Neutrino Masses and Mixings from Quark Mass Hierarchies

Sören Wiesenfeldt

ILLINOIS
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Based on work with W. Buchmüller, L. Covi, D. Emmanuel-Costa
(in preparation)

Motivation

Neutrino masses and mixings are very different from quark masses and mixings.

Quark masses are $\mathcal{O}\left(M_{\text{EW}}\right)$.

They are strongly hierarchical,

$$m_u : m_c : m_t \sim 10^{-5} : 10^{-2} : 1$$

$$m_d : m_s : m_b \sim 10^{-4} : 10^{-2} : 1$$

Neutrinos are not heavier than $\mathcal{O}(1 \text{ eV})$.

Their masses can be degenerate.

Even if the masses are hierarchical, the mass ratio of the heavier neutrino masses is constrained,

$$\frac{m_2}{m_3} > \sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} \simeq \frac{1}{5}\,.$$

Small mixing among quarks,

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}; \quad \lambda \simeq \theta_C$$

Mixing is close to ‘tribimaximal mixing’,

$$V \sim \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix}$$

Neutrinos and Grand Unification

The symmetries and the particle content of the standard model (SM) point towards GUTs as the next step in the unification of all forces.

In particular, SO(10) is a very attractive candidate:

- All quarks and leptons of one generation are unified in a single multiplet,

$$16 = (Q, u^c, d^c, L, \nu^c, e^c)\,;$$

- anomaly free;
- gauged $U(1)_{B-L}$: $B-L$ breaking gives rise automatically to Majorana masses for the singlet neutrinos and the seesaw mechanism;

$$m^\nu \sim \frac{M_{\text{EW}}^2}{M^N} \sim 0.1\ \text{eV} \quad \to \quad M^N \sim 10^{14}\ \text{GeV}\,,$$

  close to the MSSM unification scale, $M_{\text{GUT}} \sim 10^{16}$ GeV.

- SO(10) contains both SU(5) and $G_{\text{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R$ as subgroups. $\to$ left-right symmetric.

Neutrino Masses in SO(10)

Fermion masses arise from a few Yukawa couplings.

Mimimal model with large representations: $10_H,\ 126_H + \overline{126}_H,\ 210_H$

$$Y_{10}^{ij}\, 16_M^i 16_M^j 10_H + Y_{126}^{ij}\, 16_M^i 16_M^j \overline{126}_H \quad \Rightarrow\ Y_{126} = \frac{m^d - m^e}{4\,\langle H_d(\overline{126})\rangle} = \frac{m^u - m^\nu}{4\,\langle H_u(\overline{126})\rangle}$$

Y_{126}, which determines the Majorana mass matrices, is highly constrained by charged fermion masses.

Models with small representations (10_H, $16_H + \overline{16}_H$, and 45_H or 54_H) have more couplings and are less constrained / predictive.

[Albright, Barr; Babu, Pati, Wilczek 1998. . . ; SW, Willenbrock 2007]

They share the pattern

$$m^u \sim m^\nu\ , \qquad\qquad m^d \sim m^e\ .$$

Neutrino Masses in SO(10)

However, if we have additional heavy matter, this pattern is no longer valid!

Consider model with three spinorial and one vectorial matter fields, $16_{1,2,3}$ and 10_M. Then 10_M adds a fourth generation and down quarks and leptons.

m^u is 3×3 matrix, whereas m^d, m^e and m^ν are 4×4, [cf. Nomura, Yanagida 1998; Asaka 2003]

$$m^u \sim \begin{pmatrix} \mathrm{M}_{\alpha\beta}^{(3\times3)} \end{pmatrix}, \qquad m^d \sim m^e \sim m^\nu \sim \left(\begin{array}{c|c} \mathrm{M}_{\alpha\beta}^{(3\times3)} & \mathrm{M}_{\alpha 4} \\ \hline \mathrm{M}_{4\alpha} & \mathrm{M}_{44} \end{array}\right)$$

- Fourth generation acquires GUT-scale mass when the GUT symmetry is broken.
- If $\mathrm{M}_{4\alpha}$ or $\mathrm{M}_{\alpha 4}$ are comparable to M_{44}, the mixing among LH or RH states is large;
  $\rightarrow$ effective 3×3 mass matrix is much different from $m^u \sim \mathrm{M}_{\alpha\beta}^{(3\times3)}$.

SO(10) GUT in Six Dimensions

[Asaka, Buchmüller, Covi; Hall, Nomura, Okul, Smith 2001]

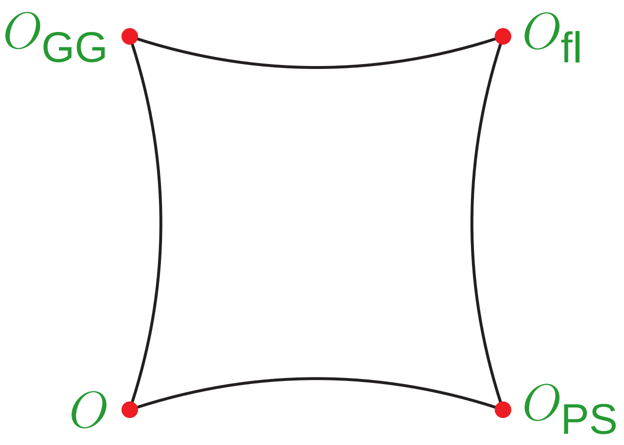
Consider 6D N=1 SUSY SO(10) compactified on $T^2/\left(\mathbb{Z}_2 \times \mathbb{Z}_2^{\text{PS}} \times \mathbb{Z}_2^{\text{GG}}\right)$.

The discrete symmetries break the extended supersymmetry and

$$P_{\text{PS}} : \text{SO(10)} \to \text{SU(4)} \times \text{SU(2)} \times \text{SU(2)}\ , \qquad P_{\text{GG}} : \text{SO(10)} \to \text{SU(5)} \times \text{U(1)}\ .$$

The breaking is localized at different points in the extra dimensions, O, O_{PS} and O_{GG}.

Physical region: 'pillow' with four fixed points (branes).



→ Four fixed points with local gauge groups SO(10), SU(5), flipped SU(5), and G_{PS}.

The unbroken gauge group of the effective 4D theory is given by the intersection of the sub-groups at the fixed points, $\text{G}_{\text{SM}} \times \text{U(1)}$.

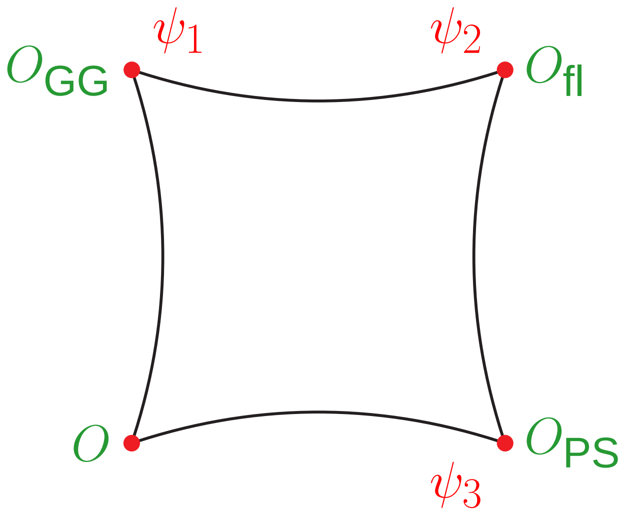
Fermion Masses and Mixing

The three families are located at different branes; they have only diagonal Yukawa couplings with the bulk Higgs fields, direct mixings are exponentially suppressed.

However, they mix with bulk field zero modes without suppression, so the zero modes

$$L_4(\phi) = \begin{pmatrix} \nu_4 \\ e_4 \end{pmatrix}, \qquad L_4^c(\phi^c) = \begin{pmatrix} \nu_4^c \\ e_4^c \end{pmatrix}, \qquad d_4^c(H_5) \qquad d_4(H_6) \ ,$$

act as a (vectorial) fourth generation of down quarks and leptons and mix with the three generations of brane fields.



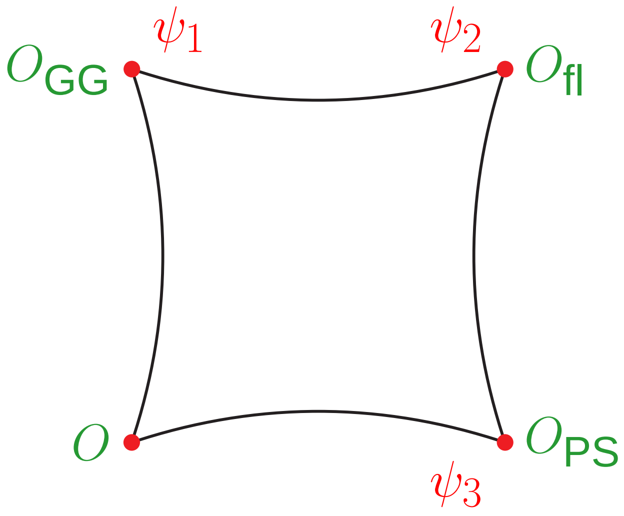
- Connect hierarchy to the location of the SM generations in the extra space-coordinates;
- the Yukawa couplings satisfy only the GUT symmetry of the local fixed point (local unification);
- the mixing between generations appears due to mixing with bulk fields zero modes with the same quantum numbers.

Fermion Masses and Mixing

General pattern of quark and lepton mass matrices: [Asaka, Buchmüller, Covi 2003]

$$\frac{1}{\tan\beta}\, m^u \sim \frac{v_1 M_*}{v_N^2}\, m^N \sim \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}, \quad m^d \sim m^e \sim m^\nu \sim \begin{pmatrix} \mu_1 & 0 & 0 & \tilde{\mu}_1 \\ 0 & \mu_2 & 0 & \tilde{\mu}_2 \\ 0 & 0 & \mu_3 & \tilde{\mu}_3 \\ \widetilde{M}_1 & \widetilde{M}_2 & \widetilde{M}_3 & \widetilde{M}_4 \end{pmatrix}$$

with (hierarchical) $\mu\,,\ \tilde{\mu} = \mathcal{O}\left(M_{\text{EW}}\right)$ and (non-hierarchical) $\widetilde{M} = \mathcal{O}\left(M_c\right)$.



Fermion Masses and Mixing

General pattern of quark and lepton mass matrices: [Asaka, Buchmüller, Covi 2003]

$$\frac{1}{\tan\beta}\, m^u \sim \frac{v_1 M_*}{v_N^2}\, m^N \sim \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}, \quad m^d \sim m^e \sim m^\nu \sim \begin{pmatrix} \mu_1 & 0 & 0 & \tilde{\mu}_1 \\ 0 & \mu_2 & 0 & \tilde{\mu}_2 \\ 0 & 0 & \mu_3 & \tilde{\mu}_3 \\ \widetilde{M}_1 & \widetilde{M}_2 & \widetilde{M}_3 & \widetilde{M}_4 \end{pmatrix}$$

with (hierarchical) $\mu\,,\ \tilde{\mu} = \mathcal{O}\left(M_{\mathrm{EW}}\right)$ and (non-hierarchical) $\widetilde{M} = \mathcal{O}\left(M_c\right)$.

- m^u can be chosen diagonal and real.
- m (m^d, m^e, m^ν) contains three phases, which can be chosen as

$$m = \begin{pmatrix} \mu_1\, e^{i\theta_1} & 0 & 0 & \tilde{\mu}_1 \\ 0 & \mu_2\, e^{i\theta_2} & 0 & \tilde{\mu}_2 \\ 0 & 0 & \mu_3\, e^{i\theta_3} & \tilde{\mu}_3 \\ \widetilde{M}_1 & \widetilde{M}_2 & \widetilde{M}_3 & \widetilde{M}_4 \end{pmatrix}.$$

Quark masses, CKM matrix, and CP violation

Assume universal Yukawa couplings at each fixed point ($\tan\beta \simeq 50$).

The "diagonal" elements μ_i are given by the hierarchy between the up quarks,

$$\mu_1 : \mu_2 : \mu_3 \sim m_u : m_c : m_t \ ,$$

$$m^u \sim \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$$

$$m^d \sim \begin{pmatrix} \mu_1 & 0 & 0 & \tilde{\mu}_1 \\ 0 & \mu_2 & 0 & \tilde{\mu}_2 \\ 0 & 0 & \mu_3 & \tilde{\mu}_3 \\ \widetilde{M}_1 & \widetilde{M}_2 & \widetilde{M}_3 & \widetilde{M}_4 \end{pmatrix}$$

while the down quark mass matrix (smaller hierarchy) is dominated by the $\tilde{\mu}_i$,

$$m_b \simeq \mu_3 \sim \tilde{\mu}_3 \ , \qquad m_s \simeq \tilde{\mu}_2 \ , \qquad \frac{m_d}{m_s} \sim \frac{\mu_2}{\tilde{\mu}_2}\frac{\tilde{\mu}_1}{\tilde{\mu}_2} \ .$$

The mixing angles are given by ratios of $\tilde{\mu}_i$,

$$V_{us} = \Theta_c \sim \frac{\tilde{\mu}_1}{\tilde{\mu}_2} \ , \qquad V_{cb} \sim \frac{\tilde{\mu}_2}{\tilde{\mu}_3} \ , \qquad V_{ub} \sim \frac{\tilde{\mu}_1}{\tilde{\mu}_3} \ .$$

Low-energy CP violation is suppressed by $\frac{m_d}{m_s}$, i.e. phases must be large,

$$J \sim -\theta_c \frac{m_d m_s}{m_b^2} \left(\sin\left(\theta_3 - \theta_2\right) + \sin\theta_2\right) \sim 10^{-5} \left(\sin\left(\theta_3 - \theta_2\right) + \sin\theta_2\right).$$

Lepton Mass Matrices

The Dirac mass matrices are very similar to the down-quark mass matrix, but the left and right handed fields are exchanged.

$\rightarrow$ large mixing between left-handed but small mixing among right-handed states.

The heavy state is effectively an SU(2)-doublet of Dirac fermions as $M^N \ll \widetilde{M} \sim M_{\text{GUT}}$.

The mass ratio of electron and muon is much smaller than the ratio of down and strange quark,

$$\frac{m_e}{m_\mu} = \left(\frac{\mu_2 \widetilde{\mu}_1}{\widetilde{\mu}_2^2}\right)_e \ll \left(\frac{\mu_2 \widetilde{\mu}_1}{\widetilde{\mu}_2^2}\right)_d \sim \frac{m_d}{m_s} \qquad \Rightarrow \quad \mu_2^e \ll \mu_2^d \quad \text{and / or} \quad \widetilde{\mu}_1^e \ll \widetilde{\mu}_1^d$$

Lepton Mass Matrices

The Dirac mass matrices are very similar to the down-quark mass matrix, but the left and right handed fields are exchanged.

$\rightarrow$ large mixing between left-handed but small mixing among right-handed states.

The heavy state is effectively an SU(2)-doublet of Dirac fermions as $M^N \ll \widetilde{M} \sim M_{\text{GUT}}$.

The mass ratio of electron and muon is much smaller than the ratio of down and strange quark,

$$\frac{m_e}{m_\mu} = \left(\frac{\mu_2 \widetilde{\mu}_1}{\widetilde{\mu}_2^2}\right)_e \ll \left(\frac{\mu_2 \widetilde{\mu}_1}{\widetilde{\mu}_2^2}\right)_d \sim \frac{m_d}{m_s} \qquad \Rightarrow \qquad \mu_2^e \ll \mu_2^d \quad \text{and / or} \quad \widetilde{\mu}_1^e \ll \widetilde{\mu}_1^d$$

Use basis, where the charged leptons are diagonal. Then the effective Dirac neutrino mass matrix reads

$$\overline{m}^\nu = \begin{pmatrix} A\overline{\rho}_1 & D\overline{\rho}_1 & \overline{\rho}_1 \\ B\overline{\rho}_2 & E\overline{\rho}_2 & \overline{\rho}_2 \\ C\overline{\rho}_3 & F\overline{\rho}_3 & \overline{\rho}_3 \end{pmatrix}, \qquad \begin{aligned} \overline{\rho}_1 &\sim \widetilde{\rho}_1 \tfrac{\widetilde{\mu}_3 - \mu_3^*}{\overline{\mu}_3}\,, \\ \overline{\rho}_2 &\sim \widetilde{\rho}_2 \tfrac{\widetilde{\mu}_3 - \mu_3^*}{\overline{\mu}_3} - \rho_2 \tfrac{\widetilde{\mu}_3 + \mu_3^*}{\overline{\mu}_3}\,, \\ \overline{\rho}_3 &\sim \widetilde{\rho}_3 \tfrac{\widetilde{\mu}_3 - \mu_3^*}{\overline{\mu}_3} - \rho_3 \tfrac{\widetilde{\mu}_3 - \mu_3^*}{\overline{\mu}_3}\,. \end{aligned}$$

$(\mu,\, \widetilde{\mu} : m^e,$
$\rho,\, \widetilde{\rho} : m^\nu)$

Neutrino Masses

The light neutrino masses result from the see-saw mechanism. The Majorana matrix for the right-handed neutrinos is diagonal, but it can have complex entries,

$$m_\nu = -\overline{m}^{\nu\top} \frac{1}{m^N} \overline{m}^\nu \,, \qquad m^N = \begin{pmatrix} M_1 e^{2i\phi_1} & 0 & 0 \\ 0 & M_2 e^{2i\phi_2} & 0 \\ 0 & 0 & M_3 e^{2i\phi_3} \end{pmatrix} .$$

Assuming $\overline{\rho}_1 : \overline{\rho}_2 : \overline{\rho}_3 \sim m_d : m_s : m_b$, we find the neutrino mass ratios

$$\frac{m_2}{m_3} \sim \frac{\overline{\rho}_2^2}{\overline{\rho}_3^2} \frac{M_3}{M_2} \sim \frac{m_s^2}{m_b^2} \frac{m_t}{m_c} \sim 0.2 \,, \qquad \frac{m_1}{m_3} \sim \frac{\overline{\rho}_1^2}{\overline{\rho}_3^2} \frac{M_3}{M_1} \sim \frac{m_d^2}{m_b^2} \frac{m_t}{m_u} \sim 0.2 \,.$$

- The weak hierarchy in the neutrino sector can be traced back to the non-perfect compensation between down and up quark hierarchies.
- The light neutrino mass spectrum is hierarchical with

$$m_1 \sim m_2 \,, \qquad \frac{m_2^2}{m_3^2} \sim \frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}} \,.$$

Neutrino Mixing

Since $A,\ C \sim \frac{\mu_2}{\overline{\mu}_2}$, $B \sim \frac{\rho_2}{\widetilde{\rho}_2}$, and $D,\ E,\ F \sim 1$, we have

$$m^\nu_{\text{eff}} \sim \begin{pmatrix} \frac{\mu_2^2}{\overline{\mu}_2^2} + \frac{\rho_2^2}{\widetilde{\rho}_2^2}\sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} & \frac{\mu_2}{\overline{\mu}_2} + \frac{\rho_2}{\widetilde{\rho}_2}\sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} & \frac{\mu_2}{\overline{\mu}_2} + \frac{\rho_2}{\widetilde{\rho}_2}\sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} \\ \frac{\mu_2}{\overline{\mu}_2} + \frac{\rho_2}{\widetilde{\rho}_2}\sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} & 1 & 1 \\ \frac{\mu_2}{\overline{\mu}_2} + \frac{\rho_2}{\widetilde{\rho}_2}\sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} & 1 & 1 \end{pmatrix} m_3 \ .$$

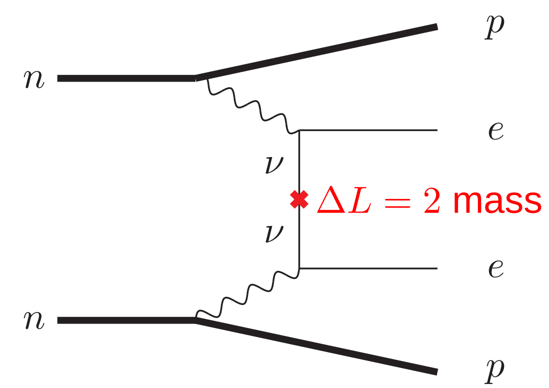
- atmospheric mixing angle θ_{23} is naturally large;
- smallness of the electron mass yields small reactor mixing angle θ_{13};
- a large solar mixing angle θ_{12} requires $\rho_2 \sim \widetilde{\rho}_2$.

$\rightarrow$ $A \sim C \sim \frac{\mu_2}{\overline{\mu}_2}$ and $B \sim D \sim E \sim F \sim 1$ fixed!

θ_{13} is rather large due to corrections $\frac{\rho_2}{\widetilde{\rho}_2}\sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}}$.

Neutrinoless Double Beta Decay

$0\nu\beta\beta$ is only allowed if neutrinos are Majorana particles.



The decay width reads

$$\Gamma = G \left|\mathcal{M}^2\right| \left|m_{ee}\right|^2,$$

where G is a phase space factor, $\mathcal{M}$ the nuclear matrix element, and $m_{ee} = \left(m^\nu_{\text{eff}}\right)_{11}$.

We can estimate the size of m_{ee},

$$m_{ee} \sim \sqrt{\Delta m^2_{\text{atm}}} \frac{\mu_2^2}{\widetilde{\mu}_2^2} e^{-2i\phi_3} + \sqrt{\Delta m^2_{\text{sol}}} e^{-2i\phi_2} \left(\frac{\mu_2}{\widetilde{\mu}_2} - \frac{\rho_2}{\widetilde{\rho}_2}\right)^2$$

in agreement with

$$m_{ee} = m_3 \sin^2\theta_{13} + \cos^2\theta_{13} \left(m_2\, e^{2i\varphi_2} \sin^2\theta_{12} + m_1\, e^{2i\varphi_1} \cos^2\theta_{12}\right).$$

CP Violation in Neutrino Oscillations

The strength of Dirac-type CP violation is obtained from

$$J_l = \frac{1}{2i} \frac{1}{\Delta \mathscr{M}_e^2} \frac{1}{\Delta \mathscr{M}_\nu^2} \; \det \left[h^\nu_{\text{eff}}, h^e \right], \qquad h^\nu_{\text{eff}} = \left(m^\nu_{\text{eff}} \right)^\dagger m^\nu_{\text{eff}}$$

$$\sim |B|^2 \frac{m_2^3}{m_2^2} \frac{m_3^3}{m_3^4} \sim \frac{\sqrt{\Delta m^2_{\text{sol}}}}{\sqrt{\Delta m^2_{\text{atm}}}} \; ,$$

provided that the phases are $\mathcal{O}\left(1\right)$.

Compared to

$$J_l = \tfrac{1}{8} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin\phi \simeq 0.1 \sin 2\theta_{13} \sin\phi \; ,$$

where ϕ is the CP violating Dirac phase in the (extended) standard model, the magnitude is enhanced by one order of magnitude.

Leptogenesis

- The decay of right-handed neutrinos generates a lepton asymmetry, which is converted to a baryon asymmetry by sphaleron processes.
- The out-of-equilibrium decay leads to the CP asymmetry [Covi, Roulet, Vissani 1996]

$$\varepsilon = -\frac{3}{16\pi} \sum \frac{\mathrm{Im}\left(\mathsf{M}_{1i}^2\right)}{\mathsf{M}_{11} v_u^2} \frac{M_1}{M_i}, \qquad \mathsf{M} = \overline{m}^\nu \left[\overline{m}^\nu\right]^\dagger$$

- We estimate the size of ϵ,

$$\varepsilon \sim 10^{-6} \left[\sin \Delta\phi_{13} - \sin\left(2\Delta\phi_{13} + \delta_3\right) + \sin 2\left(\Delta\phi_{13} + \delta_3\right) + \sin 2\Delta\phi_{12}\right],$$

  where ϕ_i are the phases of m^N, δ_3 the phase of ρ_3.
- Then the size of the baryon asymmetry is $\eta_B \sim 10^{-10}$.

  $\rightarrow$ Right order of magnitude with large phases.

Summary

Why are masses and mixings of quarks and neutrinos so different and how does this happen in grand unified theories?

- The CKM mixings are small because LH down-quarks are pure brane states. The large mixings of RH down quarks, together with the down-quark mass hierarchy, leads to small mixings for LH quarks.
- The MNS mixings are large because neutrinos are mixtures of brane and bulk states, which are unrelated to quark and charged lepton masses and therefore not suppressed by small mass ratios.
- Neutrinos have a small mass hierarchy because of the seesaw mechanism and the mass relations $m^d \sim m^\nu$, $m^u \sim m^N$,

$$m_\nu = -\left(\overline{m}^\nu\right)^\top \frac{1}{m^N}\, \overline{m}^\nu \sim -\left(\overline{m}^d\right)^\top \frac{1}{m^u}\, \overline{m}^d \, .$$

  The ‘squared’ down-quark hierarchy is almost canceled by the larger up-quark hierarchy.

Neutrino phenomenology is fixed in terms of quark masses and mixings.

Summary

These results are generally valid in the case where

$$m^u \sim m^N \sim \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}, \qquad m^d \sim m^e \sim m^\nu \sim \begin{pmatrix} \mu_1 & 0 & 0 & \tilde{\mu}_1 \\ 0 & \mu_2 & 0 & \tilde{\mu}_2 \\ 0 & 0 & \mu_3 & \tilde{\mu}_3 \\ \widetilde{M}_1 & \widetilde{M}_2 & \widetilde{M}_3 & \widetilde{M}_4 \end{pmatrix},$$

with (hierarchical) $\mu, \widetilde{\mu} = \mathcal{O}(M_{\text{EW}})$ and (non-hierarchical) $\widetilde{M} = \mathcal{O}(M_{GUT})$.

[cf. Asaka 2003]

For simplicity, we assumed degenerate $\widetilde{M}$ and universal Yukawa couplings μ and $\widetilde{\mu}$;

exception: μ_2 (ρ_2), where

$$\begin{cases} m_d : m_s & \longrightarrow \quad \mu_2^d : \widetilde{\mu}_2 \simeq 1 : 4 \\ m_e : m_\mu & \longrightarrow \quad \mu_2^e : \widetilde{\mu}_2 \sim 1 : 10 \\ \tan 2\theta_{12} \sim 1 & \longrightarrow \quad \rho_2 \sim \widetilde{\rho}_2 \end{cases}$$