Neutrino time travel

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Abstract. We discuss causality properties of extra-dimensional theories allowing for effectively superluminal bulk shortcuts. Such shortcuts for sterile neutrinos have been discussed as a solution to the puzzling LSND and MiniBooNE neutrino oscillation results. We focus here on the sub-category of asymmetrically warped brane spacetimes and argue that scenarios with two extra dimensions may allow for timelike curves which can be closed via paths in the extra-dimensional bulk. In principle sterile neutrinos propagating in the extra dimension may be manipulated in a way to test the chronology protection conjecture experimentally.

PACS. 04.20.Gz Spacetime topology, causal structure – 14.60.St String and brane phenomenology – 11.25.Wx Sterile neutrinos

1 Introduction

In the Marvel superhero comics, the time keepers are unfriendly, greenish looking guys with insect faces, which

"were born at the end of time, entrusted with the safety of Time by He Who Remains. The Time Keepers were meant to watch over the space time continuum just outside of Limbo, and make sure the universe thrived." [1]

On planet Earth, the representative of the time keepers is Stephen W. Hawking, who announced in his chronology protection conjecture that

"It seems that there is a chronology protection agency which prevents the appearance of closed time-like curves and so makes the universe safe for historians" [2].

Hawking had good reasons for this statement: the ideas that exist in the literature for spacetimes which allow for closed timelike curves (CTCs), including wormholes, Kerr- or Kerr-Newman black holes and Tipler cylinders (for an overview see [3]), typically are found to suffer from one or more of the following obstacles:

– negative energy densities violating the so-called energy conditions may be required to warp spacetime into the desired geometry. Such negative energy seems to be unstable to small perturbations, at least in semi-classical calculations (for a recent discussion see [4]).
– quantum corrections to the stress-energy tensor seem to diverge close to the chronology horizon which separates spacetime regions allowing for CTCs from the ones in which they are forbidden, again at least semi-classically [2].
– nobody has ever spotted a wormhole or the region inside the black hole horizon, and Tipler cylinders have to be unreasonably large.

While these facts do not constitute a proof that the corresponding spacetimes or time travel in general are impossible – after all no complete theory of quantum gravity which could decide the issue exists – they make these possibilities unlikely in the view of most physicists.

Here we discuss the example of an extra-dimensional brane universe with CTCs, which alleviates at least some of these problems. Moreover, what is particular attractive: this scenario is in principle experimentally testable in neutrino experiments.

2 Asymmetrically warped brane universes and CTCs

In his SUSY'07 presentation one of us (HP) discussed the possibility, that the LSND [5] neutrino oscillation anomaly and the MiniBooNE null result [6] can be explained by oscillations into a sterile neutrino taking shortcuts in extra dimension [7]. This scenario also potentially fits [8] the anomalous resonance-like structure seen in the low-energy bins of the MiniBooNE data [6]. All simple scenarios of this type are causally stable. This includes scenarios where the shortcut is realized by asymmetrical warping of an extra dimension, described by a spacetime metric of the type [9, 10] [1]

$$ds^2 = dt^2 - \sum_i \alpha^2(u)\,(dx^i)^2 - du^2, \qquad (1)$$

with a warp factor $\alpha^2(u) < 1$ allowing for effectively superluminal propagation in the extra dimension. Here

[1] Such spacetimes have also been discussed as a solution to the cosmological horizon and dark energy problems.

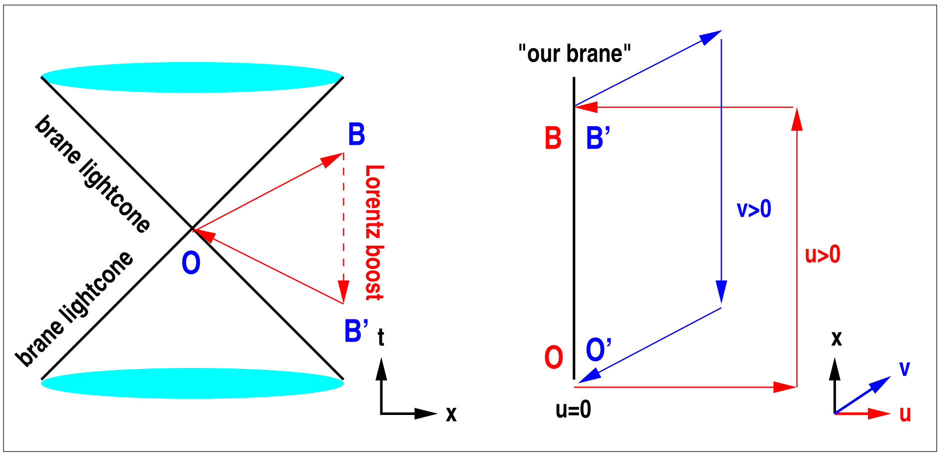

Fig. 1. Closed timelike curve in an asymmetrically warped universe: (i) A signal takes a space-like shortcut via a path of constant $u = u_1$ with $v = 0$ from point O to point B. (ii) A Lorentz boost transforms B into B' with negative time coordinate. (iii) A return shortcut at constant $v = v_1$ with $u = 0$ closes the timelike curve.

our brane is located at the $u = 0$ sub-manifold and the index i runs over the spatial dimensions $i = 1, 2, 3$ parallel to the brane. In this note, however, we keep the promise made as a concluding remark in the SUSY'07 talk: "if you are desperate to have a neutrino time machine I'll get you one."

In order to do so we consider two asymmetrically warped extra dimensions "u" and "v" with warp factors $\alpha(u)$ and $\eta(v)$, respectively [11]. We assume that the u- and v dimensions assume the simple form (1) in different Lorentz frames. Note that this is natural for any spacetime with two or more extra dimensions, as there is no preferred Lorentz frame from the viewpoint of the brane. In the following we construct this 6-dimensional metric explicitly. We denote the relative velocity between the two Lorentz frames, in which the u and v dimensions assume the simple form (1) as β_{uv}.

It is easy to show, then, that the full 6-dimensional metric assumes the form

$$\begin{aligned} ds^2 = \gamma^2 \big[&(1-\beta_{uv}^2\eta^2(v))dt^2 - 2\beta_{uv}\alpha(u)(\eta^2(v)-1)dxdt \\ &-\alpha^2(u)(\eta^2(v)-\beta_{uv}^2)dx^2\big] - du^2 - dv^2 . \end{aligned} \tag{2}$$

For $v = 0$ (2) reduces to (1), for $u = 0$ (2) reduces to (1) boosted by β_{uv}, and for $u = v = 0$, i.e. on the brane, to 4-dimensional Minkowski spacetime. By boosting the metric (2) with $\beta = -\beta_{uv}$ the v-dimension assumes the simple form (1) and the metric for the u-dimension becomes non-diagonal.

If the x coordinate would be periodic, the $u = 0$ slice of the metric (2) would allow for a simple mapping into the Tipler-van-Stockum spacetime, which is well known to accommodate CTCs [12]. However, in the case of a boosted asymmetrically warped extra dimension, the variable x is not periodic (unless our universe has the topology of a flat torus). It is thus required to construct an explicit return path to the spacetime point of origin, to close the CTC.

In the following we consider a signal following a particular path as given in Fig. 1. The signal leaves our brane at the spacetime point $O = (t = 0, x = 0, u = 0)$, and propagates on the hypersurface at u_1 for a travel time t with the limiting velocity $(\alpha(u_1))^{-1} \equiv \alpha_1^{-1}$. We will assume that $0 < \alpha_1 < 1$, so that the travel speed in the bulk is superluminal relative to travel speed on our metric. At later time t, the signal may reenter our brane. In the limit $u_1 \ll \alpha_1^{-1}t$, which is always fulfilled for sufficiently large t, the reentry point on our brane is $B^\mu \approx (t, x = \alpha_1^{-1}t, u = 0)$. Since the distance to the reentry point B^μ is space-like (i.e. outside the brane's lightcone), it may be transformed to negative time by a boost on our brane. The boosted point B'^μ has coordinates

$$x' = \gamma\, t\left(\alpha_1^{-1} - \beta\right), \qquad t' = \gamma\, t\left(1 - \beta\alpha_1^{-1}\right). \tag{3}$$

It is clear that for

$$0 < \alpha_1 < \beta < 1 \tag{4}$$

an observer in the boosted frame on our brane sees the signal arrive in time with $t' < 0$, i.e., before it was emitted. However, this result alone does not imply any conflict with causality. In particular, it does not necessarily imply that the spacetime is blessed with CTCs. To close the timelike curve, one has to show that the time t' during which the signal traveled backwards in time, is sufficiently large to allow a return from the spacetime point $B'^\mu = (t', (x = \alpha_1^{-1}t)', 0)$ to the spacetime point of origin, $O = O' = (0, 0, 0)$. The speed required to close the lightlike curve of the signal, as seen by the boosted observer on the brane, is

$$c'_{\rm req} = \frac{(x = \alpha_1^{-1}t)'}{|t'|} = \frac{1 - \beta\alpha_1}{\beta - \alpha_1}, \tag{5}$$

where the latter expression results from using (3). It is easy to show that the condition $0 < \alpha_1 < \beta < 1$ implies that $c'_{\rm req}$ itself is superluminal. Thus there is no return path on our brane which leads to a CTC.

We now benefit from the fact that the spacetime considered has two extra dimensions and that we have the possibility to use the v dimension to chose an alternative return path from the boosted point B' to O,

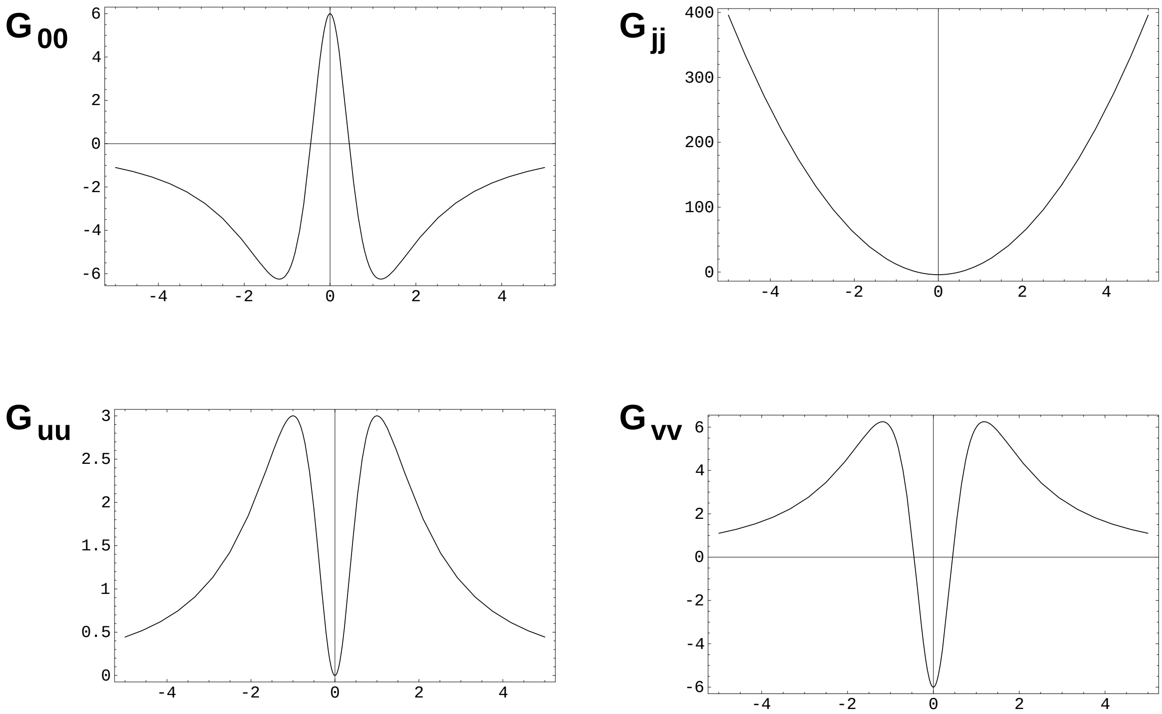

Fig. 2. Nonzero elements of the Einstein tensor $G^{\mu\nu}$ on the $v = 0$ slice, as a function of u, for warp factors $\alpha(u) = 1/(u^2 + c^2)$ and $\eta(v) = 1/(v^4 + c^2)$, with $c = 1$. While the null, weak and dominant energy conditions are violated in the bulk, they are conserved on the brane.

allowing for $|c'_{\rm bulk}| > |c'_{\rm req}|$. For simplicity we choose $\beta = -\beta_{uv}$ and the return path in the $u = 0$, $v > 0$ plane, where the $x - v$ slice of the metric takes the simple form (1). Now any suitably chosen warp factor $\eta(v)$ with $(\eta(v_1))^{-1} \geq c'_{\rm req}$ in combination with $\beta_{uv} > \alpha(u_1)$ generates a CTC.

3 Stress energy tensor, energy conditions and other pathologies

In view of the stability pathologies related to the requirement of negative energy densities for spacetimes with CTCs known in the literature, it is interesting to analyze, whether the stress-energy tensor

$$T_{\mu\nu} = \frac{1}{8\,\pi\,\mathrm{G_N}}\,G_{\mu\nu} \tag{6}$$

for the extra-dimensional metric (2) fulfills the energy conditions. The null, weak, strong and dominant energy conditions are defined as follows,

$$\mathrm{NEC}: \rho + p^j \geq 0 \quad \forall j; \tag{7}$$

$$\mathrm{WEC}: \rho \geq 0 \quad \text{and} \quad \forall j, \quad \rho + p^j \geq 0; \tag{8}$$

$$\mathrm{SEC}: \forall j, \quad \rho + p^j \geq 0 \quad \text{and} \quad \rho + \Sigma_j p^j \geq 0; \tag{9}$$

$$\mathrm{DEC}: \rho \geq 0 \quad \text{and} \quad \forall j, \quad p^j \in [\rho, -\rho]. \tag{10}$$

While it is in principle straightforward to calculate the Christoffel symbols from the metric (2), the general expressions are complicated. Thus we constrained ourselves to a numerical analysis of specific examples for the warp factors $\alpha(u)$ and $\eta(v)$.

It is not difficult to find a functional form for the warp factors α and η , which conserves some of the energy conditions, at least on the brane. One such example is given by $\alpha(u) = 1/(u^2 + c^2)$ and $\eta(v) = 1/(v^4 + c^2)$. For this case the elements of the Einstein tensor on the $v = 0$ slice are shown as a function of u in Fig. 2. The null, weak and dominant energy conditions are conserved on the brane, while the strong energy condition is violated on the brane (see Fig. 2).

This property of good energy conditions on the brane favors the scenarios discussed over most previous examples of spacetimes with CTCs. Moreover, one could argue that also instabilities due to quantum corrections are less likely to diverge in a spacetime of higher dimension. Finally, due to the ubiquity of the extra dimension this scenario has the particular advantage that the ambiguity due to the lack of a full quantum gravity treatment of the problem can be solved: it allows - at least in principle - for an experimental test.

4 Neutrino time machine

As mentioned before, a natural and well motivated candidate for a bulk fermion in string theory is a gauge singlet “sterile” neutrino. In many particle theories, these sterile neutrinos would mix with the Standard Model neutrinos, and will be generated by neutrino oscillations when neutrinos propagate in space and time. Sterile neutrinos thus provide an accessible probe for the causality properties of extra dimensions. An experiment testing for such properties of spacetime could effectively transform active neutrinos into sterile neutrinos by using the resonant conversion in an increasing matter density known as Mikheev-Smirnov-Wolfenstein (MSW) [13,14] effect. A neutrino beam of suitably chosen energy could be generated in a beam-dump experiment at one of the Earth's poles and pointed into the ground, traversing a pit with a slowly changing matter density profile. Once being converted inside the Earth's core, the sterile neutrinos will avail shortcuts in extra dimension to extremize the action of the propagation path, and thus effectively propagate a spacelike distance. On their way out of the Earth's interior the sterile neutrinos are reconverted into active flavors, which could be observed by a neutrino detector at the equator. If the neutrinos have advanced a spacelike distance, the Earth's spin at the equator will transform the signal into a moving reference frame and thus reverse the order of emission and detection. It could be possible to send the signal back to the point of origin (the pole lab) before it had been sent off.

5 Conclusions

In this paper we discussed the causality properties of a class of brane universes with two asymmetrically warped extra dimensions. Such scenarios are attractive in view of the LSND and MiniBooNE anomalies, as well as due to their potential to solve or alleviate the cosmological horizon and dark energy problems [9,10]. We have shown that rather generic examples of such spacetimes have closed timelike curves without violating the null, weak and dominant energy conditions on the brane. Moreover, neutrino oscillations may provide a unique possibility to test Hawking's chronology protection conjecture, should such spacetimes really be realized in Nature. A realistic description of this process, however, would require a quantum field theoretic treatment similar to the one preformed in [15]. We conclude that neutrinos might not only serve as a probe for physics beyond the Standard model and cosmology, but also for the understanding of the deepest foundations of causality and time.

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