SUSY2007 @Karlsruhe Univ.
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# Graphical Representation of SUSY and Application to QFT

S. Ichinose

Univ. of Shizuoka, SFNS, Japan

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# Sec.1 Introduction

How to denote a mathematical (physical) quantity often affects the understanding of its meaning

Ex. A Vector Analysis

$$\nabla \cdot \mathbf{E} \leftrightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\nabla \times \mathbf{E} \leftrightarrow (\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y})$$

Ex. B Differential Form

$$dA \wedge dB \leftrightarrow \epsilon_{ijk} \partial^j A \partial^k B$$

# Ex.C Penrose's spinor notation ('71)

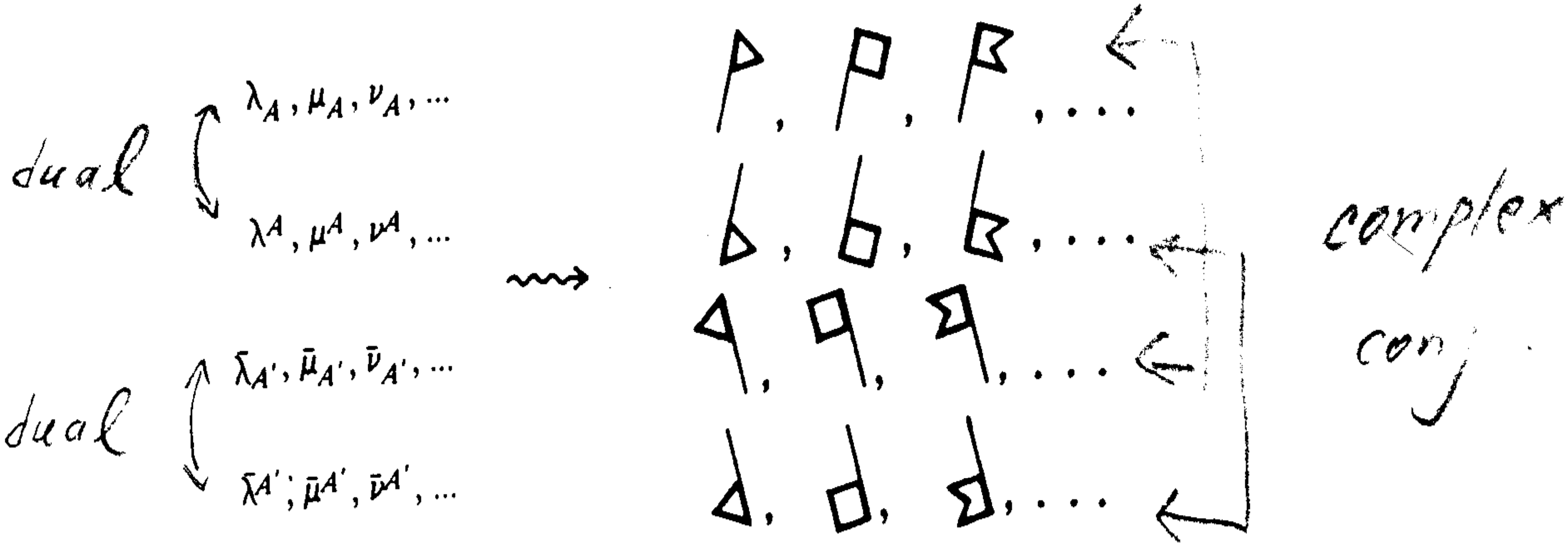



Fig. A-11. Spin-vectors; duals, complex conjugates.

Ex. D Graph. Rep. of Riemann Tensor (S.I. ‘95)

Riemann Tensor $R_{\mu\nu\lambda\sigma}$

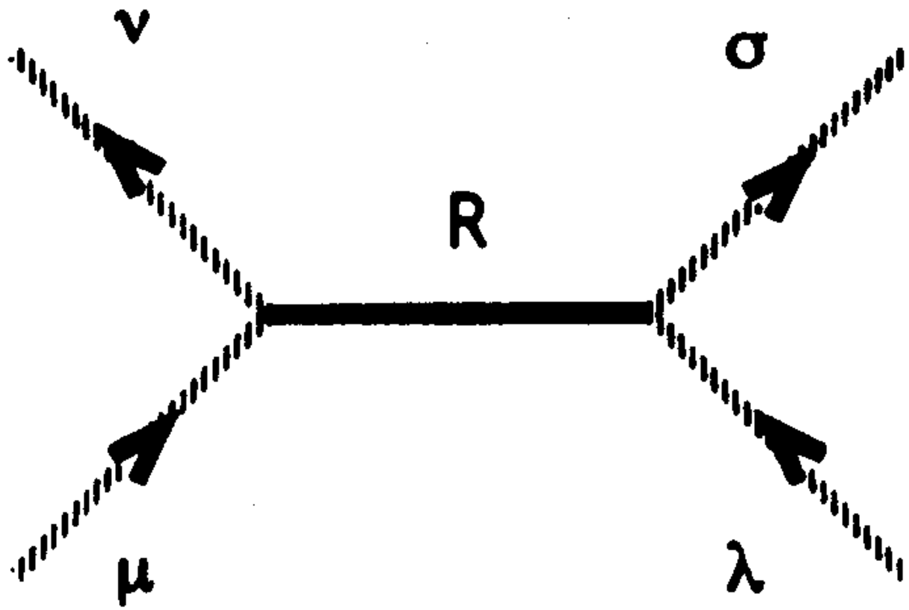



**Figure 1. Graphical representation for the Riemann tensor $R_{\mu\nu\lambda\sigma}$.**

Generally, as the No of suffixes increases, the suffix-free notation is technically advantageous.

# RRR-invariants 1 (on-shell vanishing)

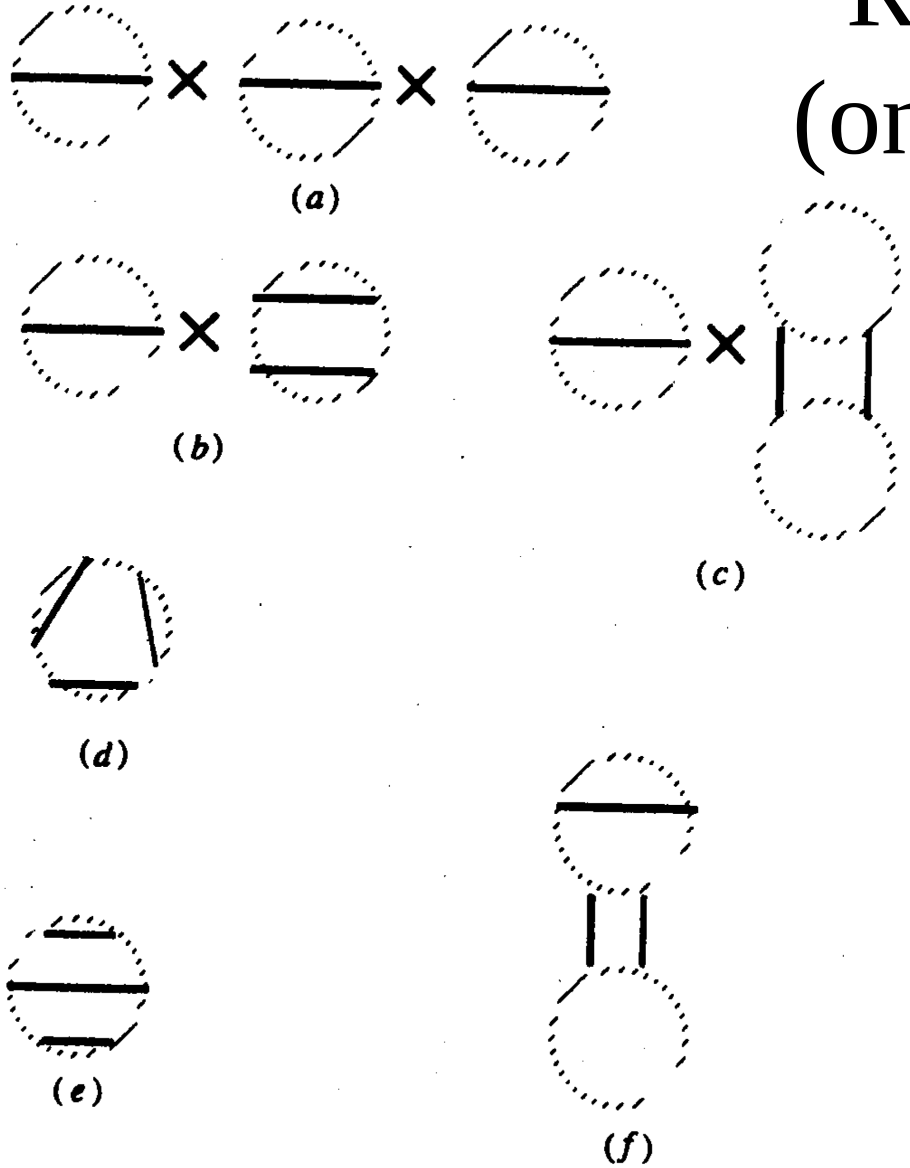



**Figure 15.** The invariants: (*a*) $P_1 = RRR$; (*b*) $P_2 = RR_{\mu\nu}R^{\mu\nu}$; (*c*) $P_3 = RR_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$; (*d*) $P_4 = R_{\mu\nu}R^{\nu\lambda}R_{\lambda}{}^{\mu}$; (*e*) $P_5 = R_{\mu\nu\lambda\sigma}R^{\mu\lambda}R^{\nu\sigma}$; (*f*) $P_6 = R_{\mu\nu\lambda\sigma}R_{\tau}{}^{\nu\lambda\sigma}R^{\mu\tau}$.

# RRR-invariants2

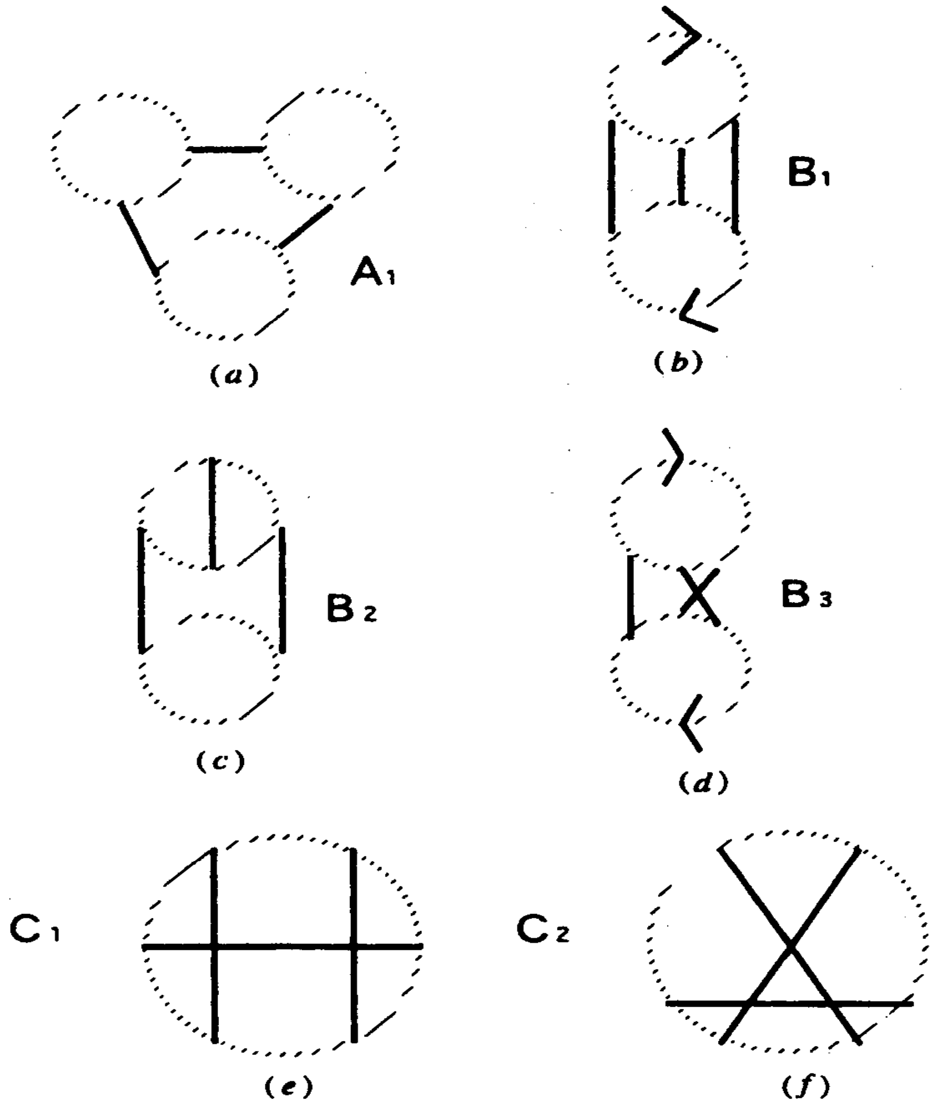



**Figure 16.** The invariants: (*a*) $A_1 = R_{\mu\nu\lambda\sigma}R^{\sigma\lambda}{}_{\tau\omega}R^{\omega\tau\nu\mu}$; (*b*) $B_1 = R_{\mu\nu\tau\sigma}R^{\nu}{}_{\lambda\omega}{}^{\tau}R^{\lambda\mu\sigma\omega}$; (*c*) $B_2 = R_{\mu\nu\omega\tau}R_{\lambda\sigma}{}^{\tau\omega}R^{\sigma\mu\nu\lambda}$; (*d*) $B_3 = R_{\mu\nu\omega\tau}R^{\nu}{}_{\lambda}{}^{\tau}{}_{\sigma}R^{\lambda\mu\sigma\omega}$; (*e*) $C_1 = R_{\mu\nu\sigma\tau}R^{\nu}{}_{\lambda\omega}{}^{\mu}R^{\lambda\sigma\tau\omega}$; (*f*) $C_2 = R_{\mu\nu\sigma\tau}R^{\nu}{}_{\lambda}{}^{\tau}{}_{\omega}R^{\lambda\sigma\omega\mu}$ .

# New Off-shell Relations

*Off-shell relation 1.* Let us consider the identity of figure 17. This idea was explicitly noticed in [GS,FKWC]. The identity figure 17 holds true because each greek suffix runs from 0 to 3 (or from 1 to 4 for Euclidean gravity) in four-dimensional spacetime. This identity turns out to be, by use of a computer†,

$$-P_2 + \tfrac{1}{2}P_3 + 2P_4 - 4P_5 - 5P_6 + A_1 - 2B_1 = 0. \tag{5.2}$$

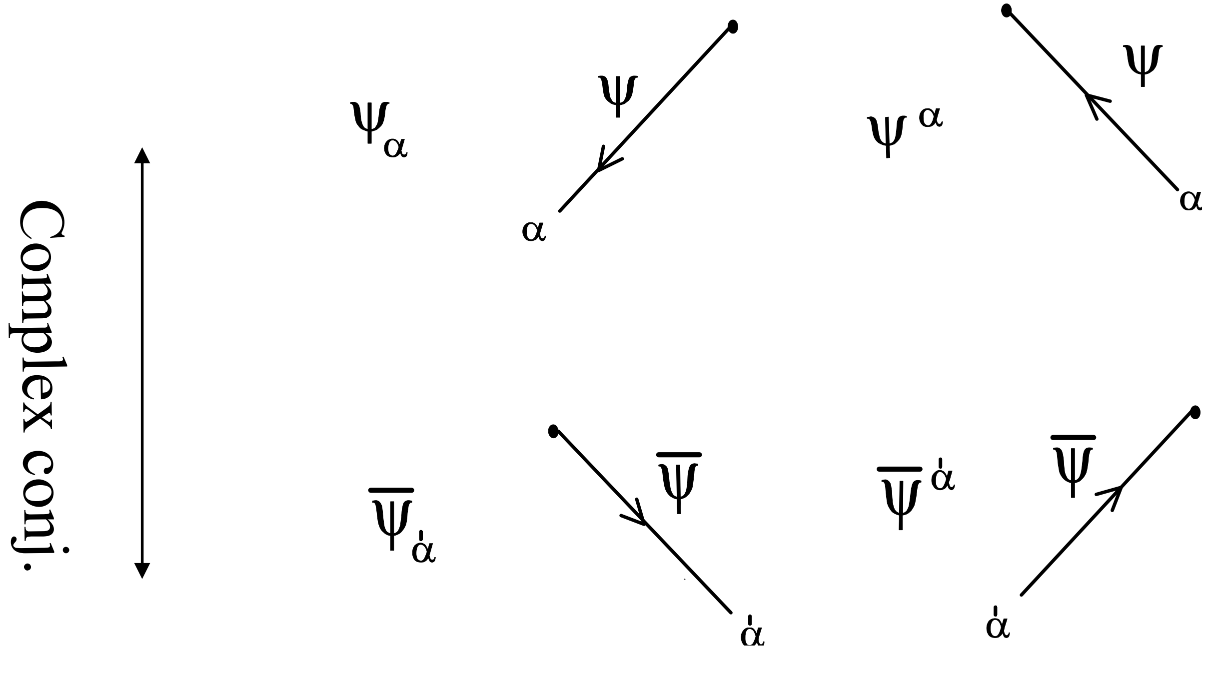
The on-shell case of (5.2), $A_1 = +2B_1$, was obtained in [VW] by use of the spinor formalism.

*Off-shell relation 2.* Similarly we can consider the identity of figure 18. This identity turns out to be, by use of a computer,

$$I \equiv 8(-P_1 + 12P_2 - 3P_3 - 16P_4 + 24P_5 + 24P_6 - 4A_1 + 8B_1) = 0. \tag{5.3}$$

The on-shell case of (5.3) again gives $A_1 = +2B_1$. At the off-shell level, however, (5.2) and (5.3) are independent relations.

# Sec.2 Spinor (Weyl Fermion)




$\epsilon_{\alpha\beta}, \epsilon_{\dot\alpha\dot\beta}$

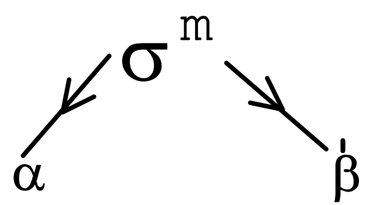
SL(2,C) dual, suffix up-down

# Sigma Matrices

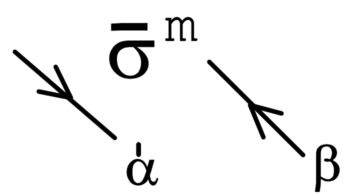
2 by 2 Hermite matrices

Connection between chiral world
and space-time (vector) world

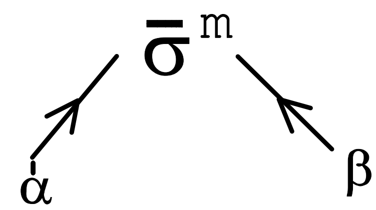
$$(\sigma^m)_{\alpha\dot{\beta}}$$



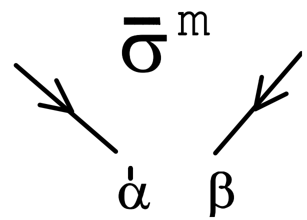

$$(\bar{\sigma}^m)_{\dot{\alpha}}{}^{\beta}$$




$$(\bar{\sigma}^m)^{\dot{\alpha}\beta}$$

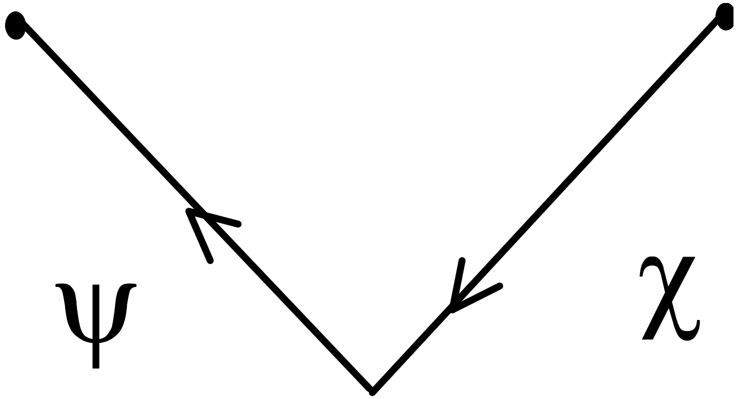



$$(\bar{\sigma}^m)_{\dot{\alpha}\beta}$$

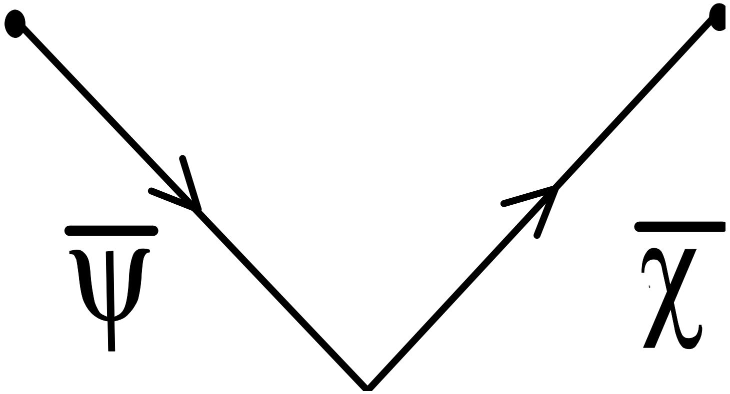


# Spinor Suffix Contraction

$$\psi^{\alpha}\chi_{\alpha}$$



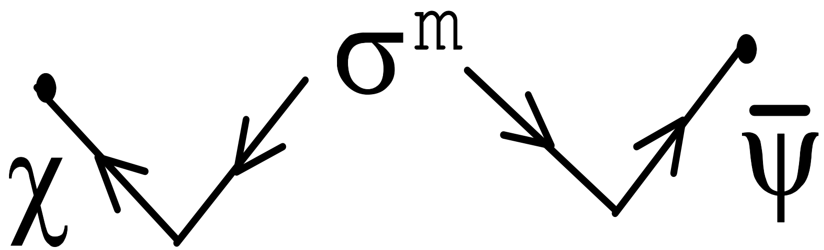

$$\bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}$$



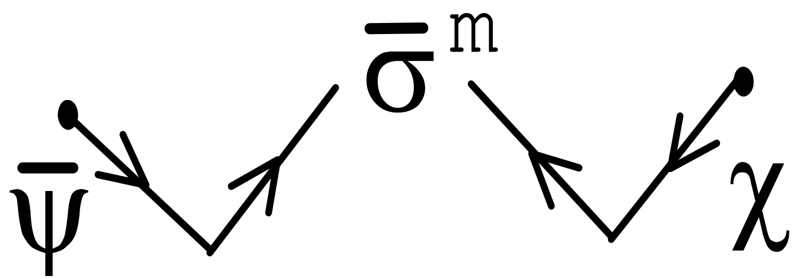

Wedge structure

# Lorenz Vector

$$\chi^{\alpha}(\sigma^{m})_{\alpha\dot{\beta}}\bar{\psi}^{\dot{\beta}}$$



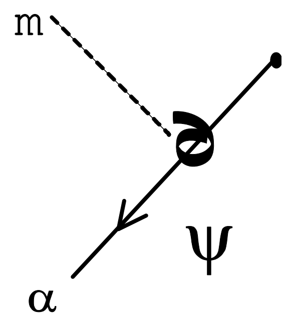

$$\bar{\psi}_{\dot{\alpha}}(\bar{\sigma}^{m})^{\dot{\alpha}\beta}\chi_{\beta}$$



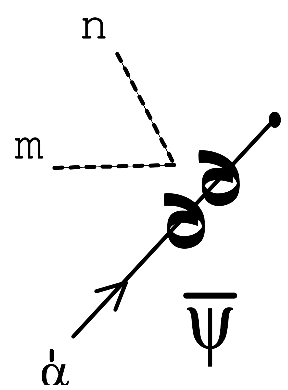

Double wedge structure

# Derivatives of Fermions

$\partial_m \psi_\alpha$

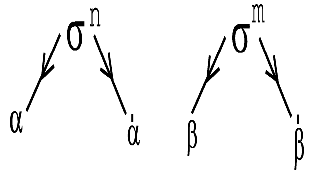



$\partial_m \partial_n \overline{\psi}^{\dot{\alpha}}$

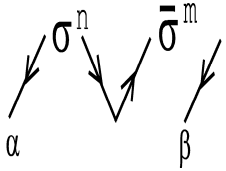


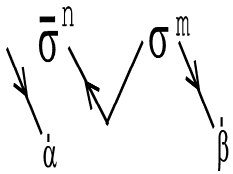
# Sec.3 Graph Relations

## Fierz Identity

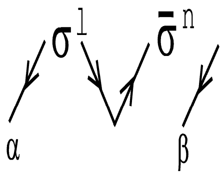


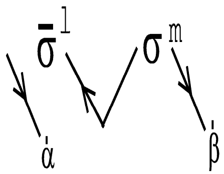

$=$

$\frac{1}{4}\Big\{-$


$\epsilon_{\dot{\alpha}\dot{\beta}} +$


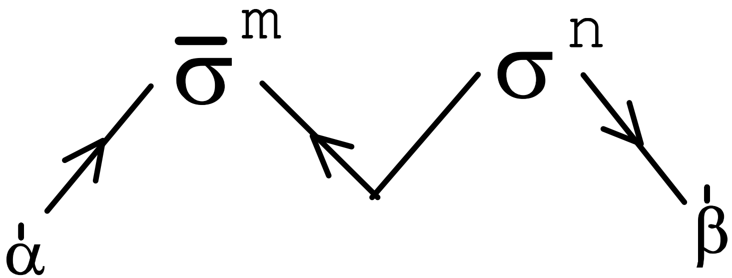
$\epsilon_{\alpha\beta} - m \leftrightarrow n\Big\}$

$-\frac{1}{2}\eta^{nm}\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}} - \frac{1}{8}\Big\{$


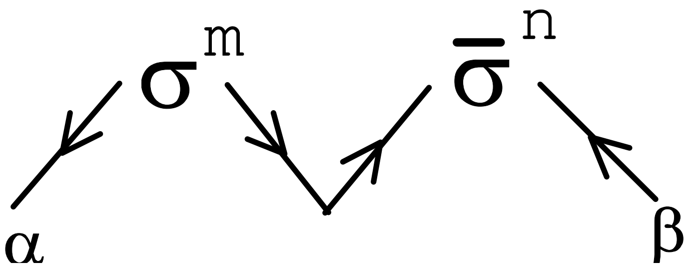
$- l \leftrightarrow n\Big\}\Big\{$


$- l \leftrightarrow m\Big\},$

# 2sigma's

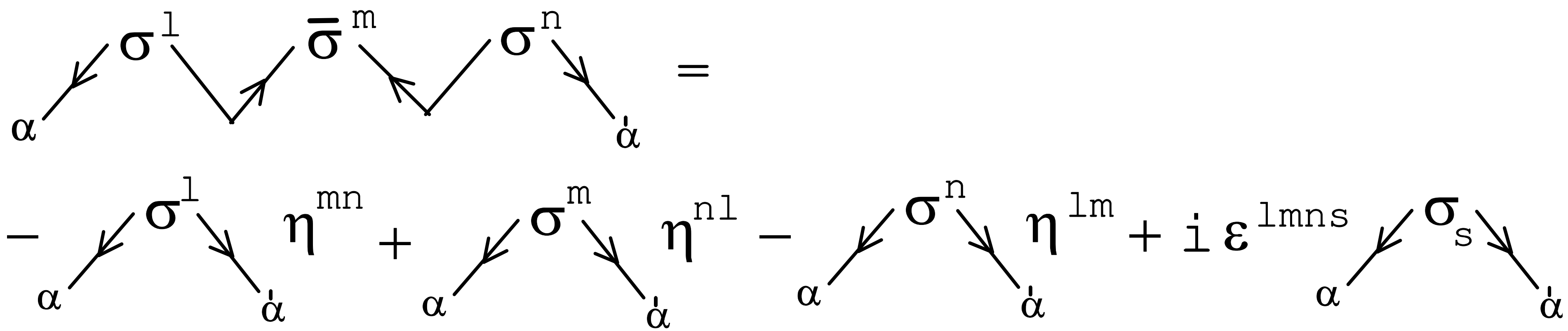



$$+ \quad m \; <--> \; n \; = -2\eta^{mn}\delta^{\dot{\alpha}}_{\dot{\beta}}$$

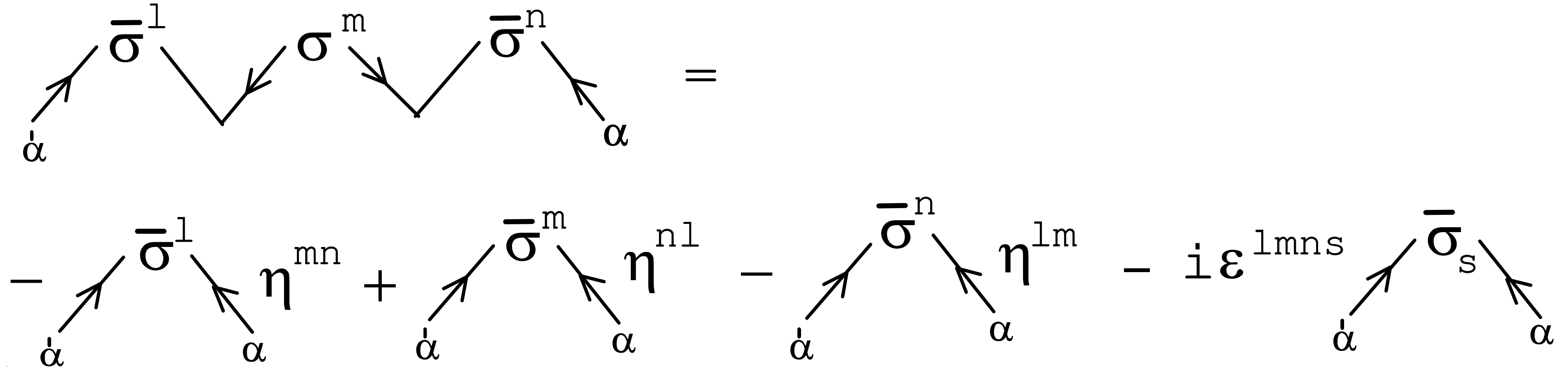



$$+ \quad m \; <--> \; n \; = -2\eta^{mn}\delta^{\beta}_{\alpha}$$

# 3sigma's



σ l σ̄ m σ n
α α̇ =
− σ l η mn + σ m η nl − σ n η lm + i ε lmns σ s
α α̇ α α̇ α α̇ α α̇

σ̄ l σ m σ̄ n
α̇ α =
− σ̄ l η mn + σ̄ m η nl − σ̄ n η lm − i ε lmns σ̄ s
α̇ α α̇ α α̇ α α̇ α

# Sec.4 Graph Indices

Advantage 0. We are free from suffixes

Advantage 1. We can calculate using Graphical Relations

Advantage 2. Graphs are identified by their Graph Indices

Left Chiral Number, Right Chiral Number

Left Up-Down Number, Right Up-Down Number

Left Wedge Number, Right Wedge Number

Dotted Line Number

# Indices of SQED

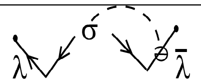
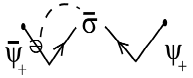
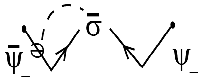
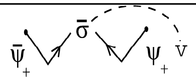
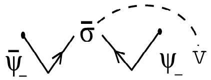
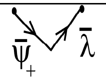
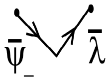
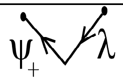
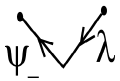
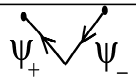
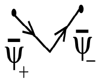
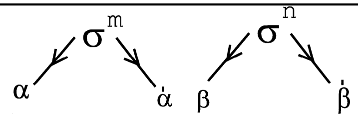
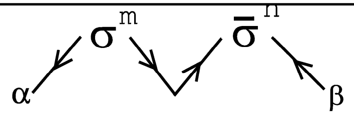
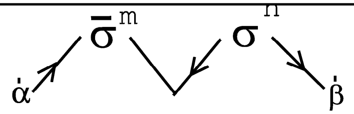
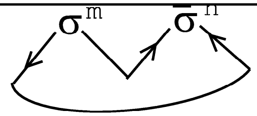
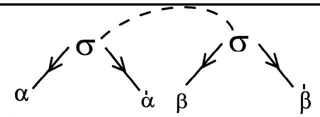
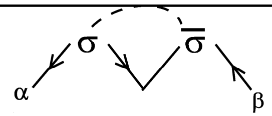
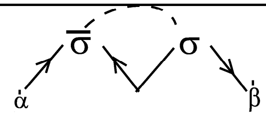
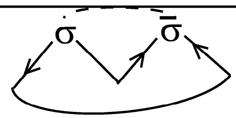
| | | (LCN,RCN) =(LWN,RWN) | (LUDN,RUDN) | DIF | Fields |
|---|---|---|---|---|---|
| 1 | $-i$  | $(1,1)$ | $(0,0)$ | 1 | $\lambda, \bar{\lambda}$ |
| 2 | $i$  | $(1,1)$ | $(0,0)$ | 1 | $\psi_+, \bar{\psi}_+$ |
| 3 | $i$  | $(1,1)$ | $(0,0)$ | 1 | $\psi_-, \bar{\psi}_-$ |
| 4 | $\frac{e}{2}$  | $(1,1)$ | $(0,0)$ | 0 | $\psi_+, \bar{\psi}_+, v^m$ |
| 5 | $-\frac{e}{2}$  | $(1,1)$ | $(0,0)$ | 0 | $\psi_-, \bar{\psi}_-, v^m$ |
| 6 | $-\frac{ie}{\sqrt{2}}A_+$  | $(0,1)$ | $(0,0)$ | 0 | $A_+, \bar{\psi}_+, \bar{\lambda}$ |
| 7 | $+\frac{ie}{\sqrt{2}}A_-$  | $(0,1)$ | $(0,0)$ | 0 | $A_-, \bar{\psi}_-, \bar{\lambda}$ |
| 8 | $+\frac{ie}{\sqrt{2}}A^*_+$  | $(1,0)$ | $(0,0)$ | 0 | $A^*_+, \psi_+, \lambda$ |
| 9 | $-\frac{ie}{\sqrt{2}}A^*_-$  | $(1,0)$ | $(0,0)$ | 0 | $A^*_-, \psi_-, \lambda$ |
| 10 | $-m$  | $(1,0)$ | $(0,0)$ | 0 | $\psi_+, \psi_-$ |
| 11 | $-m$  | $(0,1)$ | $(0,0)$ | 0 | $\bar{\psi}_+, \bar{\psi}_-$ |

Table 3 List of indices for all spinor operators in the super QED Lagrangian. $(\lambda, \bar{\lambda})$: photino; $v^m$: photon; $(\psi_+, \bar{\psi}_+)$: $+e$ chiral fermion; $(\psi_-, \bar{\psi}_-)$: $-e$ chiral fermion.

Classification of 2 Sigma's

| DLN | LWN | RWN | figure |
|---|---|---|---|
| 0 | 0 | 0 |  |
| | 0 | 1 |  |
| | 1 | 0 |  |
| | 1 | 1 |  $= -2\eta^{mn}$ |
| 1 | 0 | 0 |  $= -2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}$ |
| | 0 | 1 |  $= -4\delta_{\alpha}^{\beta}$ |
| | 1 | 0 |  $= -4\delta^{\dot{\alpha}}_{\dot{\beta}}$ |
| | 1 | 1 |  $= -8$ |

TAB 4 Class. of product of 2 sigma's (nsi=2).

# Sec.5 Further Extension

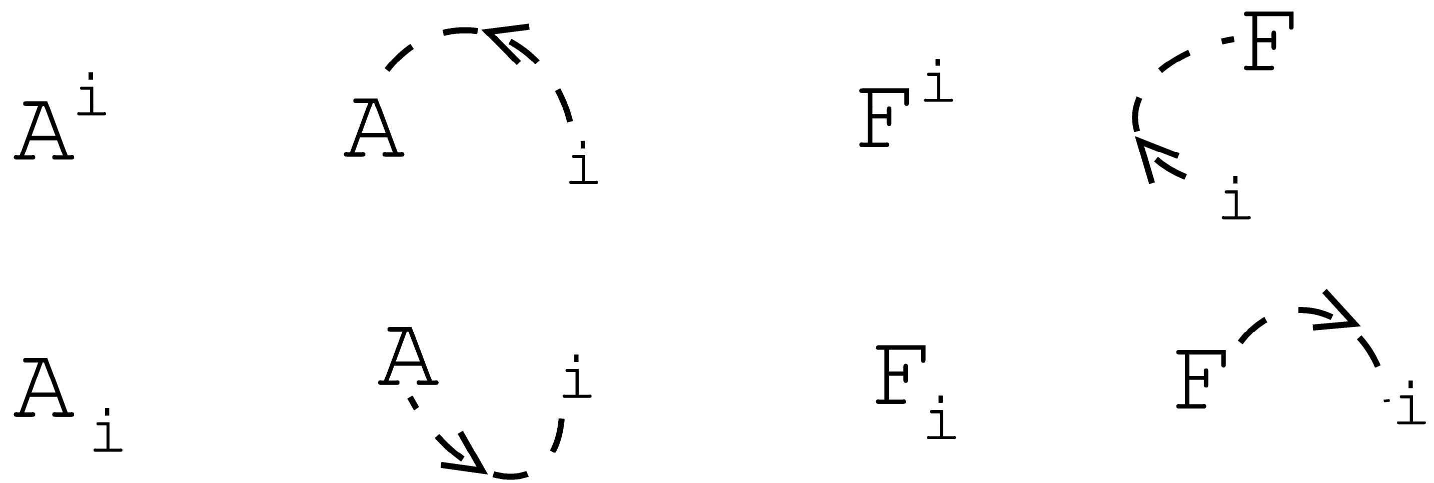
Non Abelian

Higher dimension

5D Hypermultiplet $(A^i, \chi, F_i)$

$A^i$, $F^i$: SU(2)$_R$ doublet of complex scalars

$A_i = \epsilon_{ij} A^j, F^i = \epsilon^{ij} F_j$

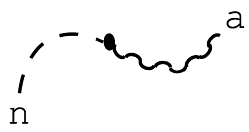


# Gravitational theory

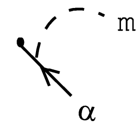
vier-bein

Rarita-Schwinger

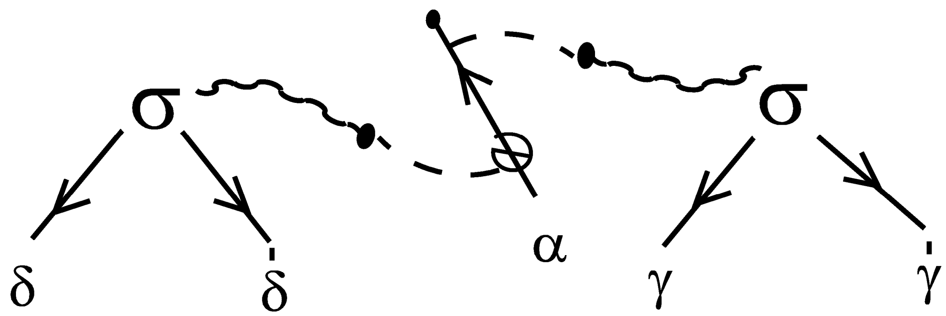
$e^n{}_a$ :




$\psi_m{}^\alpha$ :




$$\psi_{\delta\dot{\delta}\gamma\dot{\gamma}\alpha} = (\sigma^d)_{\delta\dot{\delta}}(\sigma^c)_{\gamma\dot{\gamma}} e_d{}^n e_c{}^m (\psi_{nm})_\alpha, \ (\psi_{nm})^\alpha = \partial_n \psi_m^\alpha + \cdots - n \leftrightarrow m.$$




$$+ \cdots \quad ,$$

# Sec.6 Input-Output Sample

## Input

2

6 $\Phi^\dagger$

0 -1 4 t 0 0 1 s 0 1 1 1 1 2 51 t 1 0 2 B 1 51

1 0 5 t 0 0 1 t 0 1 1 t 1 1 2 t 1 0 2 C 1 1

2 0 2 t 1 1 2 p 1 0 2

0 1 5 t 1 1 2 t 1 0 2 t 0 0 1 s 0 1 1 1 1 4 51 q 1 0 4 51

1 0 3 t 1 1 2 t 1 0 2 F 1 1

1 0 1 A 1 1

6 $\Phi$

0 -1 4 t 0 0 1 s 0 1 1 1 1 2 51 t

1 0 2 B 1 51

$$\Phi^\dagger = -i\theta^1(\sigma^{51})_{12}\bar{\theta}^2\ \partial_{51}A^* + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\ \partial^2 A^* + \cdots$$

# Output

T[0]=2 T[1]=3

****** TERMSCOMBINE ****
th*th*thbar*thbar-term

$$\theta^2\bar{\theta}^2$$

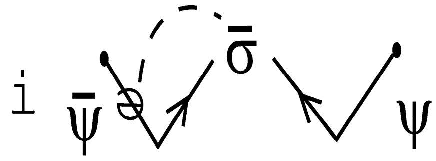
****** SORTOUTthBth ****
lab50 at thBthBthth, Final result
weight= 0+i(-2)  PlusMinus= 4  Sign=0  Nthth=1
NthBthB=1  Nhalf=1

$$(-2i) \times (-1)^{4+0} \times \frac{1}{2} = -i$$

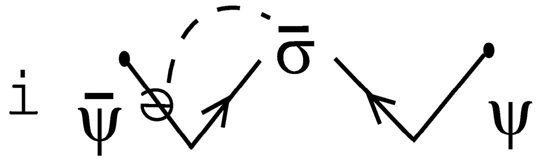
****** SigmaContraction ****
SigGraN=1  FinalOutPut: MultiFac=1 + i(0)
type2[c=4]= s  si2[c=4,0,1]=7  si2[c=4,1,1]=2
siv2[c=4]=52  type2[c=5]= p  psi2[c=5,1,0]=2
type2[c=6]= q  dps2[c=6,0,0]=7  dpsv2[c=6]=52

T[0]=3 T[1]=2

$$1 \times \sigma^{52}{}_{72}\bar{\psi}^2\partial^{52}\psi^7 =$$



# Final Output

$\mathcal{L} =$ 

$+ A^* \partial^2 A + F^* F$

$$= i \partial_n \bar{\psi}_{\dot{\alpha}} (\bar{\sigma}^n)^{\dot{\alpha}\beta} \psi_\beta + A^* \partial^2 A + F^* F$$

# Sec.6 Conclusion

We report the present status of Graphical Representation of SUSY. Outline is finished. However it still needs further development of programming for the ‘public’ use.

GOAL

1. It can do the transformation between the superfield expression and the component expression.

2. It can do the SUSY transformation of various quantities. In particular it can confirm the SUSY-invariance of the Lagrangian in the graphical way and give the final total divergence.

3. It can do algebraic SUSY calculation involving $D_\alpha, \bar{D}^{\dot{\alpha}}, Q_\alpha$ and $\bar{Q}^{\dot{\alpha}}$.