

# Some Theoretical Developments in Supersymmetry

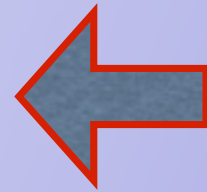
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## Topics

- Metastable vacua at strong coupling;
- Non-Abelian flux tubes, confined monopoles
- One step beyond Seiberg's duality
- Planar equivalence
- B theory (multileg/multiloop amplitudes)
- $\mathcal{N} = (2, 0)$  sigma model (Heterotic flux tubes of Edalati-Tong)



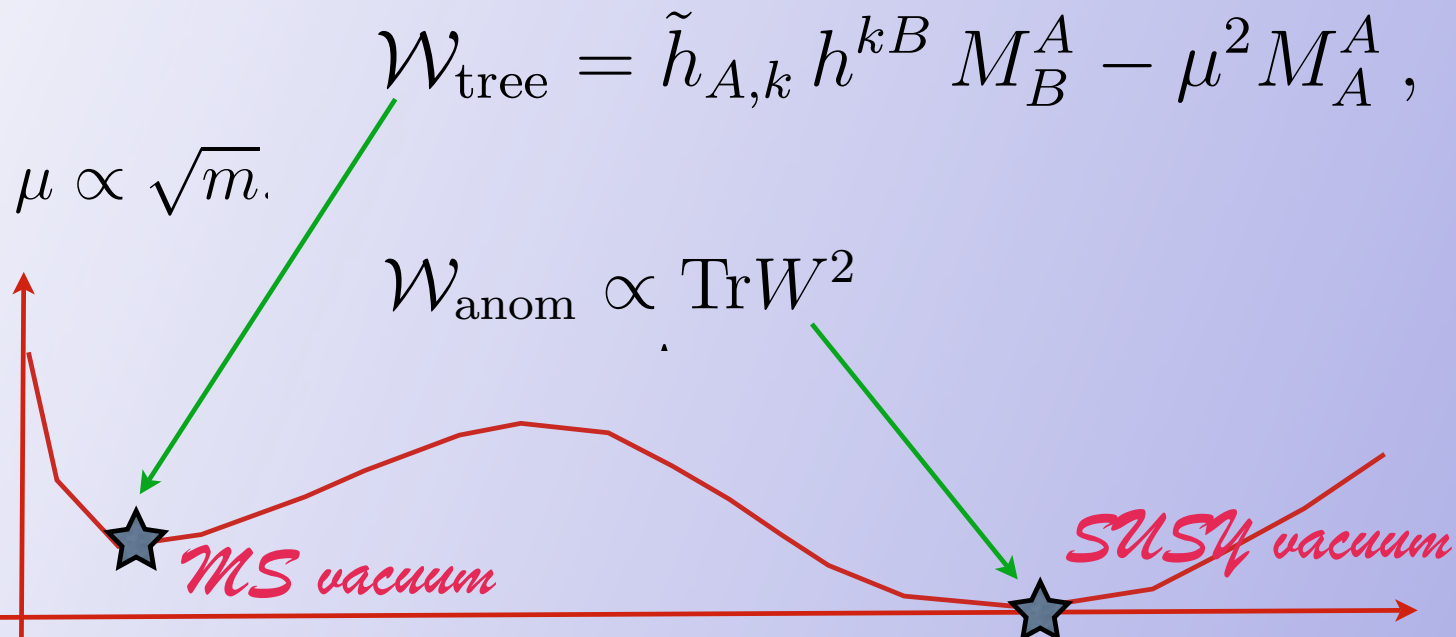
# Intriligator, Seiberg, Shih '06 *Metastable vacua & stuff*

$\mathcal{N} = 1$   $SU(N_c)$  SQCD with  $N_c + N$  flavors ( $N < N_c/2$ )

$$\mathcal{W}_{\text{electric}} = m \tilde{Q}Q, \quad m \ll$$

Magnetic dual: color  $SU(N)$ , dual quarks  $h$ , meson field  $M$

$$\mathcal{W} = \mathcal{W}_{\text{tree}} + \mathcal{W}_{\text{anom}},$$



☞ Lesson: small deformations of electric theory lead to drastic changes on the magnetic side of duality

☞ Applications of the idea (incl. other than MS vacua)

*e.g. gluino condensate, next slide*



The mystery of 5/4:  $\langle \lambda\lambda \rangle_{\text{weak c.}} = (5/4)^{1/2} \langle \lambda\lambda \rangle_{\text{str.c.}}$  ← NSVZ, 1985

?

KS '97: Chirally symmetric vacuum  $\langle \lambda\lambda \rangle = 0$  fixes strong coupling

Cachazo, Douglas, Seiberg, Witten, 2002 → proof of no chirally symmetric SUSY vacuum

ISS metastable vacuum has  $\langle \lambda\lambda \rangle = 0$  !

It lives long at  $m \ll \Lambda$ ; if a minimum survives at  $m \gtrsim \Lambda$ , it may play a role in strong coupling calculation!

+ Douglas, Shelton, Torroba

Eto et al. 2006 → Flux tubes in ISS metastable vacua  $SO(N)+SU(N)$  with baryon  $U(1)$  gauged

- ★ Seiberg & Witten '94 → First demonstr. of dual Meissner effect in  $N = 2$ ;
- ★ Hanany, Strassler, Zaffaroni '97 → SW=Abelian strings, "wrong" confinement;
- ★ 1997-2003 ← In search of Non-Abelian Flux Tubes (strings)!
- ★ Hanany & Tong; Auzzi et al. 2003 → Non-Abelian strings found in  $N = 2$   $U(2)$  SQCD with  $N_f = N_c$

Benchmark Model : gauge  $SU(N) \times U(1)$

Vector multiplet :  $A_\mu, A_\mu^a, \lambda^{1,2}, \lambda^{1,2,a}, a, a^a$

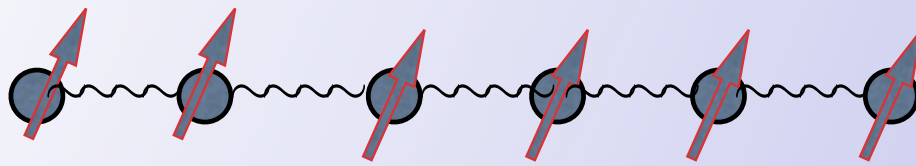
Quarks multiplets :  $q^{kA}, \tilde{q}_{Ak}, \psi^{kA}, \tilde{\psi}_{Ak}, (A \text{ flavor})$

+ Fayet – Iliopoulos term  $\xi$

+ quark mass terms  $m_A$



- ☛ If  $\Delta m \neq 0$ , magnetic flux in Cartan subalgebra of  $SU(N) \rightarrow Z_N$  strings;
- ☛ If  $\Delta m = 0 \rightarrow$  orientational zero modes;
- ✈  $CP(N-1)$  model on the string world sheet!!!

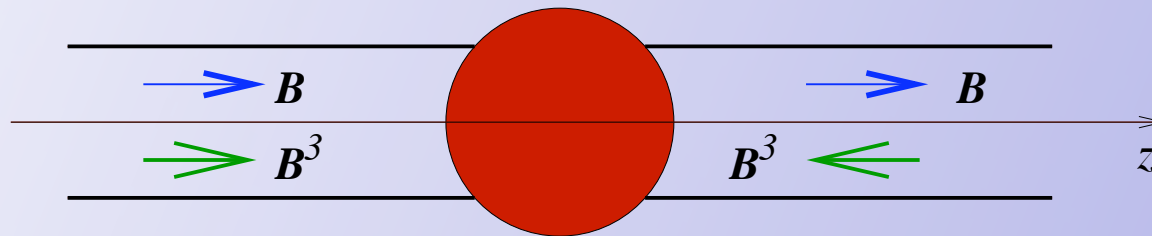


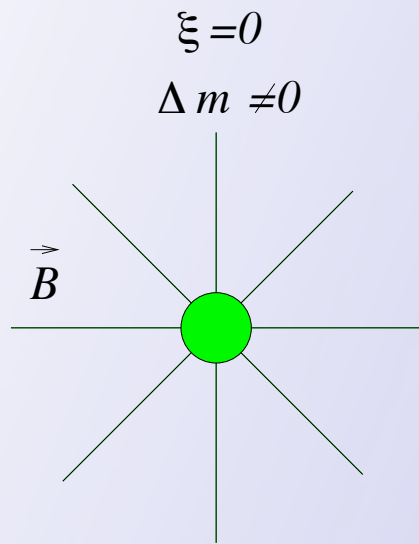
CP(2) model with  
4 Q's

$\Delta m = 0$

$\Delta m \neq 0, \Delta m \gg \Lambda$

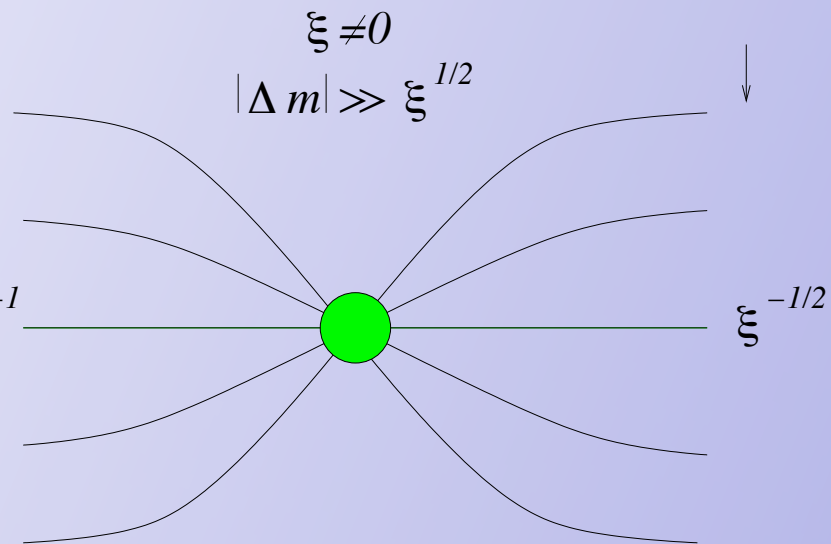
$Z_2$  string junction



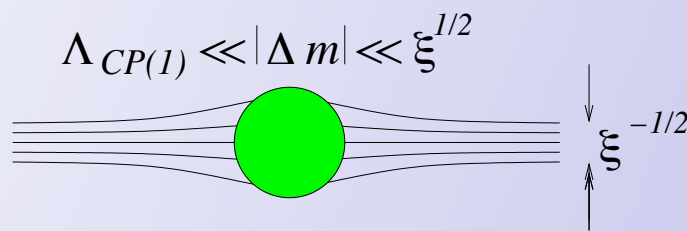


*The 't Hooft-Polyakov monopole*

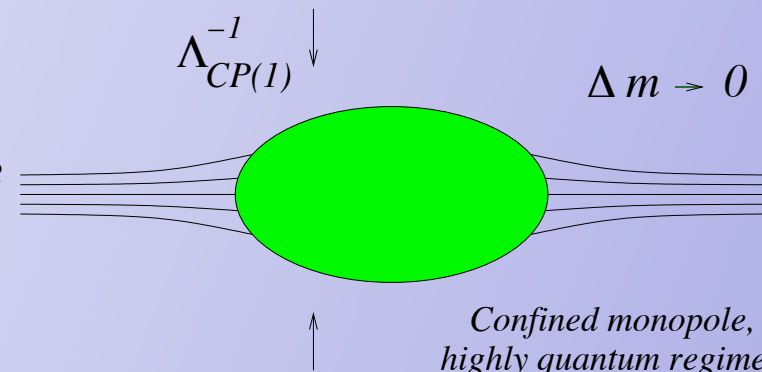
$|\Delta m|^{-1}$



*Almost free monopole*



*Confined monopole, quasiclassical regime*



*Confined monopole, highly quantum regime*

Less supersymmetry:  $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$

**M model**

GSY 2007

$\mathcal{N} = 1$  SQCD with the gauge group  $U(N_c)$  and  $N_c$  quark flavors

$$\mathcal{W} = QMQ + \tilde{\mu} \mathcal{A}^2$$

$\mu \rightarrow$  , no massless modes in the bulk! Non-abelian strings almost intact (1/2 BPS-ness is lost)

$\mathcal{N} = 1$  SQCD

$N_c + N$  flavors

Deform & dualize

Extra  $U(1)$

keep  $N_c$  flavors massless

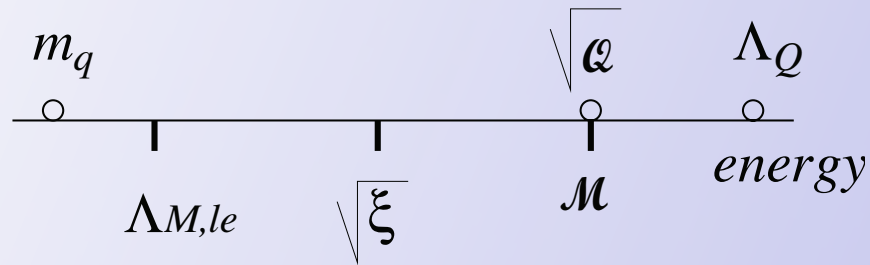
$$\langle Q\tilde{Q} \rangle \neq 0$$

$N$  flavors are endowed with a mass term  $m_q$

Dual quarks massive can be integr. out

Dual th. fully Higgsed





Secondary color or ...



Eto et al.  $SO(N)$  instead of  $U(N)$

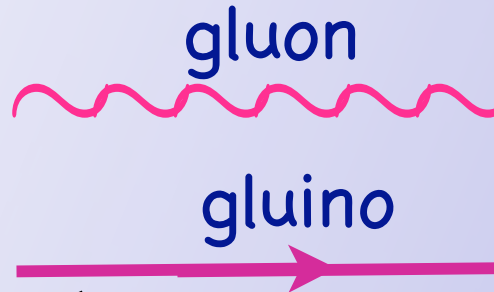
$$\pi_1(SO(N)) = Z_2$$

Spinor probe quarks are not screened!

The question of scale is still there .....

Planar Equivalence  
2007

$$\mathcal{L} = -\frac{1}{4g^2} G_{\mu\nu}^a G^{\mu\nu a} + \frac{i}{2} \bar{\lambda} \not{D} \lambda$$



supersymmetric  
gluodynamics

Orienti:  $\lambda \rightarrow$  Dirac  $\Psi^{ij}$

$Z_2$  Orbi Dirac bifundamental

$SU(2N) \rightarrow SU(N) \times SU(N)$

$g_D^2 = 2g_P^2$

Orienti-AS: at  $N=3$  one-flavor QCD  $\rightarrow$  quark condensate,...

Kovtun, Ünsal, Yaffe: necessary and sufficient  
cond.  $\Rightarrow$  nonbreaking of discrete symmetries:  
C for orienti and  $Z_2$  for orbi

Sannino; Ünsal  $\Rightarrow$   
T dependence of  
planar equivalence

C in QCD-like theories? P proof  $\leftarrow$  vafa, Witten

Discrete symmetry nonbreaking = convergence of expansion in fermion loops in pure Yang-Mills  $\rightarrow$  ASV, '06

Witten: in pure YM vacuum is unique at  $\theta=0$

$\rightarrow$  Convergence = uniqueness of vacuum

Polyakov's criterion:  $Z_N$  center in orienti at  $N \rightarrow \infty$

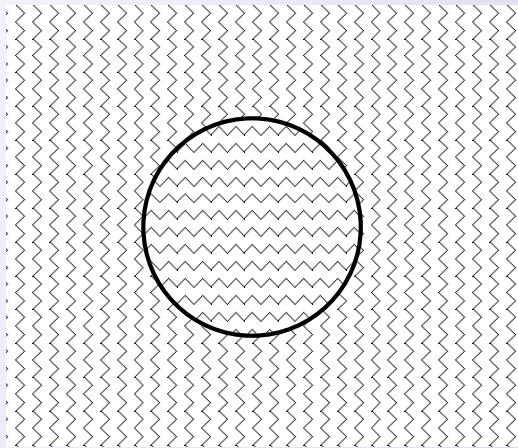
$R_3 \times S_1$  compactification  $\left\langle P \exp \left( \int_{S_1} iA_\mu dx^\mu \right) \right\rangle$

*Polyakov line = 0  $\rightarrow Z_N$  unbroken  $\rightarrow$  confinement*

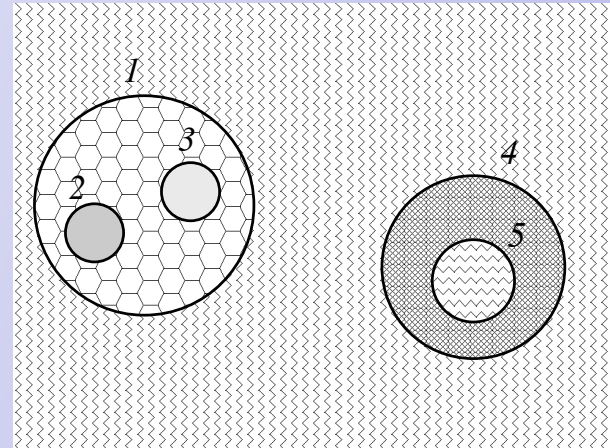
*Polyakov line  $\neq 0 \rightarrow Z_N$  broken  $\rightarrow$  deconfinement*

$\Psi^{ij} \rightarrow Z_2$  at most at even  $N$  ????

\*\*\* At  $N \rightarrow \infty$  we find  $Z_N$  center in orienti \*\*\*



Two SU(N)'s, one condition



Six SU(N)'s, five conditions

\*\*\* Developments in field-theoretic D branes \*\*\* DS '96

- ☞ Sakai, Tong  $\rightarrow$  generic boojums
- ☞ Eto et al.  $\rightarrow$  Moduli matrix method for multiwalls & multistrings



# \*Multileg/multiloop amplitudes in pert. YM theories\*

- ☞ '90s, Bern, Dixon, Kosower → string methods in SUSY theor.
- ☞ 2003, Witten → twistor variables in gluon amplitudes
- ☞ 2005, BCFW → on-shell amplitudes from recursion relations

## \*B theory ???\*

$$\mathcal{A}(2 \text{ gluons} \rightarrow 2 \text{ gluons}) = \mathcal{A}(2 \text{ gluons} \rightarrow 2 \text{ gluons})_{\text{tree}} \times$$

$$\exp \left[ (\text{IR divergent}) + \frac{f(\lambda)}{8} \left( \ln \frac{s}{t} \right)^2 + \text{const.} \right]$$

↑ cusp anom. dim.

← '05 Bern, Dixon, Smirnov, weak coupl. conjecture

N=4 Yang-Mills

- ☞ 2007, Alday & Maldacena, gauge-gravity duality, strong coupling. At  $\lambda \rightarrow \infty$  single class. string conf. with BC depending on momenta. BDS confirmed !

## \*\*\* Edalati-Tong heterotic flux tubes \*\*\*

☞ Return to M model,  $N=2$  broken to  $N=1$

☞ Bosonic part of string worldsheet model intact,  $CP(N-1)$

★ Fermionic part ? Supersymmetrization? ★

☞ Four supercharges  $\rightarrow N=(2,2)$  standard SUSY  $CP(N-1)$

✈ In fact, we have  $C \times CP(N-1)$ ; supersymmetrization with two supercharges possible  $\rightarrow$  chiral  $N=(2,0)$  SUSY  $CP(N-1)$ !!!  
Left-handed fermions interact differently from right-handed on the worldsheet

## Conclusions

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