Topics

- Metastable vacua at strong coupling;
- Non-Abelian flux tubes, confined monopoles
- One step beyond Seiberg’s duality
- Planar equivalence
- B theory (multileg/multiloop amplitudes)
- $\mathcal{N} = (2, 0)$ sigma model (Heterotic flux tubes of Edalati-Tong)
Intriligator, Seiberg, Shih '06 *Metastable vacua & stuff*

\[ \mathcal{N} = 1 \text{ SU}(N_c) \text{ SQCD with } N_c + N \text{ flavors } (N < N_c/2) \]

\[ \mathcal{W}_{\text{electric}} = m \, \tilde{Q} Q, \quad m \ll \Lambda \]

Magnetic dual: color SU(N), dual quarks h, meson field M

\[ \mathcal{W} = \mathcal{W}_{\text{tree}} + \mathcal{W}_{\text{anom}}, \]

\[ \mathcal{W}_{\text{tree}} = \tilde{h}_{A,k} \, h^{kB} \, M^A_B - \mu^2 M^A_A, \]

\[ \mu \propto \sqrt{m}. \]

\[ \mathcal{W}_{\text{anom}} \propto \text{Tr} W^2. \]

MS vacuum \[\rightarrow\] SUSY vacuum

M. Shifman
Lesson: small deformations of electric theory lead to drastic changes on the magnetic side of duality

Applications of the idea (incl. other than MS vacua)

E.g. gluino condensate, next slide
The mystery of 5/4: $\langle \lambda \lambda \rangle_{\text{weak c.}} = (5/4)^{1/2} \langle \lambda \lambda \rangle_{\text{str.c.}}$ \quad \text{NSVZ, 1985}

KS ‘97: Chirally symmetric vacuum $\langle \lambda \lambda \rangle = 0$ fixes strong coupling

Cachazo, Douglas, Seiberg, Witten, 2002 \rightarrow proof of no chirally symmetric SUSY vacuum

ISS metastable vacuum has $\langle \lambda \lambda \rangle = 0$!

It lives long at $m < \langle \Lambda \rangle$; if a minimum survives at $m \gtrsim \Lambda$, it may play a role in strong coupling calculation!

+ Douglas, Shelton, Torroba

Eto et al. 2006 \rightarrow Flux tubes in ISS metastable vacua $\text{SO}(N) + \text{SU}(N)$ with baryon $U(1)$ gauged
Seiberg & Witten ’94 — First demonstr. of dual Meissner effect in $\mathbb{N} = 2$;
Hanany, Strassler, Zaffaroni ’97 — SW=Abelian strings, “wrong” confinement;
1997-2003 — In search of Non-Abelian Flux Tubes (strings)!
Hanany & Tong; Auzzi et al. 2003 — Non-Abelian strings found in $\mathbb{N} = 2 \, U(2)$
SQCD with $N_f = N_c$

**Benchmark Model** : gauge $SU(N) \times U(1)$

**Vector multiplet** : $A_\mu, A^a_\mu, \lambda^{1,2}, \lambda^{1,2,a}, a, a^a$

**Quarks multiplets** : $q^{kA}, \tilde{q}_{Ak}, \psi^{kA}, \tilde{\psi}_{Ak}$, (A flavor)

+ Fayet – Iliopoulos term $\xi$
+ quark mass terms $m_A$

- If $\Delta m \neq 0$, magnetic flux in Cartan subalgebra of $SU(N) \to \mathbb{Z}_N$ strings;
- If $\Delta m = 0$ → orientational zero modes;
- CP(N-1) model on the string world sheet!!!
CP(2) model with 4 Q's

$\Delta m = 0$

$\Delta m \neq 0, \quad \Delta m \gg \Lambda$

$Z_2$ string junction

$B$, $B^3$, $B^3$
The 't Hooft–Polyakov monopole

\[ \xi = 0 \]
\[ \Delta m \neq 0 \]

Almost free monopole

\[ \xi \neq 0 \]
\[ |\Delta m| \gg \xi^{1/2} \]

Confined monopole, quasiclassical regime

\[ \Lambda_{CP(1)} \ll |\Delta m| \ll \xi^{1/2} \]

Confined monopole, highly quantum regime

\[ \Lambda_{CP(1)}^{-1} \]
\[ \Delta m \rightarrow 0 \]
Less supersymmetry: $N = 2 \rightarrow N = 1$

$\mathcal{N} = 1$ SQCD with the gauge group $U(N_c)$ and $N_c$ quark flavors

$$\mathcal{W} = QMQ + \sim A^2$$

$\rightarrow \infty$, no massless modes in the bulk! Non-abelian strings almost intact (1/2 BPS-ness is lost)

$\mathcal{N} = 1$ SQCD

$N_c + N$ flavors

keep $N_c$ flavors massless

$\langle Q\tilde{Q} \rangle \neq 0$

$N$ flavors are endowed with a mass term $m_q$

Extra $U(1)$

Deform & dualize

Dual quarks massive can be integr. out

Dual th. fully Higgsed
Eto et al. SO($N$) instead of U($N$)

$$\pi_1(SO(N)) = \mathbb{Z}_2$$

Spinor probe quarks are not screened!
The question of scale is still there ......

Secondary color or ...

\textbullet\textbullet\textbullet\textbullet
$\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu a} + \frac{i}{2} \bar{\lambda} D\lambda$

Orienti: $\lambda \rightarrow \text{Dirac } \Psi^{ij}$

Orienti-AS: at $N=3$ one-flavor QCD $\Rightarrow$ quark condensate,...

Kovtun, Ünsal, Yaffe: necessary and sufficient cond. $\Rightarrow$ nonbreaking of discrete symmetries:
$C$ for orienti and $Z_2$ for orbi

Sannino; Ünsal $\Rightarrow$
$T$ dependence of planar equivalence

In QCD-like theories? Proof $\leftarrow$ Vafa, Witten
Discrete symmetry nonbreaking = convergence of expansion in fermion loops in pure Yang-Mills ➔ ASV, ‘06

Witten: in pure YM vacuum is unique at $\theta=0$

Convergence = uniqueness of vacuum

Ploykov's criterion: $Z_N$ center in orienti at $N\to\infty$

$R_3 \times S_1$ compactification
$$\left\langle P \exp \left( \int_{S_1} iA_\mu dx^\mu \right) \right\rangle$$

Polyakov line $= 0 \rightarrow Z_N$ unbroken $\rightarrow$ confinement

Polyakov line $\neq 0 \rightarrow Z_N$ broken $\rightarrow$ deconfinement

$\Psi^{ij} \rightarrow Z_2$ at most at even $N$ ?????
At $N \to \infty$ we find $Z_N$ center in orienti

Two SU(N)'s, one condition

Six SU(N)'s, five conditions

Developments in field-theoretic D branes

- Sakai, Tong \(\rightarrow\) generic boojums
- Eto et al. \(\rightarrow\) Moduli matrix method for multiwalls & multistrings

M. Shifman
Multileg/multiloop amplitudes in pert. YM theories

'90s, Bern, Dixon, Kosower→string methods in SUSY theor.
2003, Witten→twistor variables in gluon amplitudes
2005, BCFW→on-shell amplitudes from recursion relations

B theory ???

\[ A(2 \text{ gluons} \to 2 \text{ gluons}) = A(2 \text{ gluons} \to 2 \text{ gluons})_{\text{tree}} \times \]
\[
\exp \left[ (\text{IR divergent}) + \frac{f(\lambda)}{8} \left( \ln \frac{s}{t} \right)^2 + \text{const.} \right]
\]
↑ cusp anom. dim.

'05 Bern, Dixon, Smirnov, weak coupl. conjecture

N=4 Yang-Mills

At \( \lambda \to \infty \) single class. string conf. with BC depending on momenta.
BDS confirmed!
Edalati-Tong heterotic flux tubes

Return to M model, N=2 broken to N=1

Bosonic part of string worldsheet model intact, CP(N-1)

Fermionic part? Supersymmetrization?

Four supercharges → N=(2,2) standard SUSY CP(N-1)

In fact, we have $C \times CP(N-1)$; supersymmetrization with two supercharges possible → chiral $N=(2,0)$ SUSY CP(N-1)!!!

Left-handed fermions interact differently from right-handed on the worldsheet
Conclusions

- Metastable vacua at strong coupling;
- Non-Abelian flux tubes, confined monopoles
- One step beyond Seiberg’s duality
- Planar equivalence
- B theory (multileg/multiloop amplitudes)
- $\mathcal{N} = (2, 0)$ sigma model (Heterotic flux tubes of Edalati-Tong)