INFLATION &

UNIFICATION

- INTRODUCTION
- NON-SUPERSYMMETRIC MODELS
- SUPERSYMMMETRIC MODELS
- LEPTOGENESIS
- SUMMARY

Books / Reviews

Kolb & Turner	Lyth & Riotto
Linde	Lazarides
Liddle & Lyth	

Talks at this meeting:

Kolb, Mazumdar, Takahashi, Yamaguchi, ...

Standard Model (SM)

+ Einstein' GR

⇒ Hot Big Bang Cosmology

Predictions

- Existence of CMB;
- Redshift (Galaxies);
- Primordial Nucleosynthesis.

BUT IT FAILS TO EXPLAIN:

1) Observed Isotropy of CMB

(COBE)

2) Origin of $\delta T/T$ – COBE, ..., WMAP

3) $\Omega_{TOTAL} = 1$ (critical density)

4) $\Omega_{CDM} \approx 0.22$ (non-baryonic DM)

5) $n_b/n_\gamma \approx 10^{-10}$ (baryon asymmetry)

- If GR stays intact, an extension of the SM is needed.

(Dark Energy ?)

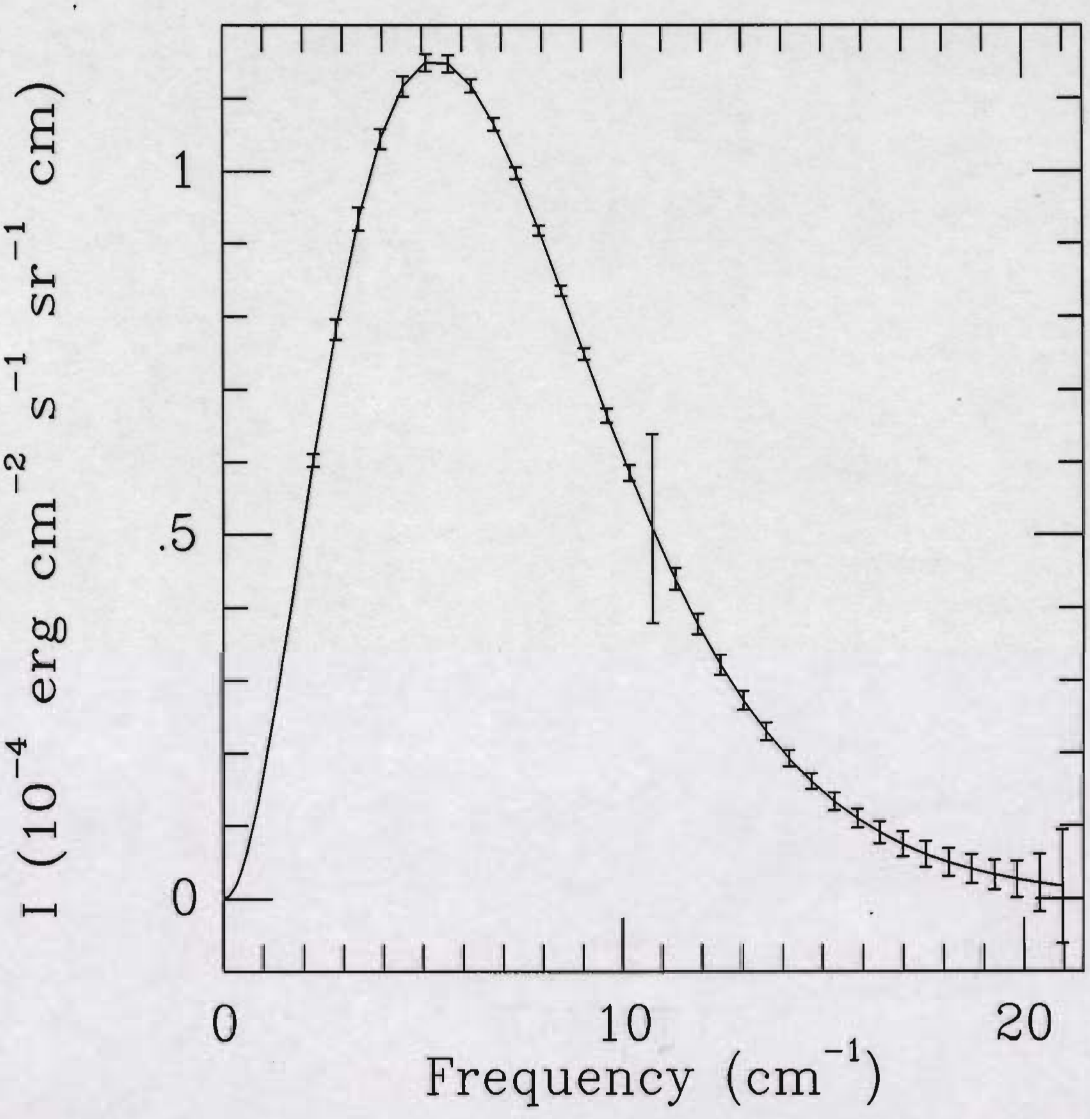


Figure 4: Spectrum of the Cosmic Microwave Background Radiation as measured by the FIRAS instrument on COBE and a black body curve for $T = 2.7277\,\mathrm{K}$. Note, the error flags have been enlarged by a factor of 400. Any distortions from the Planck curve are less than 0.005% (see Fixsen *et al.* , 1996).

Hinshaw et al. 2006

(ΛCDM)

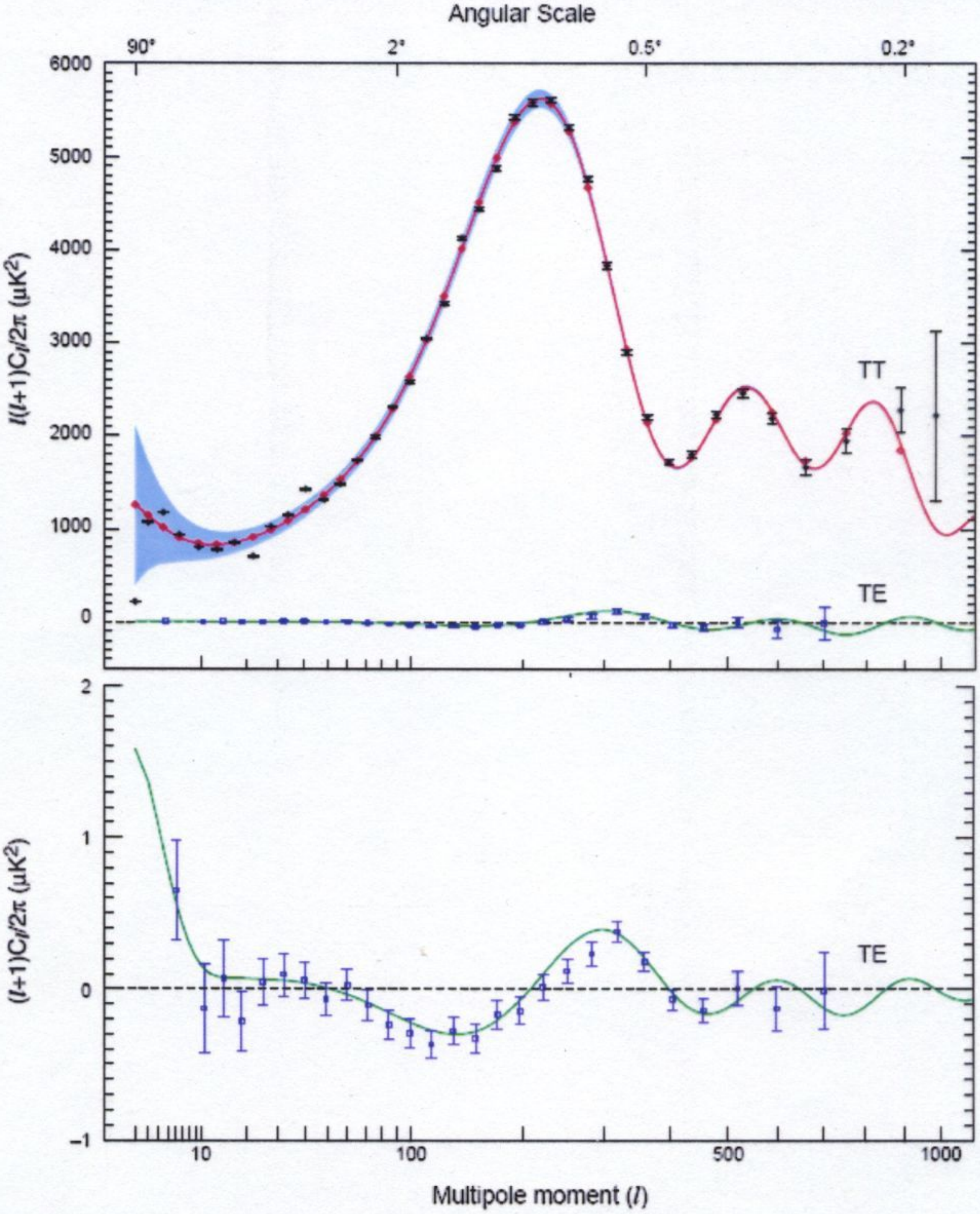


Fig. 22.— Angular power spectra C_l^{TT} & C_l^{TE} from the three-year *WMAP* data. *top*: The TT data are as shown in Figure 16. The TE data are shown in units of $l(l+1)C_l/2\pi$, on the same scale as the TT signal for comparison. *bottom*: The TE data, in units of $(l+1)C_l/2\pi$. This updates Figure 12 of Bennett et al. (2003b).

6) Indeed, Neutrino Oscillations also require an extension of the SM :

Atmospheric ν Oscillations :

$$\Rightarrow |\Delta m_{ATM}| \approx 0.05\,eV,$$

Solar ν Oscillations :

$$|\Delta m_{SOLAR}| \approx 0.01\,eV\,;$$

But $|\Delta m_{\nu}|_{SM} \lesssim 0.00001\,eV$

(Dim 5 Ops. $(LH)^2$))

- Inflationary Cosmology can take care of (1), (2), (3), and an inflation model can be called realistic if it can explain (4) → CDM and (5) → n_b/n_γ. Some models also provide a link with (6) → neutrino physics.

- Testable predictions?

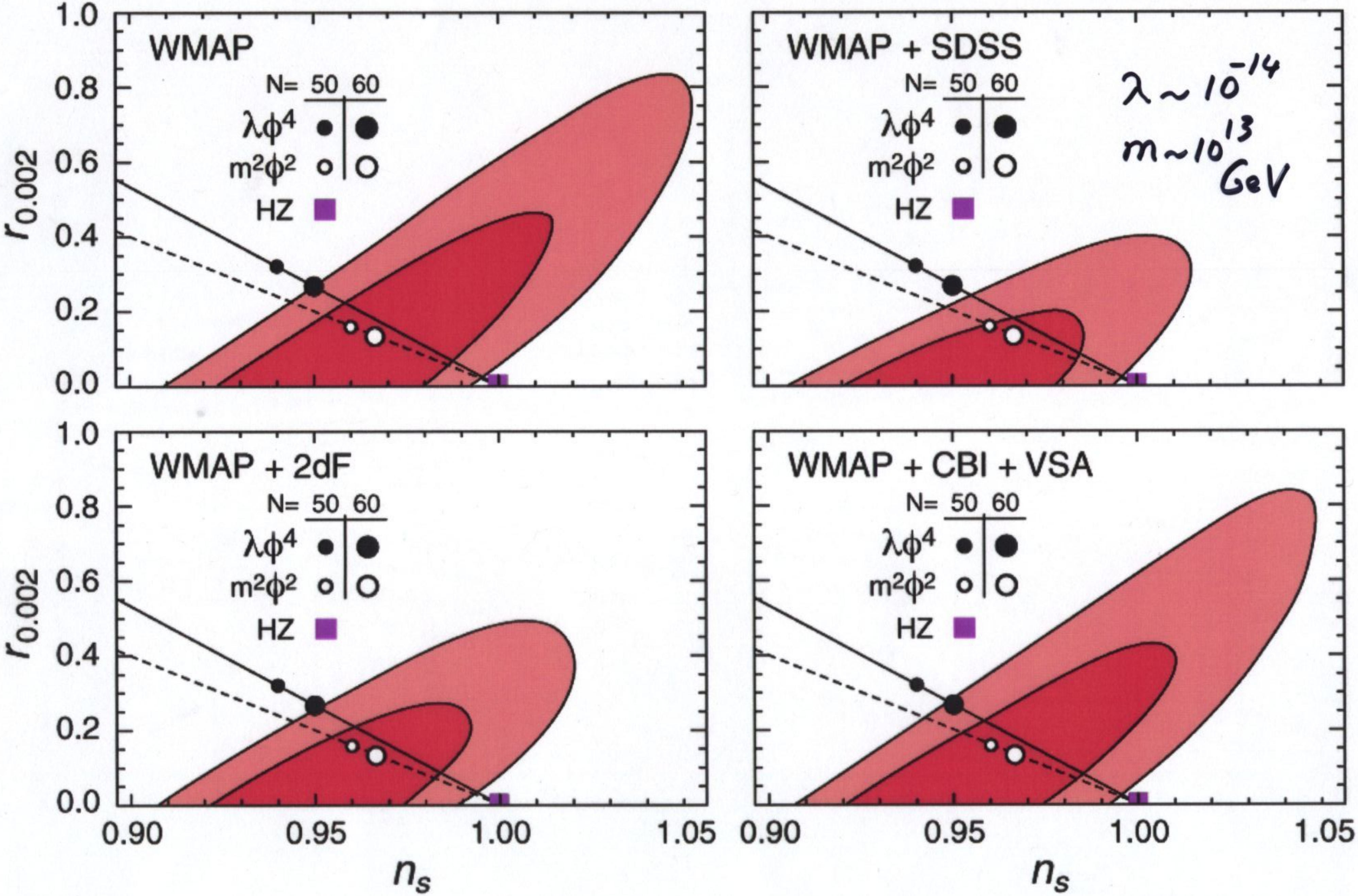
- One key parameter in cosmology is the scalar spectral index n_s.

According to Harrison & Zeldovich (HZ), $n_s = 1$ is the most 'natural' value, referred to as the scale invariant value.

- The most recent analysis from WMAP yields $n_s \approx 0.95 \pm 0.03$

($WMAP\,1$: $n_s \approx 0.99 \pm 0.04$)

A far more precise determination of n_s is crucial for distinguishing inflation models.



$\lambda \sim 10^{-14}$

$m \sim 10^{13}$ GeV

- Inflation Models come in a variety of flavors. These include

  Chaotic Inflation (Linde, ..., Murayama, ... Yanagida)

  New Inflation (Linde, Albrecht, Steinhardt, ..., Senoguz, ...)

  Hybrid Inflation (non-susy, susy)

  Supergravity Inflation

  Brane Inflation (Dvali, Tye, ...)

  Compactification (Arkani-Hamed et. al, ... Schmidt et. al, ...)

  Quintessence/Inflation

  ⋮

In this talk I will assume that inflation is associated with some symmetry breaking (phase transition) in the early universe. This is motivated as follows:

- SM gives rise to phase transitions:

  Electroweak ($T_c \sim 100$ GeV) $\to SU(2) \times U(1) \to U(1)_{em}$

  QCD ($T_c \sim 100$ MeV) $\hookrightarrow$ Confinement

- SM gauge symmetry is part of some larger symmetry. Examples:

  SU(5), $L \leftrightarrow R$ models, Extra dimensions;

  Global symmetries ($U(1)_{axion}, \ldots$)

To keep the discussion as simple as possible, consider $U(1)_{B-L}$, an accidental global symmetry of the SM.

(Similar discussion holds if $U(1)_{B-L} \to U(1)_{axion}$)

- We require that $U(1)_{B-L}$ is spontaneously broken and introduce the coupling

$$y_{ij} N_i N_j \phi \quad ;$$

Here $\langle \phi \rangle$ breaks $U(1)_{B-L}$ and also provides masses for the right-handed neutrinos N_i.

- ϕ is going to be the inflaton and, since it couples to N_i, the observed baryon asymmetry arises via leptogenesis.

Quartic (CW) Potential (non-SUSY)

$$V(\phi) = A\phi^4\left[\ln\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + \frac{AM^4}{4}$$

gauge singlet

$$V(\phi = M) = 0\ ;\ V(\phi=0) = AM^4/4 \equiv V_0$$

$$V(\phi \ll M) \approx \frac{AM^4}{4} - b\phi^4$$

- For $V_0^{1/4} < 10^{16}$ GeV, $\phi < m_p$ ($\simeq 2.4\times 10^{18}$ GeV)

Model behaves as for $V \simeq V_0(1 - (\phi/\mu)^4)$

$$n_s \simeq 1 - \frac{3}{N_0}\ ,\quad \alpha \simeq (n_s - 1)/N_0$$

e-folds for $k_0 = 0.002\ \mathrm{Mpc}^{-1}$

($V_0^{1/4} > 10^5$ GeV to avoid conflict with WMAP)

- For $V_0^{1/4} \gtrsim 10^{16}$ GeV, $\phi > m_P$ during observable inflation.

Predictions approach that of ϕ^2 potential, with

$$n_s = 1 - \frac{2}{N_0} \simeq 0.96$$

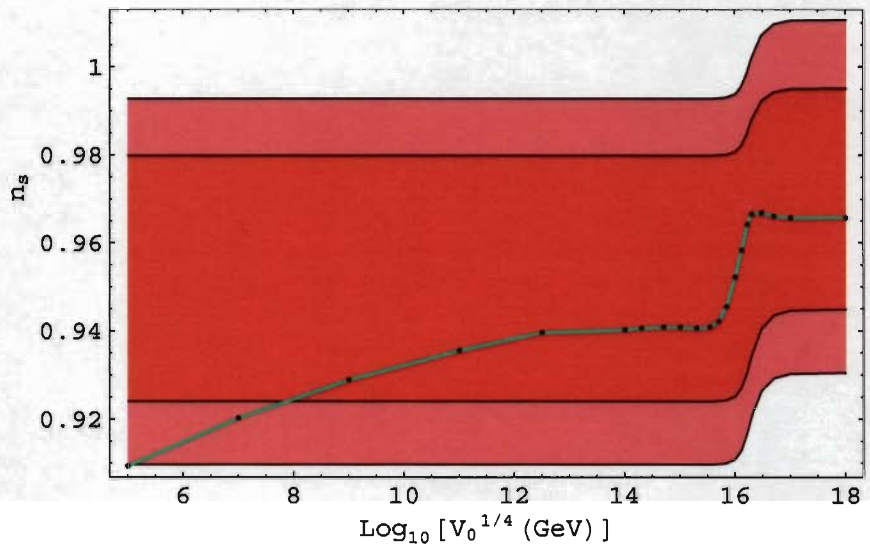
$$r \simeq 0.13$$

$$\alpha \simeq -0.6 \times 10^{-3}$$

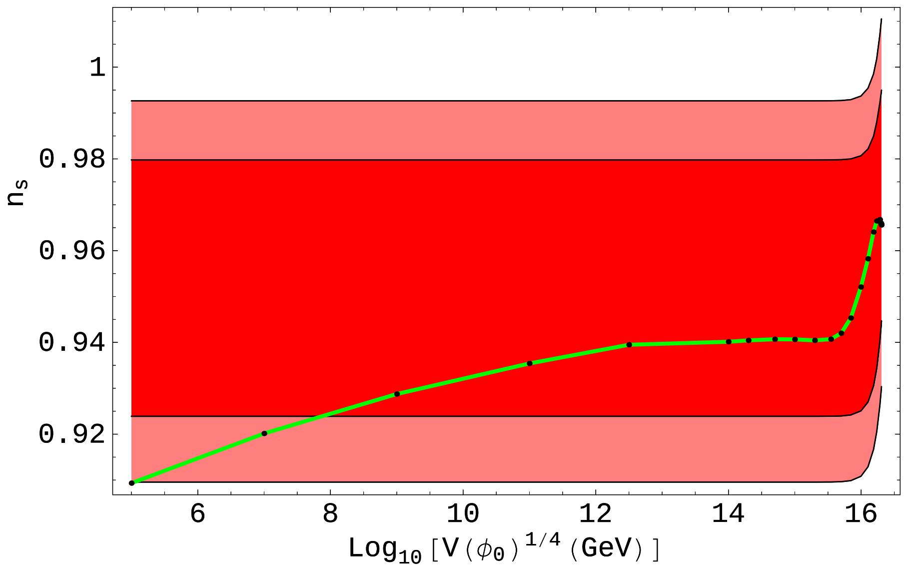
? Where does ϕ come from

Breaks global $U(1)_{B-L}$ (SM)

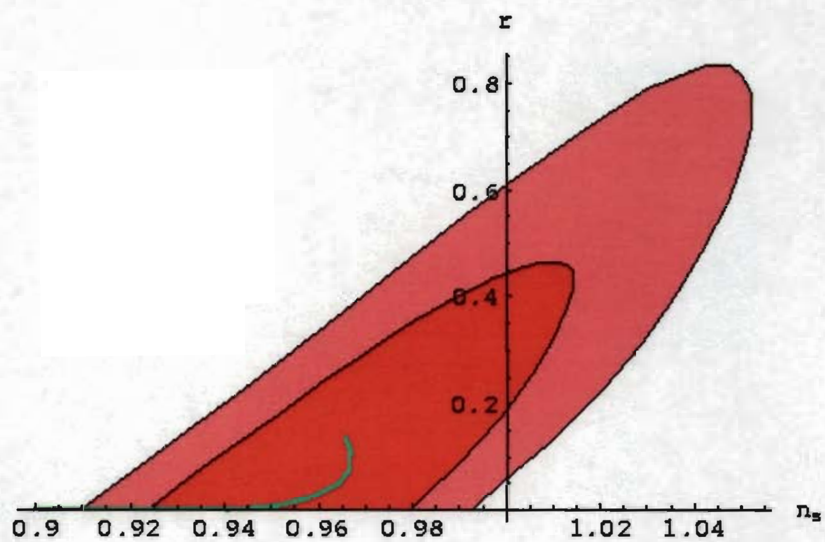
$U(1)_{PQ}$



The spectral index n_s vs $\log[V_0^{1/4}\,(\mathrm{GeV})]$ for the Coleman-Weinberg potential (green curve), compared with the WMAP range for n_s (68% and 95% confidence levels, taken from Spergel *et al.*, astro-ph/0603449). Note that the tensor to scalar ratio $r \approx 0$ for $V_0^{1/4} \ll 10^{16}$ GeV and $r \approx 0.14$ for $V_0^{1/4} \gg 10^{16}$ GeV.



The spectral index n_s vs $\log[V(\phi_0)^{1/4}\,(\mathrm{GeV})]$ for the Coleman-Weinberg potential (green curve), compared with the WMAP range for n_s (68% and 95% confidence levels, taken from Spergel *et al.*, astro-ph/0603449). Note that the tensor to scalar ratio $r \approx 0$ for $V(\phi_0)^{1/4} \ll 10^{16}$ GeV and $r \approx 0.14$ for $V(\phi_0)^{1/4} \approx 2 \times 10^{16}$ GeV.



The tensor to scalar ratio r vs the spectral index n_s for the Coleman-Weinberg potential (green curve). The WMAP contours (68% and 95% confidence levels) are taken from Spergel *et al.*, astro-ph/0603449.

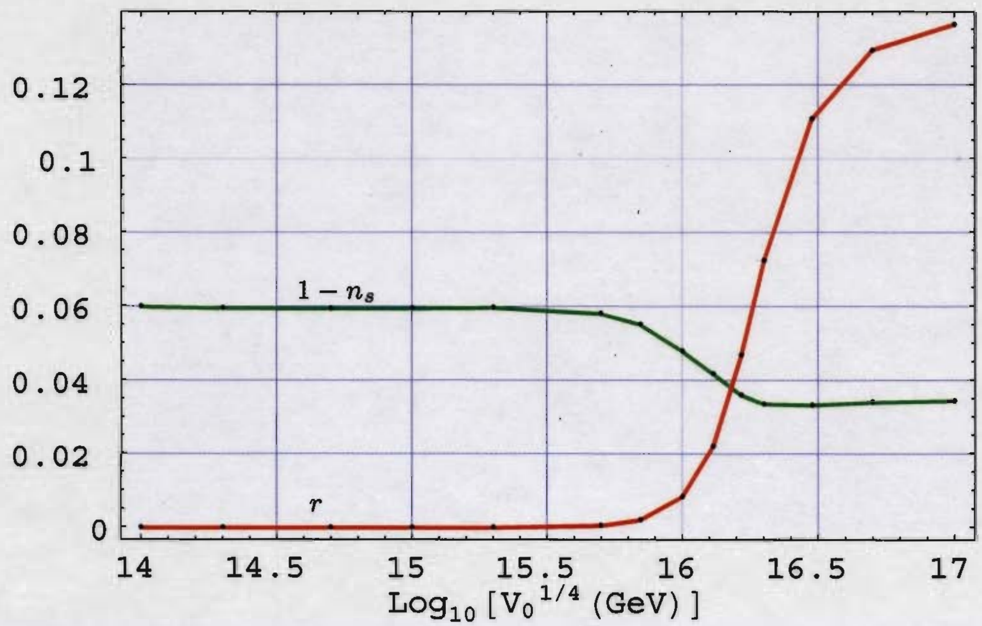


FIG. 1: $1-n_s$ and r vs. $\log[V_0^{1/4}\,(\mathrm{GeV})]$ for Coleman-Weinberg potential.

$$r \equiv (\Delta T/T)_T^2 \,/\, (\Delta T/T)_S^2$$

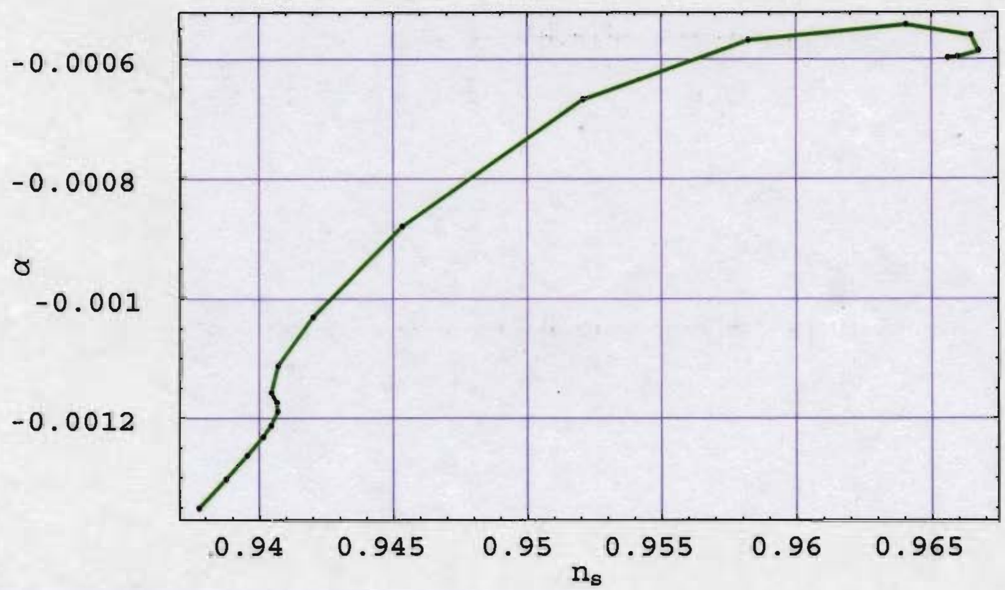


FIG. 2: α vs. n_s for the Coleman-Weinberg potential.

$$\alpha \equiv dn_s / d\ln k$$

- For the next inflationary model, let us take $U(1)_{B-L}$ to be a local (instead of global) symmetry.

- It is also helpful (perhaps even necessary?), to introduce susy in order to construct realistic models;
  {when local symmetries are associated with inflation.}

Potential Hierarchy Problem(s)

- $M_W \ll M_{P}$ (lanck)
- $M_{B-L} \ll M_P$ (ν Osc)
- f_a(axion) $\sim 10^{11} - 10^{12}$ GeV
- $m_{inflaton} \ll M_{GUT}$ ($\delta\rho/\rho$, T_r, ...)

MSSM

A compelling extension of the SM:

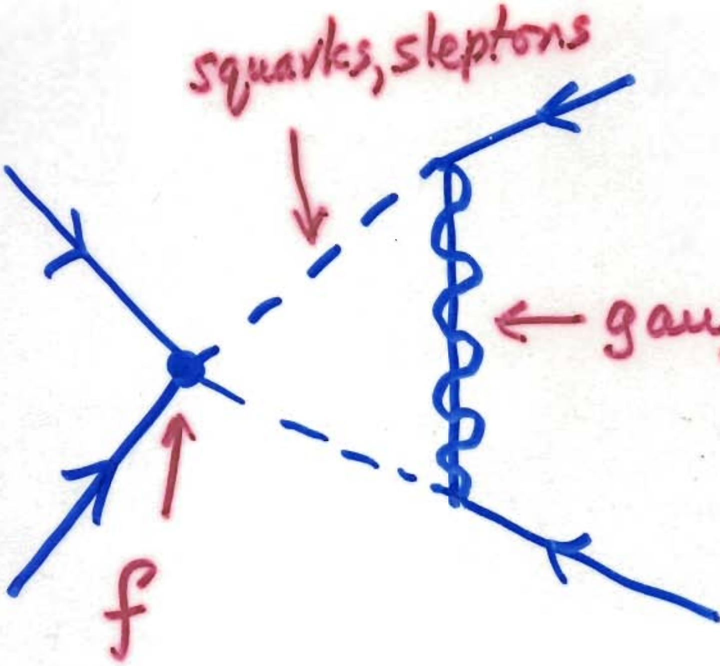
- 'Resolution' of the gauge hierarchy problem;
- Dark matter candidate (LSP);
- Unification of gauge couplings;
- (Rich phenomenology (LHC))
- → Inflation plausible (Bellido et. al.; $n_s \approx 0.92$)
- Baryogenesis from MSSM?

- Z_2 'matter' parity eliminates rapid proton decay and delivers cold dark matter (LSP).

- But dim 5 p decay is still problematic because of superpotential terms such as

$$\frac{f}{M_P} Q\,Q\,Q\,L \quad \left(\text{allowed by } U(1)_{B-L}\right)$$

$$\left\{ \text{Cf: } \frac{1}{M_P^2}\, q\,q\,q\,\ell \text{ in SM} \Rightarrow \tau_p \gtrsim 10^{42}\,\text{yr} \right\}$$



Require $\tau_p \gtrsim 10^{33}$ yrs

$\Rightarrow$ Coefficient $f \lesssim 10^{-7}$ (how?)

One solution:

$$\mathbb{Z}_2 \subset U(1)_R$$

Under R: Q, L carry charge $1/2$
& W (superpotential)
has unit charge.

(... invariant)

- Another good reason to include $U(1)_R$:

It excludes the MSSM term $\mu H_u H_d$ (μ, in principle, of order M_P).

(Clearly, matter parity does not.)

BOTTOM-UP APPROACH

Gauged $U(1)_{B-L}$ (SM: global $U(1)_{B-L}$)

- Anomaly cancellation
  $\Rightarrow$ 3 (right handed) ν_R

- Seesaw Mechanism (light ν masses)
  $\downarrow$
- Leptogenesis $\rightarrow$ baryon asymmetry

? B-L breaking scale

Working with MSSM $\times$

$U(1)_{B-L} \times U(1)_R$,

one can

- easily realize inflation;

- determine M_{B-L} (from $\delta T/T$)

  not possible from gauge coupling unification?

- put bounds on right handed ν masses;

- resolve μ problem;

- explain n_b/s via leptogenesis.

Starting theory quite simple:

$$W = W_{MSSM} + W_{B-L}$$

(except for μ term)

$$W_{B-L} \supset \kappa S(\bar{\phi}\phi - M_{B-L}^2)$$

(Consistent with $U(1)_R$)

Higher order terms can be added, and indeed are useful if topological defects pose a problem. This leads to:

Shifted inflation ($W \supset S(\bar{\phi}\phi)^2/M_*^2$)

Smooth inflation

Inflation (101)

(inflation linked to some phase transition)

Recall that we deal with spontaneously broken gauge theories:

Example: $\mathcal{U}(1)_{B-L} \xrightarrow{\langle\phi\rangle} \mathbb{1}$

e.g. supercond.

Consider the potential energy density

dimensionless

$$V \sim \kappa^2 |\phi^*\phi - M^2_{B-L}|^2 + 2\kappa^2 S^2 |\phi|^2$$

req'd by susy

Minimum corresponds to

$$|\phi| = M_{B-L}, \quad \langle S \rangle = 0 \quad (\Rightarrow V = 0)$$

$\Rightarrow \mathcal{U}(1)$ spontaneously broken (today!)

To realize an inflationary epoch, some fields must be displaced from their present positions.

Think of S as temperature (≈ 0 at present).

For $S \gg M_{B-L}$ (very early universe), we should have $|\phi| = 0$, so that

$$V \sim \kappa^2 M_{B-L}^4$$

symmetry restoration (Cf: superconductor)

Thus, we expect to have gauge symmetry restoration above some 'critical' temperature $S_c > M_{B-L}$

A non-zero vacuum energy density $\Rightarrow$ exponential expansion of the universe $\Rightarrow$ inflation.

? How does inflation terminate

- Through quantum corrections that are calculable;
- Soft susy breaking terms;

In computing the potential

one includes:

Radiatuie corrections,

SUGRA corrections / Softterms;

(remarkably, with minimal

Kähler potential, the S field

remains light ($\ll H$) so that

inflation can be realized.)

[For $\kappa \lesssim 10^{-4}$
soft terms may
dominate the
inflationary potential]

With minimal Kähler potential the inflationary potential is given by

SUGRA

soft terms

$$V = \kappa^2 M_{B-L}^4 \Big[1 + \frac{|S|^4}{2 m_p^4} + a m_{3/2} \kappa M_{B-L}^2 |S|$$

$$+ \frac{\kappa^2 \mathcal{N}}{32\pi^2} \Big(2 \ln \frac{\kappa^2 |S|^2}{\Lambda^2} + (z+1)^2 \ln (1+z^{-1})$$

radiative corr^ns

$$+ (z-1)^2 \ln (1-z^{-1}) \Big) \Big],$$

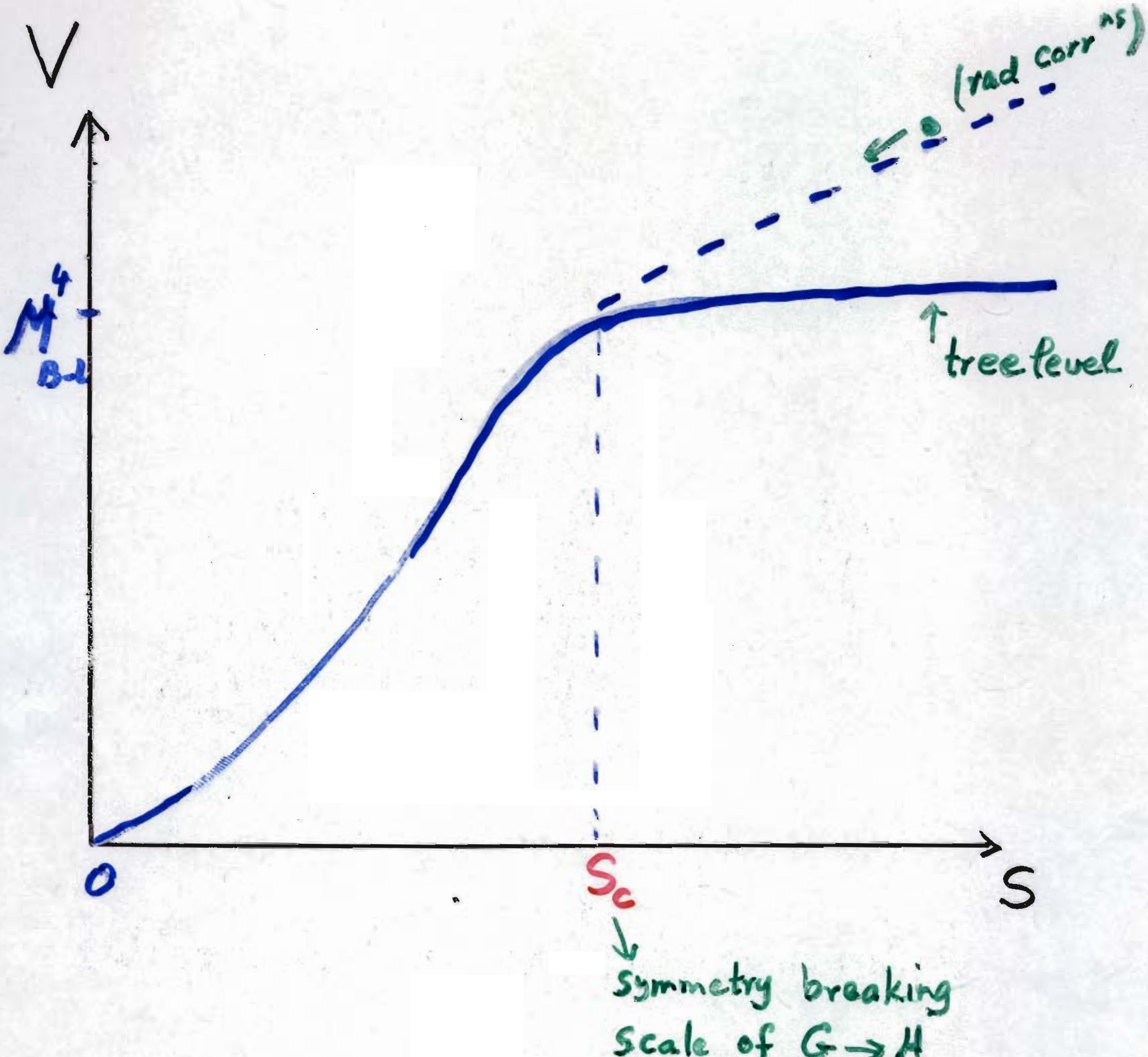
$z = |S|^2/M^2$

$\mathcal{N} = 1$ (for $U(1)_{B-L}$)

$|a m_{3/2}| \sim$ TeV

$|S| \ll M_{planck}$ during inflation

$M_{B-L} \to$ scale of $U(1)_{B-L}$ breaking, say



For $S > S_c$, (& κ not too small)

$$V_{eff}(S) \sim \kappa^2 M^4 \left[1 + \frac{N \kappa^2}{32\pi^2} \ln\left(\frac{\kappa^2 S^2}{\Lambda^2}\right) \right]$$

Use to calculate

$\propto (M/M_P)^2 \leftarrow \delta T/T$, n, S/T $\left(n \simeq 1 - \frac{1}{N_\ell} \simeq 0.98\right)$

For $10^{-3} \lesssim \kappa \lesssim 10^{-1}$, the radiative corrections dominate, so that

no. of e foldings

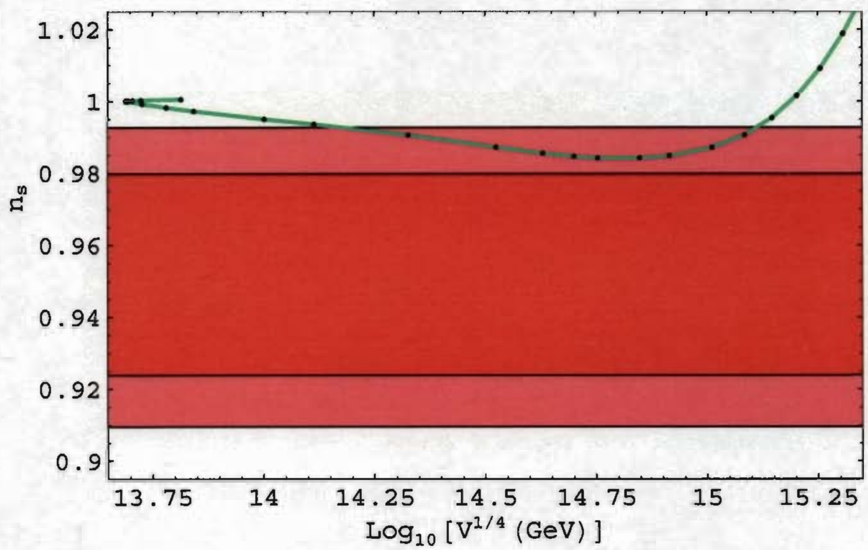
$$\frac{\delta T}{T} \approx \left(\frac{N_\ell}{45 \mathcal{N}}\right)^{1/2} \left(\frac{M}{M_P}\right)^2$$

(=1 for $U(1)_{B-L}$)

Taking $\frac{\delta T}{T} \approx 6 \times 10^{-5}$, say,

$$M \simeq (6 \times 10^{15})\, \mathcal{N}^{1/4} \text{ GeV}$$

- Knowing M, one could 'predict' $\delta T/T$.



The spectral index n_s vs $\log[V^{1/4}\,(\mathrm{GeV})]$ for SUSY hybrid inflation with $\mathcal{N} = 1$ (green curve), compared with the WMAP range for n_s (68% and 95% confidence levels, taken from Spergel *et al.*, astro-ph/0603449).

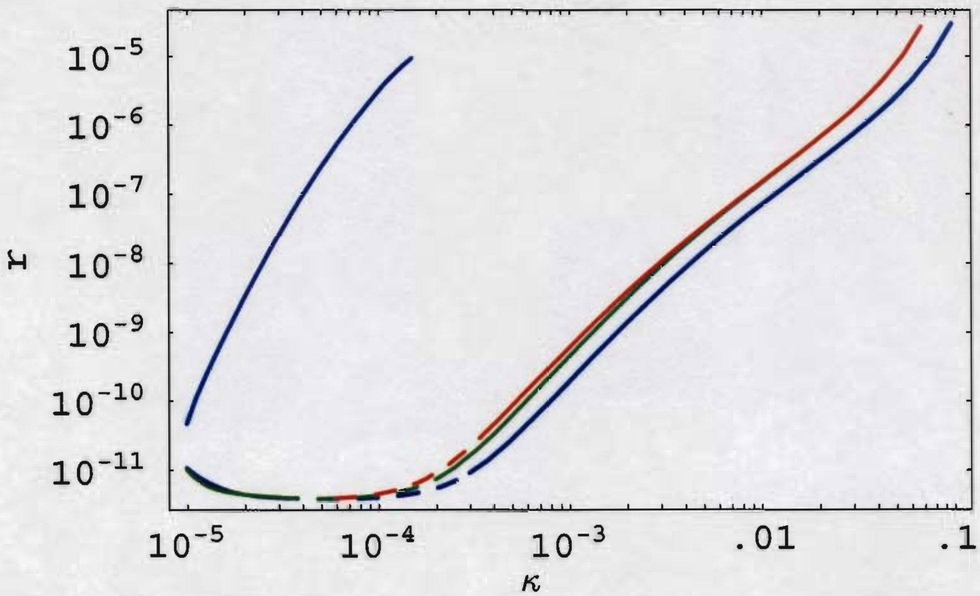


FIG. 5: The tensor to scalar ratio r vs. the allowed range of κ, for SUSY hybrid inflation with $\mathcal{N} = 1$ (blue), with $\mathcal{N} = 2$ (green), and for shifted hybrid inflation with $M_S = m_P$ (red). The dashed segments denote the range of κ for which the change in $\arg S$ is significant.

$$r \equiv (\Delta T/T)_T^2 \Big/ (\Delta T/T)_S^2 \; ;$$

$$(\Delta T/T)_T^2 \simeq \frac{V}{M_P^4} \sim \frac{H^2}{M_P^2}$$

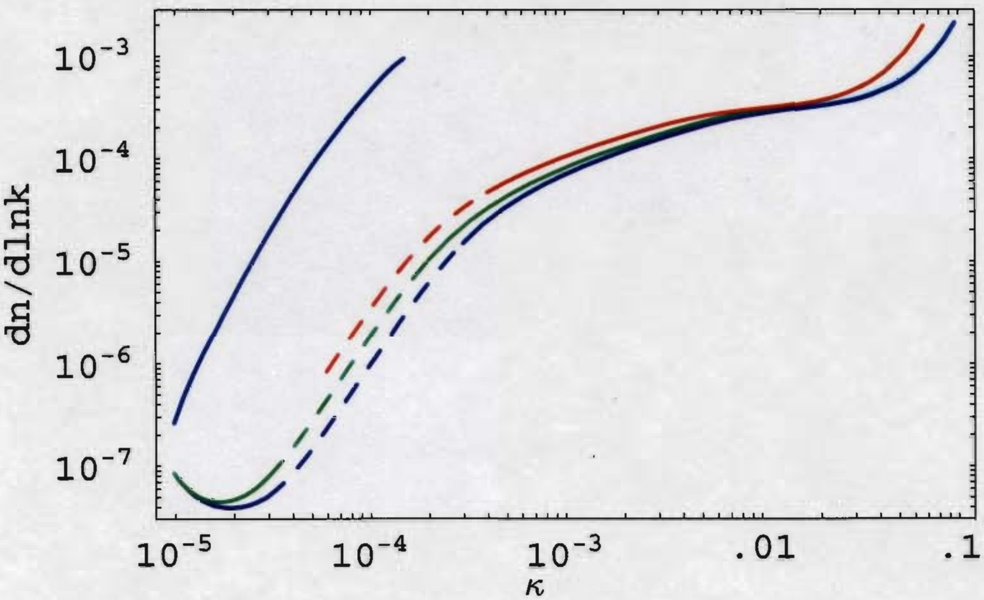


FIG. 4: $dn_s/d\ln k$ vs. the allowed range of κ, for SUSY hybrid inflation with $\mathcal{N} = 1$ (blue), with $\mathcal{N} = 2$ (green), and for shifted hybrid inflation with $M_S = m_P$ (red). The dashed segments denote the range of κ for which the change in $\arg S$ is significant.

- For 'regular' and 'shifted' hybrid inflation, one finds

$$n_s \gtrsim 0.98$$

- For 'smooth' inflation

$$n_s \gtrsim 0.97$$

Consistent with WMAP 1.

Not with WMAP 3?

Non-minimal Models

right handed sneutrino

$$K = |S|^2 + |\phi|^2 + |\bar{\phi}|^2 + |N|^2$$

$$+ \kappa_S \frac{|S|^4}{4m_P^2} + \kappa_{S\phi} \frac{|S|^2|\phi|^2}{m_P^2} + \kappa_{SN} |S|^2 |N|^2 + \cdots$$

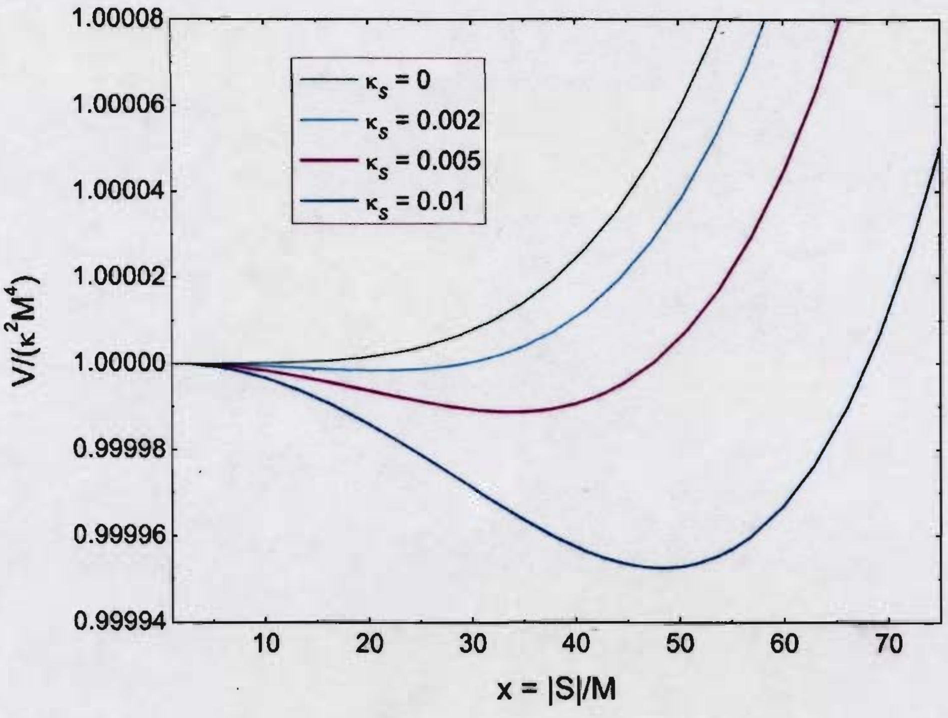
- 'regular' hybrid inflation with ϕ & N at origin during inflation, but V_{infl} picks up a term

$$(-\kappa_S)\, \kappa^2 M^4 \frac{S^2}{m_P^2}$$, such that

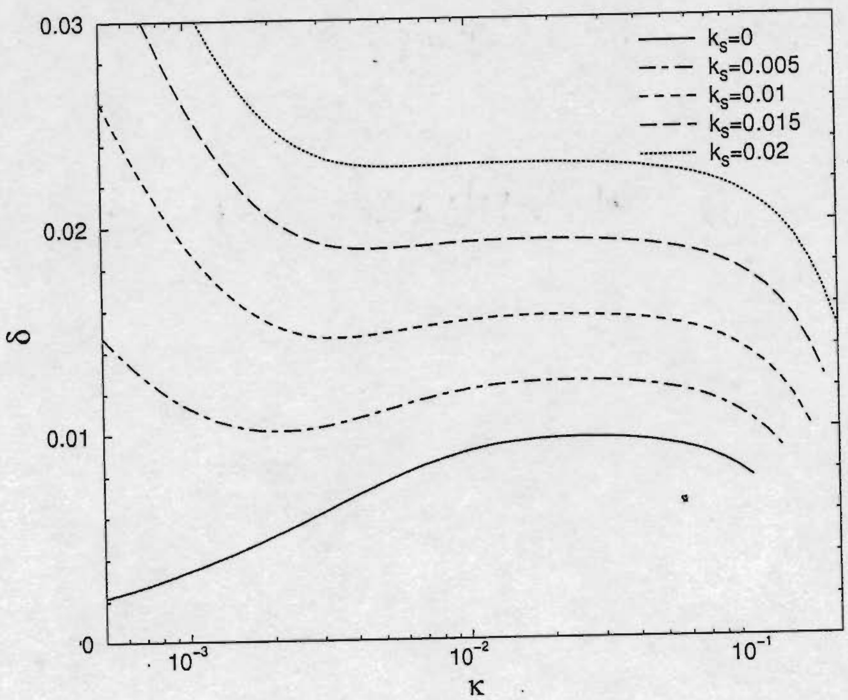
$$n_s \simeq 1 - 2\delta - 2\kappa_S$$

radiative corrections

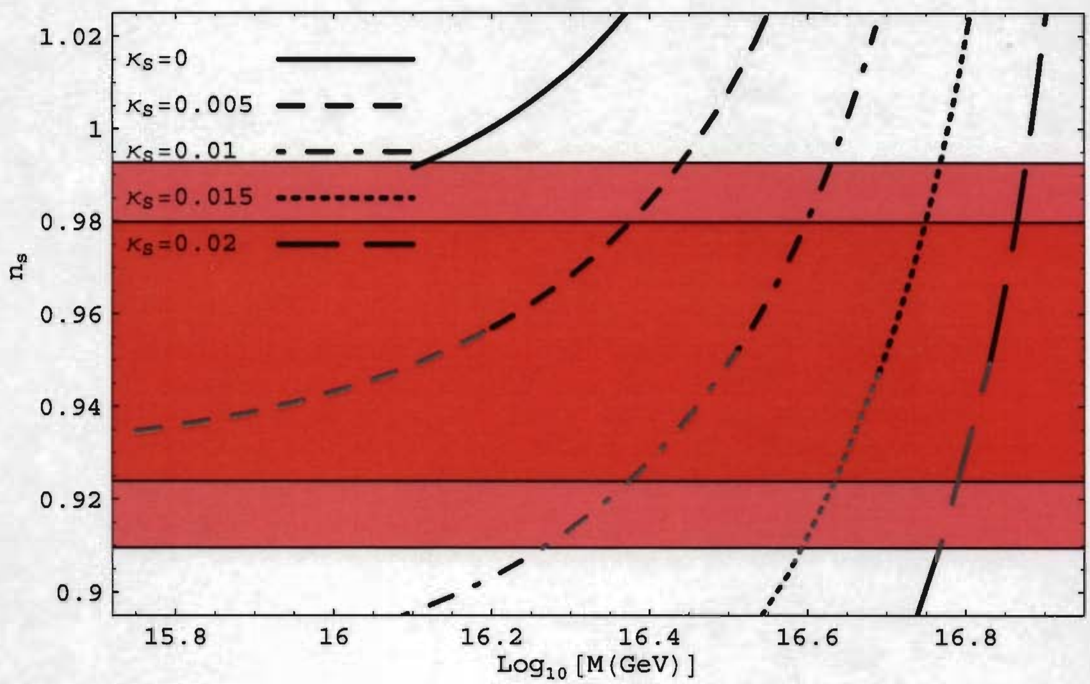
non-minimal contribution



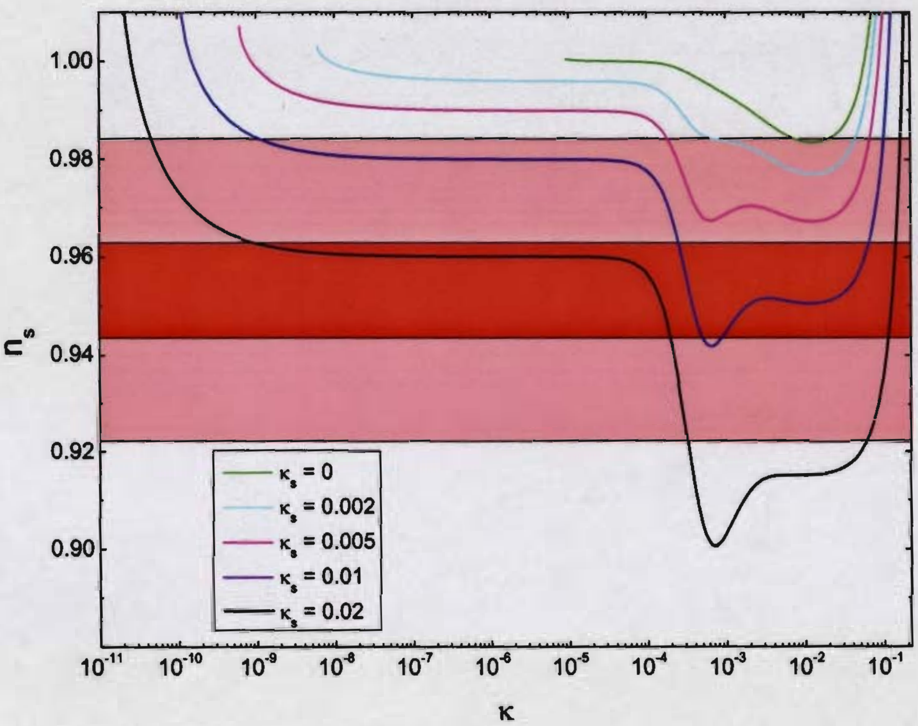
1.00008
1.00006
1.00004
1.00002
1.00000
0.99998
0.99996
0.99994
10
20
30
40
50
60
70
V/(κ²M⁴)
x = |S|/M
κ_s = 0
κ_s = 0.002
κ_s = 0.005
κ_s = 0.01



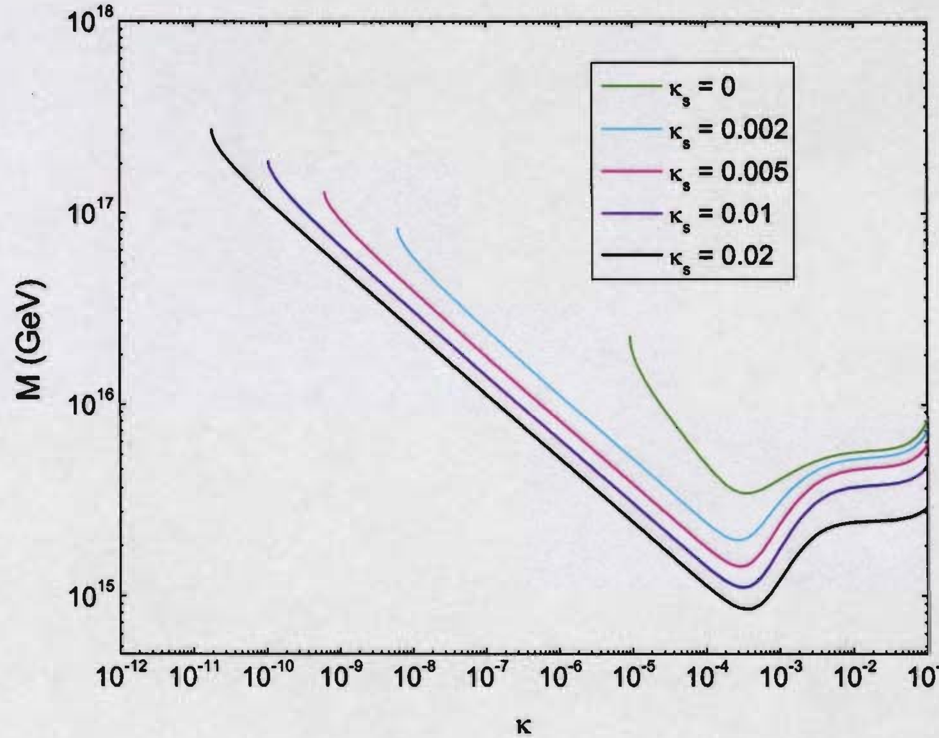
0.03
0.02
0.01
0
δ
10^{-3}
10^{-2}
10^{-1}
κ
k_s=0
k_s=0.005
k_s=0.01
k_s=0.015
k_s=0.02



The spectral index n_s as a function of the gauge symmetry breaking scale M for smooth hybrid inflation, compared with the WMAP range for n_s (68% and 95% confidence levels, taken from Spergel *et al.*, astro-ph/0603449). The gray sections indicate that the field is initially close to a local maximum.



n_s as a function of κ and κ_S ($\mathcal{N} = 1$).

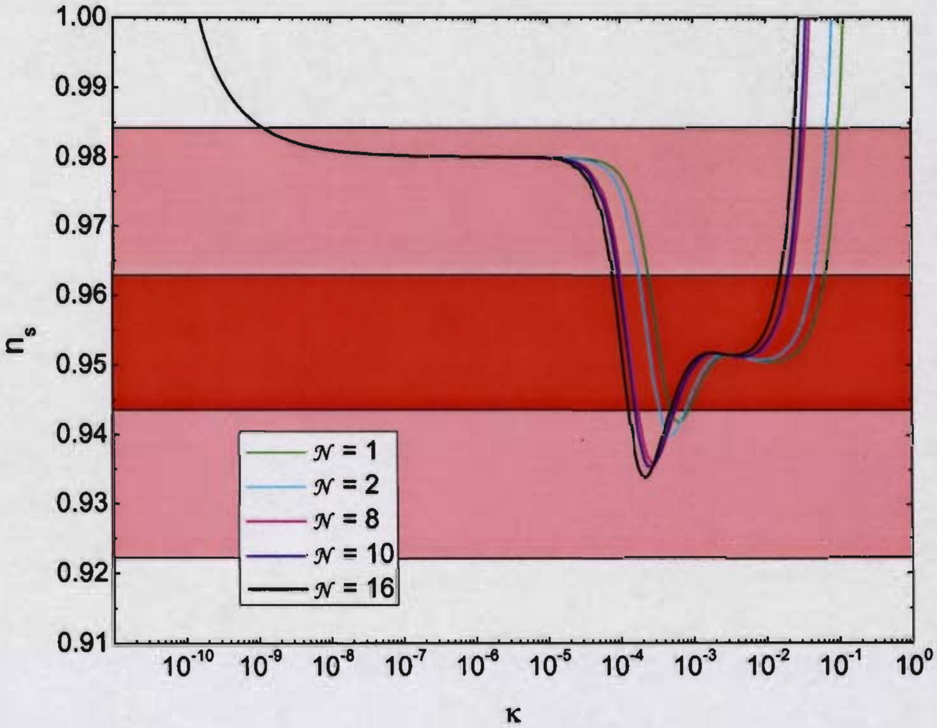


M as a function of κ and κ_S ($\mathcal{N} = 1$).

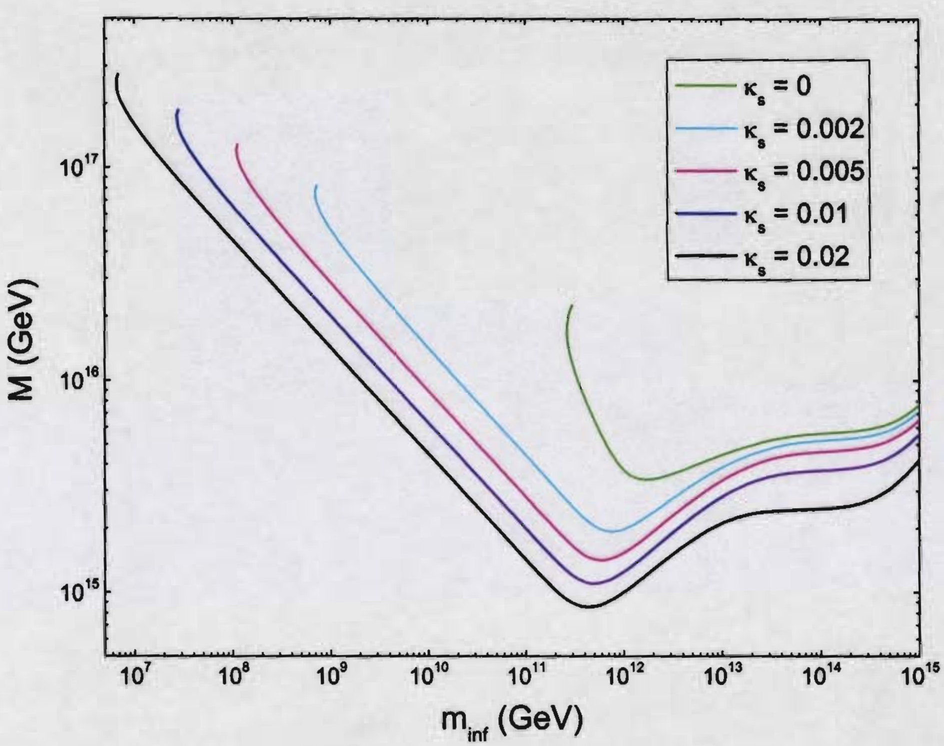
↑

symmetry breaking scale

(Inflation 'scale' ~ $\kappa^{1/2} M$)



n_s as a function of κ for different values of $\mathcal{N}$ ($\kappa_S = 0.01$).

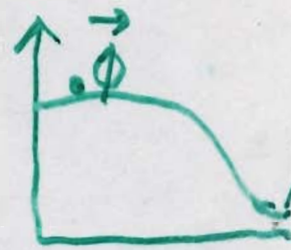


M as a function of $m_{\rm inf}$ and κ_S ($\mathcal{N} = 1$).

Kawasaki et al
Senoguz, QS
⋮

- ## 'New' inflation

Here S and N stay at zero during inflation now driven by ϕ ($W = S\left(-\mu^2 + \frac{(\bar{\Phi}\phi)^m}{M_*^{2m-2}}\right)$)



During inflation,

$$V \simeq \mu^4\left(1 - \frac{\beta}{2}\frac{\phi^2}{m_p^2} + \cdots\right)$$

(with $\beta \equiv \kappa_{S\phi} - 1 \geq 0$.)

$$n_s \simeq 1 - 2\beta\,,\ \text{for}\ \beta \gg 1/[(2m-2)N_\ell]$$

$$\simeq 1 - [2(2m-1)/(2m-2)N_\ell]\,,\ \text{for}\ \beta \simeq 0\,.$$

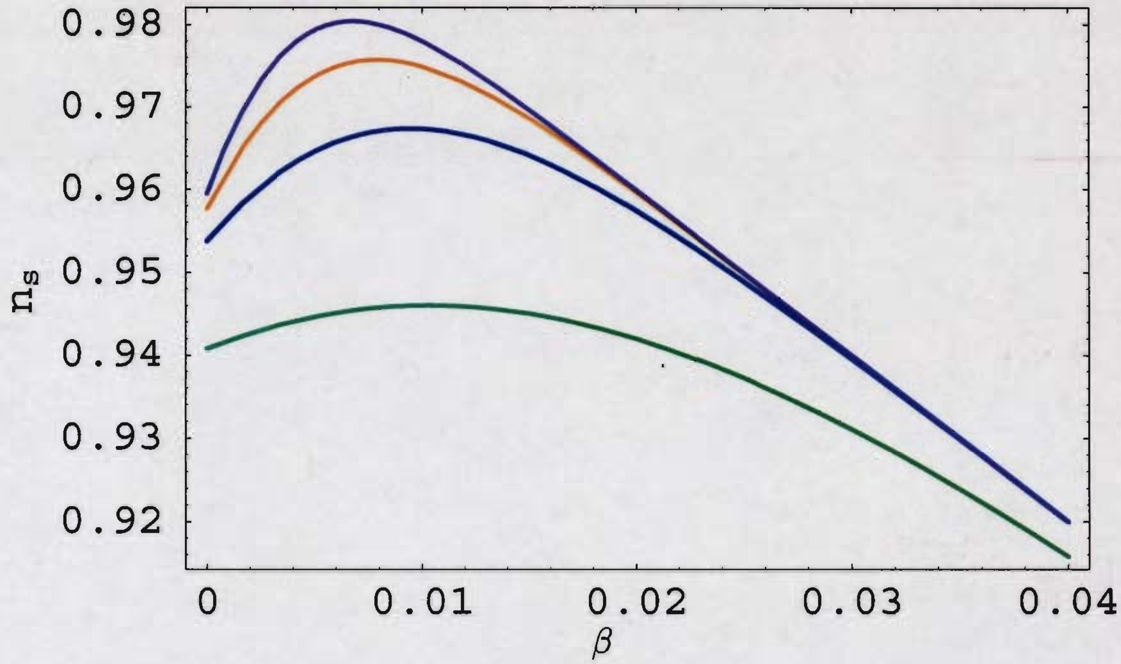


FIG. 1: The spectral index n_s vs. β, for $m = 2$ (green), $m = 3$ (blue), $m = 4$ (orange) and $m = 5$ (purple).

Sneutrino Hybrid Inflation

Antusch, Bastero-Gil et al

Here the right-handed sneutrino behaves as inflaton.

$$n_s \simeq 1 - 2\gamma \qquad (\gamma \equiv \kappa_{S_N} - 1)$$

$$r \ll \gamma^2$$

$$dn_s/d\ln k \lesssim -\gamma\left(N^2/m_p^2\right)$$

For $\gamma \approx 0.02$, the model is consistent with WMAP3.

T_{reheat} (T_r) / Leptogenesis

- Inflaton consists of S and/or $\phi, \bar{\phi}$ fields;

- Relevant couplings include

$$\begin{cases} N_i N_j \bar{\phi}\bar{\phi} & \text{(masses for right handed } \nu \text{)} \\ S H_u H_d & \leftarrow \text{suppose this is absent} \\ S \bar{\phi} \phi & \end{cases}$$

then

inflaton decays into right handed ν & $\tilde{\nu}$.

If $S H_u H_d$ present then T_r is somewhat higher.

Then

$$T_r \sim \left(\frac{1}{10} - \frac{1}{50}\right) M_i$$

$(2M_i \leq m_{inf})$

Thus, we require at least one $M_i \lesssim 10^{10} - 10^{11}$ GeV.

Suppose $M_3 \sim 10^{14}$ GeV $(16_3\, 16_3\, \overline{16}\, \overline{16})$, so that $m_3 \sim m_D^2/M_3 \sim 10^{-1}$ eV (Δm_{ATM})

Then, we could have

$$T_r \sim 10^9 \text{ GeV}$$

$$m_\phi \sim 10^{12} - 10^{13} \text{ GeV}$$

$$M_1, M_2 \sim 10^{10} - 10^{11} \text{ GeV}$$

$\Rightarrow$ non-thermal leptogenesis possible.

With $T_r \ll M_1, M_2 \ll M_3$,

$$n_L/s \lesssim 3\times 10^{-10}\left(\frac{T_r}{m_\phi}\right)\left(\frac{M_i}{10^6\,\mathrm{GeV}}\right)\left(\frac{m_{\nu_3}}{0.05\,\mathrm{eV}}\right)$$

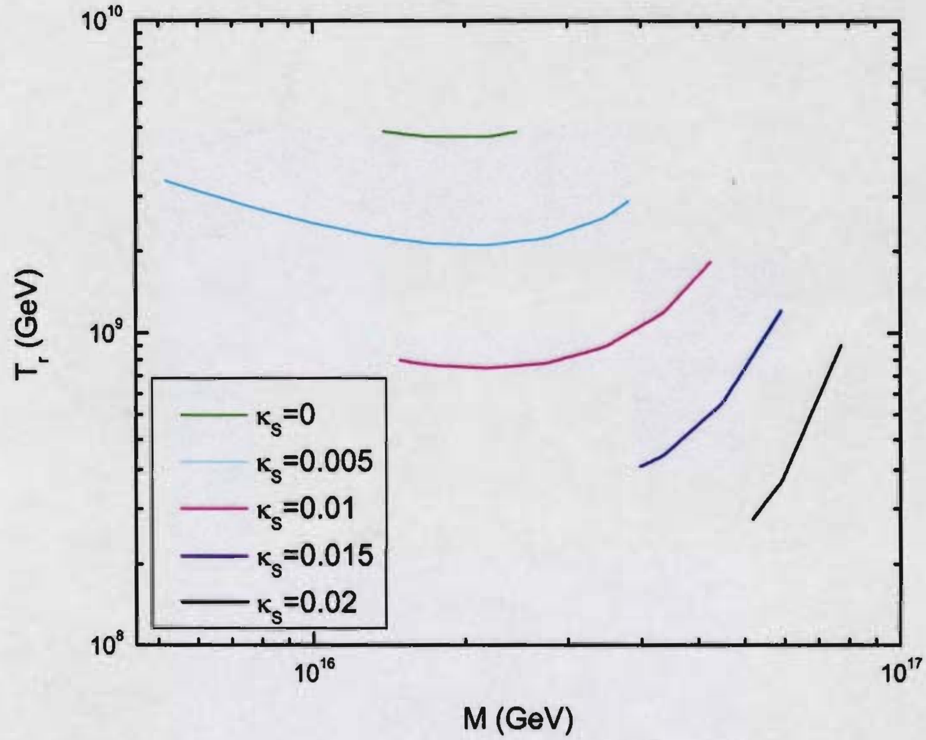
$$\Rightarrow\ T_r \gtrsim 2\times 10^{7}\,\mathrm{GeV}\left(\frac{10^{16}\,\mathrm{GeV}}{M}\right)^{1/2}\left(\frac{m_\phi}{10^{11}\,\mathrm{GeV}}\right)^{3/4}\left(\frac{.05\,\mathrm{eV}}{m_{\nu_3}}\right)^{1/2}$$

For the simplest models, $T_r \gtrsim 3\times 10^{7}\,\mathrm{GeV}$.

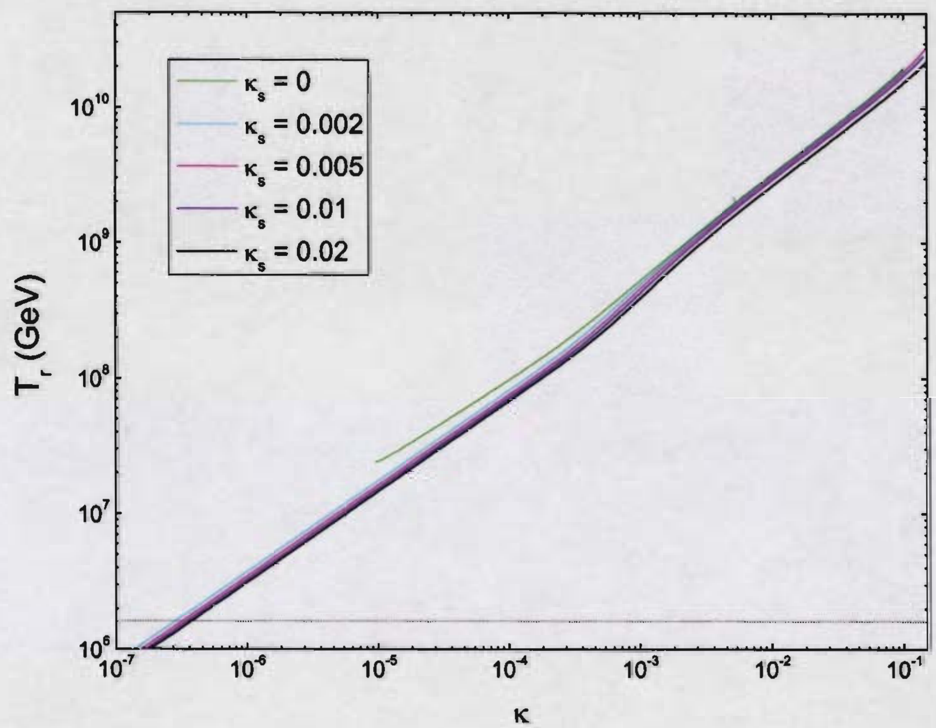
Note: For quasi-degenerate right handed neutrinos, the lepton asymmetry ϵ per neutrino decay can be of order unity, so that $T_r \gtrsim 10^2\,\mathrm{GeV}$ (TeV scale leptogenesis)

$$\boxed{n_L/s \sim \frac{T_r}{m_\phi}\epsilon}$$

Pilaftsis, ...



The lower bound on the reheat temperature T_r vs. the symmetry breaking scale M for smooth hybrid inflation. Only those sections satisfying $M_* > \sigma_0$ and $0.9 < n_s < 1.02$ are shown.



T_r as a function of κ and κ_S ($\mathcal{N} = 1$).

SUMMARY

- Precision Cosmology can play an important role in the search for new physics beyond the SM.

- Challenge for PLANCK & other ongoing / future expts: Determine
  n_s, n_T, $dn_s/d\ln k$, r, w_{DE}, …
  to a high degree of precision

- Find DARK MATTER
  (LSP, axion, majoran, KK, …)

$\Rightarrow$ help discover standard model of inflation.