$B_s - \bar{B}_s$ mixing and $B_s \to KK$ decays within supersymmetry

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Abstract. We consider the constraint of $B_s - \bar{B}_s$ mass difference, Δm_s, on an MSSM scenario with large flavor mixing. Even with this constraint, we show that a large deviation from the SM in CP asymmetries of $B_s \to KK$ decays is still possible, making this channel promising in search for supersymmetry.

PACS. 12.60.Jv Supersymmetric models – 13.25.Hw Decays of bottom mesons

1 Introduction

The flavor changing processes in the $s - b$ sector are sensitive probe of new physics (NP) beyond the standard model (SM) because they are experimentally the least constrained. In the minimal supersymmetric standard model (MSSM), however, the flavor mixing in the chirality flipping down-type squarks, $\widetilde{s}_{L(R)} - \widetilde{b}_{R(L)}$, is already strongly constrained by the measurement of $BR(B \to X_s + \gamma)$. On the other hand, large flavor mixing in the chirality conserving $\widetilde{s}_{L(R)} - \widetilde{b}_{L(R)}$ has been largely allowed. Especially the large mixing scenario in the $\widetilde{s}_R - \widetilde{b}_R$ sector has been drawing much interest because it is well motivated by the measurement large neutrino mixing and the idea of grand unification [1].

The DØ and CDF collaborations at Fermilab Tevatron reported the results on the measurements of $B_s - \overline{B}_s$ mass difference [2,3]

$$\begin{aligned} 17 \text{ ps}^{-1} < \Delta m_s < 21 \text{ ps}^{-1} \quad (90\% \text{ CL}), \\ \Delta m_s = 17.33^{+0.42}_{-0.21} \pm 0.07 \text{ ps}^{-1}, \end{aligned} \tag{1}$$

respectively. These measured values are consistent with the SM predictions [4,5]

$$\begin{aligned} \Delta m_s^{\rm SM}(\text{UTfit}) = 21.5 \pm 2.6 \text{ ps}^{-1}, \\ \Delta m_s^{\rm SM}(\text{CKMfit}) = 21.7^{+5.9}_{-4.2} \text{ ps}^{-1}, \end{aligned} \tag{2}$$

which are obtained from global fits, although the experimental measurements in (1) are slightly lower. Therefore (1) impose strong constraints which predict large $b - s$ mixings [6].

Another $b \to s$ dominating processes, $B \to \pi K$ decays, have been extensively studied [7]. The current measurements of branching ratios (BRs) and CP-asymmetries (CPAs) in the four $B \to \pi K$ channels show some interesting discrepancy from the SM predictions [7,8]. We argue that this "$B \to \pi K$ puzzle" manifests itself in the CP-violating observables like the difference between $A_{\rm CP}(B^+ \to \pi^+ K^0)$ and $A_{\rm CP}(B^0 \to \pi^- K^+)$ or $S_{CP}(B^0 \to \pi^0 K^0)$ from its SM predictions. The puzzle can be solved if we introduce NP in the electroweak penguin sector [9].

We'd like to stress that even with the constraint given by (1) there are still much room for NP contributions in $b \to s$ transitions. We demonstrate that the CPAs in $B \to KK$ decays can be very different from the values expected in the SM [10]. In addition, if the NP appears in the electroweak penguin sector as required by the $B \to \pi K$ puzzle, [9] the predictions in the two modes, $B \to K^+ K^-$ and $B \to K^0 \bar{K}^0$, can be very different.

2 $B_s - \bar{B}_s$ mixing

We consider the implications of (1) on an MSSM scenario with large mixing in the LL and/or RR sector. We do not consider flavor mixing in the LR(RL) sector because they are i) are already strongly constrained by $BR(B \to X_s \gamma)$ and ii) therefore relatively insensitive to $B_s - \overline{B}_s$ mixing. We neglect mixing between the 1st and 2nd generations which are tightly constrained by K meson decays and $K - \overline{K}$ mixing, and mixing between the 1st and 3rd generations which is also known to be small by the measurement of $B_d - \overline{B}_d$ mixing. And the down-type squark mass matrix is given by

$$M^2_{\tilde{d},LL} = \begin{pmatrix} \widetilde{m}^{d,2}_{L_{11}} & 0 & 0 \\ 0 & \widetilde{m}^{d,2}_{L_{22}} & \widetilde{m}^{d,2}_{L_{23}} \\ 0 & \widetilde{m}^{d,2}_{L_{32}} & \widetilde{m}^{d,2}_{L_{33}} \end{pmatrix}, \quad M^2_{\tilde{d},LR(RL)} \equiv 0_{3\times 3} \tag{3}$$

The $M^2_{\tilde{d},RR}$ can be obtained from $M^2_{\tilde{d},LL}$ by exchanging $L \leftrightarrow R$. We note that this kind of scenario is orthogonal to the one with flavor violation controlled only by CKM matrix minimal flavor violation model [11], where large flavor violation in $s - b$ is impossible a priori. Defining

$$M^s_{12} \equiv M^{s,\rm SM}_{12}(1 + R), \tag{4}$$

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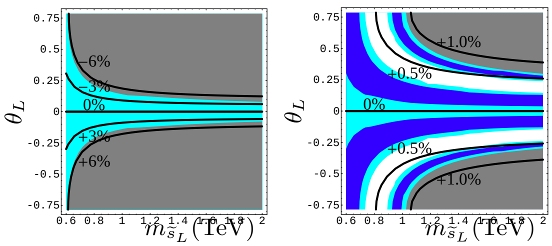
we obtain the following constraint,

$$|1+R| = 0.77^{+0.02}_{-0.01}(\text{exp}) \pm 0.19(\text{th}). \tag{5}$$



(a) (b)

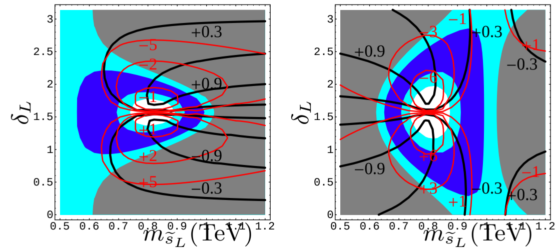
Fig. 1. Contour plots for $|1+R|$ in ($m_{\tilde{s}_L}$,θ_L) plane. Sky blue region represents 2σ allowed region ($0.39 \le |1+R| \le 1.15$), blue 1σ allowed region ($0.58 \le |1+R| \le 0.96$), and white (grey) region is excluded at 95% CL by giving too small (large) Δm_s. The labeled thick lines represent the constant $\Big(BR^{\text{tot}}(B \to X_s\gamma) - BR^{\text{SM}}(B \to X_s\gamma)\Big)/BR^{\text{SM}}(B \to X_s\gamma)$ contours. Only LL mixing is assumed to exist. The fixed parameters are $m_{\tilde{g}} = 0.5$ (TeV), $m_{\tilde{b}_L} = 0.5$ (TeV), (a) δ_L=0, (b) $\delta_L = \pi/2$.

The larger the mass splitting between $\tilde{s}$ and $\tilde{b}$, the larger the SUSY contributions are. Therefore we expect that Δm_s^{exp} constrains the mass splitting when the mixing angle $\theta_{L(R)}$ is large. This can be seen in Figure 1 where we show filled contour plots for $|1+R|$ in ($m_{\tilde{s}_L}$,θ_L) plane: sky blue region represents 2σ allowed region ($0.39 \le |1+R| \le 1.15$), blue 1σ allowed region ($0.58 \le |1+R| \le 0.96$), and white (grey) region is excluded at 95% CL by giving too small (large) Δm_s. For these plots we assumed that only LL mixing exists and fixed $m_{\tilde{g}} = 0.5$ TeV, $m_{\tilde{b}_L} = 0.5$ TeV. In Figure 1(a), we fixed $\delta_L = 0$. We can see that the SUSY interferes with the SM contribution constructively (*i.e.* the SUSY contribution has the same sign with the SM), and when the mixing angle is maximal, *i.e.* $\theta_L = \pm\pi/4$, $m_{\tilde{s}_L} - m_{\tilde{b}_L}$ cannot be greater than about 150 GeV. In Figure 1(b), we set $\delta_L = \pi/2$. The SUSY contribution can interfere destructively (*i.e.* in opposite sign) with the SM and much larger mass splitting is allowed. Therefore we can see that the allowed parameters are sensitive to the CPV phase. We see that the $B(b \to s\gamma)$ constraint is not important in this case.

The CPV phase in the $B_s - \overline{B}_s$ mixing amplitude will be measured at the LHC in the near future through the time-dependent CP asymmetry

$$\frac{\Gamma(\overline{B}_s(t) \to \psi\phi) - \Gamma(B_s(t) \to \psi\phi)}{\Gamma(\overline{B}_s(t) \to \psi\phi) + \Gamma(B_s(t) \to \psi\phi)} \equiv S_{\psi\phi}\sin(\Delta m_s t) \tag{6}$$

In the SM, $S_{\psi\phi}$ is predicted to be very small, $S_{\psi\phi}^{\text{SM}} = -\sin 2\beta_s = 0.038 \pm 0.003$ ($\beta_s \equiv \arg[(V_{ts}^* V_{tb})/(V_{cs}^* V_{cb})]$).



(a) (b)

Fig. 2. Contour plots for $|1+R|$ in ($m_{\tilde{s}_L}$,δ_L) plane. The $S_{\psi\phi}$ predictions are also shown as thick contour lines. The thin red lines are constant $A_{SL}^s[10^{-3}]$ contours assuming $\text{Re}(\Gamma_{12}^s/M_{12}^s)^{\text{SM}} = -0.0040$. (a) Only LL mixing is assumed to exist. We fixed $m_{\tilde{g}} = m_{\tilde{b}_L} = 0.5$ TeV, $\delta_L = \pi/4$. (b) Both LL and RR mixing are assumed to exist simultaneously. We fixed $m_{\tilde{g}} = 2$ TeV, $m_{\tilde{b}_L} = m_{\tilde{b}_R} = 1$ TeV, $m_{\tilde{s}_R} = 1.1$ TeV, $\theta_R = \pi/4$, $\delta_L = \pi/4$, and $\delta_R = \pi/2$. The rest is the same with Figure 1.

If the NP has additional CPV phases, however, the prediction

$$S_{\psi\phi} = -\sin(2\beta_s + \arg(1+R)) \tag{7}$$

can be significantly different from the SM prediction.

In Figure 2, we show $|1+R|$ constraint and the prediction of $S_{\psi\phi}$ in ($m_{\tilde{s}_L}, \delta_L$) plane. However, the $B \to X_s\gamma$ prediction is not shown from now on because it is irrelevant as mentioned above. For Figure 2(a), we assumed the scenario with LL mixing only and maximal mixing $\theta_L = \pi/4$. We fixed $m_{\tilde{g}} = 0.5$ TeV, $m_{\tilde{b}_L} = 0.5$ TeV. For Figure 2(b), we allowed both LL and RR mixing simultaneously, while fixing $m_{\tilde{g}} = 2$ TeV, $m_{\tilde{b}_L} = m_{\tilde{b}_R} = 1$ TeV, $m_{\tilde{s}_R} = 1.1$ TeV, $\theta_R = \pi/4$, $\delta_L = \pi/4$, and $\delta_R = \pi/2$. In both cases we can see that large $S_{\psi\phi}$ is allowed for large mass splitting between $m_{\tilde{b}_L}$ and $m_{\tilde{s}_L}$. At the moment, $S_{\psi\phi}$ can take any value in the range $[-1, 1]$ even after imposing the current Δm_s^{exp} constraint.

3 The $B \to \pi K$ puzzle

The $B \to \pi K$ decays, dominated by $b \to s$ transitions, are one of the most promising candidates where large NP contributions can be probed. The current data shown in Tab. 1, can be analyzed using the diagrammatic amplitudes [8]:

$$\begin{aligned}
A^{+0} &= -P'_{tc} + P'_{uc}e^{i\gamma} - \frac{1}{3}P'^{C}_{EW}\,, \\
\sqrt{2}A^{0+} &= -T'e^{i\gamma} - C'e^{i\gamma} + P'_{tc} \\
&\quad - P'_{uc}e^{i\gamma} - P'_{EW} - \frac{2}{3}P'^{C}_{EW}\,, \\
A^{-+} &= -T'e^{i\gamma} + P'_{tc} - P'_{uc}e^{i\gamma} - \frac{2}{3}P'^{C}_{EW}\,, \\
\sqrt{2}A^{00} &= -C'e^{i\gamma} - P'_{tc} + P'_{uc}e^{i\gamma} - P'_{EW} - \frac{1}{3}P'^{C}_{EW}\,.
\end{aligned} \tag{8}$$

Mode	$BR[10^{-6}]$	A_{CP}	S_{CP}
$B^+ \to \pi^+ K^0$	23.1 ± 1.0	0.009 ± 0.025	
$B^+ \to \pi^0 K^+$	12.8 ± 0.6	0.047 ± 0.026	
$B_d^0 \to \pi^- K^+$	19.7 ± 0.6	-0.093 ± 0.015	
$B_d^0 \to \pi^0 K^0$	10.0 ± 0.6	-0.12 ± 0.11	0.33 ± 0.21

Table 1. Branching ratios, direct CP asymmetries A_{CP}, and mixing-induced CP asymmetry S_{CP} (if applicable) for the four $B \to \pi K$ decay modes.

Neglecting P'_{uc} and P'^{C}_{EW} which are expected to give small contributions, we can fit (8) to the data in Tab. 1. The ratio $|C'/T'| = 1.6 \pm 0.3$ is required here (we stress that correlations have been taken into account in obtaining this ratio). This value is much larger than the naive estimates, the NLO pQCD prediction [12], $|C'/T'| \sim 0.3$, and the maximal SCET (QCDf) prediction [13], $|C'/T'| \sim 0.6$. Thus, if one takes this theoretical input seriously – as we do here – this shows explicitly that the $B \to \pi K$ puzzle is still present, at $\sim 3\sigma$ level. (The abnormally large value of $|C'/T'| = 1.6 \pm 0.3$ found here is partially due to S_{CP}. Without it we obtain $|C'/T'| = 0.8 \pm 0.1$.) In Ref. [14] (2004), $|C'/T'| = 1.8 \pm 1.0$ was found. We thus see that the puzzle has gotten much worse in 2006. In passing we note that the similar problem in $B \to \pi\pi$ decays can be solved if we can separate the P_{uc} component from the T and C amplitude using, for example, the measurements of $B \to KK$ decays [15].

If we include NP, the NP contribution in the electroweak penguin amplitude, $\mathcal{A}'^{,comb} e^{i\Phi'}$, fits the data best: $\chi^2_{min}/d.o.f. = 0.6/3$ (90%). For this fit, we set other NP amplitudes to be zero. This is the same conclusion as that found in Ref. [14]. Thus, not only is the $B \to \pi K$ puzzle still present, but it is still pointing towatds the same type of NP, $\mathcal{A}'^{,comb} e^{i\Phi'} \neq 0$. For this (good) fit, we find $|T'/P'| = 0.09$, $|\mathcal{A}'^{,comb}/P'| = 0.24$, $\Phi' = 85°$. We therefore find that the NP amplitude must be sizeable, with a large weak phase.

4 Large SUSY contributions to $B \to KK$ decays

In the SM the $B_s \to KK$ decays can be parameterized as

$$\begin{aligned} \mathcal{A}(B_s^0 \to K^+K^-) &\simeq V_{ub}^* V_{us} [T' + (P'_u - P'_t)] \\ &\quad + V_{cb}^* V_{cs} (P'_c - P'_t) \\ &\equiv V_{ub}^* V_{us} T^{s\pm} + V_{cb}^* V_{cs} P^{s\pm} , \\ \mathcal{A}(B_s^0 \to K^0 \bar{K}^0) &\simeq V_{ub}^* V_{us} T^{s0} + V_{cb}^* V_{cs} P^{s0} . \quad (9) \end{aligned}$$

The amplitudes $P^{s\pm}$, P^{s0}, $T^{s\pm}$ and T^{s0} can be determined from the measurements of $B_d^0 \to K^0 \bar{K}^0$ decay [16]. The amplitude for $B_d^0 \to K^0 \bar{K}^0$ can be written

$$\mathcal{A}(B_d^0 \to K^0 \bar{K}^0) \simeq V_{ub}^* V_{ud} T^{d0} + V_{cb}^* V_{cd} P^{d0} . \quad (10)$$

The three unknown physical quantities in T^{d0} and P^{d0} are determined from the three conditions: *i)*

$$BR(B_d \to K^0 \bar{K}^0) = (0.96 \pm 0.25) \times 10^{-6}, \quad (11)$$

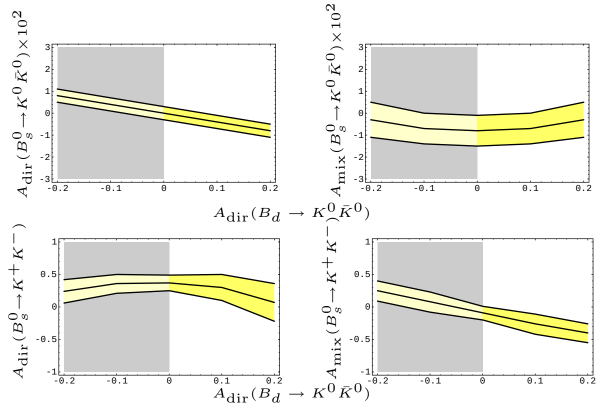


Fig. 3. SM predictions for the CP asymmetries in $B_s^0 \to K^0 \bar{K}^0$ (up) and $B_s^0 \to K^+K^-$ (down) as a function of $A_{\rm dir}(B_d \to K^0 \bar{K}^0)$. As explained in the text, the preferred range is the non-shadowed half of the plots $[A_{\rm dir}(B_d \to K^0 \bar{K}^0) \geq 0]$.

ii) a quantity free from IR cutoff in QCDF [17,18],

$$\Delta_d = (1.09 \pm 0.43) \times 10^{-7} + i(-3.02 \pm 0.97) \times 10^{-7} \text{ GeV}, \quad (12)$$

where $\Delta_d \equiv T^{d0} - P^{d0}$, and *iii)* the fact that only values $-0.2 \leq A^{d0}_{dir} \leq 0.2$ are consistent with the measured value of $BR(B_d \to K^0 \bar{K}^0)$ and the theoretical value of Δ_d [16]. From these conditions we obtain

$$\begin{aligned} |T^{d0}| &= (1.1 \pm 0.8) \times 10^{-6} \text{ GeV}, \\ |P^{d0}/T^{d0}| &= 1.2 \pm 0.2 , \\ \arg(P^{d0}/T^{d0}) &= (-1.6 \pm 6.5)° . \quad (13) \end{aligned}$$

Now we can relate the parameters in $B_d \to K^0 \bar{K}^0$ decays to those in the $B_s \to KK$ decays using SU(3) symmetry. We impose the factorizable SU(3)-breaking effect

$$f = \frac{M^2_{B_s^0} F_0^{B_s^0 \to \bar{K}^0}(0)}{M^2_{B_d^0} F_0^{B_d^0 \to \bar{K}^0}(0)} = 0.94 \pm 0.20 . \quad (14)$$

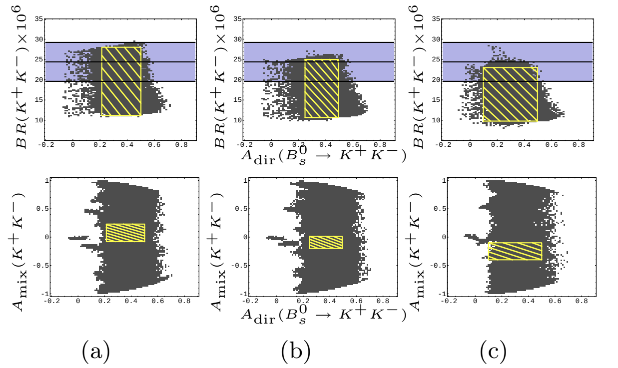
We can predict the observables in $B_s \to KK$ decays shown in Fig. 3

The NP contribution can be parameterized as

$$\begin{aligned} \mathcal{A}(B_s^0 \to K^+K^-) &= \mathcal{A}^{s\pm}_{SM} + \mathcal{A}^u e^{i\Phi_u} , \\ \mathcal{A}(B_s^0 \to K^0 \bar{K}^0) &= \mathcal{A}^{s0}_{SM} + \mathcal{A}^d e^{i\Phi_d} . \quad (15) \end{aligned}$$

If the NP conserves isospin, we have $\mathcal{A}^u = \mathcal{A}^d$ and $\Phi_u = \Phi_d$, but in general this need not be the case. Especially in our NP model [10] described in Section 2, there can be large isospin violation [9]. To see how large the NP contributions can be we scanned in the following SUSY parameter space:

- $m_{\tilde{u}_L} = m_{\tilde{d}_{L,R}} = m_{\tilde{b}_{L,R}} = m_{\tilde{g}} = 250$ GeV
- $250 \text{ GeV} < m_{\tilde{u}_R}, m_{\tilde{s}_{R,L}} < 1000$ GeV
- $-\pi < \delta_{L,R} < \pi$
- $-\pi/4 < \theta_{L,R} < \pi/4$



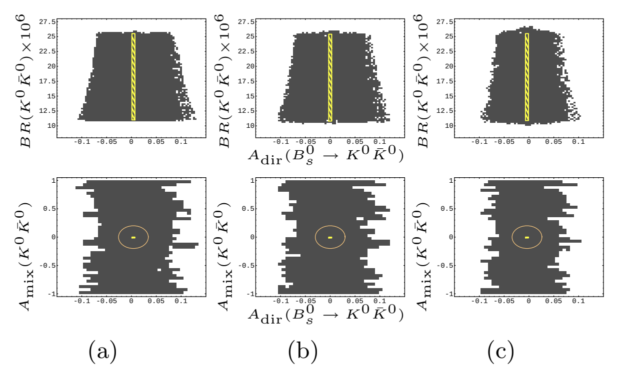


Fig. 4. Predictions, in the form of scatter plots, for the correlations between $BR(B_s^0 \to K^+K^-)-A_{\rm dir}(B_s^0 \to K^+K^-)$ (up) and $A_{\rm mix}(B_s^0 \to K^+K^-) - A_{\rm dir}(B_s^0 \to K^+K^-)$ (down) in the presence of SUSY, for a) $A_{dir}^{d0} = -0.1$, (b) $A_{dir}^{d0} = 0$ and (c) $A_{dir}^{d0} = 0.1$. The dashed rectangles correspond to the SM predictions. The horizontal band shows the experimental value for $BR(B_s^0 \to K^+K^-)$ at 1σ.

Fig. 5. Predictions, in the form of scatter plots, for the correlations between $BR(B_s^0 \to K^0\bar{K}^0) - A_{\rm dir}(B_s^0 \to K^0\bar{K}^0)$ (up) and $A_{\rm mix}(B_s^0 \to K^0\bar{K}^0) - A_{\rm dir}(B_s^0 \to K^0\bar{K}^0)$ (down) in the presence of SUSY, for (a) $A_{dir}^{d0} = -0.1$, (b) $A_{dir}^{d0} = 0$ and (c) $A_{dir}^{d0} = 0.1$. The dashed rectangles correspond to the SM predictions. These are quite small in the three lower plots, so they are indicated by a circle.

We imposed $BR(B \to X_s\gamma) = (3.55 \pm 0.26) \times 10^{-4}$ and Δm_s constraints considered in Section 2.

As can be seen in Figs. 4 and 5, there can be huge deviations from the SM predictions, in $A_{\rm mix}(B_s^0 \to K^+K^-)$, $A_{\rm dir,mix}(B_s^0 \to K^0\bar{K}^0)$.

5 Conclusions

We have seen that the $\Delta m_s^{\rm exp}$ gives strong constraints on large $b-s$ mixing in NP. On the other hand the nonleptonic $B \to \pi K$ decays seem to require NP contributions. We have shown that even with the $\Delta m_s^{\rm exp}$ constraint the NP still allows large $b-s$ leaving observable effects, for example, in $B_s \to KK$ decays.

Acknowledgements:
The author thanks David London, Joaquim Matias, and Javier Virto for collaborations. This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD) No. KRF-2007-359-C00009.

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