Very Light Cosmological Scalar Fields from a Tiny Cosmological Constant

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New picture of the Universe

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Scalar fields in Cosmology

- The phenomenology of scalar fields in the standard model can be rather exotic, especially the Higgs sector is a gateway to hidden sectors.

- What about cosmology?

- Scalar fields are used to explain different phenomena (dark energy, inflation…)

- Let me have a different perspective and raise the following question: given what we know of cosmology (ΛCDM), what does it take for a scalar field to fluctuate today and hence to impact our universe today?

- The expansion of a scalar field in a expanding universe is given by

\[
\ddot{\phi} + 3H\dot{\phi} + m^2\phi + ... = 0
\]

- Deriving this equation is trivial: assume Robertson-Walker metric and use Einstein’s equations.

- Finding a solution is easy:

\[
\phi(t) = \text{Re}(c_1 \exp(w_1 t) + c_2 \exp(w_2 t))
\]

with

\[
w_{1/2} = -3/2H \pm \sqrt{9/4H^2 - m^2}
\]
• Thus oscillations at time $H$ are possible iff $m > 3/2H$.

• Note that if the mass is much bigger than $H$, the field has reached a minimum a long time ago and will not impact our present universe.

• However today $H \sim 10^{-33}\text{eV}$

• How do we get such a small scalar mass?

• A regular mass term $m^2 \phi \phi$ will not do the work!

• Let us study the operator

$$\alpha \int d^4x \sqrt{-g} R\phi\phi$$
• The action we are considering is given by:

\[ \int d^4x \sqrt{-g} \left( -\frac{1}{16\pi G} (R - 2\Lambda) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \alpha R \phi^2 \right) \]

• The corresponding field equations are:

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi G S_{\mu\nu} \]

\[ g^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \alpha R \phi = 0 \]

with

\[ S_{\mu\nu} = \left( \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial_\rho \phi \partial^\rho \phi - \alpha \phi^2 R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \alpha \phi^2 R - \alpha (g_{\mu\rho} g^{\alpha\beta} \phi^2_{;\alpha\beta} - \phi^2_{;\mu\nu}) \right) \]

where \( \alpha R \) plays the role of a mass term.
• It is useful to rewrite the field equation as

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \frac{G_{\text{eff}}}{G}\Lambda g_{\mu\nu} = -8\pi G_{\text{eff}} \left( \partial_\mu \phi \partial_\nu \phi - \frac{1}{2}g_{\mu\nu} \partial_\rho \phi \partial^\rho \phi - \alpha (g_{\mu\nu} g^{\alpha\beta} \phi_{;\alpha\beta} - \phi_{;\mu\nu}^2) \right) \]

• With

\[ G_{\text{eff}} = \frac{G}{1 - 8\pi G\alpha \phi^2} \]

• Newton’s constant is space-time dependent, this could easily lead to a time dependence of the couplings of the standard model.
• Using the contracted Einstein equation, we get
\[ g^{\mu\nu} \nabla_\mu \nabla_\nu \phi + 4\alpha \Lambda \phi - 8\pi G \alpha \phi (\partial_\mu \phi \partial^\mu \phi - \alpha \phi^2 R + 3\alpha \nabla_\mu \nabla^\mu \phi^2) = 0 \]

• The scalar field is now massive!
• Using \( \Lambda = 8\pi G \rho_{\text{vac}} \) and \( \rho_{\text{vac}} \sim (2.4 \times 10^{-3} \text{ eV})^4 \) we find:
\[ m = 4.7 \times 10^{-33} \text{ eV} \]

  where we assumed \( \alpha = 1 \) we thus find \( m \sim 3/2H \)
• This scalar is thus relevant in today’s universe!
• For the time change of the Newton constant we obtain:
\[
\frac{G_{\text{eff}}(t_0) - G_{\text{eff}}(0)}{G_{\text{eff}}(t_0)} = -8\pi G \alpha \frac{\Delta \phi^2}{1 - 8\pi G \alpha \phi^2}
\]
• Our action can be mapped to a Jordan-Brans-Dicke action:

\[
\int d^4x \sqrt{-g} \frac{1}{16\pi} \left( \Phi R + \omega g^{\mu\nu} \frac{\partial_\mu \Phi \partial_\nu \Phi}{\Phi} \right)
\]

• with a space-time dependent parameter

\[
\omega = \frac{(1 - 8\pi \phi^2 G \alpha)}{(32\pi \phi^2 G \alpha^2)}
\]

• If we assume that the scalar field oscillates slowly we can use the bound on the parameter of the JBD-theory ( \( \omega > 500 \) ) and obtain:

\[
\phi / \Lambda_{Planck} < 4 \times 10^{-3}
\]

• and thus the time change since the Big Bang of the Newton constant is bounded

\[
|\Delta G / G| < 4 \times 10^{-4}
\]
• A consequence can be a time variation of physical “constants”.
• If the controversial observation of Webb et al:  

\[ \frac{\Delta \alpha}{\alpha} = (-0.57 \pm 0.10) \times 10^{-5} \]

\[ z \approx 0.5 \ldots 3.5 \]

turned out to be correct, a natural way to describe it is a very light scalar field.
• We could then interpret the time variation as a renormalization effects (the details depend on the unification scheme).

\[ \frac{1}{\alpha_i} \frac{\dot{\alpha}_i}{\alpha_i} = \left[ \frac{1}{\alpha_u} \frac{\dot{\alpha}_u}{\alpha_u} - \frac{b_i}{2\pi} \frac{\dot{\Lambda}_G}{\Lambda_G} \right] \]

• This effect is expected in Kaluza-Klein models as shown by Marciano in 1984.
• Let us now look at the theory in the Einstein frame:

\[ g_{\mu\nu} = \cosh^2 \left( \frac{\phi \sqrt{\alpha}}{M_r} \right) \hat{g}_{\mu\nu} \]

\[ \phi = \sqrt{\frac{1}{\alpha} M_r \tanh \left( \frac{\phi \sqrt{\alpha}}{M_r} \right)} \]

where \( M_r = \sqrt{1/(8\pi G)} \)

• One gets:

\[ \int d^4x \sqrt{-\hat{g}} \left( \frac{1}{16\pi G} \left( \hat{R} - 2\Lambda \cosh^4 \left( \frac{\phi \sqrt{\alpha}}{M_r} \right) \right) + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) \]

• Note that physics is not identical in both frames.
• Let us now expand the cosh term:

\[
\int d^4x \sqrt{-\hat{g}} \left( \frac{1}{16\pi G} \left( \hat{R} - 2\Lambda \right) - 2\alpha \Lambda \hat{\phi}^2 - \frac{5}{24\pi G} \alpha^2 \Lambda \hat{\phi}^4 - \mathcal{O} \left( \frac{\hat{\phi}}{M_r} \right)^6 + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} \right)
\]

• Could we in principle have a Higgs effect? Let us assume for a second that the scalar field is gauged.

• If \( \Lambda > 0 \) (de Sitter) and \( \alpha > 0 \) : no Higgs effect

• If \( \Lambda < 0 \) (anti-de Sitter) and \( \alpha > 0 \) or \( \alpha < 0 \) : no Higgs effect

• If \( \Lambda > 0 \) (de Sitter) and \( \alpha < 0 \) : Higgs effect possible

• Note that we could have introduced a self-interaction term: \( \lambda \phi^4 \)

• In that case Higgs mechanism is possible both in anti-de Sitter and de Sitter cases.
So far we had to rely on fine-tuning to obtain a small scalar mass. However local conformal symmetry can be imposed in the scalar sector:

$$\int d^4 x \frac{1}{2} \sqrt{-g} \left( g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{6} R \phi^2 \right)$$

Self-interaction term is also possible $\sqrt{-g} \lambda_\phi \phi^4$ however it does not introduce interesting effects.

We are assuming that conformal invariance is broken in the gravity sector, this will induce a conformal symmetry breaking in the scalar sector. This is rather exotic physics. Let us thus have a model independent approach and assume only that we are living in an expanding universe.

The expansion of our scalar field in a Robertson-Walker universe is given by:

$$\ddot{\phi} + 3H \dot{\phi} + (1 - q) H^2 \phi = 0$$

with the deceleration parameter given by:

$$q(z) = \frac{3}{2} \frac{\sum_i \Omega_i^0 (1 + \omega_i)(1 + z)^3(1+\omega_i)}{\sum_i \Omega_i^0 (1 + z)^3(1+\omega_i)} - 1$$
• We thus obtain:

\[ m(z) = \sqrt{1 - q(z)} H(z) \]

• using the input \( \Omega_m^0 = 0.3 \) and \( \Omega_{\Lambda} = 0.7 \) i.e. \( q(0) = -0.55 \)

• we find:

\[ m(0) = 1.9 \times 10^{-33} \text{ eV} \]

• In other words: because the cosmological constant is of the same order of magnitude as today’s Hubble time, a scalar field coupled in a conformal manner to gravity would have a mass term of the order of the Hubble time and thus will be active in today’s universe.
How does the scalar field couple to SM?

- Coupling to Higgs field is dangerous!

\[ h^\dagger h\phi^2 \]

- It thus has to couple only gravitationally to the SM which is fine since it’s a gauge singlet.

- Local conformal invariance is required to maintain a light scalar field. What about conformal invariance in other sectors? Much progress on conformal invariance in the Higgs sector: Coleman-Weinberg does not work in the minimal SM but does work if a singlet is added (e.g. Meissner and Nicolai).
• Much progress has been done as well in conformal gravity (e.g. Mannheim):

\[-\alpha_g \int d^4 x (-g)^{1/2} C_{\lambda \mu \nu \kappa} \dot{C}^{\lambda \mu \nu \kappa}\]

• This action leads to a fourth order PDG. However ghost is not an issue if you look at the quantum theory from the PT symmetric point of view (Bender and Mannheim): spectrum is real.

• Do we have any hint of how scales are introduced in the SM? Maybe: typical thought experiments lead to two bounds:
  – QM:
    \[\Delta x \equiv \max [\Delta x(0), \Delta x(t)] \geq \sqrt{\frac{t}{2M}}\]
  – Gravitational bound:
    \[t > R > M\]

• Could nature be described by a theory which is scale invariant at tree level?
Conclusions

• We have considered a scalar field coupled in a non minimal way to the Ricci scalar.
• This mechanism naturally leads to a very light cosmological scalar field which is active today and could lead for example to a time variation of the Newton constant.
• The reason is that the cosmological constant is of the same order of magnitude as today’s Hubble time: we live at an interesting time.
• There has been some interesting progress in conformal gravity (see recent papers by Mannheim, Bender and Mannheim): developments in PT-symmetric quantum mechanics open the door to a viable alternative to Einstein’s gravity. If this mechanism is correct, one typically ends up with scalar fields couple in a non-minimal way to gravity.
• Thank you for your attention!
Backup
A minimal length from QM and GR

Claim: GR and QM imply that no operational procedure exists which can measure a distance less than the Planck length.

Assumptions:
- Hoop Conjecture (GR): if an amount of energy $E$ is confined to a ball of size $R$, where $R < E$, then that region will eventually evolve into a black hole.
- Quantum Mechanics: uncertainty relation.

Minimal Ball of uncertainty:
Consider a particle of Energy $E$ which is not already a Black hole. Its size $r$ must satisfy:

$$ r \gtrsim \max \left[ \frac{1}{E}, E \right] $$

where $1/E$ is the Compton wavelength and $E$ comes from the Hoop Conjecture. We find:

$$ r \sim l_P $$
Could an interferometer do better?

Our concrete model:
We assume that the position operator has discrete eigenvalues separated by a distance $l_p$ or smaller.
• At least one of the uncertainties $\Delta x(0)$ or $\Delta x(t)$ must be larger than:

$$\sqrt{\frac{t}{2M}}$$

• A measurement of the discreteness of $x(0)$ requires two position measurements, so it is limited by the greater of $\Delta x(0)$ or $\Delta x(t)$:

$$\Delta x \equiv \max [\Delta x(0), \Delta x(t)] \geq \sqrt{\frac{t}{2M}}$$

• This is the bound we obtain from Quantum Mechanics.
• To avoid gravitational collapse, the size $R$ of our measuring device must also grow such that $R > M$.
• However, by causality $R$ cannot exceed $t$.
• GR and causality imply: $t > R > M$

• Combined with the QM bound, they require $\Delta x > 1$ in Planck units or

$$\Delta x > l_P$$

• This derivation was not specific to an interferometer - the result is device independent: no device subject to quantum mechanics, gravity and causality can exclude the quantization of position on distances less than the Planck length.