Constructing Textures in Extended Quark-Lepton Complementarity

Florian Plentinger

Universität Würzburg

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In collaboration with: Gerhart Seidl and Walter Winter

Outline

- Observations
- Extended Quark-Lepton Complementarity
- Results
- Extension to seesaw mechanism
- Outlook

Observations

($\varepsilon \sim \theta_C$)

- *Quarks:*

$$m_u : m_c : m_t = \varepsilon^6 : \varepsilon^3 : 1$$
$$m_d : m_s : m_b = \varepsilon^4 : \varepsilon^2 : 1$$

$$V_{CKM} \sim \begin{pmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}$$

$\theta_C^2 = m_d / m_s$ (Gatto et al.,1968)

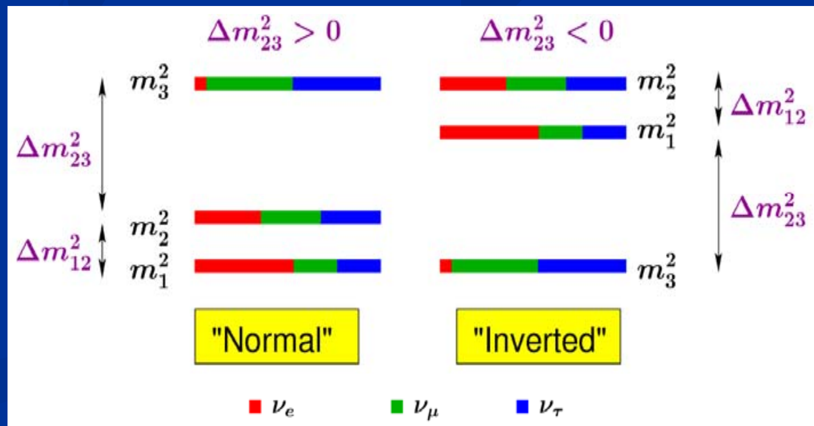
$\theta_{12} \approx \varepsilon$, $\theta_{13} \approx \varepsilon^3$, $\theta_{23} \approx \varepsilon^2$, $\delta \approx 63^o$

- *Leptons:*

$$m_e : m_\mu : m_\tau = \varepsilon^4 : \varepsilon^2 : 1$$

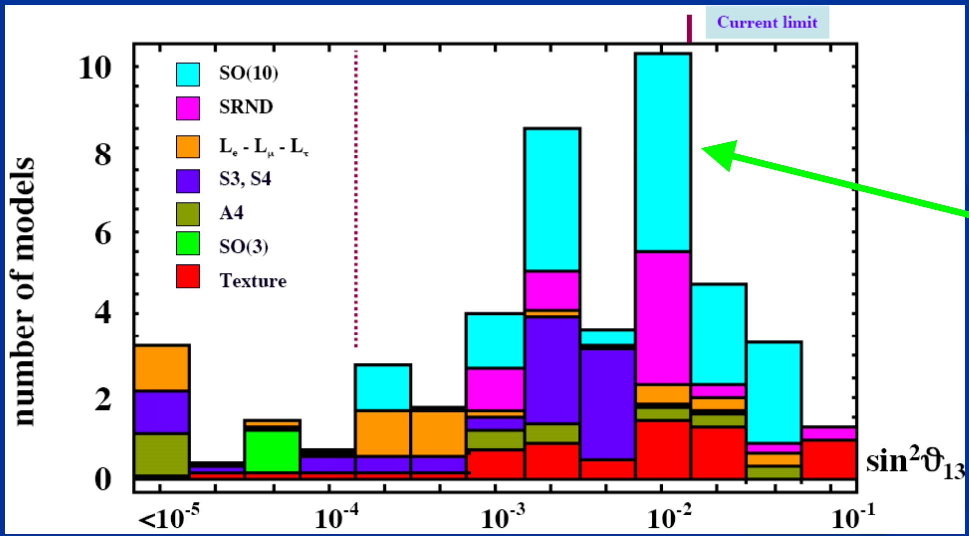
$$\Delta m^2_{12} : \Delta m^2_{23} \sim \varepsilon^2$$

$\theta_{23} \approx \pi/4$, $\theta_{12} \approx \pi/4 - \varepsilon$, $\theta_{13} \leq \varepsilon$



Usual Methods

- Symmetries (GUTs, flavor,…), anarchy arguments etc. used to predict/explain observations
- Example: literature research on θ_{13}



Generic or biased peak?

(Albright,Chen,2006)

New Systematic Approach

- Model independent bottom-up approach
- Systematic reconstruction of Yukawa couplings
- Extremely simple and efficient
- Extensive extraction of realizations and textures

Unbiased exploration of parameter space

Extended QLC

$(\varepsilon \sim \theta_C \approx 0.2)$

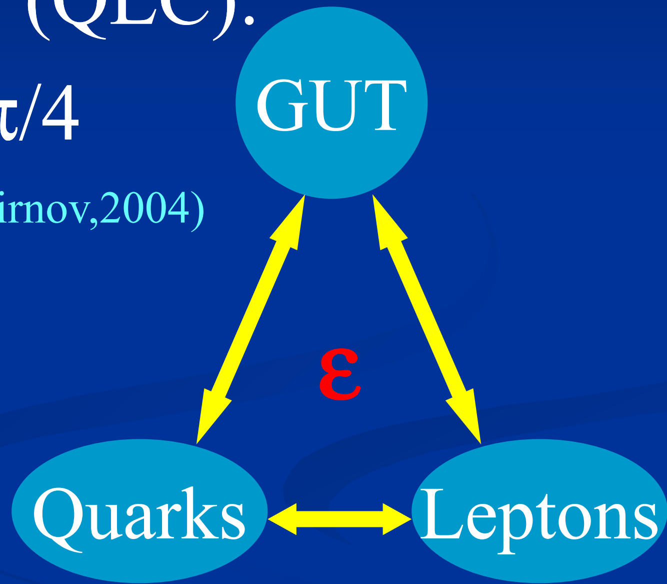
- Quark-Lepton Complementarity (QLC):

$$\theta_{12}+\theta_C \approx \pi/4, \qquad \theta_{23}+\theta_{cb} \approx \pi/4$$

(Petcov,Smirnov,1993; Smirnov,2004; Raidal,2004; Minakata,Smirnov,2004)

E.g. from $U_{PMNS} \sim V_{CKM}^{+} U_{bimax}$

$(U_{bimax}: \theta_{12}=\theta_{23}=\pi/4)$



- *ε parameterizes everything?!*

→ Extended QLC:

- All mass ratios are powers of ε
- All mixing angles from $\{\pi/4, \varepsilon, \varepsilon^2, .., 0\}$

Extended QLC Results

- All real possibilities with mixing angles $\{\pi/4, \varepsilon, \varepsilon^2, 0\}$
  $\longrightarrow$ 262.144 possibilities
- 2.468 *realizations* in agreement with experiments
- In 10 year limit: 20 *textures*

$\xi = \{0, \pi\}$

#		Normal Hierarchy		Inverted Hierarchy		Degenerate	$(s^\ell_{12}, s^\ell_{13}, s^\ell_{23})$ $(s^\nu_{12}, s^\nu_{13}, s^\nu_{23})$ $(\delta^\ell, \delta^\nu, \widehat{\varphi}_1, \widehat{\varphi}_2)$	$(\theta_{12}, \theta_{13}, \theta_{23})$
	M_ℓ	M_ν^{Maj}	M_ν^{Dirac}	M_ν^{Maj}	M_ν^{Dirac}	M_ν^{Dirac}		
17	$\begin{pmatrix} 0 & \epsilon^2 & \epsilon \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \\ 0 & \epsilon & 0 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 0 \\ 1 & \epsilon & \epsilon \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & 0 \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\frac{1}{\sqrt{2}}, \epsilon, \frac{1}{\sqrt{2}})$ $(\epsilon, \frac{1}{\sqrt{2}}, 0)$ $(\pi, \xi, \pi, \xi+\pi)$	$(35.2°, 3.8°, 50.8°)$
18	$\begin{pmatrix} 0 & \epsilon^2 & \epsilon \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \\ 0 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & 1 & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 0 \\ 1 & \epsilon & \epsilon \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & \epsilon^2 \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\frac{1}{\sqrt{2}}, \epsilon, \frac{1}{\sqrt{2}})$ $(\epsilon, \frac{1}{\sqrt{2}}, \epsilon^2)$ $(\pi, \pi, \pi, 0)$	$(33.6°, 3.1°, 52.2°)$

(FP, Seidl, Winter, hep-ph/0612169)

Results - Textures

- New textures:

Example: “Diamond” textures

“Diamond”: $\theta_{13}=\pi/4$

#	M_ℓ	Normal Hierarchy M_ν^{Maj}	Normal Hierarchy M_ν^{Dirac}	Inverted Hierarchy M_ν^{Maj}	Inverted Hierarchy M_ν^{Dirac}	Degenerate M_ν^{Dirac}	$(s_{12}^\ell, s_{13}^\ell, s_{23}^\ell)$ $(s_{12}^\nu, s_{13}^\nu, s_{23}^\nu)$ $(\delta^\ell, \delta^\nu, \widehat{\varphi}_1, \widehat{\varphi}_2)$	$(\theta_{12}, \theta_{13}, \theta_{23})$
17	$\begin{pmatrix} 0 & \epsilon^2 & \epsilon \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \\ 0 & \epsilon & 0 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 0 \\ 1 & \epsilon & \epsilon \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & 0 \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\frac{1}{\sqrt{2}}, \epsilon, \frac{1}{\sqrt{2}})$ $(\epsilon, \frac{1}{\sqrt{2}}, 0)$ $(\pi, \xi, \pi, \xi+\pi)$	$(35.2°, 3.8°, 50.8°)$
18	$\begin{pmatrix} 0 & \epsilon^2 & \epsilon \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon^2 & \epsilon^2 & 1 \\ 0 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & 1 & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 0 \\ 1 & \epsilon & \epsilon \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & \epsilon^2 \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\frac{1}{\sqrt{2}}, \epsilon, \frac{1}{\sqrt{2}})$ $(\epsilon, \frac{1}{\sqrt{2}}, \epsilon^2)$ $(\pi, \pi, \pi, 0)$	$(33.6°, 3.1°, 52.2°)$

$\xi=\{0,\pi\}$

(FP, Seidl, Winter, hep-ph/0612169)

Results - Sum Rules

- Diamond textures reveal also non-trivial sum rules.
- New sum rules:

$$\theta_{12} + \frac{3}{5+2\sqrt{2}}\,\epsilon = \arctan(2-\sqrt{2}).$$

$$\theta_{13} = \arcsin\left(\frac{1}{4}(2-\sqrt{2})\right) - \frac{1}{\sqrt{5+2\sqrt{2}}}\,\epsilon\ ,$$

$$\theta_{23} = \arctan\left(1+\frac{1}{\sqrt{2}}\right) + \frac{1}{17}(2-11\sqrt{2})\,\epsilon\ .$$

(FP, Seidl, Winter, hep-ph/0612169)

QLC:

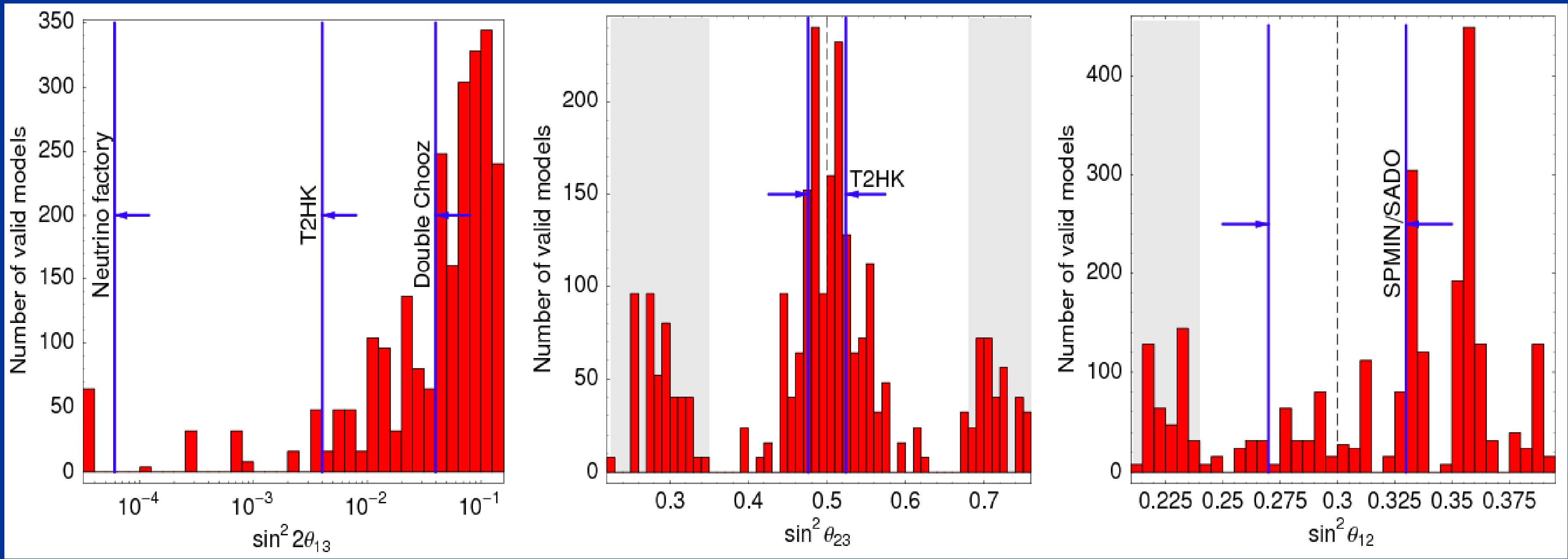
$\theta_{12}+\varepsilon=\pi/4$

$\theta_{13}=\mathcal{O}(\varepsilon)$

$\theta_{23}=\pi/4+\mathcal{O}(\varepsilon^2)$

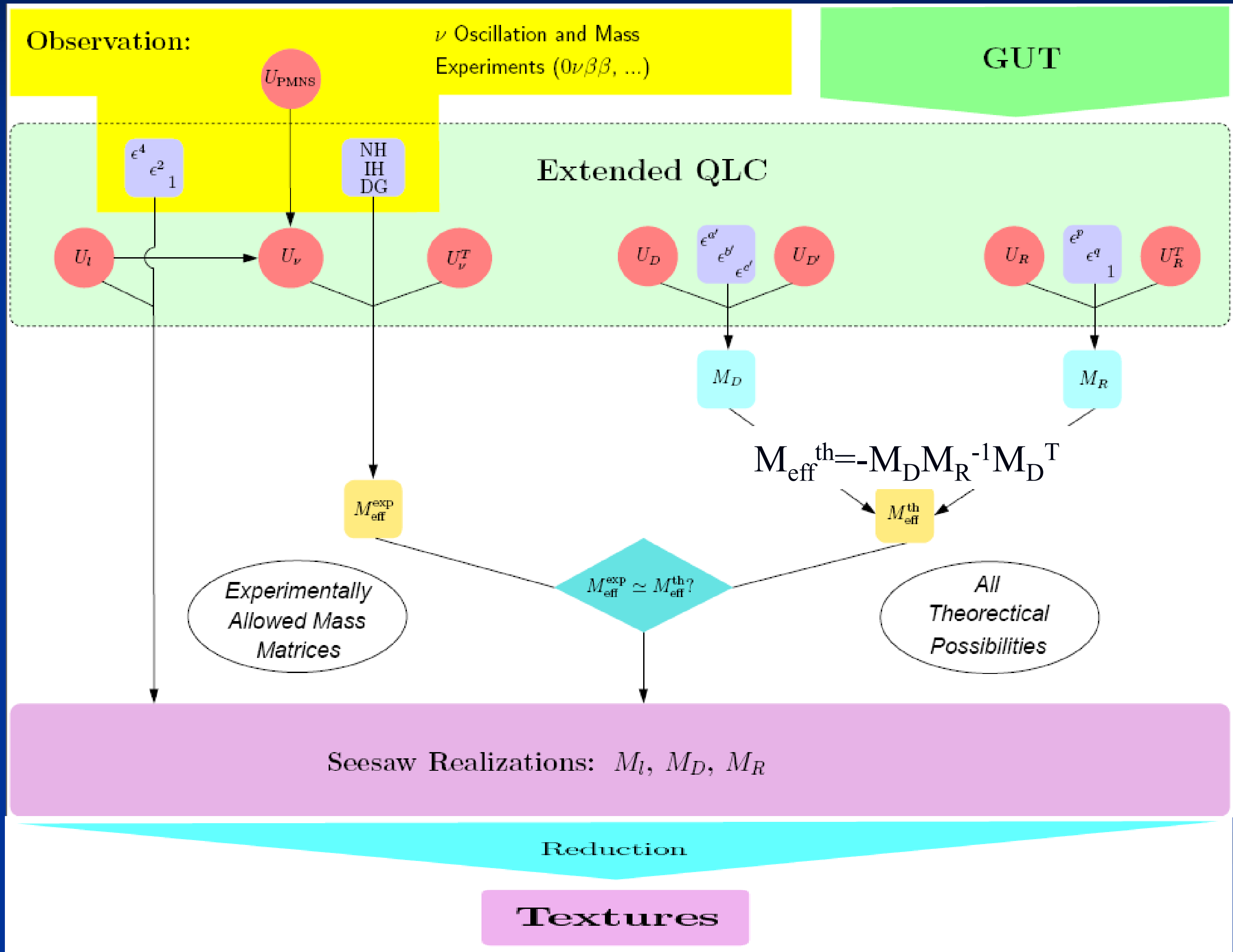
Results – Mixing Angles

- Generic distributions:
  E.g. large θ_{13} preferred



(FP, Seidl, Winter, hep-ph/0612169)

Extension to Seesaw Mechanism



(FP, Seidl, Winter, arXiv:0707.2379)

Seesaw Results - Textures

- Over 20 trillion possibilities
- 1.981 textures found

$\xi = \{0, \varepsilon^2\}$

#	M_ℓ	M_D	M_R	$M_D^{\rm diag}/m_D$ $M_R^{\rm diag}/M_{B-L}$	$(\theta^\ell_{12}, \theta^\ell_{13}, \theta^\ell_{23})$ $(\theta^D_{12}, \theta^D_{13}, \theta^D_{23})$ $(\theta^{D'}_{12}, \theta^{D'}_{13}, \theta^{D'}_{23})$ $(\theta^R_{12}, \theta^R_{13}, \theta^R_{23})$
3	$\begin{pmatrix} 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \epsilon^2 & \epsilon \\ \epsilon^2 & 1 & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & \epsilon \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\epsilon, 1, \epsilon)$ $(\epsilon, 1, 1)$	$(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4})$ $(0, \frac{\pi}{4}, \epsilon)$ $(0, 0, \epsilon)$ $(\epsilon, \frac{\pi}{4}, 0)$
12	$\begin{pmatrix} 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & 0 & \xi \\ 0 & \epsilon & \epsilon \\ \xi & \epsilon & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \xi & \xi \\ \xi & 1 & 0 \\ \xi & 0 & 1 \end{pmatrix}$	$(\epsilon, \epsilon, 1)$ $(\epsilon, 1, 1)$	$(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{4})$ (ξ, ξ, ϵ) (ξ, ξ, ϵ) (ξ, ξ, ϵ)
33	$\begin{pmatrix} 0 & 0 & \xi \\ 0 & \epsilon^2 & \xi \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \xi & \xi \\ \xi & 1 & 0 \\ \xi & 0 & 1 \end{pmatrix}$	$(\epsilon, 1, \epsilon)$ $(\epsilon, 1, 1)$	(ξ, ξ, ξ) $(\xi, \frac{\pi}{4}, \frac{\pi}{4})$ $(0, \epsilon, \frac{\pi}{4})$ (ξ, ξ, ξ)

$\theta_{23} \gtrsim 50^{\circ}$

#	$(\delta^l, \alpha^l_1, \alpha^l_2)$	$(\delta^D, \varphi^D_1, \varphi^D_2, \varphi^D_3)$	$(\delta^{D'}, \alpha^{D'}_1, \alpha^{D'}_2)$	$(\delta^R, \varphi^R_1, \varphi^R_2, \varphi^R_3)$	$(\theta_{12}, \theta_{13}, \theta_{23})$	χ^2	Cases
3	$(\pi, 0, 0)$	$(0, 0, \pi, 0)$	$(0, 0, 0)$	$(\pi, 0, \pi, 0)$	$(33.5^\circ, 0.2^\circ, 51.3^\circ)$	4.9	26
12	$(0, \pi, \pi)$	$(0, 0, 0, 0)$	$(0, \pi, 0)$	$(\pi, 0, 0, \pi)$	$(33.4^\circ, 0.0^\circ, 51.3^\circ)$	4.81	475
33	(π, π, π)	$(\pi, 0, 0, 0)$	$(0, 0, 0)$	$(0, 0, \pi, 0)$	$(33.6^\circ, 0.1^\circ, 51.5^\circ)$	5.31	83

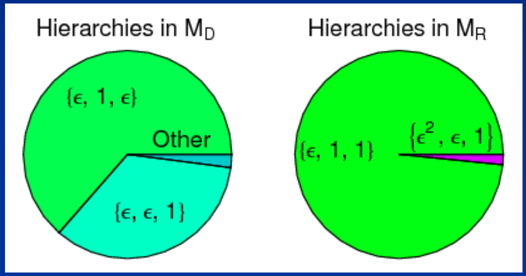
(FP, Seidl, Winter, arXiv:0707.2379)

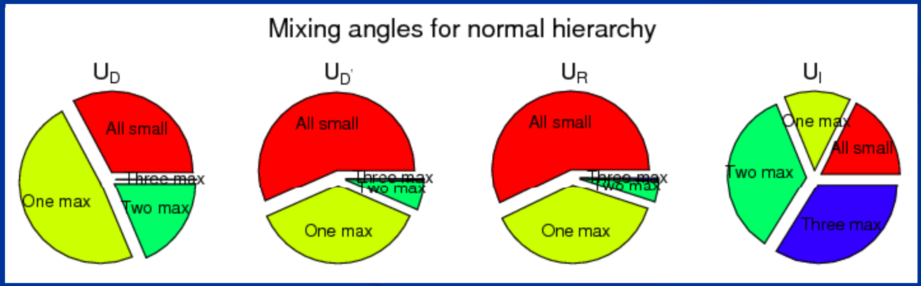
Seesaw Results - Distributions

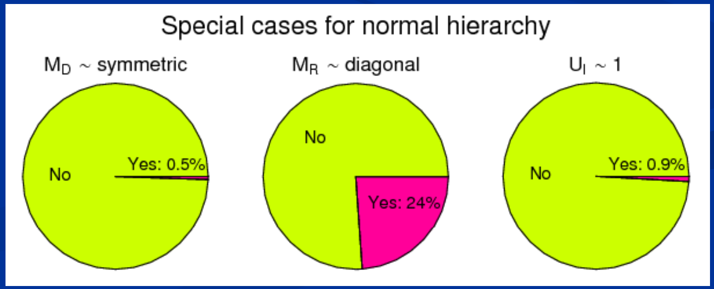
- Mostly mild hierarchies in M_R
- Charged lepton mixing is, in general, not small
- Diagonal M_R is not rare

(FP, Seidl, Winter, arXiv:0707.2379)

Normal hierarchy:







Example – Z_n Flavor Symmetries

- $G_F = Z_4^{(1)} \times Z_4^{(2)} \times \ldots \times Z_4^{(7)}$
- 2 Flavon fields f_i, f_i\` per $Z_4^{(i)}$ with charges:

  $f_i \sim 1$, f_i\`~ 2 under $Z_4^{(i)}$
- $\langle f_i \rangle \approx \langle f_i` \rangle \approx v = \varepsilon M_f$

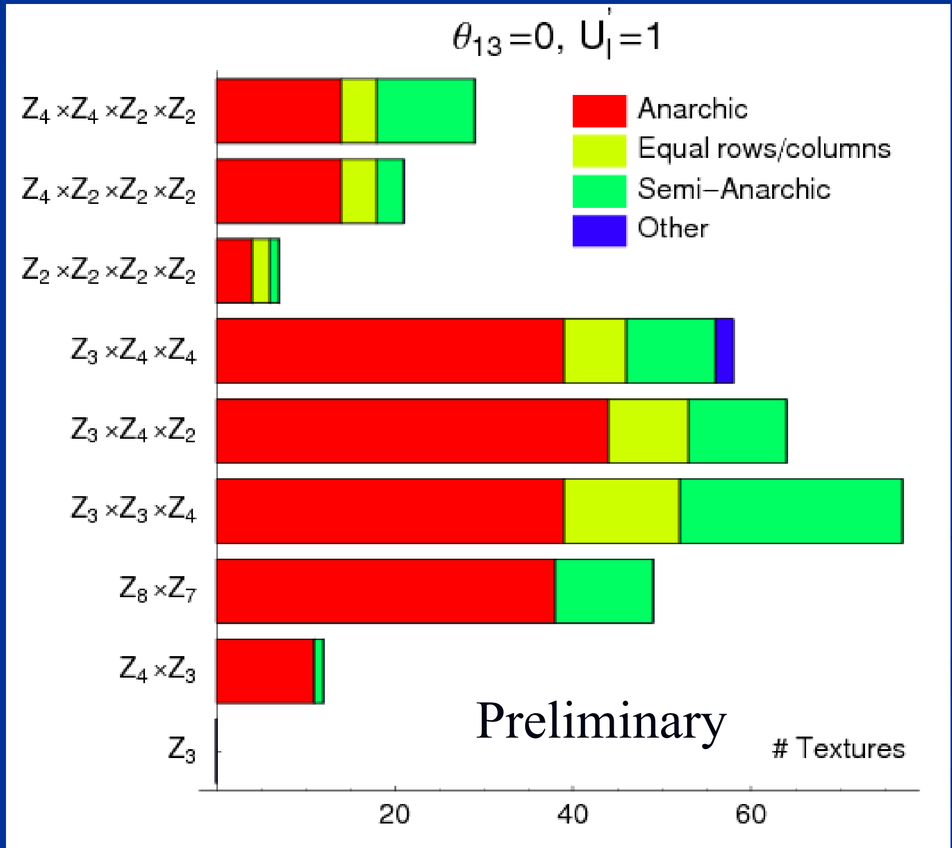
(FP, Seidl, Winter, arXiv:0707.2379)

Field	Model 1	Model 2
ν_1^c	(0,0,0,1,0,1,1)	(2,0,0,2,0,0,1)
ν_2^c	(2,0,0,1,0,1,1)	(2,0,0,2,0,0,1)
ν_3^c	(0,0,0,1,0,1,1)	(0,2,0,0,2,1,0)
l_1	(0,2,0,0,1,0,1)	(2,0,2,2,2,1,0)
l_2	(0,0,0,0,1,1,0)	(2,2,0,2,2,1,0)
l_3	(0,0,2,1,0,0,1)	(0,2,2,2,2,0,1)
e_1^c	(2,2,2,1,1,1,1)	(0,0,0,0,0,1,1)
e_2^c	(2,2,2,0,0,1,0)	(2,2,2,0,0,3,3)
e_3^c	(0,2,2,0,0,0,0)	(2,2,2,2,2,3,3)

#	M_ℓ	M_D	M_R	$M_D^{\rm diag}/m_D$ $M_R^{\rm diag}/M_{B-L}$	$(\theta^\ell_{12},\theta^\ell_{13},\theta^\ell_{23})$ $(\theta^D_{12},\theta^D_{13},\theta^D_{23})$ $(\theta^{D'}_{12},\theta^{D'}_{13},\theta^{D'}_{23})$ $(\theta^R_{12},\theta^R_{13},\theta^R_{23})$
17	$\begin{pmatrix} 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & \epsilon \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \epsilon^2 & \epsilon \\ 1 & \epsilon & 1 \\ 1 & \epsilon & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & \epsilon & 1 \\ \epsilon & 1 & \epsilon \\ 1 & \epsilon & 1 \end{pmatrix}$	$(\epsilon,\epsilon,1)$ $(\epsilon,1,1)$	$(\frac{\pi}{4},\frac{\pi}{4},\epsilon)$ $(\epsilon,0,\frac{\pi}{4})$ $(0,\frac{\pi}{4},0)$ $(\epsilon,\frac{\pi}{4},0)$
18	$\begin{pmatrix} 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \\ 0 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} \epsilon & \epsilon & 0 \\ \epsilon & \epsilon & \epsilon \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$(\epsilon,\epsilon,1)$ $(\epsilon,1,1)$	$(\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{4})$ $(\frac{\pi}{4},0,\epsilon)$ $(0,0,0)$ $(\frac{\pi}{4},\epsilon,\epsilon)$

Outlook

- Automated model building:
  How much complexity is needed to reproduce textures?



(FP, Seidl, Winter, in preparation)

Summary

- Generalization of QLC to *Extended QLC*:
  $\varepsilon \sim \theta_C$ *parameterizes everything* (masses, mixings)
- *Efficient* way to reconstruct Yukawa matrices
- *Systematic* generation of possibilities:
  Complete extraction of textures (*e.g.* diamond texture)
  Reveal general features (*e.g.* new sum rule)
- *Automated* model generation possible

References

- FP, G. Seidl and W. Winter,
  "Systematic Parameter Space Search of Extended Quark-Lepton-Complementarity",
  hep-ph/0612169 (submitted to Nucl.Phys.B)
- http://theorie.physik.uni-wuerzburg.de/%7Ewinter/Resources/Textures/index.html
- FP, G. Seidl and W. Winter,
  "The Seesaw Mechanism in Quark-Lepton Complementarity",
  arXiv:0707.2379 [hep-ph]
- http://theorie.physik.uni-wuerzburg.de/~winter/Resources/SeeSawTex/index.html

Example - $V_{CKM}^{+}\, U_{bimax}$

$$M_l = U_l\, M_l^{diag}\, U_{l'}^{+} \qquad U_{l'}^{+} \rightarrow \mathbb{1}_{3x3}$$

$$M_\nu^{Maj} = U_\nu\, M_\nu^{diag}\, U_\nu^{T}$$

$$M_\nu^{Dirac} = U_\nu\, M_\nu^{diag}\, U_{\nu'}^{+} \qquad U_{\nu'}^{+} \rightarrow \mathbb{1}_{3x3}$$

$$U_{PMNS} = U_l^{+} U_\nu$$

- *Mixing angles:*

$(\theta^l_{12},\theta^l_{13},\theta^l_{23})=(\varepsilon,0,\varepsilon^2)$, $\quad (\theta^\nu_{12},\theta^\nu_{13},\theta^\nu_{23})=(\pi/4,\varepsilon,\pi/4)$

- *PMNS mixing angles:*

$$U_l^{+}U_\nu \longrightarrow (\theta_{12},\theta_{13},\theta_{23})=(36.5^o,3.6^o,43.8^o)$$

Example - $V_{CKM}^{+}\, U_{bimax}$

$$M_l = U_l\, M_l^{diag}$$
$$M_\nu^{Maj} = U_\nu\, M_\nu^{diag}\, U_\nu^T$$
$$M_\nu^{Dirac} = U_\nu\, M_\nu^{diag}$$

$$\left\{\begin{array}{l} M_l^{diag} = m_\tau\ \mathrm{diag}(\varepsilon^4,\varepsilon^2,1) \\ M_\nu = m_3\ \mathrm{diag}(\varepsilon^2,\varepsilon,1) \\ (\theta^l_{12},\theta^l_{13},\theta^l_{23}) = (\varepsilon,0,\varepsilon^2) \\ (\theta^\nu_{12},\theta^\nu_{13},\theta^\nu_{23}) = (\pi/4,\varepsilon,\pi/4) \end{array}\right.$$

$$(\theta_{12},\theta_{13},\theta_{23})=(\, 36.5^o,3.6^o,43.8^o)$$

$$M_\ell = m_\tau \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon^2 & \epsilon^2 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon^2 & \epsilon^2 \\ 0 & 0 & 1 \end{pmatrix},$$

$$M_\nu^{\mathrm{Maj}} = m_3 \begin{pmatrix} \frac{\epsilon}{2} + \frac{3\epsilon^2}{2} & \frac{3\epsilon}{2\sqrt{2}} - \frac{\epsilon^2}{\sqrt{2}} & \frac{\epsilon}{2\sqrt{2}} \\ \frac{3\epsilon}{2\sqrt{2}} - \frac{\epsilon^2}{\sqrt{2}} & \frac{1}{2} + \frac{\epsilon}{4} - \frac{3\epsilon^2}{4} & \frac{1}{2} - \frac{\epsilon}{4} - \frac{3\epsilon^2}{4} \\ \frac{\epsilon}{2\sqrt{2}} & \frac{1}{2} - \frac{\epsilon}{4} - \frac{3\epsilon^2}{4} & \frac{1}{2} + \frac{\epsilon}{4} + \frac{\epsilon^2}{4} \end{pmatrix} \to \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix},$$

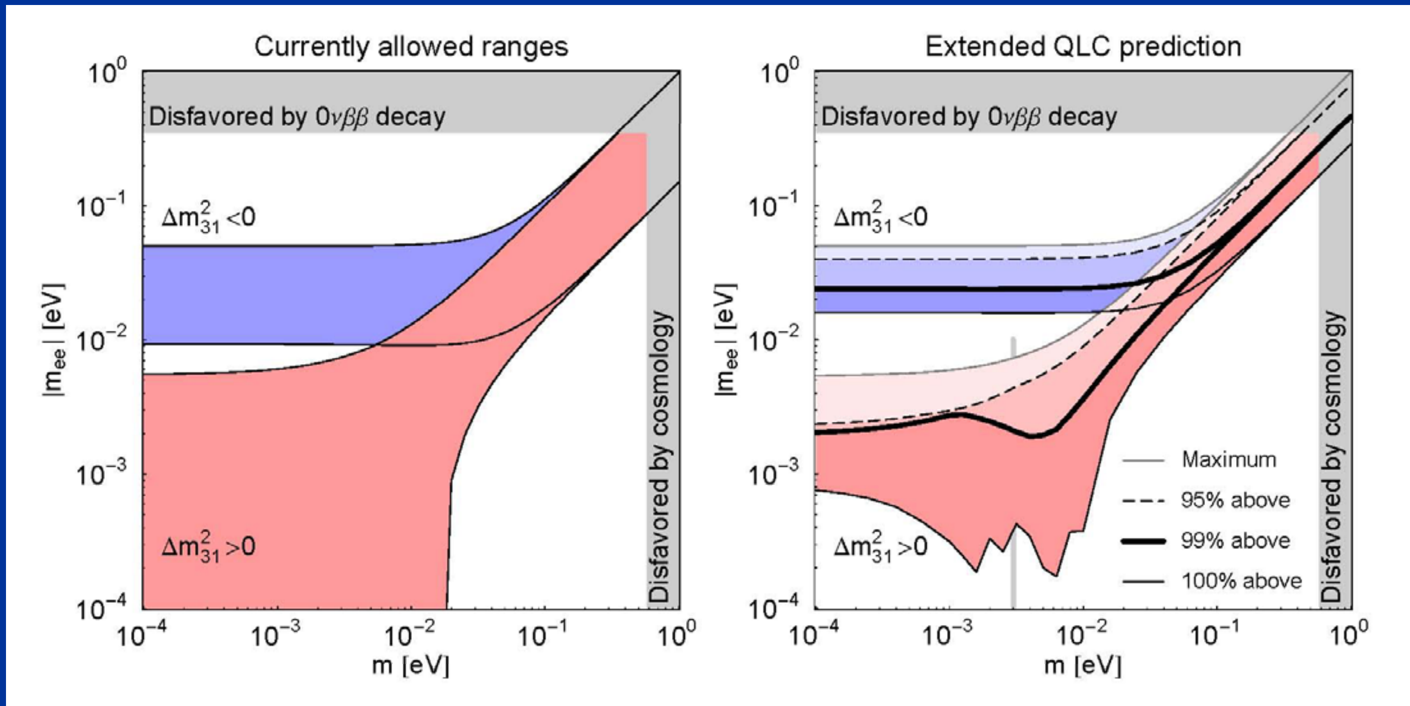
$$M_\nu^{\mathrm{Dirac}} = m_3 \begin{pmatrix} -\frac{\epsilon^2}{\sqrt{2}} & -\frac{\epsilon}{\sqrt{2}} & \epsilon \\ \frac{\epsilon^2}{2} & -\frac{\epsilon}{2} + \frac{\epsilon^2}{2} & \frac{1}{\sqrt{2}} - \frac{\epsilon^2}{2\sqrt{2}} \\ -\frac{\epsilon^2}{2} & \frac{\epsilon}{2} + \frac{\epsilon^2}{2} & \frac{1}{\sqrt{2}} - \frac{\epsilon^2}{2\sqrt{2}} \end{pmatrix} \to \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon^2 & \epsilon & 1 \\ \epsilon^2 & \epsilon & 1 \end{pmatrix},$$

(FP,Seidl,Winter,2006)

Results – 0νββ Decay

- Rate of 0νββ decay is proportional to

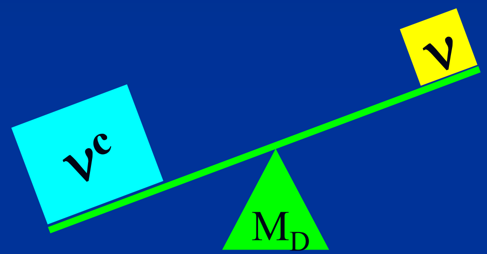
$m_{ee}=|m_1 c_{12}{}^2 c_{13}{}^2| + |m_2 s_{12}{}^2 c_{13}{}^2| e^{2i(\Phi\ -\Phi\ )} + m_3 s_{13}{}^2 e^{-2i\Phi}$



(FP,Seidl,Winter,2006)

Extension to Seesaw Mechanism

- Seesaw (Type I): $M_D = U_D\, M_D^{diag}\, U_{D'}^{+}$ $\quad M_R = U_R\, M_R^{diag}\, U_R^{+}$



$$\mathcal{L}_{mass} \sim \left(\nu, \nu^c\right) \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix}$$

$$M_l = U_l\, M_l^{diag}\, U_{l'}^{+} \qquad U_{l'}^{+} \to \mathbb{1}_{3x3}$$

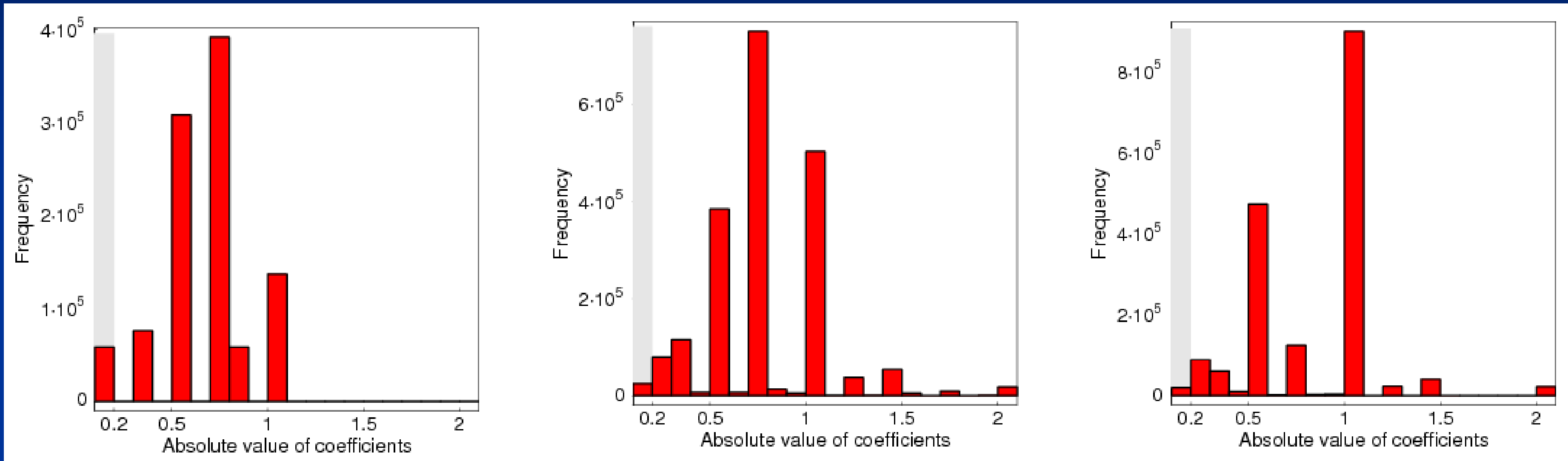
$$\Rightarrow \quad M_{eff}^{Maj} = -M_D\, M_R^{-1}\, M_D^T$$

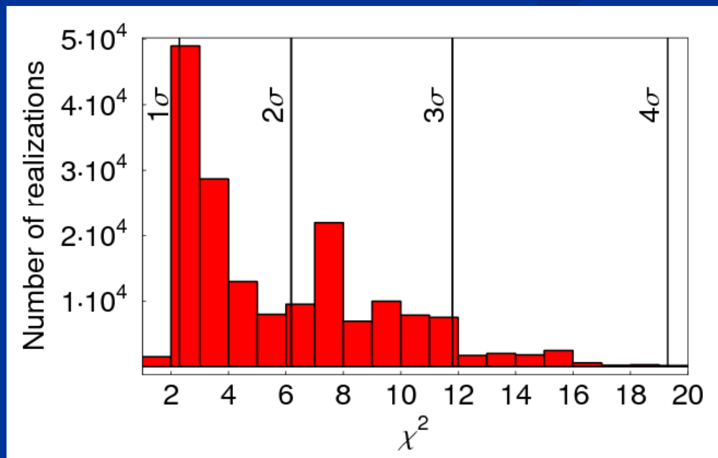
$$U_{PMNS} = U_l^{+} U_\nu$$

$\xrightarrow{\text{ok}}$ Viable realization

$\xrightarrow{\text{not ok}}$ Drop it

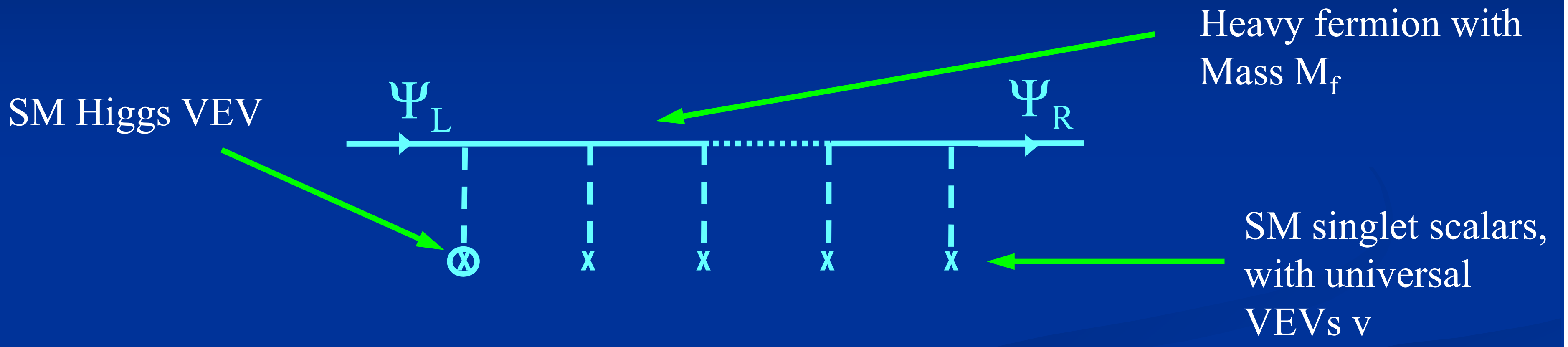
Seesaw Performance





(FP,Seidl,Winter,2007)

Froggatt-Nielsen Mechanism



- $\Psi_{L/R}$ are SM fermions
- v breaks flavor symmetry successively
- After integrating out heavy fermions:

$$\mathcal{L}^{m}{}_{eff} = \langle H \rangle \varepsilon^{n} \overline{\Psi}_{L} \Psi_{R}, \qquad \varepsilon = v/M_{f}$$