1. Introduction: Supersymmetry, gauge and Yukawa couplings

2. Some comments about ILC

3. Phenomenological analysis for LHC

4. Results for three cases:
   - Maximal model assumptions
   - Medium assumptions
   - (Almost) no assumptions
Supersymmetric couplings

Fundamental relation in supersymmetry:

\[ g = \text{Yukawa coupling } \tilde{g} \]

\[ \rightarrow \text{not broken by SUSY breaking} \]
\[ \text{required to resolve hierarchy problem} \]

Establish SUSY experimentally:

- Find new particles
- Measure their spin, masses, ...
- **Test SUSY coupling relations**
  \[ \rightarrow \text{compare precise cross-section measurements} \]
  \[ \text{with theoretical predictions} \]
SUSY couplings in the electroweak sector

Electroweak couplings can be probed at $\%$ level in

- Neutralino production

  \[
  e^+ \rightarrow \tilde{g}, \tilde{g}' \rightarrow \tilde{\chi}_j^0 \rightarrow \tilde{e}^- \rightarrow \tilde{\chi}_k^0 \rightarrow e^-
  \]

  Choi, Kalinowski, Moortgat-Pick, Zerwas '01

- Slepton production

  \[
  e^+ \rightarrow \gamma, Z \rightarrow \tilde{\mu}^+ \rightarrow g, g' \rightarrow e^-
  \]

  Freitas, v.Manteuffel, Zerwas '03

\[ g' \text{ U(1) coupl.} \]
\[ g \text{ SU(2) coupl.} \]
Testing SUSY-QCD couplings at ILC vs. LHC

Difficult at $e^+e^-$ colliders: Brandenburg, Maniatis, Weber '02

$e^+e^- \rightarrow q\tilde{q}\tilde{g}$

- Need large center-of-mass energy $\mathcal{O}(2 \text{ TeV})$
- Small cross-section $\mathcal{O}(\text{fb})$ for $q\tilde{q}\tilde{g}$ production
  → including BRs the statistics very low compared to background

Alternative: Measure QCD production process at LHC

$pp \rightarrow \tilde{q}\tilde{q}(\ast), \tilde{q}(\ast)\tilde{g}, \tilde{g}\tilde{g}$

gauge coupling $g_s$

Yukawa coupling $\tilde{g}_s$
Signal processes at LHC

Maximal information when tracking squark charges

Tagging of squark charge through chargino decay chain:

\[ \tilde{u}_L \rightarrow d \tilde{\chi}^+_1 \rightarrow d l^+ \nu_l \tilde{\chi}^0_1 \]
\[ \tilde{d}_L \rightarrow u \tilde{\chi}^-_1 \rightarrow u l^- \tilde{\nu}_l \tilde{\chi}^0_1 \]

Signature: Two same-sign leptons, two hard jets, missing energy

Reduces SM background

Contributing processes:

\[ pp \rightarrow \tilde{q}_L \tilde{q}_L \]
\[ pp \rightarrow \tilde{q}_L \tilde{g} \]
\[ pp \rightarrow \tilde{g} \tilde{q}_L \]

\[ \tilde{g} \rightarrow q \tilde{q}_L \]

Problem: Separate \( \tilde{q} \) from \( \tilde{g} \) production

→ Gluinos produce extra (hard) jet:

\[ \tilde{g} \rightarrow q \tilde{q}_L \]

Assume here that \( m_{\tilde{g}} - m_{\tilde{q}_L} \) sufficiently large to cut on extra jet !!
**Benchmark scenario**

Scenario similar to SPS1a, but with larger gluino mass

\[
\begin{align*}
M_1 &= 99 & m_L &= 197 & m_{Q1} &= 540 \\
M_2 &= 193 & m_R &= 136 & m_{U1} &= 522 \\
M_3 &= 700 & \tan \beta &= 10 & m_{D1} &= 520 \\
\mu &= 352 & A_\tau &= -254
\end{align*}
\]

\[
\begin{align*}
m_{\tilde{u}_L} &= 537 & m_{\tilde{\chi}_0^0} &= 96 \\
m_{\tilde{d}_L} &= 543 & m_{\tilde{\chi}_2^0} &= 177 \\
m_{\tilde{\tau}_1} &= 133 & m_{\tilde{\chi}_1^\pm} &= 176 \\
m_{\tilde{g}} &= 700 & m_{\tilde{\chi}_{3,4}^0} &\sim 360
\end{align*}
\]

Interesting decay chain:

\[
\tilde{u}_L \xrightarrow{65\%} u \tilde{\chi}_1^+ \xrightarrow{100\%} u \tau^+ \nu_\tau \tilde{\chi}_1^0 \xrightarrow{35\%} u l^+ + \not{E}_T, \quad l = e, \mu
\]

LHC backgrounds:

- \( t\bar{t} \)
- \( W^\pm W^\pm jj \)
- \( (W^\pm Z) \)

Cuts:

- veto on bottom jets
- 2 jets with \( p_{T,jets} > 200 \text{ GeV} \)
- \( \not{E}_T > 300 \text{ GeV} \)
Analysis I: Total cross-section

Cross-sections after cuts:

\[ \tilde{q}_L \tilde{q}_L \quad \tilde{q}_L \tilde{q}_L^* \quad \tilde{q}_L \tilde{g} \quad \tilde{g}\tilde{g} \quad \text{SM} \]

\begin{align*}
6.1 \text{ fb} & \quad 3.1 \text{ fb} & \quad 5.8 \text{ fb} & \quad 0.8 \text{ fb} & \quad 0.6 \text{ fb} \\
& \quad \text{with } 300 \text{ fb}^{-1}: \\
\sim 5000 \text{ signal events}
\end{align*}

Interpretation in terms of Yukawa coupling \( \hat{g}_s \):

Use cross-section formulae with \( \hat{g}_s \) as variable parameter

\[
\delta[\hat{g}_s/gs]
\]

<table>
<thead>
<tr>
<th></th>
<th>\delta[\hat{g}_s/gs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics for 300 fb(^{-1})</td>
<td>0.6%</td>
</tr>
<tr>
<td>PDF uncertainty</td>
<td>1.4%</td>
</tr>
<tr>
<td>NNLO corrections(^*)</td>
<td>2.0%</td>
</tr>
<tr>
<td>Mass measurements ( \Delta m_{\tilde{q}} = 10 \text{ GeV} )</td>
<td>2.0%</td>
</tr>
<tr>
<td></td>
<td>1.0%</td>
</tr>
<tr>
<td>( \Delta m_{\tilde{g}} = 12 \text{ GeV} )</td>
<td>3.4%</td>
</tr>
</tbody>
</table>

\(^*\) NLO corrections available \text{ BEENAKKER, HÖPKER, SPIRA, ZERWAS '96}
Analysis II: Cross-section ratios

Measure not absolute cross-section, but ratio of cross-sections

Use all processes that can lead to same-sign lepton signal

\[ \tilde{g} \rightarrow q \tilde{\nu}_L \]

gauge coupling \( g_s \)

Yukawa coupling \( \tilde{g}_s \)

\[
\begin{align*}
\sigma[\tilde{q}\tilde{q}] & \sim \tilde{g}_s^4 \\
\sigma[\tilde{q}\tilde{q}^*] & \sim A\tilde{g}_s^4 + Bg_s^4 \\
\sigma[\tilde{g}\tilde{g}] & \sim \tilde{g}_s^2 g_s^2 \\
\sigma[\tilde{g}\tilde{g}] & \sim A'\tilde{g}_s^4 + B'g_s^4
\end{align*}
\]
Analysis II: Cross-section ratios

Can distinguish processes by dependence on extra jets

\[ \sigma \ [\text{fb}] \]

\[ N_{\text{jets}} \text{ with } p_T > 50 \text{ GeV} \]
Analysis II: Cross-section ratios

- CP and weak isospin invariance:
  \[
  \Rightarrow \quad \text{BR}[\tilde{u}_L \rightarrow d\tilde{\chi}^+_1] \approx \text{BR}[\tilde{d}_L \rightarrow u\tilde{\chi}^-_1]
  \]
  in SPS1a: 65% \approx 61%

- **But:** Large bino-wino mixing in neutralino sector can cause
  \[\Gamma_{\text{tot}}[\tilde{u}_L] \neq \Gamma_{\text{tot}}[\tilde{d}_L]\]
  \[\tilde{\chi}^0\] mixing is small for large hierarchy \(m_{\tilde{\chi}^\pm_1}, m_{\tilde{\chi}^0_2} \gg m_{\tilde{\chi}^0_1}\)
  (Can be tested at LHC/ILC)

  **Note:** Signal selection depends on \(m_{\tilde{g}} \gg m_{\tilde{q}} \gg m_{\tilde{\chi}^\pm_1} \gg m_{\tilde{\chi}^0_1}\)

- Can also allow new electroweak singlets (e.g. NMSSM), since they modify \(\Gamma_{\text{tot}}[\tilde{u}_L]\) and \(\Gamma_{\text{tot}}[\tilde{d}_L]\) identically
Analysis II: Cross-section ratios

- Need to know \( \text{BR}[\tilde{g} \rightarrow q \tilde{q}_L] \)

\[
\text{BR}[\tilde{g} \rightarrow q \tilde{q}_L] = \text{const.} \times \tilde{\alpha}_s \frac{(m^2_{\tilde{g}} - m^2_{\tilde{q}_L})^2}{m^3_{\tilde{g}}}
\]

Depends only on squark and gluino masses

- Decays into heavy flavour, \( \tilde{g} \rightarrow b\bar{b}, \tilde{g} \rightarrow t\bar{t} \) are difficult due to mixings
  \( \rightarrow \)Reject with b veto
3rd jet radiation

Dependence on $p_{T,j3}$:

Cuts to remove SM bkgd.:

- $p_{T,j} > 200$ GeV
- $E > 300$ GeV
- b-tagging

(for 300 fb$^{-1}$)
4th jet radiation

Dependence on $p_{T,j4}$:

$p_{T,j3} > 50$ GeV

Cuts to remove SM bkgd.:

$p_{T,j} > 200$ GeV
$E > 300$ GeV
b-tagging

(for 300 fb$^{-1}$)
Analysis II: Results

Fit independently $\hat{g}_s/g_s$ and $\text{BR}[\tilde{q}_L \rightarrow q l^{\pm} E]$.

<table>
<thead>
<tr>
<th>Error on</th>
<th>$\delta[\hat{g}_s/g_s]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics for 300 fb$^{-1}$</td>
<td>3.3%</td>
</tr>
<tr>
<td>PDF uncertainty</td>
<td>2.9%</td>
</tr>
<tr>
<td>NNLO corrections</td>
<td>3.1%</td>
</tr>
<tr>
<td>$\Delta m_{\tilde{q}} = 10$ GeV</td>
<td>1.1%</td>
</tr>
<tr>
<td>$\Delta m_{\tilde{g}} = 12$ GeV</td>
<td>2.0%</td>
</tr>
<tr>
<td>$\text{BR}[\tilde{q}<em>L \rightarrow q l^{\pm} E]/\text{BR}</em>{\text{MSSM}}$</td>
<td>5.9%</td>
</tr>
</tbody>
</table>
Analysis III: Generalized picture

- Allow arbitrary $\tilde{\chi}^0$ mixing
  
  $\rightarrow BR(\tilde{u}_L \rightarrow \tilde{\chi}^+_1)$ and $BR(\tilde{d}_L \rightarrow \tilde{\chi}^-_1)$ undetermined

- Use in addition to $p_T$ spectra also ratio of $l^+l^+ / l^-l^-$ in signal

  - $\tilde{\chi}^0_i$ give equal contribution to $l^+l^+$ and $l^-l^-$
  
  - Only $\tilde{\chi}^{\pm}_1$ create difference between $l^+l^+$ and $l^-l^-$

- Separate determination of $BR(\tilde{u}_L \rightarrow \tilde{\chi}^+_1)$ and $BR(\tilde{d}_L \rightarrow \tilde{\chi}^-_1)$ difficult
  
  $\rightarrow$ Small sensitivity to squark flavour from PDF effects
Analysis III: Results
Analysis III: Results

\[
\frac{BR[\tilde{d}_L \rightarrow uX_1^\pm]}{BR_{MSSM}[\tilde{d}_L \rightarrow uX_1^\pm]} / \frac{BR[\tilde{u}_L \rightarrow dX_1^\pm]}{BR_{MSSM}[\tilde{u}_L \rightarrow dX_1^\pm]}
\]

<table>
<thead>
<tr>
<th>Error on ( g_s / g_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics for 300 fb(^{-1} )</td>
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<tr>
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<td>( \Delta m_{\tilde{q}} = 10 ) GeV</td>
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<tr>
<td>( \Delta m_{\tilde{g}} = 12 ) GeV</td>
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</tbody>
</table>
Conclusions

• Quest for SUSY involves the test of SUSY coupling relations:
  Fundamental identity
  \( g_{\text{gauge}} = g_{\text{Yukawa}} \)

• Using a dedicated LHC analysis, the strong SUSY coupling identity can be tested to the % level
  
  I: Full knowledge about model spectrum: \( \delta \hat{g}_s \sim 3.4\% \)
  II: Assuming only \( \tilde{\chi}^0 \) mass hierarchy: \( \delta \hat{g}_s \sim 5.9\% \)
  III: Allowing (almost) general N\(^n\)MSSM: \( \delta \hat{g}_s \sim 7.5\% \)

• Encouraging prospects, but depends strongly on SUSY scenario!
Backup slides
Analysis I: Total cross-section

Assume that all squark BRs known (or from $e^+e^-$ collider)

Selection of same-sign squark signal

1. at least 2 jets with $p_{T,j} > 200$ GeV
2 same-sign leptons, $p_{T,l} > 7$ GeV

2. b-tagging to reduce $tt$ efficiency 90%, $u,d$ mistag 25%
   ATLAS TDR ‘99

3. $E_T > 300$ GeV to cut SM background
Analysis I: Total cross-section

4. \( p_{T,j1} > 200 \) GeV to cut SM background

5. \( p_{T,j3} < 50 \) GeV to reduce \( \tilde{g} \) background

Remaining cross-sections:

\[ \begin{align*}
\tilde{q}_L\tilde{q}_L & \quad 6.1 \text{ fb} \\
\tilde{q}_L\tilde{q}_L^* & \quad 3.1 \text{ fb} \\
\tilde{q}_L\tilde{g} & \quad 5.8 \text{ fb} \\
\tilde{g}\tilde{g} & \quad 0.8 \text{ fb} \\
\text{SM} & \quad 0.6 \text{ fb}
\end{align*} \]

with 300 fb\(^{-1} \):

\( \sim 5000 \) signal events

\( \Delta_{\text{stat}} = 1.5\% \) on total cross-section

Interpretation in terms of Yukawa coupling \( \tilde{g}_s \):

Use cross-section formulae with \( \tilde{g}_s \) as variable parameter

\( \rightarrow \Delta \tilde{g}_s = 0.6\% \)
Input from linear collider

Branching ratios in LHC decay chain:
\( \tilde{u}_L \rightarrow d \tilde{\chi}_1^+, \tilde{d}_L \rightarrow u \tilde{\chi}_1^-, \tilde{\chi}_1^+ \rightarrow \tau^+ \nu_\tau \)

BRs of squarks can be studied in \( \bar{q}q^* \) production at \( e^+e^- \) collider
→ Need \( \sqrt{s} > 1 \) TeV in our scenario
→ Assume \( \sqrt{s} = 1.5 \) TeV

| \( P(e^+) \) | 50\% |
| \( P(e^-) \) | 80\% |

Identify different decay products of squarks by characteristic signature:
\( \tilde{\chi}_1^+ \rightarrow \tau^+ \nu_\tau \tilde{\chi}_1^0 \) (100\%)
\( \tilde{\chi}_2^+ \rightarrow Z \tilde{\chi}_1^+ \rightarrow Z \tau^+ \nu_\tau \tilde{\chi}_1^0 \) (24\%)
Assume 80\% \( \tau \) ID eff. for hadronic decay (BR = 65\%)
\( \tilde{\chi}_2^0 \rightarrow \tau \tau \tilde{\chi}_1^0 \) (100\%)
\( \tilde{\chi}_{3,4}^0 \rightarrow W^\pm \tilde{\chi}_1^\mp \rightarrow W^\pm \tau^\mp \nu_\tau \tilde{\chi}_1^0 \) (59\%, 52\%)

Use c-tagging (eff. 40\%, purity 90\%) to differentiate u- and d-squarks
**Input from linear collider**

Dominant SM background from $t\bar{t}$ and $VV$ or $VVV$ production

Can be reduced by cuts on $E, E_j$ and $m_{jj}$

From generator-level analysis of signal and background:

\(\sqrt{s} = 1.5 \text{ TeV and } L = 500 \text{ fb}^{-1}\)

\[\tilde{u}_L \rightarrow d\tilde{\chi}_1^+ \quad 67.7 \pm 3.2 \% \quad \tilde{d}_L \rightarrow u\tilde{\chi}_1^- \quad 63.9 \pm 5.2 \%\]
**Input from linear collider**

Need also information about BRs of charginos and neutralinos.

**New technique to obtain absolute BRs:**

**Measure near threshold:** unique signal of monoenergetic particles from two-body decays

\[ \tilde{\chi}_2^0 \tilde{\chi}_3^0 \text{ threshold, } \mathcal{L} = 50 \text{ fb}^{-1}: \quad \text{BR}[\tilde{\chi}_3^0 \rightarrow W^\pm \tilde{\chi}_1^\mp] = (59 \pm 6.5) \% \]

\[ \tilde{\chi}_3^0 \tilde{\chi}_4^0 \text{ threshold, } \mathcal{L} = 50 \text{ fb}^{-1}: \quad \text{BR}[\tilde{\chi}_4^0 \rightarrow W^\pm \tilde{\chi}_1^\mp] = (52 \pm 2.5) \% \]

\[ \tilde{\chi}_2^\pm \tilde{\chi}_1^\mp \text{ threshold, } \mathcal{L} = 50 \text{ fb}^{-1}: \quad \text{BR}[\tilde{\chi}_2^+ \rightarrow Z \tilde{\chi}_1^\mp] = (24 \pm 1.3) \% \]

Together with squark production at \( \sqrt{s} = 1.5 \text{ TeV} \) and \( \mathcal{L} = 500 \text{ fb}^{-1} \):

<table>
<thead>
<tr>
<th>Process</th>
<th>BR (± Error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{u}_L \rightarrow u\tilde{\chi}_1^0 )</td>
<td>0.9 ± 0.5 %</td>
</tr>
<tr>
<td>( u\tilde{\chi}_2^0 )</td>
<td>29.0 ± 3.0 %</td>
</tr>
<tr>
<td>( u\tilde{\chi}_3^0 )</td>
<td>&lt; 1 %</td>
</tr>
<tr>
<td>( u\tilde{\chi}_4^0 )</td>
<td>&lt; 1 %</td>
</tr>
<tr>
<td>( d\tilde{\chi}_1^+ )</td>
<td>67.7 ± 3.2 %</td>
</tr>
<tr>
<td>( d\tilde{\chi}_2^+ )</td>
<td>1.4 ± 0.7</td>
</tr>
<tr>
<td>( \tilde{d}_L \rightarrow d\tilde{\chi}_1^0 )</td>
<td>1.9 ± 0.8 %</td>
</tr>
<tr>
<td>( d\tilde{\chi}_2^0 )</td>
<td>28.3 ± 4.8 %</td>
</tr>
<tr>
<td>( d\tilde{\chi}_3^0 )</td>
<td>&lt; 0.2 %</td>
</tr>
<tr>
<td>( d\tilde{\chi}_4^0 )</td>
<td>1.9 ± 0.8 %</td>
</tr>
<tr>
<td>( u\tilde{\chi}_1^- )</td>
<td>63.9 ± 5.2 %</td>
</tr>
<tr>
<td>( u\tilde{\chi}_2^- )</td>
<td>4.0 ± 1.4 %</td>
</tr>
</tbody>
</table>