# Inflation and Unification

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**Abstract.** Two distinct classes of realistic inflationary models consistent with present observations are reviewed. The first example relies on the Coleman-Weinberg potential and is readily realized within the framework of spontaneously broken global symmetries (for instance, global $U(1)_{B-L}$). Depending on the parameters either new or large field inflation is possible. The second example exploits supersymmetry which makes implementation of inflation within local gauge theories much more accessible. An example based on spontaneously broken local $U(1)_{B-L}$ is discussed. Leptogenesis is naturally realized in both cases.

**PACS.** 98.80.Cq Particle-theory and field-theory models of the early Universe

## 1 From new to large field inflation

An inflationary scenario [1,2] may be termed successful if it satisfies the following criteria:

1) The total number of $e$-folds N during inflation is large enough to resolve the horizon and flatness problems. Thus, $N \gtrsim 50$–60, but it can be somewhat smaller for low scale inflation.

2) The predictions are consistent with observations of the microwave background and large scale structure formation. In particular, the predictions for $n_s$, $r$ and $\alpha$ should be consistent with the most recent WMAP results [3] (see also [4] for a brief survey of models).

3) Satisfactory resolution of the monopole problem in grand unified theories (GUTs) is achieved.

4) Explanation of the origin of the observed baryon asymmetry is provided.

In this section[1] we review a class of inflation models which appeared in the early eighties in the framework of non-supersymmetric GUTs and employed a GUT singlet scalar field $\phi$ [6,7,8]. These (Shafi-Vilenkin) models satisfy, as we will see, the above criteria and are based on a Coleman-Weinberg (CW) potential [9]

$$V(\phi) = V_0 + A\phi^4 \left[ \ln \left( \frac{\phi^2}{M_*^2} \right) + C \right] \qquad (1)$$

where, following [6] the renormalization mass $M_* = 10^{18}$ GeV and $V_0^{1/4}$ will specify the vacuum energy. The value of $C$ is fixed to cancel the cosmological constant at the minimum. It is convenient to choose a physically equivalent parametrization for $V(\phi)$ [10,11], namely

$$V(\phi) = A\phi^4 \left[ \ln \left( \frac{\phi}{M} \right) - \frac{1}{4} \right] + \frac{AM^4}{4}, \qquad (2)$$

where $M$ denotes the $\phi$ VEV at the minimum. Note that $V(\phi = M) = 0$, and the vacuum energy density at the origin is given by $V_0 = AM^4/4$. For our discussion here one reasonable choice is to assume that the global $U(1)_{B-L}$ symmetry of the standard model is spontaneously broken by the VEV of $\phi$ (see later when we briefly discuss leptogenesis).

The potential above is typical for the new inflation scenario [2], where inflation takes place near the maximum. However, as we discuss below, depending on the value of $V_0$, the inflaton can have small or large values compared to the Planck scale during observable inflation. In the latter case observable inflation takes place near the minimum and the model mimics chaotic inflation [12].

The original new inflation models attempted to explain the initial value of the inflaton through high-temperature corrections to the potential. This mechanism does not work unless the inflaton is somewhat small compared to the Planck scale at the Planck epoch [11]. However, the initial value of the inflaton could also be suppressed by a pre-inflationary phase. Here we will simply assume that the initial value of the inflaton is sufficiently small to allow enough $e$-folds.

The slow-roll parameters may be defined as [13]

$$\epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = \left( \frac{V''}{V} \right), \quad \xi^2 = \left( \frac{V' V'''}{V^2} \right). \qquad (3)$$

(Here and below we use units $m_P = 1$, where $m_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, although sometimes we will write $m_P$ explicitly. The primes denote derivatives with respect to the inflaton $\phi$.) The slow-roll approximation is valid if the slow-roll conditions $\epsilon \ll 1$ and $\eta \ll 1$ hold. In this case the spectral index $n_s$, the tensor to scalar ratio $r$ and the running

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[1] Based on Ref. [5].

**Table 1.** The inflationary parameters for the Shafi-Vilenkin model with the potential in Eq. (2) ($m_P = 1$)

| $V_0^{1/4}$(GeV) | $A(10^{-14})$ | M | $\phi_e$ | $\phi_0$ | $V(\phi_0)^{1/4}$(GeV) | $n_s$ | $\alpha(-10^{-3})$ | $r$ |
|---|---|---|---|---|---|---|---|---|
| $10^{13}$ | 1.0 | 0.018 | 0.010 | $3.0 \times 10^{-6}$ | $\approx V_0^{1/4}$ | 0.938 | 1.4 | $9 \times 10^{-15}$ |
| $5 \times 10^{13}$ | 1.2 | 0.088 | 0.050 | $7.5 \times 10^{-5}$ | $\approx V_0^{1/4}$ | 0.940 | 1.3 | $5 \times 10^{-12}$ |
| $10^{14}$ | 1.3 | 0.17 | 0.10 | $3.0 \times 10^{-4}$ | $\approx V_0^{1/4}$ | 0.940 | 1.2 | $9 \times 10^{-11}$ |
| $5 \times 10^{14}$ | 1.9 | 0.79 | 0.51 | $7.5 \times 10^{-3}$ | $\approx V_0^{1/4}$ | 0.941 | 1.2 | $5 \times 10^{-8}$ |
| $10^{15}$ | 2.3 | 1.5 | 1.1 | 0.030 | $\approx V_0^{1/4}$ | 0.941 | 1.2 | $9 \times 10^{-7}$ |
| $5 \times 10^{15}$ | 4.8 | 6.2 | 5.1 | 0.71 | $\approx V_0^{1/4}$ | 0.942 | 1.0 | $5 \times 10^{-4}$ |
| $10^{16}$ | 5.2 | 12 | 10 | 3.2 | $9.9 \times 10^{15}$ | 0.952 | 1.0 | $8 \times 10^{-3}$ |
| $2 \times 10^{16}$ | 1.1 | 36 | 35 | 23 | $1.7 \times 10^{16}$ | 0.966 | 0.6 | 0.07 |
| $3 \times 10^{16}$ | .17 | 86 | 85 | 72 | $1.9 \times 10^{16}$ | 0.967 | 0.6 | 0.11 |
| $10^{17}$ | .001 | 1035 | 1034 | 1020 | $2.0 \times 10^{16}$ | 0.966 | 0.6 | 0.14 |

of the spectral index $\alpha \equiv dn_\mathrm{s}/d \ln k$ are given by

$$n_\mathrm{s} \simeq 1 - 6\epsilon + 2\eta \qquad (4)$$
$$r \simeq 16\epsilon \qquad (5)$$
$$\alpha \simeq 16\epsilon\eta - 24\epsilon^2 - 2\xi^2. \qquad (6)$$

The number of $e$-folds after the comoving scale $l_0 = 2\pi/k_0$ has crossed the horizon is given by

$$N_0 = \frac{1}{2} \int_{\phi_e}^{\phi_0} \frac{H(\phi)\mathrm{d}\phi}{H'(\phi)} \qquad (7)$$

where $\phi_0$ is the value of the field when the scale corresponding to $k_0$ exits the horizon and $\phi_e$ is the value of the field at the end of inflation. This value is given by the condition $2(H'(\phi)/H(\phi))^2 = 1$, which can be calculated from the Hamilton-Jacobi equation [14]

$$[H'(\phi)]^2 - \frac{3}{2}H^2(\phi) = -\frac{1}{2}V(\phi). \qquad (8)$$

The amplitude of the curvature perturbation $\mathcal{P}_\mathcal{R}^{1/2}$ is given by

$$\mathcal{P}_\mathcal{R}^{1/2} = \frac{1}{2\sqrt{3}\pi m_P^3} \frac{V^{3/2}}{|V'|}. \qquad (9)$$

To calculate the magnitude of $A$ and the inflationary parameters, we use these standard equations. We also include the first order corrections in the slow roll expansion for $\mathcal{P}_\mathcal{R}^{1/2}$ and the spectral index $n_s$ [15].[2] The WMAP value for $\mathcal{P}_\mathcal{R}^{1/2}$ is $4.86 \times 10^{-5}$ for $k_0 = 0.002$ Mpc$^{-1}$. $N_0$ corresponding to the same scale is $\simeq 53 + (2/3)\ln(V(\phi_0)^{1/4}/10^{15} \text{ GeV}) + (1/3)\ln(\mathrm{T_r}/10^9 \text{ GeV})$. (The expression for $N_0$ assumes a standard thermal history [16]. See [17] for reviews.) We assume reheating is efficient enough such that the reheating temperature $T_r = m_\phi$, where the mass of the inflaton $m_\phi = 2\sqrt{A}M$.

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[2] The fractional error in $\mathcal{P}_\mathcal{R}^{1/2}$ from the slow roll approximation is of order $\epsilon$ and $\eta$ (assuming these parameters remain $\ll 1$). This leads to an error in $n_s$ of order $\xi^2$, which is $\sim 10^{-3}$ in the present model. Comparing to the WMAP errors, this precision seems quite adequate. However, in anticipation of the Planck mission, it may be desirable to consider improvements.
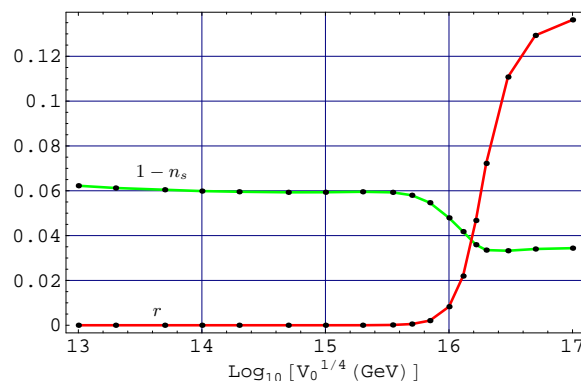


**Fig. 1.** $1 - n_s$ and $r$ vs. $\log[V_0^{1/4} \text{ (GeV)}]$ for the potential in Eq. (2).

In practice, we expect $T_r$ to be somewhat below $m_\phi$ [6].

In Table 1 and Fig. 1 we display the predictions for $n_s$, $\alpha$ and $r$, with the vacuum energy scale $V_0^{1/4}$ varying from $10^{13}$ GeV to $10^{17}$ GeV. The parameters have a slight dependence on the reheating temperature, as can be seen from the expression for $N_0$. As an example, if we assume instant reheating ($T_r \simeq V(\phi_0)^{1/4}$), $n_s$ would increase to 0.941 and 0.943 for $V_0^{1/4} = 10^{13}$ GeV and $V_0^{1/4} = 10^{15}$ GeV respectively.

For $V_0^{1/4} \lesssim 10^{16}$ GeV, the inflaton field remains smaller than the Planck scale, and the inflationary parameters are similar to those for new inflation models with $V = V_0(1 - (\phi/\mu)^4)$: $n_s \simeq 1 - (3/N_0)$, $\alpha \simeq (n_s - 1)/N_0$. As the vacuum energy is lowered, $N_0$ becomes smaller and $n_s$ deviates further from unity. However, $n_s$ remains within $2\sigma$ of the WMAP best fit value (for negligible $r$) $0.951^{0.015}_{-0.019}$ [3] even for $V_0^{1/4}$ as low as $10^5$ GeV. Inflation with CW potential at low scales is discussed in Ref. [18].

For $V_0^{1/4} \gtrsim 10^{16}$ GeV, the inflaton takes values larger than the Planck scale during observable inflation. Observable inflation then occurs closer to the minimum where the potential is effectively $V = (1/2)m_\phi^2\Delta\phi^2$, $\Delta\phi = M - \phi$ denoting the deviation of the field from the minimum. This well-known monomial model [12] predicts $m_\phi \simeq 2 \times 10^{13}$ GeV and
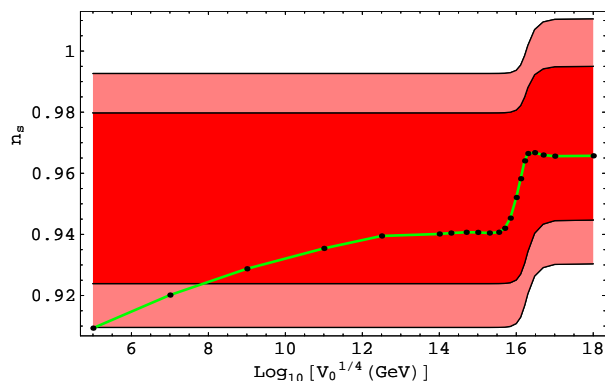
**Fig. 2.** $n_s$ vs. $\log[V_0^{1/4}$ (GeV)] for the potential in Eq. (2), shown together with the WMAP contours (68% and 95% confidence levels) [3].
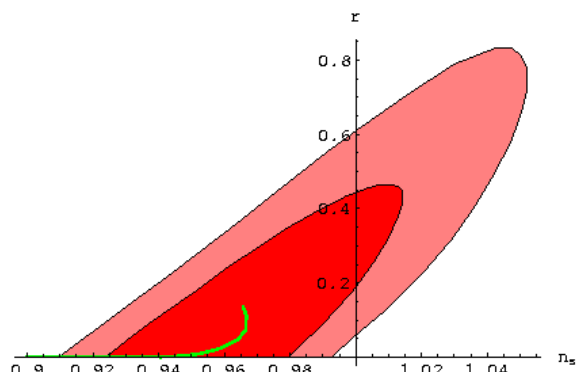


**Fig. 3.** $r$ vs. $n_s$ for the potential in Eq. (2), shown together with the WMAP contours (68% and 95% confidence levels) [3].

$\Delta\phi_0 \simeq 2\sqrt{N_0}$, corresponding to $V(\phi_0) \simeq (2 \times 10^{16}$ GeV$)^4$. For the $\phi^2$ potential to be a good approximation, $V_0$ must be greater than this value. Then the inflationary parameters no longer depend on $V_0$ and approach the predictions for the $\phi^2$ potential.

The spectral index $n_s$ and tensor to scalar ratio $r$ are displayed in Figs. 2, 3. The values are in very good agreement with the recent WMAP results [3]. The running of the spectral index is negligible, as in most inflation models (Fig. 4).

Note that the WMAP analysis suggests a running spectral index, with $|\alpha| \lesssim 10^{-3}$ disfavored at the $2\sigma$ level [19,3]. On the other hand, an analysis including the constraints from the Sloan Digital Sky Survey (SDSS) finds no evidence for running [20]. Clearly, more data is necessary to resolve this important issue. Modifications of the models discussed here, generally involving two stages of inflation, have been proposed in Refs. [21,22] and elsewhere to generate a much more significant variation of $n_s$ with $k$.

In the context of non-supersymmetric GUTs, $V_0^{1/4}$ is related to the unification scale, and is typically a factor of 3–4 smaller than the superheavy gauge boson masses due to the loop factor in the CW potential. The unification scale for non-supersymmetric GUTs is typically $10^{14}$–$10^{15}$ GeV, although it is possible to
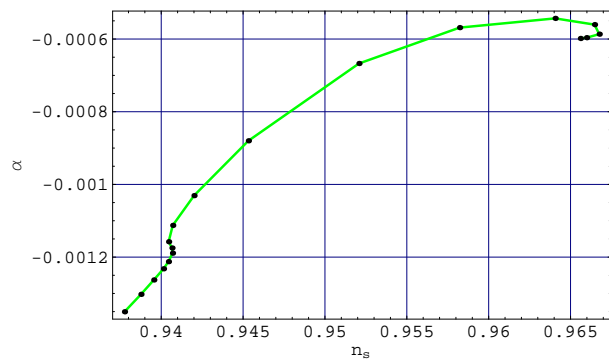


**Fig. 4.** $\alpha$ vs. $n_s$ for the potential in Eq. (2).

have higher scales, for instance associating inflation with $SO(10)$ breaking via $SU(5)$.

The reader may worry about proton decay with gauge boson masses of order $10^{14}$–$10^{15}$ GeV. In the $SU(5)$ model [23], in particular, a two-loop renormalization group analysis of the standard model gauge couplings yields masses for the superheavy gauge bosons of order $1 \times 10^{14}$–$5 \times 10^{14}$ GeV [24]. This is consistent with the SuperK proton lifetime limits [25], provided one assumes strong flavor suppression of the relevant dimension six gauge mediated proton decay coefficients. If no suppression is assumed the gauge boson masses should have masses close to $10^{15}$ GeV or higher [26].

For the Shafi-Vilenkin model in $SU(5)$, the tree level scalar potential contains the term $(1/2)\lambda\phi^2\text{Tr}\Phi^2$ with $\Phi$ being the Higgs adjoint, and $A \simeq 1.5 \times 10^{-2}\lambda^2$ [6,11]. Inflation requires $A \sim 10^{-14}$, corresponding to $\lambda \sim 10^{-6}$.

This model has been extended to $SO(10)$ in Ref. [8]. The breaking of $SO(10)$ to the standard model proceeds, for example, via the subgroup $G_{422} = SU(4)_c \times SU(2)_L \times SU(2)_R$ [27]. A renormalization group analysis shows that the symmetry breaking scale for $SO(10)$ is of order $10^{15}$ GeV, while $G_{422}$ breaks at an intermediate scale $M_I \sim 10^{12}$ GeV [28]. (This is intriguingly close to the scale needed to resolve the strong CP problem and produce cold dark matter axions.) The predictions for $n_s$, $\alpha$ and $r$ are essentially identical to the $SU(5)$ case. There is one amusing consequence though which may be worth mentioning here. The monopoles associated with the breaking of $SO(10)$ to $G_{422}$ are inflated away. However, the breaking of $G_{422}$ to the SM gauge symmetry yields doubly charged monopoles [29], whose mass is of order $10^{13}$ GeV. These may be present in our galaxy at a flux level of $10^{-16}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ [8].

As stated earlier, before an inflationary model can be deemed successful, it must contain a mechanism for generating the observed baryon asymmetry in the universe. In the $SU(5)$ case the color Higgs triplets produced by inflaton decay can generate the baryon asymmetry, provided the Higgs sector of the model has the required amount of CP violation [6].

The discovery of neutrino oscillations requires that we introduce SU(5) singlet right handed neutrinos,

presumably three of them, to implement the seesaw mechanism and generate the desired masses for the light neutrinos. In this case it is natural to generate the observed baryon asymmetry via leptogenesis [30] by introducing the couplings $N_i N_j \phi^2 / m_P$, where $N_i$ (i=1,2,3) denote the right handed neutrinos, and the renormalizable coupling to $\phi$ is absent because of the assumed discrete symmetry. By suitably adjusting the Yukawa coefficients one can arrange that the $\phi$ field decays into the right handed neutrinos. Note that the presence of the above Yukawa couplings then allows one to make the color triplets heavier, of order $10^{14}$ GeV, thereby avoiding any potential conflict with proton decay. In the $SO(10)$ model, leptogenesis is almost automatic [8].

# 2 $U(1)_{B-L}$: Neutrino Physics and Inflation

Physics beyond the Standard Model (SM) is required by the following experimental observations:

- Neutrino Oscillations: $\Delta m_{\mathrm{SM}}^2 \lesssim 10^{-10}$ eV$^2$ $\ll$ (mass difference)$^2$ needed to understand atmospheric and solar neutrino observations;
- CMB Anisotropy ($\delta T/T$): requires inflation which cannot be realized in the SM;
- Non-Baryonic Dark Matter ($\Omega_{\mathrm{CDM}} = 0.25$): SM has no plausible candidate;
- Baryon Asymmetry ($n_b/s \sim 10^{-10}$): Not possible to achieve in the SM.

Recall that at the renormalizable level the SM possesses a global $U(1)_{B-L}$ symmetry. If the symmetry is gauged, anomaly cancellation requires the existence of three right handed neutrinos. An important question therefore is the symmetry breaking scale of $U(1)_{B-L}$. Note that this scale is not fixed by the evolution of the three SM gauge couplings. Remarkably, we will be able to determine the $M_{B-L}$ by implementing inflation. With $M_{B-L}$ well below the Planck scale the seesaw mechanism enables us to realize light neutrino masses in the desired range. Furthermore, it will turn out that leptogenesis is a natural outcome after inflation is over.

The introduction of a gauge $U(1)_{B-L}$ symmetry broken at a scale well below the Planck scale exacerbates the well known gauge hierarchy problem. There are at least four potential hierarchy problems one could consider:

- $M_W \ll M_P$;
- $M_{B-L} \ll M_P$ (required by neutrino oscillations);
- $m_\chi \ll M_P$ (where $m_\chi$ denotes the inflaton mass);
- $f_a \sim 10^{10} - 10^{12}$ GeV ($\ll M_P$), where $f_a$ denotes the axion decay constant.

Supersymmetry (SUSY) can certainly help here, especially if the SUSY breaking scale in the observable sector is of order TeV. Thus, it seems that a good starting point, instead of SM $\times U(1)_{B-L}$, could be MSSM $\times$ $U(1)_{B-L}$. The $Z_2$ 'matter' parity associated with the MSSM has two important consequences. It eliminates

rapid (dimension four) proton decay, and it delivers a respectable cold dark matter candidate in the form of LSP. However, Planck scale suppressed dimension five proton decay is still present and one simple solution is to embed $Z_2$ in a $U(1)_R$ symmetry. It turns out that the R symmetry also plays an essential role in realizing a compelling inflationary scenario and in the resolution of the MSSM $\mu$ problem. Finally it seems natural to extend the above discussion to larger groups, especially to $SO(10)$ and its various subgroups.

## 2.1 Supersymmetric Hybrid Inflation Models

In this section[3] we review a class of supersymmetric hybrid inflation models [32] where inflation can be linked to the breaking of $U(1)_{B-L}$. We compute the allowed range of the dimensionless coupling in the superpotential and the dependence of the spectral index on this coupling, in the presence of canonical supergravity (SUGRA) corrections.

The simplest supersymmetric hybrid inflation model [33] is realized by the renormalizable superpotential [34]

$$W_1 = \kappa S(\Phi\overline{\Phi} - M^2) \qquad (10)$$

where $\Phi(\overline{\Phi})$ denote a conjugate pair of superfields transforming as nontrivial representations of some gauge group $G$, $S$ is a gauge singlet superfield, and $\kappa$ ($> 0$) is a dimensionless coupling. A suitable $U(1)$ R-symmetry, under which $W_1$ and $S$ transform the same way, ensures the uniqueness of this superpotential at the renormalizable level [33]. In the absence of supersymmetry breaking, the potential energy minimum corresponds to non-zero vacuum expectation values (VEVs) ($= M$) in the scalar right handed neutrino components $\left|\langle \nu_H^c \rangle\right| = \left|\langle \overline{\nu}_H^c \rangle\right|$ for $\Phi$ and $\overline{\Phi}$, while the VEV of $S$ is zero. (We use the same notation for superfields and their scalar components.) Thus, $G$ is broken to some subgroup $H$ which, in many interesting models, coincides with the MSSM gauge group.

In order to realize inflation, the scalar fields $\Phi$, $\overline{\Phi}$, $S$ must be displayed from their present minima. For $|S| > M$, the $\Phi$, $\overline{\Phi}$ VEVs both vanish so that the gauge symmetry is restored, and the tree level potential energy density $\kappa^2 M^4$ dominates the universe, as in the originally proposed hybrid inflation scenario [35,34]. With supersymmetry thus broken, there are radiative corrections from the $\Phi - \overline{\Phi}$ supermultiplets that provide logarithmic corrections to the potential which drives inflation.

In one loop approximation the inflationary effective potential is given by [33]

$$V_{\mathrm{LOOP}} = \kappa^2 M^4 \left[ 1 + \frac{\kappa^2 \mathcal{N}}{32\pi^2} \left( 2 \ln \frac{\kappa^2 |S|^2}{\Lambda^2} + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right) \right], \quad (11)$$

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[3] Based on [31].

where $z \equiv x^2 \equiv |S|^2/M^2$, $\mathcal{N}$ is the dimensionality of the $\Phi$, $\overline{\Phi}$ representations, and $\Lambda$ is a renormalization mass scale.

The scalar spectral index $n_s$ is given by Eq. (4), where primes denote derivatives with respect to the normalized real scalar field $\sigma \equiv \sqrt{2}|S|$. For relevant values of the parameters ($\kappa \ll 1$), the slow roll conditions ($\epsilon, \eta \ll 1$) are violated only 'infinitesimally' close to the critical point at $x = 1$ ($|S| = M$) [32]. So inflation continues practically until this point is reached, where it abruptly ends.

The number of $e$-folds after the comoving scale $l_0$ has crossed the horizon is given by Eq. (7), which in the slow roll approximation can also be written as

$$N_0 = \frac{1}{m_P^2} \int_{\sigma_e}^{\sigma_0} \frac{V \, d\sigma}{V'} \,. \tag{12}$$

Using Eqs. (11, 12), we obtain

$$\kappa \approx \frac{2\sqrt{2}\pi}{\sqrt{\mathcal{N} N_0}} \, y_0 \, \frac{M}{m_P} \,. \tag{13}$$

(The subscript 0 implies that the values correspond to $k_0 \equiv 0.002 \text{ Mpc}^{-1}$.) $N_0 \approx 55$ is the number of $e$-folds and

$$y_0^2 = \int_1^{x_0^2} \frac{dz}{z f(z)} \quad , y_0 \geq 0 \,, \tag{14}$$

with

$$f(z) = (z+1)\ln\left(1 + z^{-1}\right) + (z-1)\ln\left(1 - z^{-1}\right) \,. \tag{15}$$

Using Eqs. (11, 13, 9), $\mathcal{P}_{\mathcal{R}}^{1/2}$ is found to be [33,32,36]

$$\mathcal{P}_{\mathcal{R}}^{1/2} \approx 2 \left(\frac{N_0}{3\mathcal{N}}\right)^{1/2} \left(\frac{M}{m_P}\right)^2 x_0^{-1} y_0^{-1} f(x_0^2)^{-1} \,. \tag{16}$$

Up to now, we ignored supergravity (SUGRA) corrections to the potential. More often than not, SUGRA corrections tend to derail an otherwise succesful inflationary scenario by giving rise to scalar (mass)$^2$ terms of order $H^2$, where $H$ denotes the Hubble constant. Remarkably, it turns out that for a canonical SUGRA potential (with minimal Kähler potential $|S|^2 + |\Phi|^2 + |\overline{\Phi}|^2$), the problematic (mass)$^2$ term cancels out for the superpotential $W_1$ in Eq. (10) [34]. This property also persists when non-renormalizable terms that are permitted by the $U(1)_R$ symmetry are included in the superpotential.

In general, $K$ can be expanded as

$$K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 + \kappa_S \frac{|S|^4}{4m_P^2} + \dots \,, \tag{17}$$

and only the $|S|^4$ term in $K$ generates a (mass)$^2$ for $S$, which would spoil inflation for $\kappa_S \sim 1$ [37].[4]

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[4] We should note that, since the superpotential is linear in the inflaton, the presence of other fields with Planck scale VEVs would also induce an inflaton mass of order $H$. Some ways to suppress the inflaton mass are discussed in [38].

The scalar potential is given by

$$V = e^K \left[ \left(\frac{\partial^2 K}{\partial z_i \partial z_j^*}\right)^{-1} D_{z_i} W D_{z_j^*} W^* - 3|W|^2 \right] + V_D \,, \tag{18}$$

with

$$D_{z_i} W = \frac{\partial W}{\partial z_i} + \frac{\partial K}{\partial z_i} W \,, \tag{19}$$

where the sum extends over all fields $z_i$, and $K = \sum_i |z_i|^2$ is the minimal Kähler potential. The D-term $V_D$ vanishes in the D-flat direction $|\overline{\Phi}| = |\Phi|$. From Eq. (18), with a minimal Kähler potential one contribution to the inflationary potential is given by [34,39,40,21]

$$V_{\text{SUGRA}} = \kappa^2 M^4 \left[ \frac{|S|^4}{2} + \dots \right] \,. \tag{20}$$

There are additional contributions to the potential arising from the soft SUSY breaking terms. In $N = 1$ SUGRA these include the universal scalar masses equal to $m_{3/2}$ ($\sim$ TeV), the gravitino mass. However, their effect on the inflationary scenario is negligible, as discussed below. The more important term is the $A$ term $(2 - A)m_{3/2}\kappa M^2 S(+\text{h.c.})$. For convenience, we write this as $a \, m_{3/2}\kappa M^2|S|$, where $a \equiv 2|2 - A| \cos(\arg S + \arg(2 - A))$. The effective potential is approximately given by Eq. (11) plus the leading SUGRA correction $\kappa^2 M^4 |S|^4/2$ and the $A$ term:

$$\begin{aligned} V_1 = \kappa^2 M^4 &\left[ 1 + \frac{\kappa^2 \mathcal{N}}{32\pi^2} \left( 2 \ln \frac{\kappa^2 |S|^2}{\Lambda^2} \right.\right. \\ &\left. + (z+1)^2 \ln(1 + z^{-1}) + (z-1)^2 \ln(1 - z^{-1}) \right) \\ &\left. + \frac{|S|^4}{2} \right] + a \, m_{3/2}\kappa M^2 |S| \,. \end{aligned} \tag{21}$$

We perform our numerical calculations using this potential, taking $|a \, m_{3/2}| = 1$ TeV. It is, however, instructive to discuss small and large $\kappa$ limits of Eq. (21). For $\kappa \gg 10^{-3}$, $1 \gg \sigma \gg \sqrt{2}M$, and Eq. (21) becomes

$$V_1 \simeq \kappa^2 M^4 \left[ 1 + \frac{\kappa^2 \mathcal{N}}{32\pi^2} 2 \ln \frac{\kappa^2 \sigma^2}{2\Lambda^2} + \frac{\sigma^4}{8} \right] \tag{22}$$

to a good approximation. Comparing the derivatives of the radiative and SUGRA corrections one sees that the radiative term dominates for $\sigma^2 \lesssim \kappa \sqrt{\mathcal{N}}/2\pi$. From $3H\dot{\sigma} = -V'$, $\sigma_0^2 \simeq \kappa^2 \mathcal{N} N_0/4\pi^2$ for the one-loop effective potential, so that SUGRA effects are negligible only for $\kappa \ll 2\pi/\sqrt{\mathcal{N}} N_0 \simeq 0.1/\sqrt{\mathcal{N}}$. (For $\mathcal{N} = 1$, this essentially agrees with [40]).

$\mathcal{P}_{\mathcal{R}}^{1/2}$ is found from Eq. (22) to be

$$\mathcal{P}_{\mathcal{R}}^{1/2} \simeq \frac{1}{\sqrt{3}\pi} \frac{\kappa M^2}{\sigma_0^3} \,. \tag{23}$$

In the absence of the SUGRA correction, the gauge symmetry breaking scale $M$ is given by Eq. (16). For $\kappa \gg 10^{-3}$, $x_0 \gg 1$ and $x_0 \, y_0 \, f(x_0^2) \to 1^-$. $\mathcal{P}_{\mathcal{R}}^{1/2}$ in this

case turns out to be proportional to $(M/m_P)^2$[33,32]. Using the WMAP best fit $\mathcal{P}_{\mathcal{R}}^{1/2} \simeq 4.7 \times 10^{-5}$ [19], $M$ approaches the value $\mathcal{N}^{1/4} \cdot 6 \times 10^{15}$ GeV. The presence of the SUGRA term leads to larger values of $\sigma_0$ and hence larger values of $M$ for $\kappa \gtrsim 0.06/\sqrt{\mathcal{N}}$.

For $\kappa \ll 10^{-3}$, $|S_0| \simeq M$ where $S_0$ is the value of the field at $k_0$, i.e. $z \simeq 1$. (Note that due to the extreme flatness of the potential the last 55 or so $e$-folds occur with $|S|$ close to $M$.) From Eqs. (9, 21), as $z \to 1$

$$\mathcal{P}_{\mathcal{R}}^{1/2} = \frac{2\sqrt{2}\pi}{\sqrt{3}} \frac{\kappa^2 M^4}{\ln(2)\kappa^3 M\mathcal{N} + 8\pi^2\kappa M^5 + 4\pi^2 a\, m_{3/2}}. \tag{24}$$

The denominator of Eq. (24) contains the radiative, SUGRA and the $A$ terms respectively. Comparing them, we see that the radiative term can be ignored for $\kappa \lesssim 10^{-4}$. There is also a soft mass term $m_{3/2}^2|S|^2$ in the potential, corresponding to an additional term $8\pi^2 m_{3/2}^2/\kappa M$ in the denominator. We have omitted this term, since it is insignificant for $\kappa \gtrsim 10^{-5}$.

For a positive $A$ term ($a > 0$), the maximum value of $\mathcal{P}_{\mathcal{R}}^{1/2}$ as a function of $M$ is found to be

$$\mathcal{P}_{\mathcal{R}}^{1/2}{}_{\max} = \frac{1}{2^{7/10}\, 5^{3/2}\, 3\pi} \left( \frac{\kappa^6}{a\, m_{3/2}} \right)^{1/5}. \tag{25}$$

Setting $\mathcal{P}_{\mathcal{R}}^{1/2} \simeq 4.7 \times 10^{-5}$, we find a lower bound on $\kappa$ ($\simeq 10^{-5}$). For larger values of $\kappa$, there are two separate solutions of $M$ for a given $\kappa$. The solution with larger $M$ is not valid if the symmetry breaking pattern produces cosmic strings. For example, strings are produced when $\Phi$, $\overline{\Phi}$ break $U(1)_R \times U(1)_{B-L}$ to $U(1)_Y \times Z_2$ matter parity, but not when $\Phi$, $\overline{\Phi}$ are $SU(2)_R \times U(1)_{B-L}$ doublets. For $a < 0$, there are again two solutions, but for the solution with a lower value of $M$, the slope changes sign as the inflaton rolls for $\kappa \lesssim 10^{-4}$ and the inflaton gets trapped in a false vacuum.

Note that the $A$ term depends on $\arg S$, so it should be checked whether $\arg S$ changes significantly during inflation. Numerically, we find that it does not, except for a range of $\kappa$ around $10^{-4}$ [31]. For this range, if the initial value of the $S$ field is greater than $M$ by at least a factor of two or so, the $A$ term and the slope become negative even if they were initially positive, before inflation can suitably end. However, larger values of the $A$ term, or the mass term coming from a non-minimal Kähler potential (or from a hidden sector VEV) would drive the value of $M$ in that region up, allowing the slope to stay positive (see Ref. [41] for the effect of varying the $A$ term and the mass).

The dependence of $M$ on $\kappa$ is shown in Fig. 5. Note that with inflation linked to the breaking of MSSM $\times$ $U(1)_{B-L}$, $M$ corresponds to the $U(1)_{B-L}$ breaking scale, which is not fixed by the evolution of the three SM gauge couplings. The amplitude of the curvature perturbation (or, equivalently, $\delta T/T$) determines this scale to be close to the SUSY GUT scale, suggesting that $U(1)_{B-L}$ could be embedded in $SO(10)$ or its subgroups. For example, $M$ can be determined in flipped $SU(5)$ from the renormalization group evolution of the
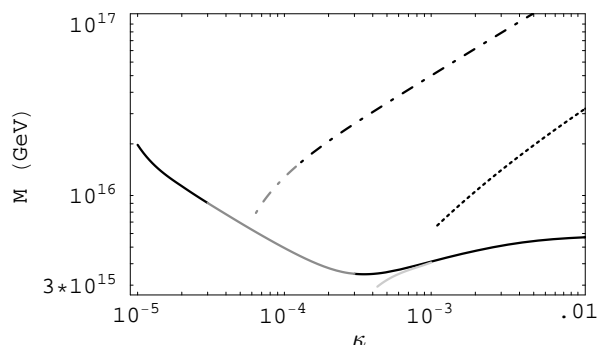


**Fig. 5.** The value of the symmetry breaking scale $M$ vs. $\kappa$, for SUSY hybrid inflation with $\mathcal{N} = 1$ (solid), and for shifted hybrid inflation (dot-dashed for $M_S = m_P$, dotted for $M_S = 5 \times 10^{17}$ GeV). Light grey portions of the curves are for $a < 0$, where only the segments that do not overlap with the solutions for $a > 0$ are shown. The grey segments denote the range of $\kappa$ for which the change in $\arg S$ is significant.

$SU(3)$ and $SU(2)$ gauge couplings. The values are remarkably consistent with the ones fixed from $\delta T/T$ considerations [42].

Here, some remarks concerning the allowed range of $\kappa$ is in order. As discussed above, a lower bound on $\kappa$ is obtained from the inflationary dynamics and the amplitude of the curvature perturbation. An upper bound on $\kappa$ is obtained from the value of the spectral index, which we discuss next. The gravitino constraint provides a more stringent upper bound ($\kappa \lesssim 10^{-2}$), as discussed in the next section. If cosmic strings form (as would be the case for $\mathcal{N} = 1$), the range of $\kappa$ is also restricted by the limits on the cosmic string contribution to $\mathcal{P}_{\mathcal{R}}^{1/2}$, however most of the range may still be allowed [41].

In the absence of SUGRA corrections, the scalar spectral index $n_s$ for $\kappa \gg 10^{-3}$ is given by [33]

$$n_s \simeq 1 + 2\eta \simeq 1 - \frac{1}{N_0} \simeq 0.98, \tag{26}$$

while it approaches unity for small $\kappa$. When the SUGRA correction is taken into account, from Eq. (22),

$$n_s \simeq 1 + 2\eta \simeq 1 + 2\left(3\sigma^2 - \frac{\kappa^2\mathcal{N}}{8\pi^2\sigma^2}\right), \tag{27}$$

which exceeds unity for $\sigma^2 \gtrsim \kappa\sqrt{\mathcal{N}}/2\sqrt{3}\pi$. For $x_0 \gg 1$, Eq. (12) yields

$$N_0 \approx \frac{\pi}{2\sigma_0^2} \frac{\kappa}{\kappa_c} \tan\left(\frac{\pi}{2}\frac{\kappa}{\kappa_c}\right), \tag{28}$$

where $\kappa_c = \pi^2/\sqrt{\mathcal{N}}N_0 \simeq 0.16/\sqrt{\mathcal{N}}$. Using Eq. (28), one finds that the spectral index $n_s$ exceeds unity for $\kappa \simeq 2\pi/\sqrt{3\mathcal{N}}N_0 \simeq 0.06/\sqrt{\mathcal{N}}$. The dependence of $n_s$ on $\kappa$ is displayed in Fig. 6. $\alpha$ is small and the tensor to scalar ratio $r$ is negligible, as shown in Fig. 7.
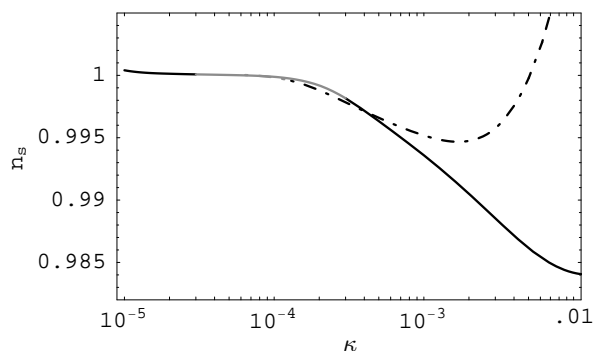
**Fig. 6.** The spectral index $n_s$ vs. $\kappa$, for SUSY hybrid inflation with $\mathcal{N} = 1$ (solid), and for shifted hybrid inflation with $M_S = m_P$ (dot-dashed). The grey segments denote the range of $\kappa$ for which the change in $\arg S$ is significant.
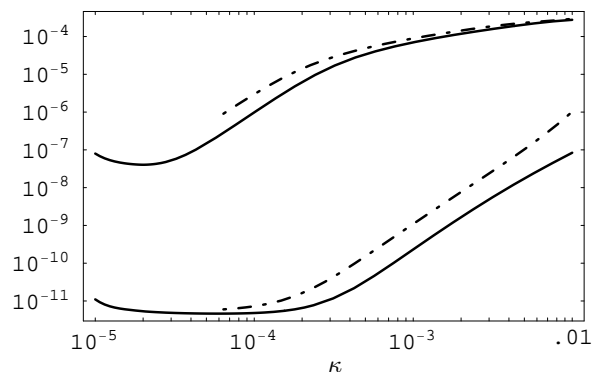


**Fig. 7.** $\alpha$ vs. $\kappa$ (top) and tensor to scalar ratio $r$ vs. $\kappa$ (bottom), for SUSY hybrid inflation with $\mathcal{N} = 1$ (solid), and for shifted hybrid inflation with $M_S = m_P$ (dot-dashed).

For negligible $r$, the WMAP three year central value for the spectral index is $n_s \approx 0.95$, and SUSY hybrid inflation with a minimal Kähler potential is disfavoured at a $2\sigma$ level [3]. It was recently shown that the spectral index for SUSY hybrid inflation can be in better agreement with the WMAP3 results in the presence of a small negative mass term in the potential. This can result from a non-minimal Kähler potential, in particular from the term proportional to the dimensionless coupling $\kappa_S$ referred to in Eq. (17) [43,44]. The spectral index $n_s$ for different values of $\kappa_S$ is displayed in Fig. 8.

The inflationary scenario based on the superpotential $W_1$ in Eq. (10) has the characteristic feature that the end of inflation essentially coincides with the gauge symmetry breaking. Thus, modifications should be made to $W_1$ if the breaking of $G$ to $H$ leads to the appearance of topological defects such as monopoles or domain walls. For instance, the breaking of $G_{PS} \equiv SU(4)_c \times SU(2)_L \times SU(2)_R$ [27] to the MSSM by fields belonging to $\Phi(\overline{4}, 1, 2)$, $\overline{\Phi}(4, 1, 2)$ produces magnetic monopoles that carry two quanta of Dirac magnetic charge [29]. As shown in [45], one simple resolution of the topological defects problem is achieved by supple-
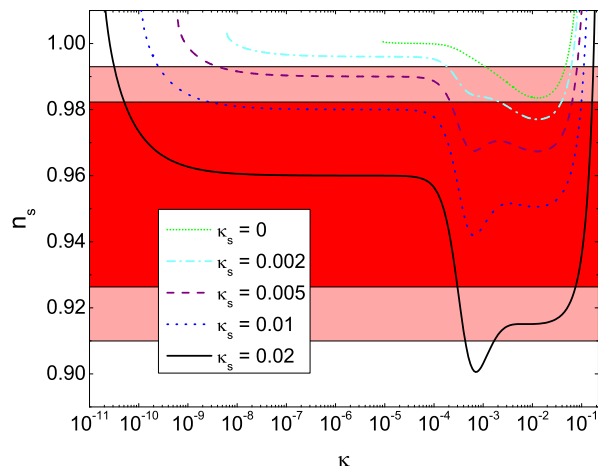


**Fig. 8.** The spectral index $n_s$ vs. $\kappa$, for SUSY hybrid inflation with $\mathcal{N} = 1$ and different values of $\kappa_S$ [44].

menting $W_1$ with a non-renormalizable term:

$$W_2 = \kappa S(\overline{\Phi}\Phi - v^2) - \frac{S(\overline{\Phi}\Phi)^2}{M_S^2}, \tag{29}$$

where $v$ is comparable to the SUSY GUT scale $M_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV and $M_S$ is an effective cutoff scale. The dimensionless coefficient of the non-renormalizable term is absorbed in $M_S$. The presence of the non-renormalizable term enables an inflationary trajectory along which the gauge symmetry is broken. Thus, in this 'shifted' hybrid inflation model the topological defects are inflated away.

The inflationary potential is similar to Eq. (21):

$$V_2 = \kappa^2 m^4 \left[ 1 + \frac{\kappa^2}{16\pi^2} \left( 2\ln\frac{\kappa^2|S|^2}{\Lambda^2} \right. \right. \tag{30}$$

$$\left. + (z+1)^2 \ln(1+z^{-1}) + (z-1)^2 \ln(1-z^{-1}) \right)$$

$$\left. + \frac{|S|^4}{2} \right] + a\, m_{3/2} \kappa v^2 |S| + \kappa^3 m^4 M_S^2 |S|^2.$$

Here $m^2 = v^2(1/4\xi - 1)$ where $\xi = v^2/\kappa M_S^2$, $z \equiv x^2 \equiv \sigma^2/m^2$, and $2 - A$ is replaced by $2 - A + A/2\xi$ in the expression for $a$. Note that the potential also contains a mass term even for minimal Kähler potential, due to the nonvanishing VEVs of $\Phi$, $\overline{\Phi}$.

The VEV $M = \left|\langle \nu_H^c \rangle\right| = \left|\langle \overline{\nu}_H^c \rangle\right|$ at the SUSY minimum is given by [45]

$$\left(\frac{M}{v}\right)^2 = \frac{1}{2\xi}\left(1 - \sqrt{1 - 4\xi}\right), \tag{31}$$

and is $\sim 10^{16} - 10^{17}$ GeV depending on $\kappa$ and $M_S$. The system follows the inflationary trajectory for $1/7.2 < \xi < 1/4$, which is satisfied for $\kappa \gtrsim 10^{-5}$ if the effective cutoff scale $M_S = m_P$. For lower values of $M_S$, the inflationary trajectory is followed only for higher values of $\kappa$, and $M$ is lower for a given $\kappa$ (Fig. 5). The spectral index is displayed in Fig. 6.

## 2.2 Leptogenesis In Supersymmetric Hybrid Inflation Models

An important constraint on SUSY hybrid inflation models arises from considering the reheating temperature $T_r$ after inflation, taking into account the gravitino problem which requires that $T_r \lesssim 10^6$–$10^{11}$ GeV [46]. This constraint on $T_r$ depends on the SUSY breaking mechanism and the gravitino mass $m_{3/2}$. For gravity mediated SUSY breaking models with unstable gravitinos of mass $m_{3/2} \simeq 0.1$–$1$ TeV, $T_r \lesssim 10^6$–$10^9$ GeV [47], while $T_r \lesssim 10^{10}$ GeV for stable gravitinos [48]. In gauge mediated models the reheating temperature is generally more severely constrained, although $T_r \sim 10^9$–$10^{10}$ GeV is possible for $m_{3/2} \simeq 5$–$100$ GeV [49]. Finally, the anomaly mediated symmetry breaking (AMSB) scenario may allow gravitino masses much heavier than a TeV, thus accommodating a reheating temperature as high as $10^{11}$ GeV [50].

After the end of inflation in the models discussed in section 2.1, the fields fall toward the SUSY vacuum and perform damped oscillations about it. The vevs of $\overline{\Phi}, \Phi$ along their right handed neutrino components $\overline{\nu}_H^c$, $\nu_H^c$ break the gauge symmetry. The oscillating system, which we collectively denote as $\chi$, consists of the two complex scalar fields $(\delta\overline{\nu}_H^c + \delta\nu_H^c)/\sqrt{2}$ (where $\delta\overline{\nu}_H^c$, $\delta\nu_H^c$ are the deviations of $\overline{\nu}_H^c$, $\nu_H^c$ from $M$) and $S$, with equal mass $m_\chi$.

We assume here that the inflaton $\chi$ decays predominantly into right handed neutrino superfields $N_i$, via the superpotential coupling $(1/m_P)\gamma_{ij}\overline{\phi}\,\phi N_i N_j$ or $\gamma_{ij}\overline{\phi}N_i N_j$, where $i, j$ are family indices (see below for a different scenario connected to the resolution of the MSSM $\mu$ problem). Their subsequent out of equilibrium decay to lepton and Higgs superfields generates lepton asymmetry, which is then partially converted into the observed baryon asymmetry by sphaleron effects [30].

The right handed neutrinos, as shown below, can be heavy compared to the reheating temperature $T_r$. Unlike thermal leptogenesis, there is then no supression factor in the lepton asymmetry, since the washout is proportional to the Boltzmann factor $e^{-z}$ (where $z = M_1/T_r$) and can be neglected for $z \gtrsim 10$ [51].[5] Without this assumption, generating sufficient lepton asymmetry would require a reheat temperature $\gtrsim 2 \times 10^9$ GeV [53].

GUTs typically relate the Dirac neutrino masses to that of the quarks or charged leptons. It is therefore reasonable to assume the Dirac masses are hierarchical. The low-energy neutrino data indicates that the right handed neutrinos will then also be hierarchical in general. As discussed in Ref. [54], setting the Dirac masses strictly equal to the up-type quark masses and fitting to the neutrino oscillation parameters generally yields strongly hierarchical right handed neutrino masses ($M_1 \ll M_2 \ll M_3$), with $M_1 \sim 10^5$ GeV. However, it is plausible that there are large radiative cor-

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[5] Lepton number violating 2-body scatterings mediated by right handed neutrinos are also out of equilibrium [52].
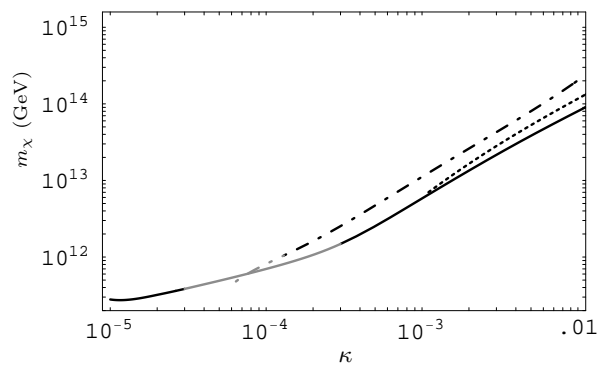


**Fig. 9.** The inflaton mass $m_\chi$ vs. $\kappa$, for SUSY hybrid inflation with $\mathcal{N} = 1$ (solid), and for shifted hybrid inflation (dot-dashed for $M_S = m_P$, dotted for $M_S = 5 \times 10^{17}$ GeV). The grey segments denote the range of $\kappa$ for which the change in $\arg S$ is significant.

rections to the first two family Dirac masses, so that $M_1$ remains heavy compared to $T_r$.

A reasonable mass pattern is therefore $M_1 < M_2 \ll M_3$, which can result from either the dimensionless couplings $\gamma_{ij}$ or additional symmetries (see e.g. [22]). The dominant contribution to the lepton asymmetry is still from the decays with $N_3$ in the loop, as long as the first two family right handed neutrinos are not quasi degenerate. Under these assumptions, the lepton asymmetry is given by [55]

$$\frac{n_L}{s} \lesssim 3 \times 10^{-10} \frac{T_r}{m_\chi} \left( \frac{M_i}{10^6 \text{ GeV}} \right) \left( \frac{m_{\nu 3}}{0.05 \text{ eV}} \right), \quad (32)$$

where $M_i$ denotes the mass of the heaviest right handed neutrino the inflaton can decay into. The decay rate $\Gamma_\chi = (1/8\pi)(M_i^2/M^2)m_\chi$ [36], and the reheating temperature $T_r$ is given by

$$T_r = \left( \frac{45}{2\pi^2 g_*} \right)^{1/4} (\Gamma_\chi\, m_P)^{1/2} \simeq 0.063 \frac{(m_P\, m_\chi)^{1/2}}{M} M_i. \quad (33)$$

From the experimental value of the baryon to photon ratio $\eta_B \simeq 6.1 \times 10^{-10}$ [19], the required lepton asymmetry is found to be $n_L/s \simeq 2.5 \times 10^{-10}$ [56]. Using this value, along with Eqs. (32, 33), we can express $T_r$ in terms of the symmetry breaking scale $M$ and the inflaton mass $m_\chi$:

$$T_r \gtrsim \left( \frac{10^{16} \text{ GeV}}{M} \right)^{1/2} \left( \frac{m_\chi}{10^{11} \text{ GeV}} \right)^{3/4}$$

$$\times \left( \frac{0.05 \text{ eV}}{m_{\nu 3}} \right)^{1/2} 1.6 \times 10^7 \text{ GeV}. \quad (34)$$

Here $m_\chi$ is given by $\sqrt{2}\kappa M$ and $\sqrt{2}\kappa M\sqrt{1-4\xi}$ respectively for hybrid and shifted hybrid inflation. The value of $m_\chi$ is shown in Fig. 9. We show the lower bound on $T_r$ calculated using this equation (taking $m_{\nu 3} = 0.05$ eV) in Fig. 10.

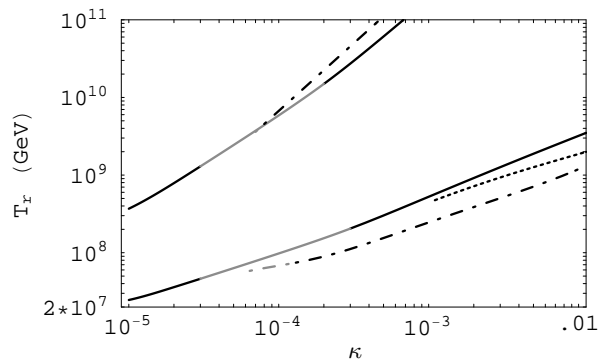Eq. (33) also yields the result that the heaviest right handed neutrino the inflaton can decay into is

**Fig. 10.** The lower bound on the reheating temperature $T_r$ vs. $\kappa$, for SUSY hybrid inflation with $\mathcal{N} = 1$ (solid) and for shifted hybrid inflation (dot-dashed for $M_S = m_P$, dotted for $M_S = 5 \times 10^{17}$ GeV). The segments in the top left part of the figure correspond to the bounds in the presence of a $\lambda Sh^2$ coupling. The grey segments denote the range of $\kappa$ for which the change in $\arg S$ is significant.

about 400 (6) times heavier than $T_r$, for hybrid inflation with $\kappa = 10^{-5}$ ($10^{-2}$). For shifted hybrid inflation, this ratio does not depend on $\kappa$ as strongly and is $\sim 10^2$. This is consistent with ignoring washout effects as long as the lightest right handed neutrino mass $M_1$ is also $\gg T_r$.[6]

Both the gravitino constraint and the constraint $M_1 \gg T_r$ favor smaller values of $\kappa$ for hybrid inflation, with $T_r \gtrsim 2 \times 10^7$ GeV for $\kappa \sim 10^{-5}$. Similarly, the gravitino constraint favors $\kappa$ values as small as the inflationary trajectory allows for shifted hybrid inflation, and $T_r \gtrsim 5 \times 10^7$ GeV for $M_S = m_P$.

So far we have not addressed the $\mu$ problem and the relationship to $T_r$ in the present context. The MSSM $\mu$ problem can naturally be resolved in SUSY hybrid inflation models in the presence of the term $\lambda Sh^2$ in the superpotential, where $h$ contains the two Higgs doublets [58]. (The 'bare' term $h^2$ is not allowed by the $U(1)$ R-symmetry.) After inflation the VEV of $S$ generates a $\mu$ term with $\mu = \lambda \langle S \rangle = -m_{3/2}\lambda/\kappa$, where $\lambda > \kappa$ is required for the proper vacuum. The inflaton in this case predominantly decays into higgses (and higgsinos) with $\Gamma_h = (1/16\pi)\lambda^2 m_\chi$. As a consequence the presence of this term significantly increases the reheating temperature $T_r$. Following Ref. [59], we calculate $T_r$ for the best case scenario $\lambda = \kappa$. We find a lower bound on $T_r$ of $4 \times 10^8$ GeV in hybrid inflation, see Fig. 10. $T_r \gtrsim 4 \times 10^9$ GeV for shifted hybrid inflation with $M_S = m_P$. An alternative resolution of the $\mu$ problem in SUSY hybrid inflation involves a Peccei-Quinn (PQ) symmetry $U(1)_{\mathrm{PQ}}$ [60,45].

The lower bounds on $T_r$ are summarized in Table 2. There is some tension between the gravitino constraint and the reheating temperature required to generate sufficient lepton asymmetry, particularly for gravity

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[6] If $M_1 < T_r$, part of the asymmetry created by decays of the next-to-lightest right handed neutrino will be washed out. However, the asymmetry that survives the washout can still be sufficient to account for the observed baryon asymmetry [57].

**Table 2.** Lower bounds on the reheating temperature (GeV)

|  | without $\lambda Sh^2$ | with $\lambda Sh^2$ |
|---|---|---|
| SUSY hybrid inflation | $2 \times 10^7$ | $4 \times 10^8$ |
| Shifted hybrid inflation | $5 \times 10^7$ | $4 \times 10^9$ |

mediated SUSY breaking models, and if hadronic decays of gravitinos are not suppressed. However, we should note that having quasi degenerate neutrinos would increase the lepton asymmetry per neutrino decay $\epsilon$ [61] and thus allow lower values of $T_r$ corresponding to lighter right handed neutrinos. Provided that the neutrino mass splittings are comparable to their decay widths, $\epsilon$ can be as large as $1/2$ [62]. The lepton asymmetry in this case is of order $T_r/m_\chi$ where $m_\chi \sim 10^{11}$ GeV for $\kappa \sim 10^{-5}$, and sufficient lepton asymmetry can be generated with $T_r$ close to the electroweak scale.

Finally, it is worth noting that new inflation models have also been considered in the framework of supersymmetric GUTs, taking account of supergravity corrections. In Ref. [22], for instance, it is shown that the spectral index $n_s$ is less than 0.98. Furthermore, reheating temperatures as low as $10^4$–$10^6$ GeV can be realized to satisfy the gravitino constraint.

### Acknowledgements

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