

# Supersymmetry Breaking and Radius Stabilization by Constant Boundary Superpotentials in Warped Space

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**Abstract.** Supersymmetry breaking and radius stabilization by constant superpotentials localized at boundaries is studied in a supersymmetric warped space model where a hypermultiplet, a compensator and a radion multiplet are taken into account. Soft mass induced by the anomaly mediation can be of the order of 100GeV and can be dominant compared to that mediated by bulk fields. A lighter physical mode composed of the radion and the moduli can have mass of the order of a TeV and the gravitino mass can be of the order of  $10^7$  GeV. The radius is stabilized by the presence of the constant boundary superpotentials. We also find that the mass splitting has an interesting dependence on the bulk mass parameter  $c$ .

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## 1 Introduction

Supersymmetry is a well-motivated extension to the standard model, which plays a crucial role in solving the gauge hierarchy problem [1]. Extra dimensions are also an alternative solution to the gauge hierarchy problem [2]. Considering both ingredients is natural in the context of the string theory and is often taken as the starting point in the phenomenological model of the brane world scenarios. In such a setup, we have to compactify extra dimensions and break supersymmetry to obtain realistic four-dimensional physics. One of the simple ways to realize it is the Scherk-Schwarz mechanism of supersymmetry breaking [3]. It is known that the Scherk-Schwarz supersymmetry breaking is equivalent to the supersymmetry breaking by a constant superpotential in flat space [4,5]. It is natural to ask whether this equivalence still holds in warped space. This issue has been discussed in the literature [6,7].

We present a brief summary of our study of supersymmetry-breaking effects and radius stabilization in a warped model with supersymmetry broken by constant boundary superpotentials [8,9]. Taking the hypermultiplet and including the compensating multiplet and the radion multiplet, we show that the radius is stabilized by the presence of the constant boundary superpotentials. It is also found that the mass spectrum depends on the bulk mass parameter in addition to the strength of the constant boundary superpotential. A lighter physical mode composed of the radion

and the moduli can have masses of the order of a TeV and that the gravitino mass can be of the order of  $10^7$  GeV. It is also shown that induced mass mediated by anomaly can be of the order of 100GeV and can be dominant compared to that mediated by bulk fields.

## 2 Model

We consider a five-dimensional supersymmetric model of a single hypermultiplet on the Randall-Sundrum background, whose metric is

$$ds^2 = e^{-2R\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2, \quad \sigma(y) \equiv k|y|, \quad (1)$$

where  $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ ,  $R$  is the radius of  $S^1$  of the orbifold  $S^1/Z_2$ ,  $k$  is the  $AdS_5$  curvature scale, and the angle of  $S^1$  is denoted by  $y$  ( $0 \leq y \leq \pi$ ). In terms of superfields for four manifest supersymmetry, our Lagrangian reads [4]

$$\begin{aligned} \mathcal{L}_5 = & \int d^4\theta \frac{1}{2} \varphi^\dagger \varphi (T + T^\dagger) e^{-(T+T^\dagger)\sigma} \\ & \times (\Phi^\dagger \Phi + \Phi^c \Phi^{c\dagger} - 6M_5^3) \\ & + \int d^2\theta \left[ \varphi^3 e^{-3T\sigma} \left\{ \Phi^c \left[ \partial_y - \left( \frac{3}{2} - c \right) T \sigma' \right] \Phi \right. \right. \\ & \left. \left. + W_b \right\} + \text{h.c.} \right], \end{aligned} \quad (2)$$

where the compensator chiral supermultiplet  $\varphi$  (of supergravity), and the radion chiral supermultiplet  $T$  are

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denoted as  $\varphi = 1 + \theta^2 F_\varphi$  and  $T = R + \theta^2 F_T$ , respectively, and the chiral supermultiplets representing the hypermultiplet is denoted as  $\Phi, \Phi^c$ . The  $Z_2$  parity is assigned to be even (odd) for  $\Phi(\Phi^c)$ . The derivative with respect to  $y$  is denoted by  $'$ , such as  $\sigma' \equiv d\sigma/dy$ . The five-dimensional Planck mass is denoted as  $M_5$ . Here we consider a model with a constant (field independent) superpotential localized at the fixed point  $y = 0$

$$W_b \equiv 2M_5^3 w_0 \delta(y), \quad (4)$$

where  $w_0$  is a dimensionless constant.

### 3 Radius stabilization

#### 3.1 Background solutions

The background solutions for the scalar components at the leading order of  $w_0$  are given by

$$\phi(y) = N_2 \exp \left[ \left( \frac{3}{2} - c \right) R \sigma \right], \quad (5)$$

$$\begin{aligned} \phi^c(y) = \hat{\epsilon}(y) & \left( \frac{\phi^\dagger \phi}{6M_5^3} - 1 \right)^{-1} \left( \frac{\phi^\dagger \phi}{6M_5^3} \right)^{\frac{5/2-c}{3-2c}} \\ & \times \left[ c_1 + c_2 \left( \frac{\phi^\dagger \phi}{6M_5^3} \right)^{-\frac{1-2c}{3-2c}} \left( \frac{\phi^\dagger \phi}{6M_5^3} + \frac{2}{1-2c} \right) \right] \end{aligned} \quad (6)$$

where  $c \neq 1/2, 3/2$ , and

$$\hat{\epsilon}(y) \equiv \begin{cases} +1, & 0 < y < \pi \\ -1, & -\pi < y < 0 \end{cases}. \quad (7)$$

The solution contains three complex integration constants:  $c_1, c_2$  are the coefficients of two independent solutions for  $\phi^c$ , and the overall complex constant  $N_2$  for the flat direction  $\phi$ . Two out of these three complex integration constants are determined by the boundary conditions. The single remaining constant (which we choose as  $N_2$ ) is determined through the minimization of the potential (stabilization).

#### 3.2 Potential

With the backgrounds (5) and (6), the potential is obtained as

$$\begin{aligned} V = & \frac{3M_5^3 k w_0^2}{2} \\ & \times \left\{ \frac{-2(1-2c)}{(1-2c)(e^{2Rk\pi} - 1)\hat{N} + 2(e^{(2c-1)Rk\pi} - 1)} \right. \\ & \times \hat{N}^{4-2c-\frac{1}{3-2c}} \\ & \left. + \frac{\hat{N}}{1-\hat{N}} \left( -4c^2 + 12c - 6 + \frac{3-2c}{3(1-\hat{N})} \right) \right\}. \end{aligned} \quad (8)$$

where  $\hat{N} \equiv |N_2|^2 / (6M_5^3)$ . We need to require the stationary condition for both modes  $R$  and  $N_2$ ,  $\partial V / \partial R = 0$  and  $\partial V / \partial \hat{N} = 0$ . From the stationary condition, we find that there is a unique nontrivial minimum with a finite value of the radius  $R$  and the normalization  $N_2$  for the flat direction  $\phi$  provided  $c < c_{\text{cr}}$  with

$$c_{\text{cr}} \equiv \frac{17 - \sqrt{109}}{12}. \quad (9)$$

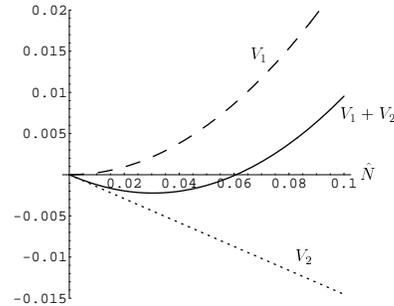
At the critical value of the mass parameter  $c_{\text{cr}}$ , the minimum occurs at infinite radius and vanishing normalization  $N_2$ ,  $\hat{N}(c_{\text{cr}}) = 0$ ,  $R(c_{\text{cr}}) = \infty$ . To examine the stabilization for  $c < c_{\text{cr}}$  more closely, we parameterize  $c = c_{\text{cr}} - \Delta c$  with a small  $\Delta c$ . After using the stationary condition solution  $\hat{N} = e^{-(3-2c)Rk\pi}$ , we find that the potential (8) for  $c = c_{\text{cr}} - \Delta c$  at the leading order of  $\Delta c$  and  $\hat{N}$  consists of two pieces

$$V \approx \frac{3M_5^3 k w_0^2}{2} (V_1 + V_2), \quad (10)$$

$$V_1 \equiv \frac{2(2c_{\text{cr}} - 1)}{3 - 2c_{\text{cr}}} \hat{N}^{\frac{4c_{\text{cr}} - 12c_{\text{cr}} + 10}{3 - 2c_{\text{cr}}}}, \quad (11)$$

$$V_2 \equiv -\hat{N} \left( -8c_{\text{cr}} + \frac{34}{3} \right) \Delta c. \quad (12)$$

The potential  $V$  and its pieces  $V_1, V_2$  are depicted as a function of  $\hat{N}$  in Fig.1. It is now obvious that a



**Fig. 1.** Potential for  $c = c_{\text{cr}} - \Delta c$

unique minimum occurs at finite values of  $\hat{N}$  provided  $\Delta c > 0$  ( $c < c_{\text{cr}}$ ) and that the minimum point approaches  $\hat{N} \rightarrow 0$  as  $\Delta c \rightarrow 0$  ( $c \rightarrow c_{\text{cr}}$ ). Actually the Fig.1 demonstrates only the stability along the direction of  $\hat{N}$ , after the other variable  $R$  is eliminated by the stationary condition. We have checked that this minimum point gives a true minimum of the potential  $V(R, \hat{N})$  as a function of two variables, establishing the stability in both directions. The stationary point at the leading order of  $\Delta c$  is obtained as

$$\begin{aligned} R \approx & \frac{-1}{[2(1-c_{\text{cr}})(3-2c_{\text{cr}}) + 1] k \pi} \\ & \times \ln \left[ \frac{(3-2c_{\text{cr}}) \left( \frac{17}{3} - 4c_{\text{cr}} \right)}{2(2c_{\text{cr}} - 1) \left( 2 - c_{\text{cr}} - \frac{1-c_{\text{cr}}}{3-2c_{\text{cr}}} \right)} \Delta c \right] \\ \approx & \frac{1}{10k} \left( \ln \frac{1}{\Delta c} - 3.4 \right), \end{aligned} \quad (13)$$

which means that the radius is stabilized with the size of  $R > 1/k$  for  $\Delta c < 10^{-6}$ .

At the stationary point the potential becomes

$$V \approx -10^{37}(kw_0)^2(\Delta c)^{1.2}. \quad (14)$$

We can show that the cosmological constant can be cancelled by an F term contribution and a D term contribution for supersymmetry breaking and that the contributions of these sectors to the soft mass and gravitino mass are small.

## 4 Mass spectrum

### 4.1 Soft mass by anomaly mediation

In a supersymmetric Randall-Sundrum model, anomaly-mediated scalar mass is given by  $\tilde{m}_{\text{AMSB}} \sim (g^2/16\pi^2) \cdot \langle F_\omega/\omega \rangle$  [10]. Here the superfield  $\omega$  is defined as a rescaled compensator multiplet  $\omega = \varphi e^{-T\sigma}$  and we denoted its lowest component also as  $\omega$ , and  $g$  is gauge coupling constant for visible sector fields. In our model, the anomaly-mediated scalar mass becomes

$$\tilde{m}_{\text{AMSB}} \sim \frac{g^2}{16\pi^2} (F_\varphi - F_T \sigma) \Big|_{y=\pi}. \quad (15)$$

Using the the hyperscalar background (5), (6) and the stationary condition, we obtain the anomaly-mediated scalar mass as

$$\tilde{m}_{\text{AMSB}} \sim \mathcal{O}(10^{-4}) \times g^2 kw_0 \quad (16)$$

For  $g^2 kw_0 \sim 10^6 \text{GeV}$ , we obtain

$$\tilde{m}_{\text{AMSB}} \sim 100 \text{GeV}, \quad (17)$$

which is a typical soft mass. We can show that soft masses by mediation of Kaluza-Klein modes in our model are smaller than that of anomaly mediation. Therefore our model passes the flavor-changing neutral current constraint also with respect to bulk field mediation while  $\tilde{m}_{\text{AMSB}} \sim 100 \text{GeV}$ .

For gaugino mass, anomaly mediation is also dominant as long as additional interactions with gauge singlets are not included. The gaugino mass is of the same order as the scalar mass.

### 4.2 Radion and moduli masses

We calculate the masses for the quantum fluctuations of the radion and moduli in our model. Without loss of generality, we can choose the phase of the background classical solution in Eq.(5) as

$$N_2 = N_2^\dagger. \quad (18)$$

We now introduce quantum fluctuation fields around the background classical solution to define the radion  $\tilde{R}$  and the moduli field  $\tilde{N}_2$  :

$$R + \tilde{R}, \quad N_2 + \tilde{N}_2, \quad \tilde{N}_2 = \tilde{N}_{2R} + i\tilde{N}_{2I}. \quad (19)$$

Substituting Eq.(5) into the Lagrangian and diagonalizing the kinetic term and mass-squared matrix, we find that at the leading order of  $e^{-Rk\pi}$  the lighter physical mode is almost exclusively made of the radion

$$m_{\text{light}}^2 \approx k^2 w_0^2 0.38(3.4 + \ln \Delta c)^2 (\Delta c)^{1.7}. \quad (20)$$

The heavier eigenmode is found to be exclusively made of the real part of moduli field

$$m_{\text{heavy}}^2 \approx k^2 w_0^2 0.47 (\Delta c)^{0.70}. \quad (21)$$

The imaginary part of the moduli field has the same mass as the real part of the moduli field in this approximation. We estimate the mass of the lighter physical mode (almost exclusively made of the radion), and that of the heavier mode (almost exclusively made of the complex moduli field) as

$$m_{\text{light}} \sim 1 \text{TeV}, \quad m_{\text{heavy}} \sim 100 \text{TeV} \quad (22)$$

for  $w_0 \sim (10^7 \text{GeV}/k)$  and  $\Delta c \sim 10^{-6}$ .

### 4.3 Gravitino mass

The other superparticles affected by  $w_0$  are gravitino and hyperscalar. The relevant gravitino Lagrangian in the bulk is given by [6]

$$\mathcal{L}_{\text{bulk}} = M_5 \sqrt{-g} \left[ i\bar{\Psi}_M^i \gamma^{MNP} D_N \Psi_P^i - \frac{3}{2} \sigma' \bar{\Psi}_M^i \gamma^{MN} (\sigma_3)^{ij} \Psi_N^j \right], \quad (23)$$

$$\begin{aligned} \Psi_M^1 &= (\psi_{M\alpha}^1, \bar{\psi}_M^{\dot{\alpha}1})^T, \quad \Psi_M^2 = (\psi_{M\alpha}^2, -\bar{\psi}_M^{\dot{\alpha}1})^T, \quad (24) \\ D_M &= \partial_M + \omega_M, \quad \omega_M = (\omega_\mu, \omega_4) = (\sigma' \gamma_4 \gamma_\mu / 2, 0), \end{aligned} \quad (25)$$

where the 5D curved indices are labelled by  $M, N = 0, 1, 2, 3, 4$ . The gamma matrix with curved indices is defined through 5D vielbein as  $\gamma^M = e_A^M \gamma^A$ , where  $A$  denote tangent space indices. In the second term in Eq.(23),  $SU(2)_R$  indices are contracted by  $(\sigma_3)$ .

Boundary terms for gravitino are also contained in the term with the boundary superpotential  $W_b$  in the superfield Lagrangian in Eq.(3). By restoring the fermionic part, we find  $\mathcal{L}_{\text{bound sup}} = \int d^2\theta \varphi^3 e^{-3T\sigma} W_b = 3 \left[ F_\varphi - \frac{1}{M_5^2} \psi_\mu^1 \sigma^{[\mu} \bar{\sigma}^{\nu]} \psi_\nu^1 + \text{h.c.} \right] W_b + \dots$ , using  $\varphi = 1 + \theta^2 F_\varphi$  [11]. Therefore we obtain a boundary mass term for gravitino associated to the boundary superpotential

$$\mathcal{L}_{\text{boundary}} = -\frac{3W_b}{M_5^2} \left[ \psi_\mu^1 \sigma^{[\mu} \bar{\sigma}^{\nu]} \psi_\nu^1 + \bar{\psi}_\mu^1 \bar{\sigma}^{[\mu} \sigma^{\nu]} \bar{\psi}_\nu^1 \right] \quad (26)$$

Here we assumed the  $Z_2$  parity of  $\psi_\mu^{1(2)}$  to be even (odd).

From the Lagrangian given above, we calculate mass spectrum. For the lightest mode  $m_n e^{Rk\pi} \ll k$ , we find

$$m_{\text{lightest}} \approx 6w_0 k, \quad (27)$$

which can be  $10^7 \text{GeV}$  for  $w_0 \sim (10^7 \text{GeV}/k)$ . This shows that the four-dimensional gravitino (lightest mode) is much heavier than the radion as well as scalars of the visible sector. This is similar to the supersymmetry-breaking mediation model considered previously by Ref.[10]. For heavier Kaluza-Klein modes of gravitino, we find for  $m_n \ll k$  and  $m_n e^{Rk\pi} \gg k$

$$m_n \approx 6w_0 k, \quad \left(n + \frac{1}{4}\right) \pi k e^{-Rk\pi} \quad (28)$$

and for  $m_n \gg k$

$$m_n \approx \left(n - \frac{6w_0}{2\pi}\right) \pi k e^{-Rk\pi}. \quad (29)$$

#### 4.4 Hyperscalar Kaluza-Klein mass

We consider  $n$ -th Kaluza-Klein effective field  $\phi_n^I(x)$  with its mode functions  $b_n^I(y)$  as  $\phi(x, y)$  component and  $b_n^c(y)$  as  $\phi^c(x, y)$  component

$$\begin{pmatrix} \phi(x, y) \\ \phi^c(x, y) \end{pmatrix} = \sum_n \sum_{I=1,2} \phi_n^I(x) \begin{pmatrix} b_n^I(y) \\ \hat{\epsilon}(y) b_n^c(y) \end{pmatrix}, \quad (30)$$

where  $I$  is the indices corresponding to the two independent effective fields eigenvalues. As for concerning this subsection, we take the constant superpotential as

$$W_b = 2M_5^3 (w_0 \delta(y) + w_\pi \delta(y - \pi)), \quad (31)$$

where  $w_0, w_\pi$  are dimensionless constants which are assumed to be  $\mathcal{O}(1)$ . After solving the equations of motion for the scalar component fields  $\phi$  and  $\phi^c$ , we find the following Kaluza-Klein mass: for  $\phi^{I=1}$ ,

$$m_n \approx k e^{-Rk\pi} \left[ \left(n + \frac{2\alpha + 1}{4}\right) \pi \pm \frac{|w_\pi|}{2\sqrt{3}} \right], \quad (32)$$

where the plus (minus) sign should be taken for  $1/2 \leq c \leq 1$  ( $c \leq -1/2$  or  $c > 1$ ),

$$m_n \approx k e^{-Rk\pi} \left[ \left(n + \frac{2\alpha + 1}{4}\right) \pi + \frac{w_\pi^2 + 12}{24 \tan c\pi} \left(1 - \sqrt{1 + \frac{w_\pi^2}{3} \tan^2 c\pi}\right) \right]. \quad (33)$$

for  $|c| < 1/2$ ; for  $\phi^{I=2}$ ,

$$m_n \approx k e^{-Rk\pi} \left[ \left(n + \frac{2\alpha + 1}{4}\right) \pi \pm \frac{|w_\pi|}{2\sqrt{3}} \right], \quad (34)$$

where the plus (minus) sign should be taken for  $1/2 \leq c \leq 1$  ( $c \leq -1/2$  or  $c > 1$ ),

$$m_n \approx k e^{-Rk\pi} \left[ \left(n + \frac{2\alpha + 1}{4}\right) \pi + \frac{w_\pi^2 + 12}{24 \tan c\pi} \left(1 - \sqrt{1 + \frac{w_\pi^2}{3} \tan^2 c\pi}\right) \right]. \quad (35)$$

for  $|c| < 1/2$ . Here  $\alpha \equiv |c + 1/2|$  and  $\beta \equiv |c - 1/2|$ . The mass splitting depends on the bulk mass parameter for  $|c| < 1/2$ , which is a new pattern of supersymmetry breaking.

#### 4.5 Conclusion

For  $w_0 \sim 10^7 \text{GeV}/k$ ,  $M_5 \sim (M_4^2 k)^{1/3}$  and  $c = c_{\text{cr}} - \Delta c$  with  $c_{\text{cr}} \simeq 0.546$  and  $\Delta c \sim 10^{-6}$  the orders of various masses are tabulated in Table 1 (the radius is stabilized at  $R \sim k^{-1}$ ).

**Table 1.** A typical mass spectrum

Soft	Gravitino	Radion	Moduli	Hyperscalar
100GeV	$10^7 \text{GeV}$	1 TeV	100TeV	$10^{-2} k$

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