

# $B_S - \bar{B}_S$ mixing and $B_S \rightarrow KK$ decays within supersymmetry



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S.B, D. London, J. Matias, J. Virto, JHEP0602, 027(2006);

S.B, JHEP0609, 077(2006);

S.B, D. London, J. Matias, J. Virto, JHEP0612, 019(2006);

S.B, D. London, work in progress

In collaboration with D. London, J. Matias, J. Virto

# Outline

- Motivation: “ $B \rightarrow \pi K$  puzzle”
- SUSY model with large isospin violation – Grossman–Neubert–Kagan (GNK) scenario
- Strong constraint from  $\Delta m_s$
- $B_s \rightarrow KK$  decays in GNK scenario
- Conclusions

# $B \rightarrow \pi K$ puzzle

In the SM,

$$A(B^+ \rightarrow \pi^+ K^0) = -P'$$

$$\sqrt{2}A(B^+ \rightarrow \pi^0 K^+) = P' - T' e^{i\gamma} - P'_{EW} - C' e^{i\gamma}$$

$$A(B^0 \rightarrow \pi^- K^+) = P' - T' e^{i\gamma}$$

$$\sqrt{2}A(B^0 \rightarrow \pi^0 K^0) = -P' - P'_{EW} - C' e^{i\gamma}$$

- In the SM:

$$\checkmark P' \gg T' \approx P'_{EW} \gg C'$$

$$R_n \equiv \frac{1}{2} \frac{BR[B_d^0 \rightarrow \pi^- K^+] + BR[\bar{B}_d^0 \rightarrow \pi^+ K^-]}{BR[B_d^0 \rightarrow \pi^0 K^0] + BR[\bar{B}_d^0 \rightarrow \pi^0 \bar{K}^0]},$$

$$\checkmark R_c \approx R_n$$

$$R_c \equiv 2 \frac{BR[B_d^+ \rightarrow \pi^0 K^+] + BR[B_d^- \rightarrow \pi^0 K^-]}{BR[B_d^+ \rightarrow \pi^+ K^0] + BR[B_d^- \rightarrow \pi^- \bar{K}^0]}.$$

$$\checkmark A_{CP}(B^+ \rightarrow \pi^0 K^+) \approx A_{CP}(B^0 \rightarrow \pi^- K^+)$$

$$\checkmark S_{CP}(B^0 \rightarrow \pi^0 K^0) \approx \sin 2\beta$$

[G. Hou's talk]

- $R_c / R_n$  problem has disappeared!!

Buras, Fleischer, Recksiegel, Schwab (04,05,06)

- $A_{\text{CP}}(\pi^0 K^+) = 0.047 \pm 0.026$

$$A_{\text{CP}}(\pi^- K^+) = -0.093 \pm 0.015$$

[G. Hou's talk]

- $S(\pi^0 K^0) = 0.33 \pm 0.21$

$$\sin(2\beta)(\text{charmionium}) = 0.675 \pm 0.026$$

- $|C'/T'| = 1.6 \pm 0.3$  mainly due to  $A_{CP}$ ,  
 $S(\pi^0 K^0)$

SB, London, [hep-ph/0701181](https://arxiv.org/abs/hep-ph/0701181)

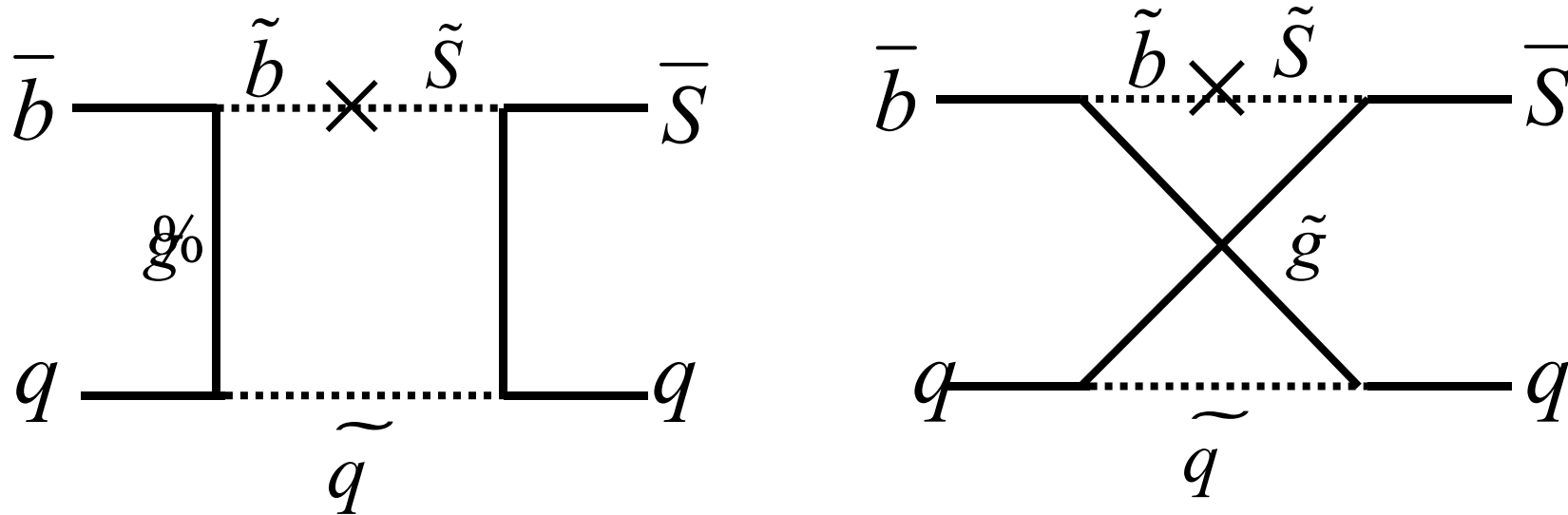
cf. In  $b \rightarrow d$  transition,  $|C/T| = 0.28 \pm 0.15$   
(Combined analysis of  $B \rightarrow \pi\pi$ ,  $B \rightarrow KK$ ).

SB, [arXiv:0707.2838](https://arxiv.org/abs/0707.2838)

→ can be solved by NP in the EW-  
penguin

# GNK scenario

- Z-mediated EW-peguin in SUSY is suppressed.
- “Trojan penguin?” Grossman, Neubert, Kagan(99)



- Large enhancement in EW-peguin sector  $\frac{\alpha_s^2 / M_{SUSY}^2}{\alpha_{ew}^2 / M_W^2}$

# Structure of squark mass matrix

Squark mass matrix:

$$M_{\tilde{d},LL}^2 = \begin{pmatrix} \tilde{m}_{L11}^{d,2} & 0 & 0 \\ 0 & \tilde{m}_{L22}^{d,2} & \tilde{m}_{L23}^{d,2} \\ 0 & \tilde{m}_{L32}^{d,2} & \tilde{m}_{L33}^{d,2} \end{pmatrix}, \quad M_{\tilde{d},LR(RL)}^2 \equiv 0_{3 \times 3}.$$

$K - \bar{K}$  (points to  $\tilde{m}_{L11}^{d,2}$ )  
 $B_d - \bar{B}_d$  (points to  $\tilde{m}_{L23}^{d,2}$ )  
 $B(B \rightarrow X_S \gamma)$  (points to the 2-3 mixing sub-block)

$$M_{\tilde{d},RR}^2 = M_{\tilde{d},LL}^2 |_{L \leftrightarrow R}$$

Large 2-3 mixing motivated by SUSY GUT.

S.B., T. Goto, Y. Okada, K. Okumura(01), Chang, Masiero, Murayama(02)

Squark mixing:

$$\Gamma_L M_{\tilde{d},LL}^2 \Gamma_L^\dagger = \text{diag}(m_{\tilde{d}_L}^2, m_{\tilde{s}_L}^2, m_{\tilde{b}_L}^2),$$

$$\Gamma_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_L & \sin \theta_L e^{i\delta_L} \\ 0 & -\sin \theta_L e^{-i\delta_L} & \cos \theta_L \end{pmatrix}.$$

$$\tilde{s}_L = \cos \theta_L \tilde{s}_L^0 - \sin \theta_L e^{-i\delta_L} \tilde{b}_L^0$$

$$\tilde{b}_L = \sin \theta_L e^{i\delta_L} \tilde{s}_L^0 + \cos \theta_L \tilde{b}_L^0$$

Oversimplified but useful in constraining 2-3 mixing.



$$H_{\text{eff}}^{NP} = \frac{G_F}{\sqrt{2}} \left[ \sum_{i,q=u,d} \left( c_i^q(\mu) O_i^q + \tilde{c}_i^q(\mu) \tilde{O}_i^q \right) + C_{8g}(\mu) Q_{8g} + \tilde{C}_{8g}(\mu) \tilde{Q}_{8g} \right],$$

3

$$\begin{aligned} O_1^q &= (\bar{b}_\alpha s_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V+A} & , & & O_2^q &= (\bar{b}_\alpha s_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V+A} , \\ O_3^q &= (\bar{b}_\alpha s_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V-A} & , & & O_4^q &= (\bar{b}_\alpha s_\beta)_{V-A} (\bar{q}_\beta q_\alpha)_{V-A} , \\ O_5^q &= (\bar{b}_\alpha q_\alpha)_{V-A} (\bar{q}_\beta s_\beta)_{V+A} & , & & O_6^q &= (\bar{b}_\alpha q_\beta)_{V-A} (\bar{q}_\beta s_\alpha)_{V+A} , \\ Q_{8g} &= (g_s/8\pi^2) m_b \bar{b} \sigma_{\mu\nu} (1 - \gamma_5) G^{\mu\nu} s . \end{aligned}$$

$$c_1^q = \frac{\alpha_s^2 \sin 2\theta_L e^{i\delta_L}}{4\sqrt{2} G_F m_{\tilde{g}}^2} \left[ \frac{1}{18} F(x_{\tilde{b}_L \tilde{g}}, x_{\tilde{q}_R \tilde{g}}) - \frac{5}{18} G(x_{\tilde{b}_L \tilde{g}}, x_{\tilde{q}_R \tilde{g}}) + \frac{1}{2} A(x_{\tilde{b}_L \tilde{g}}) + \frac{2}{9} B(x_{\tilde{b}_L \tilde{g}}) \right]$$

$$-(x_{\tilde{b}_L \tilde{g}} \rightarrow x_{\tilde{s}_L \tilde{g}})$$

$$c_2^q = \frac{\alpha_s^2 \sin 2\theta_L e^{i\delta_L}}{4\sqrt{2} G_F m_{\tilde{g}}^2} \left[ \frac{7}{6} F(x_{\tilde{b}_L \tilde{g}}, x_{\tilde{q}_R \tilde{g}}) + \frac{1}{6} G(x_{\tilde{b}_L \tilde{g}}, x_{\tilde{q}_R \tilde{g}}) - \frac{3}{2} A(x_{\tilde{b}_L \tilde{g}}) - \frac{2}{3} B(x_{\tilde{b}_L \tilde{g}}) \right]$$

$$-(x_{\tilde{b}_L \tilde{g}} \rightarrow x_{\tilde{s}_L \tilde{g}})$$

$$c_3^q = \frac{\alpha_s^2 \sin 2\theta_L e^{i\delta_L}}{4\sqrt{2} G_F m_{\tilde{g}}^2} \left[ -\frac{5}{9} F(x_{\tilde{b}_L \tilde{g}}, x_{\tilde{q}_L \tilde{g}}) + \frac{1}{36} G(x_{\tilde{b}_L \tilde{g}}, x_{\tilde{q}_L \tilde{g}}) + \frac{1}{2} A(x_{\tilde{b}_L \tilde{g}}) + \frac{2}{9} B(x_{\tilde{b}_L \tilde{g}}) \right]$$

$$-(x_{\tilde{b}_L \tilde{g}} \rightarrow x_{\tilde{s}_L \tilde{g}})$$

$$c_4^q = \frac{\alpha_s^2 \sin 2\theta_L e^{i\delta_L}}{4\sqrt{2} G_F m_{\tilde{g}}^2} \left[ \frac{1}{3} F(x_{\tilde{b}_L \tilde{g}}, x_{\tilde{q}_L \tilde{g}}) + \frac{7}{12} G(x_{\tilde{b}_L \tilde{g}}, x_{\tilde{q}_L \tilde{g}}) - \frac{3}{2} A(x_{\tilde{b}_L \tilde{g}}) - \frac{2}{3} B(x_{\tilde{b}_L \tilde{g}}) \right]$$

- Large SUSY contribution is possible when
  - small gluino mass
  - $m_{\tilde{s}_{L,R}}^2 \gg m_{\tilde{b}_{L,R}}^2$
  - $|\sin 2\theta_L|$  or  $|\sin 2\theta_R|$  not far below 1
  - $m_{\tilde{d}_R}^2 \gg m_{\tilde{u}_R}^2$  cf.  $m_{\tilde{d}_L}^2 = m_{\tilde{u}_L}^2$  by SU(2)
- $c_1^{\text{EW}}$ ,  $c_2^{\text{EW}}$ ,  $\tilde{c}_3^{\text{EW}}$  and  $\tilde{c}_4^{\text{EW}}$  can get large SUSY contributions ( $c_i^{\text{EW}} \equiv c_i^u - c_i^d$ ).

# $B_S - \bar{B}_S$ mixing

- $B_S - \bar{B}_S$  mass difference:

$$17 \text{ ps}^{-1} < \Delta m_s < 21 \text{ ps}^{-1} \quad (90\% \text{ CL}), \quad (\text{D0})$$

$$\Delta m_s = 17.33_{-0.21}^{+0.42} \pm 0.07 \text{ ps}^{-1}, \quad (\text{CDF})$$

$$\Delta m_s^{\text{SM}}(\text{UTfit}) = 21.5 \pm 2.6 \text{ ps}^{-1}, \quad \Delta m_s^{\text{SM}}(\text{CKMfit}) = 21.7_{-4.2}^{+5.9} \text{ ps}^{-1}$$

- Consistent with the SM CKM fits.
- Constrains NP models sensitive to  $\Delta m_s$  [G. Hou's talk]

# Effective Hamiltonian

$$H_{\text{eff}} = \sum_{i=1}^5 C_i O_i + \sum_{i=1}^3 \tilde{C}_i \tilde{O}_i$$

$$O_1 = (\bar{s}_L \gamma_\mu b_L) (\bar{s}_L \gamma^\mu b_L),$$

$$O_2 = (\bar{s}_R b_L) (\bar{s}_R b_L),$$

$$O_3 = (\bar{s}_R^\alpha b_L^\beta) (\bar{s}_R^\beta b_L^\alpha),$$

$$O_4 = (\bar{s}_R b_L) (\bar{s}_L b_R),$$

$$O_5 = (\bar{s}_R^\alpha b_L^\beta) (\bar{s}_L^\beta b_R^\alpha),$$

$$\tilde{O}_{i=1,\dots,3} = O_{i=1,\dots,3}|_{L \leftrightarrow R} \dots$$

$$C_1^{\text{MSSM}} = \frac{\alpha_s^2}{4m_{\tilde{g}}^2} \sin^2 2\theta_L e^{2i\delta_L} \left( f_1(x_{\tilde{b}_L, \tilde{g}}, x_{\tilde{b}_L, \tilde{g}}) - 2f_1(x_{\tilde{s}_L, \tilde{g}}, x_{\tilde{b}_L, \tilde{g}}) + f_1(x_{\tilde{s}_L, \tilde{g}}, x_{\tilde{s}_L, \tilde{g}}) \right)$$

$$C_{4(5)}^{\text{MSSM}} = \frac{\alpha_s^2}{4m_{\tilde{g}}^2} \sin 2\theta_L \sin 2\theta_R e^{i(\delta_L + \delta_R)} \left( f_{4(5)}(x_{\tilde{b}_R, \tilde{g}}, x_{\tilde{b}_L, \tilde{g}}) - f_{4(5)}(x_{\tilde{b}_R, \tilde{g}}, x_{\tilde{s}_L, \tilde{g}}) - f_{4(5)}(x_{\tilde{s}_R, \tilde{g}}, x_{\tilde{b}_L, \tilde{g}}) + f_{4(5)}(x_{\tilde{s}_R, \tilde{g}}, x_{\tilde{s}_L, \tilde{g}}) \right),$$

$$\tilde{C}_1^{\text{MSSM}} = C_1^{\text{MSSM}}|_{L \leftrightarrow R},$$

Sensitive to mass splitting of  $\tilde{s}$  and  $\tilde{b}$  and their mixing.

Only  $C_1^{\text{MSSM}} (\tilde{C}_1^{\text{MSSM}})$  is generated if only LL(RR) exists.

$$M_{12}^s = M_{12}^{s,\text{SM}}(1 + R)$$

$$\begin{aligned}\Delta m_s &= 2|M_{12}^s| \\ &= \Delta m_s^{\text{SM}}|1 + R|.\end{aligned}$$

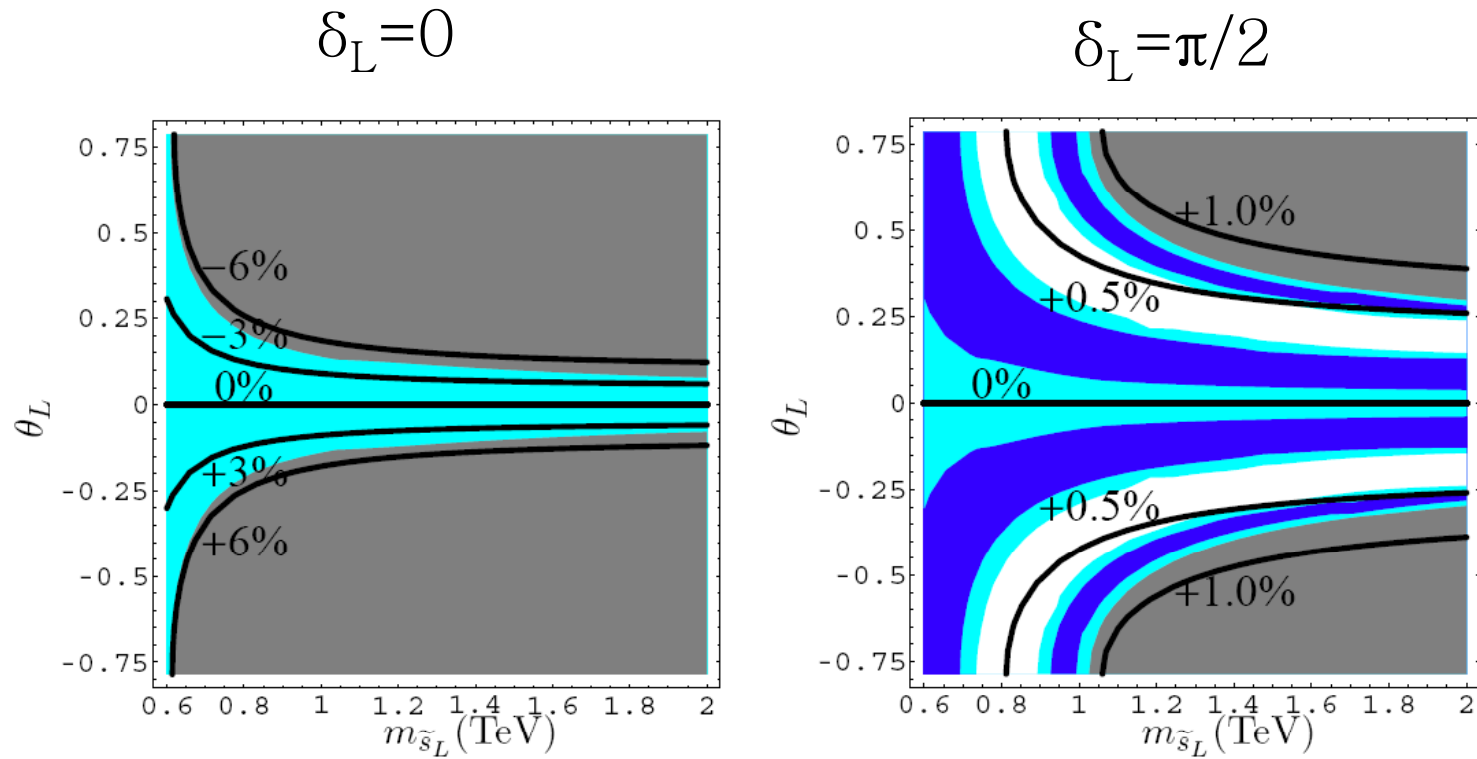
$$|1 + R| = 0.77_{-0.01}^{+0.02}(\text{exp}) \pm 0.19(\text{th})$$

$$\begin{aligned}R(\mu_b) &= \xi_1(\mu_b) + \tilde{\xi}_1(\mu_b) + \frac{3 B_4(\mu_b)}{4 B_1(\mu_b)} \left( \frac{M_{B_s}}{m_b(\mu_b) + m_s(\mu_b)} \right)^2 \xi_4 \\ &\quad + \frac{1 B_5(\mu_b)}{4 B_1(\mu_b)} \left( \frac{M_{B_s}}{m_b(\mu_b) + m_s(\mu_b)} \right)^2 \xi_5,\end{aligned}$$

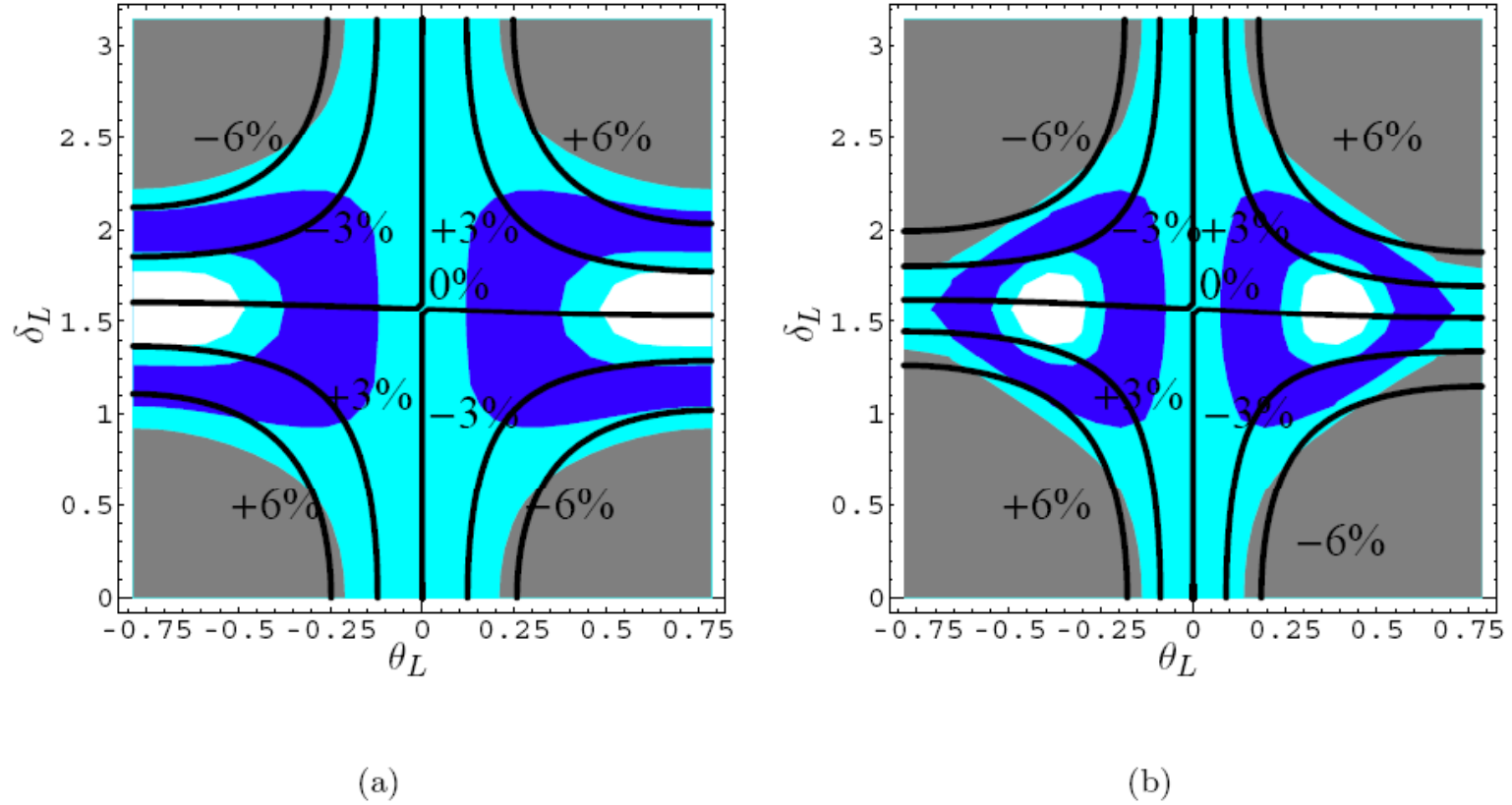
$$\xi_i(\mu_b) \equiv C_i^{\text{SUSY}}(\mu_b)/C_1^{\text{SM}}(\mu_b),$$

$$\tilde{\xi}_i(\mu_b) \equiv \tilde{C}_i^{\text{SUSY}}(\mu_b)/C_1^{\text{SM}}(\mu_b).$$

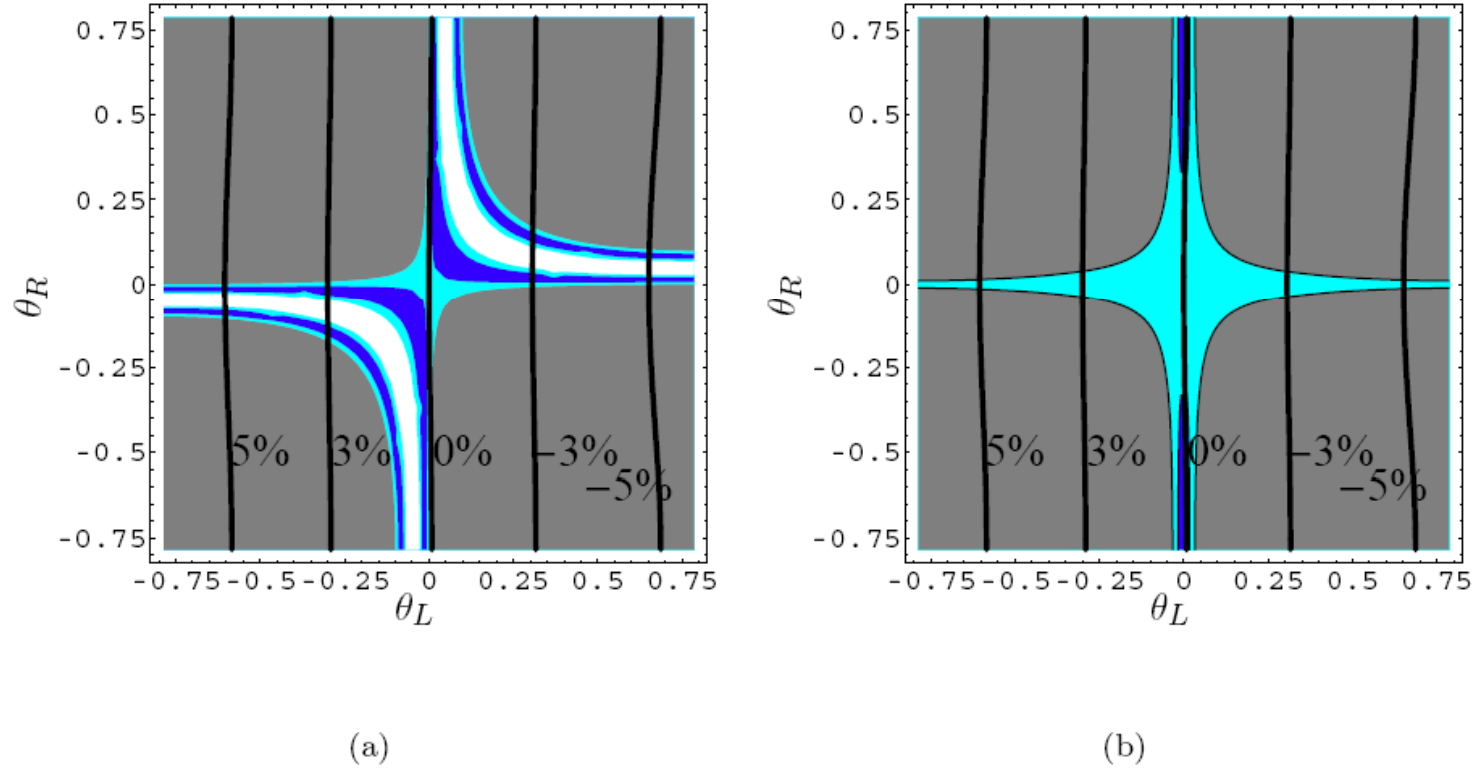
# Results for $m_{\tilde{g}} = 0.5$ (TeV), $m_{\tilde{b}_L} = 0.5$ (TeV)



$B \rightarrow X_s \gamma$  constraint  $BR^{\text{exp}}(B \rightarrow X_s \gamma) / BR^{\text{SM}}(B \rightarrow X_s \gamma) = 1.06 \pm 0.13$



**Figure 2:** Contour plots for  $|1 + R|$  in  $(\theta_L, \delta_L)$  plane. (a)  $m_{\tilde{s}_L} = 0.8$  (TeV), (b)  $m_{\tilde{s}_L} = 1.0$  (TeV).



**Figure 3:** Contour plots for  $|1 + R|$  in  $(\theta_L, \theta_R)$  plane.  $m_{\tilde{s}_L} = m_{\tilde{s}_R} = 0.6$  (TeV). (a)  $\delta_L = \delta_R = 0$   
(b)  $\delta_L = 0, \delta_R = \pi/2$ . We assume both LL and RR mixing exist. The rest is the same with figure 1.



$$B_s \text{ (R) } K^+ K^- \quad \text{and} \quad B_s \text{ (R) } K^0 \bar{K}^0$$

In the SM,

$$\begin{aligned} \mathcal{A}(B_s^0 \rightarrow K^+ K^-) &\simeq V_{ub}^* V_{us} [T' + (P'_u - P'_t)] + V_{cb}^* V_{cs} (P'_c - P'_t) \\ &\equiv V_{ub}^* V_{us} T^{s\pm} + V_{cb}^* V_{cs} P^{s\pm}, \end{aligned}$$

$$\mathcal{A}(B_s^0 \rightarrow K^0 \bar{K}^0) \simeq V_{ub}^* V_{us} T^{s0} + V_{cb}^* V_{cs} P^{s0}$$

- The SM amplitudes are fixed by  $B \rightarrow KK$  modes by flavor-SU(3) symmetry, including factorizable SU(3)-breaking effect

- $\mathcal{A}(B_d^0 \rightarrow K^0 \bar{K}^0) \simeq V_{ub}^* V_{ud} T^{d0} + V_{cb}^* V_{cd} P^{d0}$

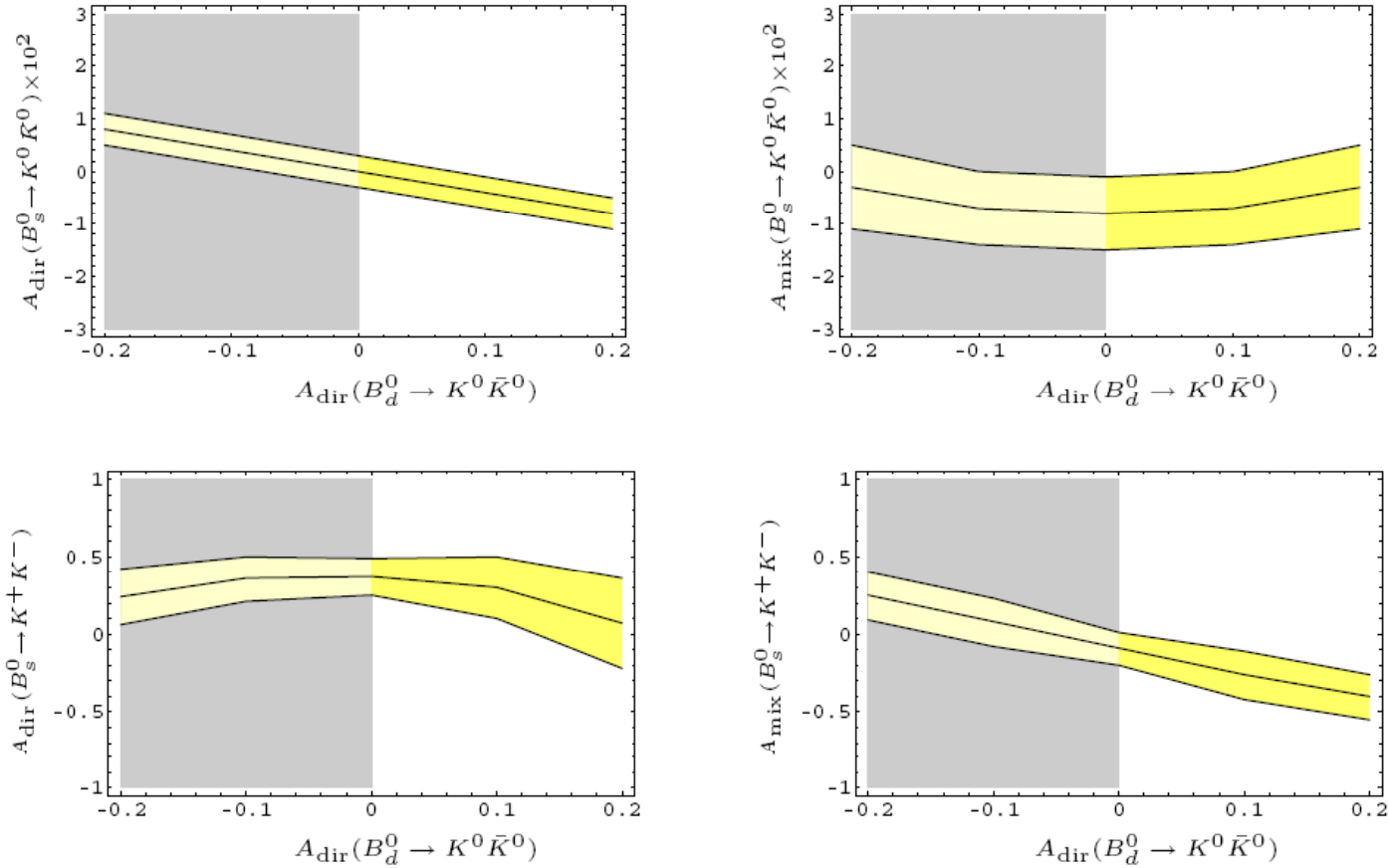
1.  $BR(B_d^0 \rightarrow K^0 \bar{K}^0) = (0.96 \pm 0.25) \times 10^{-6}$

2.  $\Delta_d \equiv T^{d0} - P^{d0}$  : QCDF, free of IR divergence and

$$\Delta_d = (1.09 \pm 0.43) \times 10^{-7} + i(-3.02 \pm 0.97) \times 10^{-7} \text{ GeV}$$

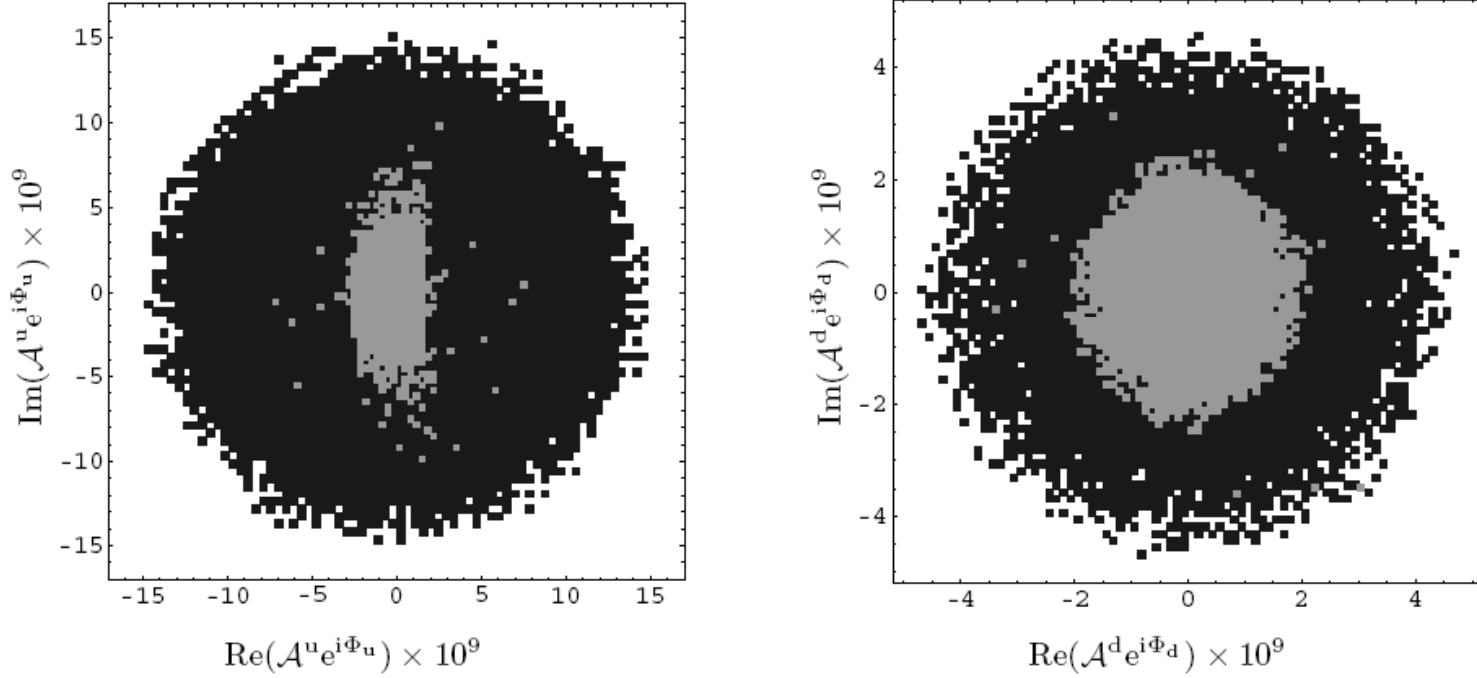
$$-0.2 \leq A_{\text{dir}}^{d0} \leq 0.2 \quad A_{\text{dir}}(B^+ \text{ @ } K^+ \bar{K}^0) = -0.12_{-0.17}^{+0.18}$$

3. cf. [S. Descotes-Genon, J. Matias, J. Virto \(06\)](#)



**Figure 1:** SM predictions for the CP asymmetries in  $B_s^0 \rightarrow K^0 \bar{K}^0$  (up) and  $B_s^0 \rightarrow K^+ K^-$  (down) as a function of  $A_{\text{dir}}(B_d^0 \rightarrow K^0 \bar{K}^0)$ . As explained in the text, the preferred range is the non-shadowed half of the plots [ $A_{\text{dir}}(B_d^0 \rightarrow K^0 \bar{K}^0) \geq 0$ ].

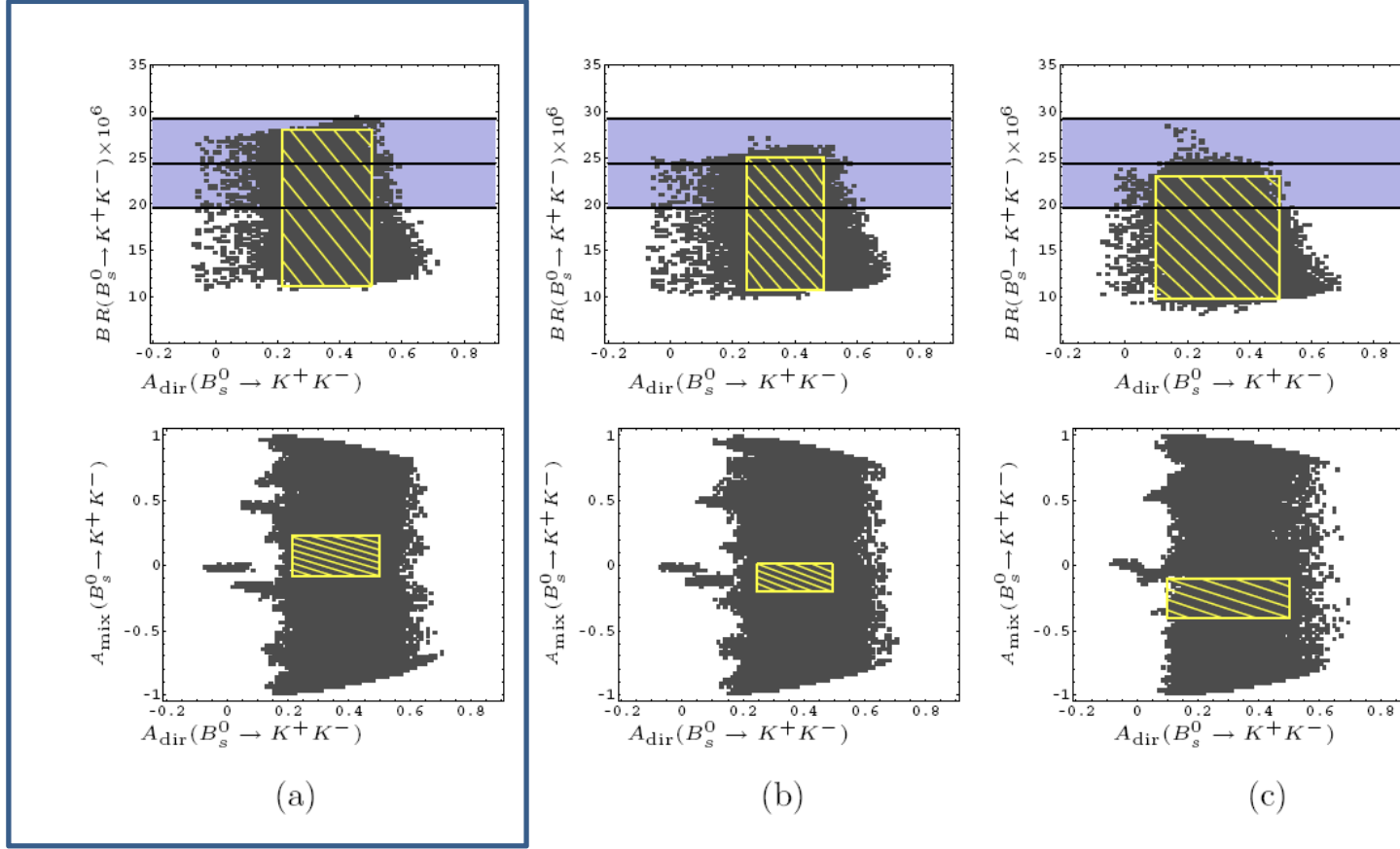
- $m_{\tilde{u}_L} = m_{\tilde{d}_{L,R}} = m_{\tilde{b}_{L,R}} = m_{\tilde{g}} = 250 \text{ GeV}$
- $250 \text{ GeV} < m_{\tilde{u}_R}, m_{\tilde{s}_{R,L}} < 1000 \text{ GeV}$
- $-\pi < \delta_{L,R} < \pi$
- $-\pi/4 < \theta_{L,R} < \pi/4$



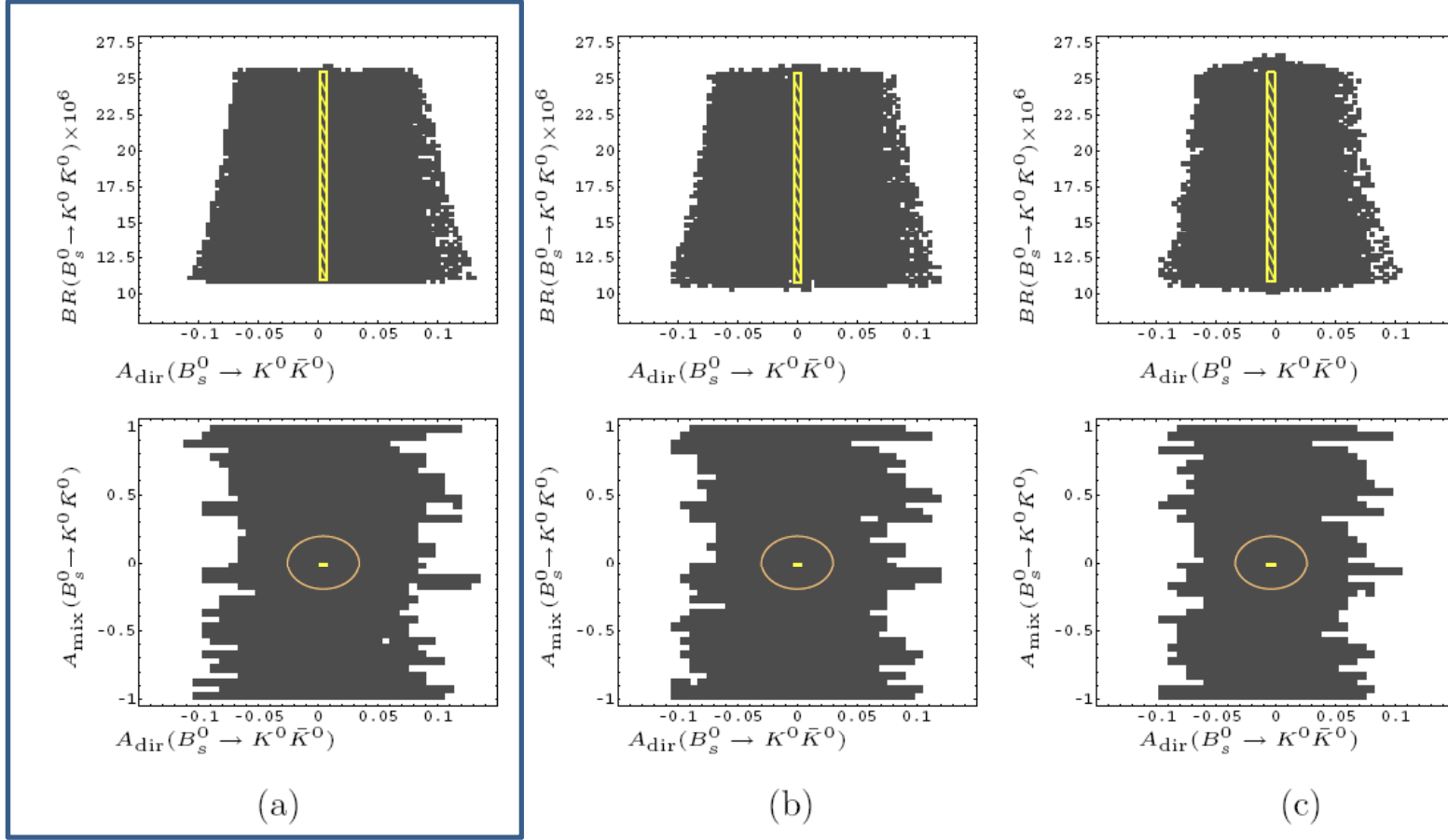
**Figure 2:** SUSY contribution to the NP amplitudes  $\mathcal{A}^u e^{i\Phi_u}$  (left) and  $\mathcal{A}^d e^{i\Phi_d}$  (right) in the scenario with simultaneous LL and RR mixings. The dark regions correspond to the variation of the SUSY parameters over the considered parameter space. The light regions satisfy the experimental bounds, including the recent measurement of  $\Delta M_s$ .

Isobreaking terms:  $P_{EW}$

$$\begin{aligned}
 -A(B_s \rightarrow K^+ K^-) &= P' + T' + \frac{2}{3} P'_{EW} + E' + PA' \\
 &= A \left[ a_5^{QCD} - a_4^{QCD} - \chi a_2^{QCD} - \lambda_u a_2 + \frac{2}{3} (a_5^{EW} - a_4^{EW} - \chi a_2^{EW}) \right] \quad (9) \\
 A(B_s \rightarrow K^0 \bar{K}^0) &= P' - \frac{1}{3} P'_{EW} + PA' \\
 &= A \left[ a_5^{QCD} - a_4^{QCD} - \chi a_2^{QCD} - \frac{1}{3} (a_5^{EW} - a_4^{EW} - \chi a_2^{EW}) \right], \quad (10)
 \end{aligned}$$



**Figure 3:** Predictions, in the form of scatter plots, for the correlations between  $BR(B_s^0 \rightarrow K^+K^-) - A_{\text{dir}}(B_s^0 \rightarrow K^+K^-)$  (up) and  $A_{\text{mix}}(B_s^0 \rightarrow K^+K^-) - A_{\text{dir}}(B_s^0 \rightarrow K^+K^-)$  (down) in the presence of SUSY, for a)  $A_{\text{dir}}^{d0} = -0.1$ , (b)  $A_{\text{dir}}^{d0} = 0$  and (c)  $A_{\text{dir}}^{d0} = 0.1$ . The dashed rectangles correspond to the SM predictions. The horizontal band shows the experimental value for  $BR(B_s^0 \rightarrow K^+K^-)$  at  $1\sigma$ .



**Figure 4:** Predictions, in the form of scatter plots, for the correlations between  $BR(B_s^0 \rightarrow K^0 \bar{K}^0) - A_{\text{dir}}(B_s^0 \rightarrow K^0 \bar{K}^0)$  (up) and  $A_{\text{mix}}(B_s^0 \rightarrow K^0 \bar{K}^0) - A_{\text{dir}}(B_s^0 \rightarrow K^0 \bar{K}^0)$  (down) in the presence of SUSY, for (a)  $A_{\text{dir}}^{d0} = -0.1$ , (b)  $A_{\text{dir}}^{d0} = 0$  and (c)  $A_{\text{dir}}^{d0} = 0.1$ . The dashed rectangles correspond to the SM predictions. These are quite small in the three lower plots, so they are indicated by a circle.



# Conclusions

- NP in the EW-penguin sector of  $B \rightarrow \pi K$  is a promising possibility.
- GNK scenario can give large enhancement to EW penguins in SUSY.
- $\Delta m_s$  gives a strong constraint.
- Large CPVs are possible in  $B_s \rightarrow K^+ K^-$  and/or  $B_s \rightarrow K^0 K^0$ . The differences between the two modes signal NP in the EW-penguin.