsf{SO}(4)$ torus are labelled by two numbers, $n_2, n_2' = 0, 1$. Points with the same value of n_2 are equivalent. At the fixed points, the $\mathbb{Z}_2$ projection conditions break the gauge group as indicated in Table 1.

At the fixed points, there are chiral multiplets from the $\mathbb{Z}_2$–twisted sector. At $n_2 = 0$ they include one standard model generation as $\bar{\mathbf{5}} \oplus \mathbf{10}$ of $\mathsf{SU}(5)$, plus further $\mathsf{SU}(5)$ singlets (and no exotics). At $n_2 = 1$, there are only exotic states and singlets.

3 Anomalies and FI Terms

In the anisotropic limit of the $\mathsf{SO}(4)$ torus being much larger than the other four internal dimensions, the effective theory is a six-dimensional supergravity on $T^2/\mathbb{Z}_2$. Such a theory faces stringent constraints from the absence of anomalies. In particular, the condition that all anomalies can be cancelled by the Green–Schwarz mechanism requires that the anomaly polynomials in the bulk and at the fixed points, I_8^{bulk} and $I_6^{n_2}$, are reducible,

$$I_8^{\text{bulk}} = X_4 Y_4\,, \qquad I_6^{n_2} = X_4\big|_{n_2} Y_2^{n_2}\,. \qquad (4)$$

Here X_4 is fixed by the variation of the 2-form field B_{MN}. Note that there are two independent fixed point anomaly polynomials since the spectrum does not depend on n_2'. These equations represent $\mathcal{O}(400)$ conditions on 33 free parameters in Y_4 and $Y_2^{n_2}$, thus the system is strongly overconstrained. Luckily, anomaly freedom is guaranteed by string theory and modular invariance conditions on the twist vectors and Wilson lines used to define the model. Nevertheless, the calculation of the anomaly polynomials is worthwhile not only as a check of the spectrum: It allows to determine the localised two-forms $Y_2^{n_2} \sim F_2^{n_2\text{an}}$, which are the field strengths of the local anomalous $\mathsf{U}(1)$'s. These in turn induce localised FI terms

$$\xi_0 = 2\frac{gM_{\text{P}}^2}{384\pi^2}\,, \qquad \xi_1 = \frac{gM_{\text{P}}^2}{384\pi^2}\,. \qquad (5)$$

These FI terms need to be cancelled for a supersymmetric vacuum configuration and thus can stabilise flat directions of the potential, albeit at a very high scale. Furthermore, they can induce nontrivial profiles of bulk vevs in the internal dimensions[5]. However, a detailed analysis of these issues is beyond this work.

4 Decoupling and Local GUT

4.1 Decoupling the Exotics

Bulk matter fields come in hypermultiplets, which, from the point of view of $\mathcal{N} = 1$ supersymmetry, split into two chiral multiplets of opposite chirality, $H = (H_L, H_R)$. At the fixed points, either H_L or H_R is projected out. For the twisted sector fields, which come in three copies, the resulting spectrum contains two left- and one right handed chiral multiplet or vice versa. Hence, two of these can always be combined to form a singlet state $H_L H_R$ and be decoupled by an effective mass term involving vevs of singlet fields (chosen such as to respect the string selection rules).

At $n_2 = 0$, there are a number of non-singlet $\mathsf{SU}(5)$ multiplets arising from the bulk $\mathsf{SU}(6)$ representations. Specifically, we obtain the following left-chiral multiplets:

$$\mathbf{35} \longrightarrow \mathbf{5} + \bar{\mathbf{5}} \qquad (6a)$$

$$9 \times (\mathbf{6} + \bar{\mathbf{6}}) \longrightarrow 8 \times \mathbf{5} + 11 \times \bar{\mathbf{5}} \qquad (6b)$$

$$\mathbf{20} \longrightarrow \mathbf{10} + \bar{\mathbf{10}} \qquad (6c)$$

Among the $\mathbf{5}$-plets originating in the $\mathbf{6}$'s of Eq. (3a), six pairs can be decoupled immediately by giving vevs to three non-Abelian singlets. Among the remaining $\mathbf{5}$-plets, two more pairs can be decoupled in a second step, so that finally only two generations of $\bar{\mathbf{5}} \oplus \mathbf{10}$ and a pair of $\mathbf{5} \oplus \bar{\mathbf{5}}$ Higgses remain. The exotics located at $n_2 = 1$ can also be decoupled by singlet vevs.

4.2 Yukawa Couplings

After decoupling, we have four standard model generations: Two in the bulk and one at each fixed point with $n_2 = 0$ (see Fig. 1). They can have a local superpotential with Yukawa couplings to the Higgses,

$$W_{\text{Yukawa}} = C_{ij}^{(u)} \mathbf{10}_i \mathbf{10}_j H_u + C_{ij}^{(d)} \bar{\mathbf{5}}_i \mathbf{10}_j H_d\,, \qquad (7)$$

where the coupling matrices $C_{ij}^{(u)}$ and $C_{ij}^{(d)}$ contain singlet vevs up to $\mathcal{O}(8)$. The choice of Higgs fields is not unique: The model can have no, partial or full gauge–Higgs unification, i.e. one can choose the Higgses to come from bulk $\mathbf{6}$'s or from the adjoint $\mathbf{35}$. An attractive feature of identifying at least H_u with the extra-dimensional component of the gauge field is that then

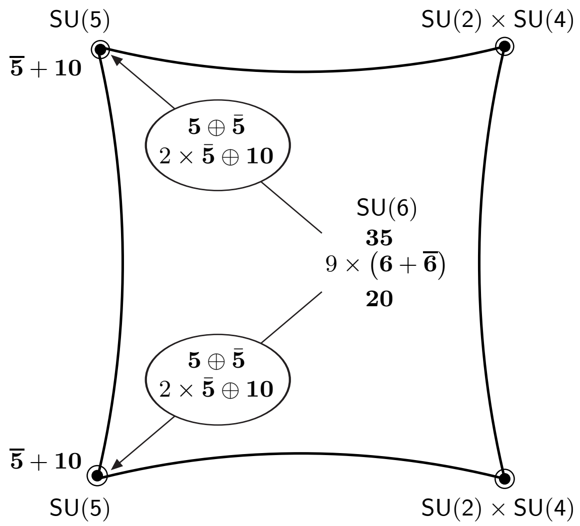

Fig. 1. The $\mathbb{Z}_2$ projection selects one pair of Higgses and two standard model generations at $n_2 = 0$, where SU(5) remains unbroken. At $n_2 = 1$, SU(5) is broken and only split multiplets survive, containing the Higgs doublets and one net generation.

the top Yukawa coupling is given by the gauge coupling at hence is naturally large.

The $\mathbb{Z}_2$ projection at the other fixed points finally breaks SU(5) and projects out one standard model generation from the two bulk $\bar{\mathbf{5}}\oplus\mathbf{10}$ fields – the remaining fields again have the quantum numbers of $\bar{\mathbf{5}}\oplus\mathbf{10}$, but the fact that they originate from different split multiplets avoids the mostly unsuccessful SU(5) mass relations. In the same way, the $\mathbb{Z}_2$ projection solves the doublet–triplet splitting problem by projecting the unwanted triplets from the Higgs **5**-plets.

5 Vacuum Configurations

To decouple unwanted states and generate Yukawa couplings, we need to give vevs to a number of singlet fields. To respect supersymmetry, the D-term potential needs to vanish. For non-anomalous symmetries, this can be accomplished by finding gauge invariant holomorphic monomials $I = \prod \phi_i^{n_i}$ in the fields. Each of these monomials defines a D-flat direction by

$$\left\langle \phi_i^\dagger \right\rangle = \frac{\partial I}{\partial \phi_i} \,. \tag{8}$$

The presence of the anomalous $U(1)$ additionally requires a monomial with negative anomalous charge. This stabilises some of the flat directions at $\sim \xi$. Indeed, we find suitable monomials such that the four-dimensional FI term is cancelled and the D-term potential vanishes. It is intriguing at this point that the scale of the FI terms is $\sim M_textGUT$.

6 Summary

We have constructed a local six-dimensional GUT from the heterotic string with the MSSM as its low-energy limit. Two standard model generations are localised at fixed points with local SU(5) symmetry, while one is composed of split bulk multiplets. Also the Higgses are realised as split multiplets, hence doublet–triplet splitting is automatic. We can find semirealistic vacuum configurations respecting supersymmetry. As opposed to a direct four-dimensional limit, the decoupling of exotic states is much more transparent due to the larger symmetry. The chosen vacuum is not phenomenologically viable: There is no R-parity forbidding proton decay, and the electron and down quark are massless. However, this model is a member of a large class ("mini-landscape") of similar models [6, 7] where these problems have been addressed and it seems likely that the phenomenology can be improved. The main theoretical questions are related to moduli stabilisation, the rôle of bulk field profiles and the blowup of singularities which will be addressed elsewhere.

Acknowledgements

I would like to thank my collaborators W. Buchmüller and J. Schmidt.

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