Susy QCD corrections in Higgs boson production via gluon fusion

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Outline

NLO SUSY-QCD corrections to $gg \rightarrow h$ (in progress...)

in collaboration with Stefan Beerli, Stefan Bucherer, Alejandro Daleo and Zoltan Kunszt

- introduction
- quark and scalar-quark two-loop amplitudes
- diagrams with gluinos and squarks (the method only - no results)
- summary

NNLO QCD corrections for $gg \rightarrow h \rightarrow W^+W^- \rightarrow l^+l^-\nu\nu$ (full calculation with all experimental cuts at the parton level)

in collaboration with Günther Dissertori and Fabian Stöckli
The gluon-gluon fusion channel in the Standard Model

- Higgs boson couples to gluons at the one-loop level, through (heavy) quarks
- Recall the large gluon density at $x \sim 10^{-4} - 10^{-2}, Q \sim 100\text{GeV}$
- Large Born cross-section at the LHC ($\sim \mathcal{O}(20)\text{pb}$)
- But not precise - scale uncertainty $\sim 40\%$
First order QCD radiative corrections

- a two-loop calculation, Spira, Djouadi, Graudenz, Zerwas
- large scale uncertainty $\sim 25\%$
- a very big correction $(\sim 70\%)$ Dawson; Spira et. al.
Gluon fusion: sensitive to BSM physics

- Anything that couples to gluons and the Higgs boson can circulate inside the production loop!
- There is a lot of room in the model building world to change the cross-section significantly.
- For example, an additional quark in a Left-Right symmetric Randall-Sundrum model could alter the Higgs cross-section from $-50\%$ to $+400\%$ without any conflict with electroweak precision data. Djouadi, Moreau
- The interpretation of the nature of the Higgs boson will also rely on the magnitude of the cross-section in various BSM models.
Two-loop corrections in BSM

- The LO cross-section is uncertain in the SM. It is the same (or worse) uncertain in BSM. Can we compute the NLO cross-section scanning a multitude of models?
- For the MSSM only, we need two loop three point diagrams with up to four different internal particles (masses)
Effective theory approach

- Heavy Quark Effective Theory is very successful for the SM Higgs boson. It will be successful in many other scenarios too. Still a formidable computation!

- NLO Wilson coefficient for heavy quarks+squarks+gluinos is known

  Harlander, Steinhauser

- It should be a very good approximation for a light MSSM Higgs boson

- ET hierarchies are not always satisfied. E.g. heavy Higgs boson? Other BSM than MSSM?
New techniques for analytic two-loop computations

- Automated reduction to master integrals
  Gehrmann, Remiddi; Laporta; CA, Lazopoulos

- Differential Equations
  Kotikov; Gehrmann, Remiddi

- Mellin-Barnes method
  Smirnov; Tausk
Two-loop diagrams with one quark or squark

Complete analytic calculation

- reduced to linear combinations of master integrals using Laporta’s algorithm
- complete set of master integrals computed with the method of differential equations (most were already known in the literature)
- analytic continuation of the master integrals above threshold

Contributions mediated by a heavy quark agree with the results of Spira et al. in the analytic form derived by Harlander and Kant

First results for the scalar quark contributions, simultaneous with Aglietti et al. and Mühlleitner and Spira
Master Integrals

All computed in terms of logs and polylogs or equivalent functions valid in all kinematic regions.
Squark contributions at two loops

\[
\frac{4x C_F}{(1-x)^2} \left\{ 5 + \frac{3x^2}{(1-x)(1+x)} H(0,x) - 3 H(1,x) + \frac{4x \zeta_2}{(1-x)^2} H(0,x) + \frac{8x(1+x^2)\zeta_2}{(1-x)^3(1+x)} H(0,0,x) + \frac{36x(1+x^2)\zeta_2^2}{5(1-x)^3(1+x)} + \frac{12x\zeta_3}{(1-x)^2} \right. \\
+ \frac{16x(1+x^2)\zeta_3}{(1-x)^3(1+x)} H(0,x) - \frac{3x(1+5x)}{(1-x)^2(1+x)} H(0,0,x) + \frac{6x}{(1-x)(1+x)} H(0,1,x) + \frac{6x}{(1-x)(1+x)} H(1,0,x) \\
+ \frac{16x}{(1-x)^2} H(0,-1,0,x) - \frac{x(-13+7x)}{(1-x)^3} H(0,0,0,x) + \frac{12x}{(1-x)^2} H(0,0,1,x) + \frac{8x}{(1-x)^2} H(0,1,0,x) \\
+ \frac{16x(1+x^2)}{(1-x)^3(1+x)} H(0,-1,0,0,x) + \frac{32x(1+x^2)}{(1-x)^3(1+x)} H(0,0,-1,0,x) \\
\left. + \frac{2x(1+x^2)}{(1-x)^3(1+x)} H(0,0,0,0,x) - \frac{8x(1+x^2)}{(1-x)^3(1+x)} H(0,0,1,0,x) + \frac{28x(1+x^2)}{(1-x)^3(1+x)} H(0,1,0,0,x) \right\} \\
\frac{4x C_A}{(1-x)^2} \left\{ 3 - \frac{32x\zeta_2^2}{5(1-x)^2} - \frac{16x\zeta_3}{(1-x)^2} - \frac{12x\zeta_3}{(1-x)^2} H(0,x) - \frac{24x\zeta_3}{(1-x)^2} H(1,x) - \frac{2x}{(1-x)^2} H(0,0,x) + \frac{2(1-7x)x}{(1-x)^3} H(0,0,0,x) \\
+ \frac{16x}{(1-x)^2} H(1,0,0,x) + \frac{4x\zeta_2}{(1-x)^2} H(0,0,x) - \frac{8x\zeta_2}{(1-x)^2} H(1,0,x) - \frac{8x}{(1-x)^2} H(0,0,-1,0,x) - \frac{2x}{(1-x)^2} H(0,0,0,0,x) \\
- \frac{16x}{(1-x)^2} H(1,0,-1,0,x) + \frac{8x}{(1-x)^2} H(1,0,0,0,x) \right\} 
\]
Many massive particle in the loops

- Analytic computations are not easy any more! Mass-thresholds and singularities in $D = 4$ dimensions.
- Could be attacked numerically; e.g. methods of Passarino and Uccirati or Spira et al.
- These methods operate on a case by case basis and may be limited in applications.
- Do we have an algorithm which can treat generic multi-loop integrals with infrared, ultraviolet, and all types of threshold singularities?
Automated numerical methods

Two general and fully automated methods to deal with multiloop integrals in a numerical way

- sector decomposition
  - extended to fully differential NNLO cross section calculations
    - Binoth, Heinrich
    - CA, Melnikov, Petriello
  - not possible to handle thresholds automatically

- numerical integration of Mellin-Barnes representations
  - works fine both in Euclidean and physical regions
  - CA, Daleo; Czakon
  - has problems in most loop integrals with internal masses
  - most probably cannot deal with thresholds

None of these two can do $gg \to h$ at two loops
Loop Singularities

\[ \sim \int dxdy \frac{1}{x^{1+\epsilon}} (M_t^2(x + y) - M_h^2 xy - i\delta)^{1+\epsilon} \]

- (Overlapping) singularities at the edges of the integration region. Regulated by \( \epsilon \)

\[ x = y = 0 \]

- Threshold singularities \( \sim i\pi \) terms

\[ M_t^2(x + y) = M_h^2 xy \]
Overlapping singularities can be factorized

- Singularity when two (or more) variables reach the same corner

\[ \lambda_1 \lambda_2 \frac{\lambda_1^\epsilon \lambda_2^\epsilon}{(\lambda_1 + \lambda_2)^2} f(\lambda_1, \lambda_2) \]

- Split into sectors

- Map each sector to \([0, 1]\)

\[ = \int_0^1 d\lambda_1 d\lambda_2 \frac{1}{\lambda_1^{1+\epsilon}} \frac{f(\lambda_1, \lambda_2 \lambda_1)}{(1 + \lambda_2)^2} + \int_0^1 d\lambda_1 d\lambda_2 \frac{1}{\lambda_2^{1+\epsilon}} \frac{f(\lambda_1 \lambda_2, \lambda_2)}{(1 + \lambda_1)^2} \]
Threshold singularities

○ Singular inside the integration region; not the edges

\[ I = \int_0^1 dx \frac{1}{x - a - i0}, \]

○ Regulator \( i0 \) is not good enough for a numerical evaluation.

○ Choose a different contour \( C : z = x - i\lambda x(1 - x) \)

\[ I = \int_C dz \frac{1}{z - a} = \int_0^1 dx \frac{\partial z}{\partial x} \frac{1}{z - a} \]

\[ = \int_0^1 dx \left[ 1 + i\lambda \left( 1 - \frac{x}{2} \right) \right] \frac{1}{x - a - i\lambda x(1 - x)} \]

○ Suitable for numerical integration!
General multi-loop integration method

- Merge the sector decomposition algorithm with an algorithm proposed by Nagy and Soper to deform the contour automatically for Feynman parameters.

- Very general method introduced by two groups:
  - Lazopoulos, Melnikov, Petriello to compute $pp \rightarrow ZZ Z$ at NLO
  - CA, Beerli, Daleo for the two-loop SUSY QCD amplitude, re-computing numerically all diagrams with (initially) only quarks or one squark.
SUSY QCD corrections to $gg \rightarrow h$

whole amplitude, including gluinos and squark-mixing, with similar precision

\[ = C \left( \frac{c_1}{\epsilon} + c_0 \right) \]
Status of the calculation

- Have computed all two-loop diagrams with quarks, squarks (and squark-mixing), gluinos.
- Renormalization + checked the infrared poles
- Performed a consistency check with our separate analytic calculation in the $m_h \to 0$ limit.
- We are checking against the computation of the Wilson coefficient of Harlander and Steinhauser
- Re-computed the real radiation $gg \to hg, \ldots$ amplitudes
- Finishing checks on the two-loop amplitude
- Writing the NLO Monte-Carlo program. Results soon (sorry!)
NNLO computation for SM $gg \rightarrow H \rightarrow WW \rightarrow l^+l^−\nu\nu$
Heavy top-quark approximation

A very good approximation for a light Higgs boson
Simplifies QCD corrections at NNLO ($3 \rightarrow 2$ loops).
NNLO effective theory Chetyrkin, Kniehl, Steinhauser
Higgs boson total cross-section through NNLO

- Slowly converging perturbative series: large NLO (70%) and smaller NNLO (30%) corrections,
- Scale variation (~15% at NNLO)
Effect of experimental cuts

- NNLO for fully differential cross-sections are not trivial
- $pp \rightarrow h, \; pp \rightarrow h \rightarrow \gamma\gamma$: First such computations for a hadron collider process
  CA, Melnikov, Petriello
- Extended the NNLO calculation to $pp \rightarrow h \rightarrow W W \rightarrow ll\nu\nu$
  CA, Dissertori, Stöckli
- For $m_h \sim 160 - 180\,\text{GeV}$ almost exclusive decay to Ws.
- No narrow peak reconstruction; large backgrounds from top and W pairs.
- Aggressive cuts (jet-veto, large missing energy, small lepton angle, restricted lepton $p_t$) to isolate a signal.
- What is the cross-section after cuts?
Accepted cross-section

- **Loose preselection cuts**

\[
\begin{array}{|c|c|c|c|}
\hline
\sigma (\text{fb}) & \text{LO} & \text{NLO} & \text{NNLO} \\
\hline
\mu = \frac{m_h}{2} & 71.63 \pm 0.07 & 126.95 \pm 0.13 & 140.73 \pm 0.45 \\
\hline
\end{array}
\]

\[M_{ll} < 80\text{GeV}, \ p_t^l > 20\text{GeV}, \ |\eta^l| < 2, \ \Delta \phi < 135^\circ, \ E_t^{\text{miss}} > 20\text{GeV}\]

- **Signal selection cuts**

\[
\begin{array}{|c|c|c|c|}
\hline
\sigma (\text{fb}) & \text{LO} & \text{NLO} & \text{NNLO} \\
\hline
\mu = \frac{m_h}{2} & 21.002 \pm 0.021 & 22.47 \pm 0.11 & 18.45 \pm 0.54 \\
\hline
\mu = 2m_h & 14.529 \pm 0.014 & 19.50 \pm 0.10 & 19.01 \pm 0.27 \\
\hline
\end{array}
\]

\[12\text{GeV} < M_{ll} < 40\text{GeV}, \ p_t^l > 25\text{GeV}, \ 30\text{GeV} < p_t^{l,\text{max}} < 55\text{GeV}, \ |\eta^l| < 2, \ \Delta \phi < 45^\circ, \ E_t^{\text{miss}} > 50\text{GeV}, \ \text{isolated, } p_t^{\text{jet}} < 25\text{GeV}\]
Conclusions

- Two-loop amplitude for $gg \rightarrow h$ in SUSY QCD:
  - Complete analytic and numerical calculation of single quark and squark loops
  - Last checks on the full amplitude including gluino and mixed squark diagrams
- NLO Monte-Carlo is also under completion
- A new numerical method for computing multi-loop divergent integrals with thresholds automatically
- Brief report on the NNLO cross-section for the SM $pp \rightarrow H \rightarrow WW \rightarrow ll\nu\nu$ at the LHC. Dramatic change of K-factors with cuts!