Robustness of quintessence models against quantum corrections

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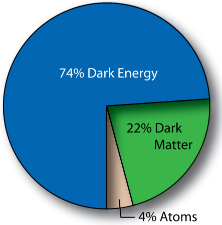
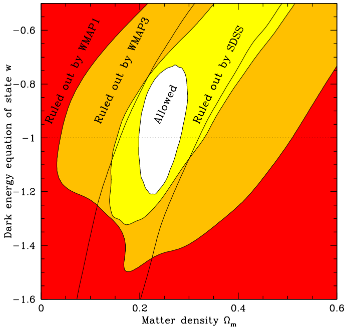
Introduction





Tegmark et.al. PRD74:123507,2006

Concordance within FRW

Geometry (CMB, SN1a, BAO)
Structure (LSS, lensing, clusters)
Homogeneous Dark Energy

$$\Omega_X = \Omega_{total} - \Omega_M \approx 0.75$$
$$\omega_X = -1 \pm 0.15$$

Explanations for DE within FRW

Cosmological Constant
Quintessence scalar field

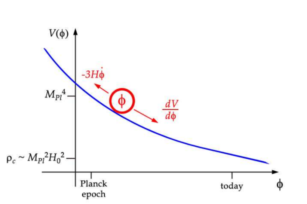
$$m_\phi \sim H_0 \sim 10^{-33}\text{eV}$$

Tracker solutions simplify coincidence problem

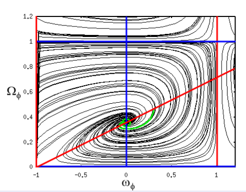
$$m_\phi(t) \approx H(t)$$

Quintessence mass scale

Tracking attractor



$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi)$$



Dynamical mass

- evolving field requires

$$m_\phi \sim H_0 \sim 10^{-33}\text{eV}$$

- dynamical mass

$$m_\phi^2(t) = V''(\phi(t))$$

- tracking attractor

$$V''(\phi^*(t)) = \frac{9}{2}\Gamma\,(1 - \omega_\phi^{*2}) \cdot H^2$$

- self-adjusting mass

$$m_\phi(t) \sim H(t)$$

- **no** fine-tuning classically

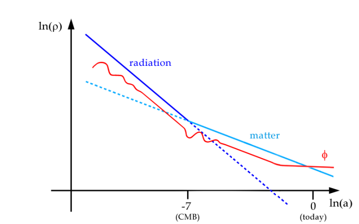
Quintessence with tracking solutions

Tracker potentials $\Gamma \equiv V''(\phi)\, V(\phi)/(V'(\phi))^2 \approx \text{const}$

Steinhardt,Wang,Zlatev PRD59:123504,1999; Wetterich NPB302:668,1988; Peebles,Ratra PRD37:3406,1988

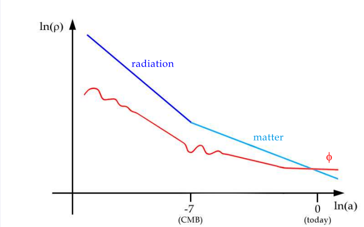
$$V(\phi) \;\propto\; \begin{cases} M^{4+\alpha}\phi^{-\alpha} & \text{for} \quad \Gamma > 1 \\ \exp\left(-\lambda \frac{\phi}{M_{pl}}\right) & \text{for} \quad \Gamma = 1 \\ \phi^n & \text{for} \quad \Gamma < 1 \qquad \text{(no cross-over)} \end{cases}$$

Exponential potential



early de $\Omega_\phi = \frac{3}{\lambda^2}$, cross-over when $\lambda(\phi) < \sqrt{3}$

Inverse power law potential



cross-over today if $M \sim 10^{-\frac{122}{4+\alpha}} M_{pl}$, SUGRA-like

Quantum fluctuations alter classical quintessence potential

How robust is $m_\phi \sim H$ against quantum corrections ?

Two sources for quantum corrections

	Uncoupled Models	Coupled Models
Qu. fluct. of	Quintessence field	SM particles
Theory is	Non-renormalizable	Renormalizable
Framework	Effective QFT	Quantum backreaction
Goal	Self-consistency	Limits on couplings

Study robustness of the **shape** of the potential ($V_{cl} \to V_{eff}$)
Old CC Problem remains untouched ($V_{eff} \to V_{eff} + \text{const}$)

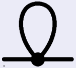
Uncoupled models

Framework: Effective QFT valid up to scale $\Lambda \lesssim M_{pl}$

One-loop analysis

Doran,Jaeckel PRD66:043519,2002; Brax,Martin PRD61:103502,2002

$$V''_{1-loop}(\phi) = V''_{cl}(\phi) + \quad \text{[diagram]} \quad + \dots$$



$$\approx V''_{cl}(\phi) + \frac{\Lambda^2}{32\pi^2} V^{(4)}_{cl}(\phi)$$

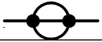
$$\rightarrow \begin{cases} m^2 + \frac{\lambda}{32\pi^2}\Lambda^2 & V_{cl} = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{c_6}{6!}\frac{1}{\Lambda^2}\phi^6 \\ V''_{cl}(\phi)\left(1 + \frac{\lambda^2}{32\pi^2}\frac{\Lambda^2}{M^2}\right) & V_{cl} = M^4 \exp\left(-\lambda\phi/M\right) \end{cases}$$

One-loop: Robust hierarchy for tracker potential

Goal: Systematic expansion scheme within effective QFT

Power Counting

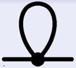
$$V(\phi) = M^4 \exp\left(-\lambda\phi/M\right)$$

	$\mathcal{V}=1$	$\mathcal{V}=2$	$\mathcal{V}=3$
$L=1$	 $\sim \Lambda^2 V^{(4)} \sim \frac{\Lambda^2}{M^2} V''$	 $\sim (V''')^2 \sim \frac{\Lambda^2}{M^2} \frac{V''}{\Lambda^2} V''$	–
$L=2$	 $\sim \Lambda^4 V^{(6)} \sim \frac{\Lambda^4}{M^4} V''$	 $\sim \frac{\Lambda^4}{M^4} \frac{V''}{\Lambda^2} V''$  $\sim \frac{\Lambda^2 V''}{M^4} \frac{V''}{\Lambda^2} V''$	$\dots$
$L=3$	 $\sim \Lambda^6 V^{(8)} \sim \frac{\Lambda^6}{M^6} V''$	$\vdots$	$\ddots$

- $\mathcal{V} \geq 2$ suppressed by $V''/\Lambda^2 \sim H^2/\Lambda^2 \lll 1$
- Curved- and time-dependent background
  $k^2 \sim \partial^2 \sim H^2 \sim V''$, $R \sim H^2 \sim V''$
- RG induced couplings $\partial^n V^{(m)} \sim \frac{\Lambda^n}{M^n} V^{(m-n)}$

Resummation of relevant loop corrections

Gap Equation in Hartree-Fock Approximation

$$M^2_{eff}(\phi) = V''_{cl}(\phi) +$$

$$+$$

$$+$$

$$+ \dots$$

$$= \exp\left[\frac{1}{2}\left(\int_q \frac{f(q/\Lambda)}{q^2 + M^2_{eff}}\right)\frac{d^2}{d\phi^2}\right] V''_{cl}(\phi)$$

$f(q/\Lambda) =$ form factor

$$\exp\left[c\,\frac{d^2}{d\phi^2}\right] = 1 + c\,\frac{d^2}{d\phi^2} + \frac{1}{2}c^2\frac{d^4}{d\phi^4} + \dots$$

Application to Liouville field theory in $d = 1+1$

Jackiw,D'Hoker PRD26:3517,1982; Goldstone; Polyakov PLB103:207,1981; . . .

$$V_{cl}(\phi) = V_0 \exp\left(-\lambda\phi\right)$$
$$V_{eff}(\phi) = V_R \exp\left(-\tilde{\lambda}\phi\right), \quad \tilde{\lambda}^{-1} = \lambda^{-1} + \lambda/(8\pi)$$

Resummed effective potential

Effective tracker potential in $d = 3 + 1$

UV scale $\int\frac{d^4q}{(2\pi)^4}\frac{f(q/\Lambda)}{q^2+V_{cl}''} = \pm\frac{\Lambda^2}{16\pi^2} + \ldots$

$$V_{eff}(\phi) = \exp\left[\frac{1}{2}\left(\pm\frac{\Lambda^2}{16\pi^2}\right)\frac{d^2}{d\phi^2}\right]V_{cl}(\phi), \quad M_{pl} \gtrsim \Lambda \gg H_{max}$$

Robustness of exponential potential

$$V_{eff}(\phi) \;=\; \exp\left[\frac{1}{2}\left(\pm\frac{\Lambda^2}{16\pi^2}\right)\frac{d^2}{d\phi^2}\right]M^4\exp\left(-\lambda\frac{\phi}{M}\right)$$

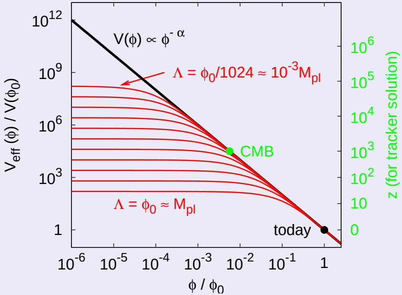
Form invariant $\Rightarrow$ tiny mass $V'' \sim H(t)^2$ robust

Curved background, NLO $V_{cl}''/\Lambda^2 \sim R/\Lambda^2 \sim H^2/\Lambda^2 \sim 10^{-120}$

Robustness of power law potential

Inverse power law potential

$$V_{cl}(\phi) = M^{4+\alpha}\phi^{-\alpha}$$



$\mathcal{V} = 1$ effective potential V_{eff} for $\alpha = 2$ and UV scale $\Lambda/M_{pl} = 1, 1/2, 1/4, \ldots, 1/1024$

Valid for $\phi/\phi_0 > 10^{-16}$

Effective potential

- 1-loop contribution

  Brax,Martin

  Requires $\phi \gg \Lambda \sim M_{pl}$

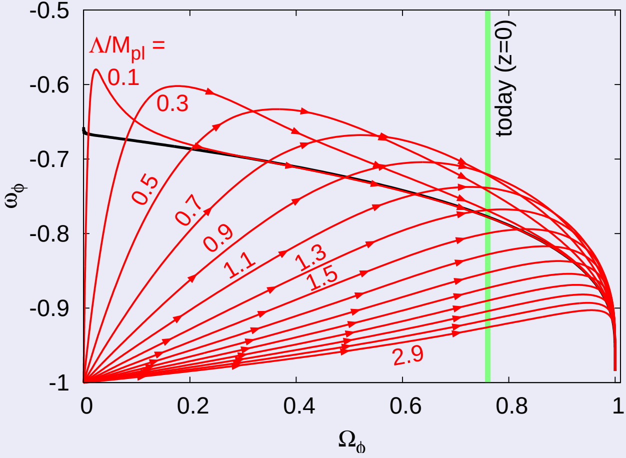
- Hartree-Fock

$$V_{eff} \rightarrow \begin{cases} V_{cl} & \phi \rightarrow \infty \\ \text{const} & \phi \rightarrow 0 \end{cases}$$

- Tracking attractor ?

Modified power law potential

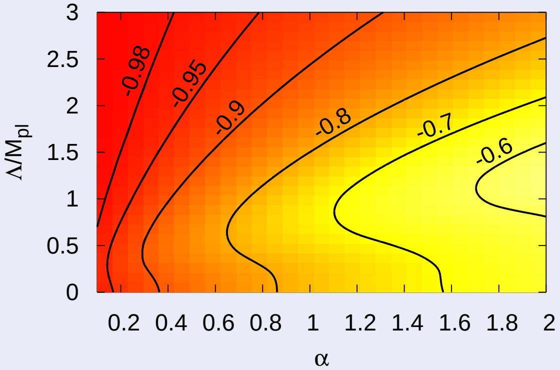
Evolution in $(\Omega_\phi, \omega_\phi)$-plane



$$V_{cl}(\phi) = M^5\phi^{-1},\ \Lambda/M_{pl} = 0.1, 0.3, \ldots, 2.9$$

Modified power law potential

Present equation of state $\omega_{de} = \omega_\phi(z = 0)$

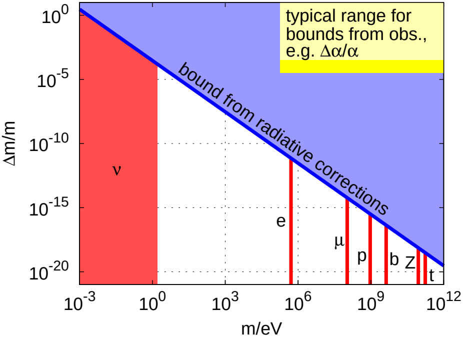


$$V_{cl}(\phi) = M^{4+\alpha}\phi^{-\alpha}$$

$$H_0 = 73\mathrm{km/sMpc},\ \Omega_{de} = 0.76$$

Bound for quasi-free species with independent mass variations since redshift $z \sim 2$

$$\frac{\Delta m_i}{m_i} \lesssim 3\ln 3 \left| \frac{d \ln m_i^2}{d \ln V''} \right| \ll 3 \ln 3 \left(\frac{1-\omega_{de}}{2} \frac{\Omega_{de}}{0.7} \right)^{\frac{1}{3}} \frac{1}{\sqrt[3]{g_i}} \left(\frac{1.3\mathrm{meV}}{m_i(\phi_0)} \right)^{\frac{4}{3}}$$



Bound on coupling

- Time variation
- Quantum backreaction
- Minimal response
- Relax renormalization conditions
  $\Rightarrow$ stronger bounds

Conclusions

Tiny mass $m_\phi \sim H$ required for dynamical DE

Classical EoM:

Self-adjusting mass $m_\phi \sim H$ for tracking solutions

Quantum EoM from effective action:

Uncoupled models

- Extension to Hartree-Fock approximation
- Cosmological solutions within effective QFT
- Hierarchy $m_\phi \sim H \lll \Lambda$ robust for tracker potentials

Coupled models

- Quantum backreaction
- $m_\phi \ll m_i(\phi)$ leads to bound on $dm_i(\phi)/d\phi$

Reference MG, PRD74:043009,2006

Coupled models

Interactions with SM suppressed by GUT/Planck/String scale

Carroll,PRL81:3067,1998; Chung,Everett,Riotto,PLB556:61,2003; Wetterich,JCAP0310:002,2003; ...

Time-dependent background

- time-varying couplings $\Delta\alpha_{em}/\alpha_{em} \sim 10^{-5}$
- mass-varying SM particles $\Delta\mu/\mu \sim 3 \cdot 10^{-5}$
- long-range force $\eta = \Delta a/a \lesssim 10^{-12}$
- MaVaN $m_\nu(\phi)$, DM-DE coupling $m_{dm}(\phi)$

Backreaction on scalar dynamics

- Classical backreaction and **quantum backreaction**
- Low energy effective theory known: renormalizable SM
- Response of total energy $E(\phi)$ to background $\phi(t)$

Quantum backreaction

Euler-Heisenberg effective action

$$E(\phi) = E_0 + E_0' \delta\phi + \frac{1}{2} E_0'' \delta\phi^2 + \frac{1}{3!} E_0''' \delta\phi^3 + \dots$$

One-loop flat and static limit, vacuum contribution

$$\begin{aligned} E(\phi) &= V(\phi) + \frac{1}{2} \sum_i \mathrm{Tr} \ln G_i^{-1}[\phi] \\ &= V(\phi) + \frac{1}{2} \sum_i (2s_i + 1)(-1)^{[2s_i]} \int \frac{d^4 q}{(2\pi)^4} \ln(q^2 + m_i(\phi)^2) \end{aligned}$$

- Counterterms $\delta V = C_4 + C_2^i m_i(\phi)^2 + C_0^i m_i(\phi)^4$
- Renormalization conditions for E_0, E_0', E_0'' (no oversubtraction)
- Conservative choice $E_0^{(k)} = V_0^{(k)}$ (minimal response)
- Demand $\Delta E / E \ll 1 \Rightarrow$ bound on $\Delta m_i / m_i$

Quantum corrections in curved background

- Non-minimal couplings between ϕ and R can be induced
- Corrections to kinetic term in time-dependent background

One-loop correction using HKE with $X \equiv \partial^2 V/\partial\phi^2 - R/6$

$$\Delta\Gamma[\phi]_{1L} = \int \frac{d^4x}{32\pi^2}\sqrt{-g}\left[-\frac{X^2}{2} - \frac{C}{120} + \frac{G}{360} - \frac{\Box R}{120} - \frac{\Box X}{6}\right] \ln\frac{X}{\mu^2} + \dots$$

$C = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$, $G = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$

- Power counting $V'' \sim R \sim \Box \sim H^2$, $C \sim G \sim H^4$
- Corrections are $\sim H^4$ **if** $\phi(t)$ close to classical solution

RGE running and improved potential $V_{LL}(\phi, R)$

$$\frac{\partial V_{LL}}{\partial t} = \frac{1}{64\pi^2}\left(\frac{\partial^2 V_{LL}}{\partial\phi^2} - \frac{R}{6}\right)^2$$