RG Evolution of neutrino masses in type I and type II seesaw scenarios

Michael A. Schmidt

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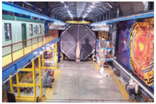
SUSY07

based on
S. Antusch, J. Kersten, M. Lindner, MS, M. Ratz [JHEP 0503:024]
M. Lindner, MS, A. Smirnov [JHEP 0507:048,2005]
MS, A. Smirnov [PRD74:113003,2006]
MS arXiv: 0705.3841 [hep-ph]

Why do we need the renormalization group?

Measurements by
low–energy experiments



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Predictions from
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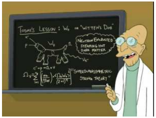
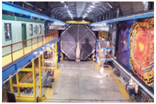
SO(10)





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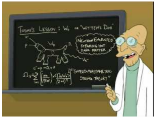

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↑

Renormalization Group Evolution

↓

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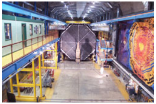

Outline

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Seesaw model

neutrino mass matrix

$$\begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix}^T \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix}$$

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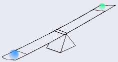

RH neutrinos decouple and give mass to light neutrinos via the seesaw mechanism

[Minkowski;Yanagida;Glashow;Gell-Mann,Ramond,Slansky;Mohapatra,Senjanovic]

$$(m_\nu)_{ij} = -\frac{v^2}{2}\frac{(Y_\nu)_{ki}(Y_\nu)_{kj}}{M_k}$$

Seesaw model

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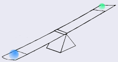
$$\begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix}^T \begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} \Rightarrow m_\nu = m_L - \frac{v^2}{2} Y_\nu^T M^{-1} Y_\nu$$



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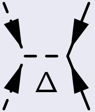
Higgs triplet $\Delta \sim (\mathbf{3}, \mathbf{1})$ [Magg, Wetterich;Lazaridis, Shafi, Wetterich; Mohapatra, Senjanovic]:



$$\Rightarrow \quad m_L = v_\Delta Y_\Delta = \frac{v^2}{2} \frac{\Lambda_6}{M_\Delta^2} Y_\Delta$$

General structure of β–function

1 loop β-function:

$P = P(Y_e,\ Y_\nu,\ Y_\Delta)$

$\alpha = \alpha(g_1,\ g_2,\ \mathrm{Tr}\, Y^\dagger Y,\ \Lambda_i)$

$$16\pi^2 \mu \frac{\mathrm{d}m_\nu}{\mathrm{d}\mu} = 16\pi^2 \beta_{m_\nu} = m_\nu P + P^T m_\nu + \alpha m_\nu$$

P.H. Chankowski, Z. Pluciennik (1993)

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	SM			MSSM		
m_ν	C_e	C_ν	C_Δ	C_e	C_ν	C_Δ
$-\frac{v^2}{4}\kappa$	$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	1	1	$\frac{3}{2}$
$-\frac{v^2}{2} Y_\nu^T M^{-1} Y_\nu$	$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	1	1	$\frac{3}{2}$
$\frac{v^2}{2}\frac{\Lambda_6}{M_\Delta^2} Y_\Delta$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	1	1	$\frac{3}{2}$

MNS mixing matrix

$$U = \mathrm{diag}(e^{i\delta_e}, e^{i\delta_\mu}, e^{i\delta_\tau}) \cdot V \cdot \mathrm{diag}(e^{-i\varphi_1/2}, e^{-i\varphi_2/2}, 1)$$

where ($s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$)

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\, e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}\, e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}\, e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}\, e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}\, e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Experimental data

T. Schwetz [Phys.Scripta T127:1-5,2006]

Parameter	Best-fit	Allowed range (3σ)
$\sin^2\theta_{12}$ [°]	0.30	0.24 . . . 0.40
$\sin^2\theta_{23}$ [°]	0.50	0.34 . . . 0.68
$\sin^2\theta_{13}$ [°]	0.000	$\leq$ 0.041
Δm^2_{21} [10^{-5} eV^2]	7.9	7.1 . . . 8.9
$\lvert\Delta m^2_{31}\rvert$ [10^{-3} eV^2]	2.5	1.9 . . . 3.2

Effective field theory

$$P = C_e Y_e^\dagger Y_e$$

$$C_e^{\mathrm{SM}} = -\frac{3}{2},\ C_e^{\mathrm{MSSM}} = 1$$

S. Antusch, J. Kersten, M. Lindner, M. Ratz [NPB674:401-433,2003]

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General structure (also applicable for phases):

Renormalization scale

$$\mu \frac{\mathrm{d}\theta_{ij}}{\mathrm{d}\mu} \propto \frac{f(m_l, \delta, \varphi_1, \varphi_2)}{m_j^2 - m_i^2} \times F^{(ij)}(y_k, \theta_{lm})$$

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- The more degenerate the spectrum is, the larger is the running.
- Phases can damp the running.

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- $F^{(ij)} \propto$ Yukawa coupling $\Rightarrow \tan\beta$ dependence
- $F^{(ij)} = 0$ for zero mixing (fixed point)

S. Antusch, J. Kersten, M. Lindner, M. Ratz [NPB674:401-433,2003]

Outline

Standard seesaw: full theory

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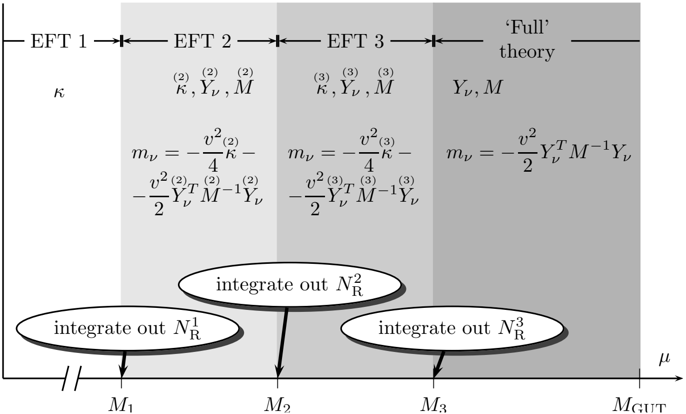
- cancellations possible
- off–diagonal terms $\to$ mixing can be generated
- GUT: Y_ν strongly hierarchical $\to$ P_{33} dominates

Thresholds



EFT 1
EFT 2
EFT 3
'Full' theory
κ
$\overset{(2)}{\kappa}, \overset{(2)}{Y_\nu}, \overset{(2)}{M}$
$\overset{(3)}{\kappa}, \overset{(3)}{Y_\nu}, \overset{(3)}{M}$
Y_ν, M
$m_\nu = -\frac{v^2}{4}\overset{(2)}{\kappa} - \frac{v^2}{2}\overset{(2)}{Y_\nu^T}\overset{(2)}{M}{}^{-1}\overset{(2)}{Y_\nu}$
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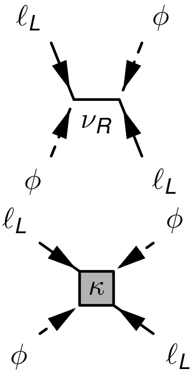
$m_\nu = -\frac{v^2}{2} Y_\nu^T M^{-1} Y_\nu$
integrate out N_R^2
integrate out N_R^1
integrate out N_R^3
μ
M_1
M_2
M_3
M_{GUT}



ℓ_L
ϕ
ν_R
ϕ
ℓ_L
ℓ_L
ϕ
κ
ϕ
ℓ_L

Between thresholds

- 2 contributions to m_ν:
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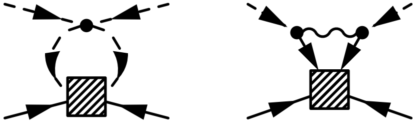

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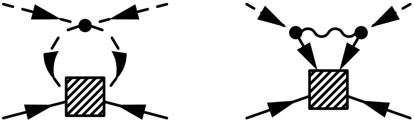

- rescaling of right–handed neutrino masses

M. Lindner, MS, A. Smirnov [JHEP 0507:048,2005]

$$m_\nu = Z_{\mathrm{ext}}^T Y_\nu^T X M^{-1} Y_\nu Z_{\mathrm{ext}}$$

additional vertex corrections

Quark–Lepton Complementarity

QLC-1: $U_e = V_{\text{CKM}} \qquad U_\nu = U_{\text{bimax}}$ H. Minakata, A. Smirnov [PRD70:073009,2004]

$\to \theta_{12} = 35.4^\circ \pm 0.3^\circ,\ \sin^2\theta_{13} = 0.026 \pm 0.008,\ 0.5 - \sin^2\theta_{23} = 0.025 \pm 0.039$

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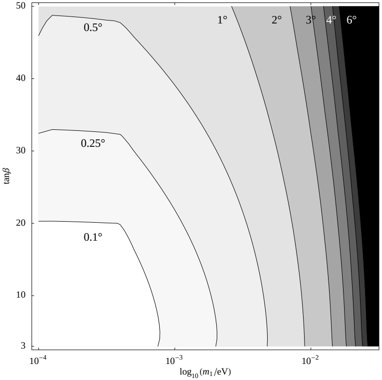

MSSM

MS, A. Smirnov [PRD74:113003,2006]

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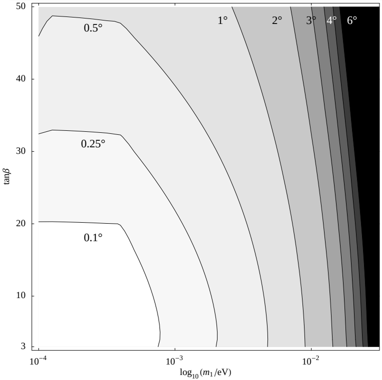

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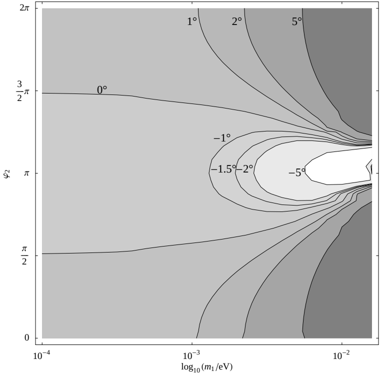

SM

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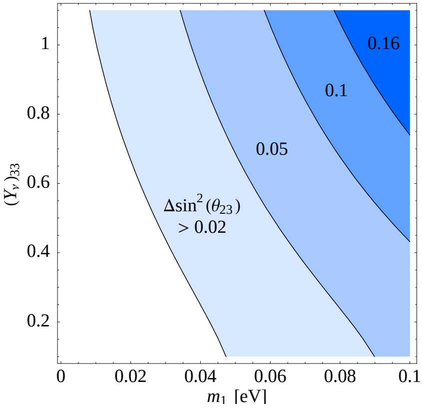


- $|0.5 - \sin^2\theta_{23}| \leq 0.16$

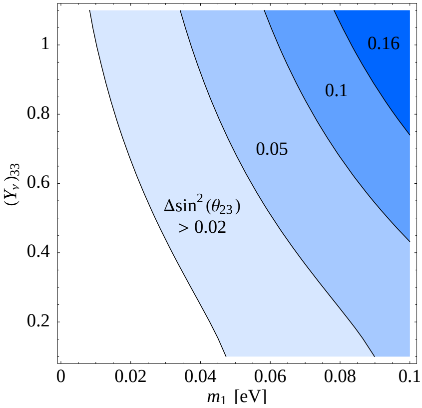
  T. Schwetz [Phys.Scripta T127:1-5,2006]

$\tan\beta = 20$, $\delta = \varphi_1 = \varphi_2 = 0$, analytic estimate

Current	Beams	T2K+NuMI	JPARC-HK	NuFact-II
0.16	0.1	0.050	0.020	0.055

P. Huber, M. Lindner, M. Rolinec, T. Schwetz, W. Winter [PRD70:073014,2004]

θ_{23}



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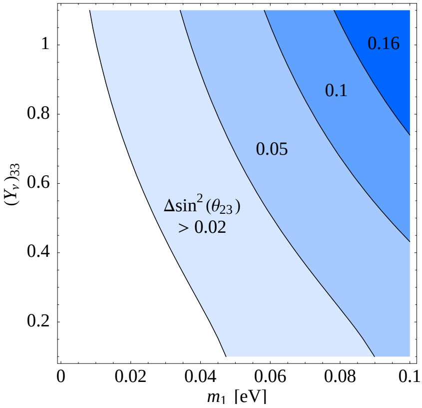

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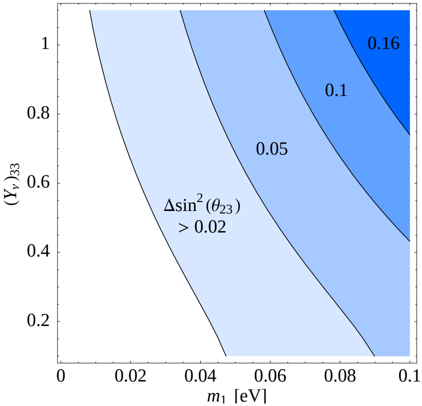

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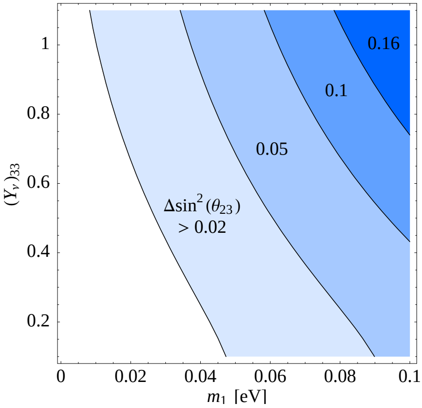

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- small deviations from maximal mixing
- running above see–saw scales
- suppression by phases possible
- RG effect comparable to precision of upcoming experiments

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0.16	0.1	0.050	0.020	0.055

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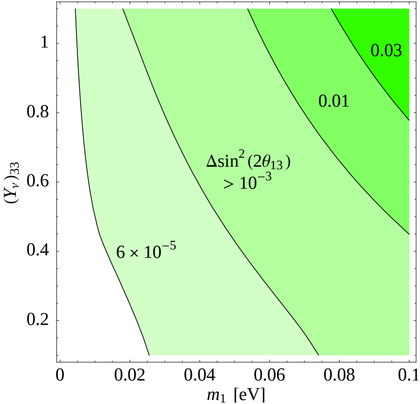
θ_{13}



- $\sin^2 2\theta_{13} \leq 0.16$

  T. Schwetz [Phys.Scripta T127:1-5,2006]
- $\theta_{13} = 0$ at GUT scale only preserved if $m_3 = 0$
- suppression by phases possible

$\tan\beta = 20$, $\delta = \varphi_1 = 0$, $\varphi_2 = \pi$, analytic estimate

Current	Beams	D-CHOOZ	T2K+NuMI	JPARC-HK	NuFact-II
0.16	0.061	0.032	0.023	10^{-3}	6×10^{-5}

P. Huber, M. Lindner, M. Rolinec, T. Schwetz, W. Winter [PRD70:073014,2004]

Outline

Only Higgs triplet

$$m_\nu = v_\Delta Y_\Delta$$

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convenient basis: $Y_\Delta = \mathrm{diag}(y_1, y_2, y_3)$ diagonal

$$16\pi^2 \dot{y}_i = 2C_\Delta y_i^3 + \alpha y_i$$
$$16\pi^2 \left(\dot{Y}_e\right)_{ij} = (Y_e)_{ij} D_\Delta y_j^2 + \alpha_e (Y_e)_{ij}$$

Chao, Zhang [PRD75:033003,2007]

General structure of RG equations I

$$16\pi^2\dot{\theta}_{12} = -\frac{D_\Delta}{2}\frac{\Delta m^2_{21}}{v^2_\Delta}\sin 2\theta_{12}$$

$$16\pi^2\dot{\theta}_{13} = -\frac{D_\Delta}{2}\frac{m_3^2-\left(m_2^2 s_{12}^2+m_1^2 c_{12}^2\right)}{v_\Delta^2}\sin 2\theta_{13}$$

$$16\pi^2\dot{\theta}_{23} = -\frac{D_\Delta}{2}\left[\frac{m_3^2-\left(m_2^2 c_{12}^2+m_1^2 s_{12}^2\right)}{v_\Delta^2}\sin 2\theta_{23}\right.$$
$$\left.+\frac{\Delta m^2_{21}}{v^2_\Delta}\cos\delta\sin 2\theta_{12}\, c_{23}^2\,\theta_{13}\right]+\mathcal{O}(\theta_{13}^2)$$

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$$16\pi^2\dot{\theta}_{23} = -\frac{D_\Delta}{2}\left[\frac{m_3^2-\left(m_2^2 c_{12}^2+m_1^2 s_{12}^2\right)}{v^2_\Delta}\sin 2\theta_{23} + \frac{\Delta m^2_{21}}{v^2_\Delta}\cos\delta\,\sin 2\theta_{12}\, c_{23}^2\,\theta_{13}\right] + \mathcal{O}(\theta_{13}^2)$$

$$16\pi^2\dot{\theta}_{ij} \approx -\frac{D_\Delta}{2}\frac{\Delta m^2_{ji}}{v^2_\Delta}\sin 2\theta_{ij}$$

General structure of RG equations II

$$16\pi^2\dot{\theta}_{12} = -\frac{1}{2}\left[D_\Delta \frac{\Delta m^2_{21}}{v^2_\Delta} + C_e y^2_\tau \frac{(m_2+m_1)^2}{\Delta m^2_{21}}\sin\theta_{23}\right]\sin 2\theta_{12}$$

$$16\pi^2\dot{\theta}_{13} = -\frac{C_e}{2} y^2_\tau \frac{(m_2-m_1)m_3}{(m_3-m_1)(m_3-m_2)}\cos\delta \sin 2\theta_{12} \sin 2\theta_{23}$$

$$16\pi^2\dot{\theta}_{23} = -\frac{1}{2}\left[D_\Delta\left(\frac{m_3^2}{v_\Delta^2} - \frac{m_1^2}{v_\Delta^2} s^2_{12} - \frac{m_2^2}{v_\Delta^2} c^2_{12}\right) + C_e y^2_\tau \frac{m_3^2 - m_1 m_2 + (m_2-m_1)\, m_3 \cos 2\theta_{12}}{(m_3-m_2)\,(m_3-m_1)}\right]\sin 2\theta_{23}$$

$$16\pi^2\dot{\delta} = \frac{C_e}{2}\frac{(m_2-m_1)m_3}{(m_3-m_1)(m_3-m_2)} y^2_\tau \sin\delta \sin 2\theta_{12} \sin 2\theta_{23}\ \theta_{13}^{-1}$$

General structure of RG equations III

- $F^{(ij)} \propto$ Yukawa coupling $\Rightarrow \tan\beta$ dependence

- almost no running for $\theta_{ij} = 0$

$$\mu \frac{\mathrm{d}\theta_{ij}}{\mathrm{d}\mu} \propto \frac{f(m_l,\delta)}{m_j^2 - m_i^2} \times F^{(ij)}(y_k, \theta_{lm}) - \frac{D_\Delta}{2} \frac{\Delta m_{ji}^2}{v_\Delta^2} \times \sin 2\theta_{ij}$$

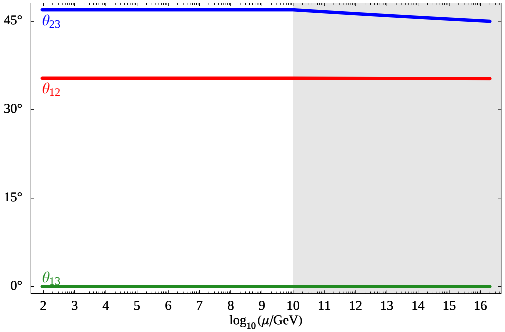
- The more degenerate the spectrum is, the larger is the running.

- The more hierarchical the spectrum is, the larger is the running.

- independence of Majorana phases
- zero mixing is fixed point

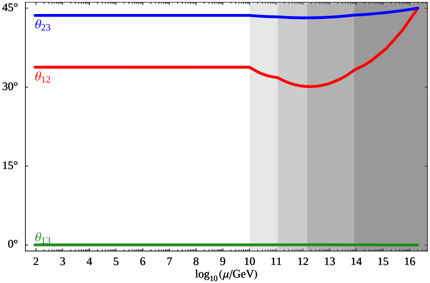
RG evolution of θ_{23}

$$16\pi^2\dot{\theta}_{23} \approx -\frac{D_\Delta}{2}\frac{m_3^2-\left(m_2^2 c_{12}^2+m_1^2 s_{12}^2\right)}{v_\Delta^2}\sin 2\theta_{23}$$



n.h., $m_1 = 0\,\mathrm{eV}$, $\langle\Delta\rangle \sim 0.15\,\mathrm{eV}$, $Y_\Delta \sim \mathcal{O}(0.1-1)$, $D_\Delta = \frac{3}{2}$

RG evolution in models with type I and type II seesaw contribution



n.h., $m_1 = 0.02\,\mathrm{eV}$, $Y_\Delta \sim \mathcal{O}(10^{-5})$, $Y_\nu = 0.37\,\mathrm{diag}(0.01, 0.1, 1)$

REAP/MixingParameterTools: `http://www.ph.tum.de/~rge`

Outline

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- RG effect proportional to mass squared difference
- No damping due to phases
- Sizable RG effect on θ_{23}