Abstract. The main production mechanism for supersymmetric Higgs particles at hadron colliders crucially depends on the size of their Yukawa couplings to bottom quarks. For sufficiently large $\tan \beta$ the total cross section for some of the neutral Higgs bosons in the MSSM is dominated by bottom-quark fusion. After an introduction to bottom-associated Higgs production, we discuss the complete $\mathcal{O}(\alpha)$ electroweak and $\mathcal{O}(\alpha_s)$ strong corrections for the $b\bar{b}$-fusion channel in the MSSM. Choosing proper renormalization and input-parameter schemes, an improved Born approximation, constructed from previously known results, can absorb the bulk of the corrections so that the remaining non-universal corrections are typically of the order of a few per cent. Numerical results are discussed for the SPS benchmark scenarios.


1 Introduction

The Higgs mechanism is a cornerstone of the Standard Model (SM) and its supersymmetric extensions. Thus, Higgs bosons are intensively searched for at the upgraded proton–antiproton collider Tevatron, followed in the near future by the proton–proton collider LHC. In this talk, we concentrate on the precise prediction of the total Higgs-boson production cross section at the LHC based on the results in Ref. [1].

In the SM, the total production cross section for Higgs bosons $H$ at the LHC is dominated by gluon fusion. Higgs radiation off bottom quarks [2]

$$pp/p\bar{p} \rightarrow b\bar{b}\phi^0+X$$

with $\phi^0=H$, is a negligible contribution. The relevant bottom Yukawa coupling $\lambda_b^{\text{SM}}$ is known to be small because it is determined by the ratio of the small bottom-quark mass $m_b$ and the known vacuum expectation value (VEV) $v$ of the SM Higgs field, $\lambda_b^{\text{SM}} = m_b/v$.

In contrast, in the MSSM, bottom-associated production of neutral Higgs bosons, $\phi^0 = h^0, H^0, A^0$, can dominate the total cross section at large $\tan \beta$. Two different Higgs doublets are needed to generate masses for up- and down-type fermions. These two Higgs doublets $H_u$ and $H_d$ acquire VEVs $v_u$ and $v_d$, respectively, and one defines $\tan \beta = v_u/v_d$. While $v^2 = v_u^2 + v_d^2$ is fixed by the gauge-boson masses, the ratio $\tan \beta$ is a free parameter. For large $\tan \beta$ the down-type VEV $v_d$ is small and the Yukawa coupling of the down-type Higgs doublet is enhanced with respect to its SM value. The mass $m_b$ is not small due to a small Yukawa coupling, on the contrary, the relevant VEV $v_d$ is small. For $\tan \beta \sim \mathcal{O}(50)$, the bottom Yukawa coupling is as big as the top Yukawa coupling in the SM.

The couplings of the CP-even Higgs-boson mass eigenstates are determined by the mixing of up- and down-type Higgs fields characterized by the mixing angle $\alpha$. For the Yukawa couplings to $b$ quarks one finds in the MSSM

$$\lambda_b^0 = -\lambda_b^{\text{SM}} \frac{\sin \alpha}{\cos \beta},$$

$$\lambda_b^{H_0} = \lambda_b^{A_0} = \lambda_b^{\text{SM}} \frac{\cos \alpha}{\cos \beta},$$

(2)

For sizeable mixing in the Higgs sector, the cross sections $\sigma$ for the $b$-associated production of all neutral MSSM Higgs bosons are enhanced for large $\tan \beta$, i.e. $\sigma \propto \tan^2 \beta$. However, one finds $\sin \alpha \rightarrow -\cos \beta$ if the mass $M_A$ of the pseudoscalar Higgs boson, the second input parameter of the MSSM Higgs sector, is large compared to the Z-boson mass. In this limit, the lighter CP-even Higgs boson $h^0$ is SM like and shows no enhanced bottom Yukawa coupling. Nevertheless, the total cross section for the two heavy neutral Higgs bosons $H^0$ and $A^0$ is dominated by $b$-associated production.

Current searches for bottom–Higgs associated production in the MSSM at the Fermilab Tevatron exclude values $\tan \beta \gtrsim 50$ for light $M_A \approx 100$ GeV [3]. For a recent sensitivity study at the LHC see Ref. [4].

The theoretical description for $b\bar{b}\phi^0$ production can start from different initial states for the hard scattering process. The $b$ quarks are either generated from
gluon splittings within the hard process or they are considered to be part of the proton, i.e. the gluon splitting is factorized from the hard process.

In the so-called four-flavour number scheme (4FNS) with no b quarks in the initial state, the lowest-order QCD production processes are gluon–gluon fusion and quark–antiquark annihilation, $gg \rightarrow b\bar{b}\phi^0$ and $q\bar{q} \rightarrow b\bar{b}\phi^0$, respectively. In this framework the splitting of gluons into $b\bar{b}$ pairs is treated retaining the full dependence on the bottom mass. The complete kinematics of the $2 \rightarrow 3$ process is available so that the bottom jets in the final state can be used for tagging and background suppression.

However, for a hard process involving a large scale, e.g. the Higgs-boson mass, the $b$ quark is effectively almost massless. Higgs production is thus dominated by events where gluons split into nearly collinear $b\bar{b}$ pairs. Consequently, the inclusive cross section for $gg \rightarrow b\bar{b}\phi^0$ contains large logarithms $\ln(\mu_F/m_b)$, where the large scale $\mu_F \sim M_{\phi^0}$ corresponds to the upper limit of the collinear region up to which factorization is valid. Hence, the perturbative expansion in $\ln(\mu_F/m_b)$, will eventually break down for large Higgs masses and the perturbation series has to be reorganized. The logarithms $\ln(\mu_F/m_b)$ can be summed to all orders in perturbation theory by introducing bottom parton densities. This defines the so-called five-flavour number scheme (5FNS) [5]. In this scheme, the leading-order (LO) process for the inclusive $b\bar{b}\phi^0$ cross section is $b\bar{b}$ fusion,

$$b\bar{b} \rightarrow \phi^0.$$ (3)

The next-to-leading order (NLO) cross section in the 5FNS includes $O(\alpha_s)$ corrections to $b\bar{b} \rightarrow \phi^0$ and tree-level processes like $gb \rightarrow b\phi^0$. For developments on corrections to the latter subprocess see Ref. [6].

The use of bottom distribution functions is based on the collinear approximation, i.e. outgoing $b$ quarks are considered to have small transverse momentum and to be part of the proton remnant. There is no theoretical control over additional $b$ jets at LO.

To all orders in perturbation theory the four- and five-flavour schemes are identical, but the way of ordering the perturbative expansion is different, and the results do not match exactly at finite order. However, numerical comparisons between calculations of inclusive Higgs production in the two schemes [7][8][9][10] show that the two approaches agree within their respective uncertainties, once higher-order QCD corrections are taken into account.

There has been considerable progress in improving the cross-section predictions for inclusive associated $b\bar{b}\phi^0$ production by calculating NLO-QCD [7][9] and SUSY-QCD [11] corrections in the four-flavour scheme, and NNLO QCD corrections [12] in the five-flavour scheme. In the 5FNS, the QCD scale uncertainties have been reduced to the 10% level such that radiative effects from the electroweak sector become of interest.

The complete one-loop QCD and electroweak corrections for the decay of MSSM Higgs bosons to bottom quarks have been presented in Ref. [13]. Recently, complete supersymmetric QCD and electroweak corrections to the hadronic production cross section have been presented in Ref. [1]. These results and their relation to known universal corrections are discussed in the following sections. For technical details and derivations we refer the reader to Ref. [1].

### 2 Radiative corrections

In $b$-quark fusion, $b\bar{b} \rightarrow \phi^0$, there are universal radiative corrections which lead to the definition of the improved Born approximation for the partonic cross section

$$\hat{\sigma}_{\text{IBA}} = \hat{\sigma}_{\text{SM}} \left\{ \begin{array}{l} \sin^2 \alpha_{\text{eff}} \cos^2 \beta \left( \frac{1 - \Delta_b/(\tan \beta \tan \alpha_{\text{eff}})}{1 + \Delta_b} \right)^2 \\
\cos^2 \alpha_{\text{eff}} \cos^2 \beta \left( \frac{1 + \Delta_b \tan \alpha_{\text{eff}} / \tan \beta}{1 + \Delta_b} \right)^2 \\
\tan^2 \beta \left( \frac{1 - \Delta_b / \tan^2 \beta}{1 + \Delta_b} \right)^2, \end{array} \right.$$ (4)

where

$$\hat{\sigma}_{\text{SM}} = \frac{\sqrt{2} \pi G_f m_b(\mu_R)^2}{6 M_{\phi^0}^2} \delta(1 - M_{\phi^0}/s).$$ (5)

We denote the partonic CMS energy by $\sqrt{s}$, $M_{\phi^0}$ is the mass of the produced Higgs boson, $G_f$ is the Fermi constant, and $m_b(\mu_R)$ is the running bottom mass at the renormalization scale $\mu_R$. Potentially large radiative corrections are encoded in the parameters $\Delta_b$ and $\alpha_{\text{eff}}$ to be briefly explained in the following.

While $b$ quarks do not couple to the Higgs field $H_u$ at tree level, this interaction is radiatively induced at the one-loop level, e.g. via the bottom coupling to $H_u$ in SUSY-QCD loops. This induces a shift $\Delta_b$ in the relation between the $b$ mass and the respective Yukawa coupling. This shift is proportional to $\tan \beta$ and, thus, for large $\tan \beta$ the corresponding correction is sizeable. It has been shown [14] that the correction can be resummed and that it affects the cross section according to eq. (4).

Radiative corrections can also have a large impact on the mixing of the Higgs fields to form the CP-even mass eigenstates $h^0$ and $H^0$. The bulk of these corrections can be absorbed in a loop-corrected, process-independent effective mixing angle $\alpha_{\text{eff}}$ which replaces its tree-level counterpart in eq. (4).

Precise definitions of $\Delta_b$ as well as $\alpha_{\text{eff}}$ are given in Ref. [1], where we also describe in detail the calculation of the complete SUSY-QCD and electroweak corrections including the renormalization of the MSSM Higgs sector. The bottom mass has been renormalized in such a way that the corrections due to $\Delta_b$ are absorbed into the input value for $m_b$. Thus, the numerical value for this effective $m_b$ quantifies the $\tan \beta$ enhanced corrections. This procedure automatically avoids double counting for the $\Delta_b$ corrections. When we relate the corrections from the full calculation to $\hat{\sigma}_{\text{IBA}}$ we also carefully avoid double counting with respect to corrections already contained in $\alpha_{\text{eff}}$. 

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**Colliders - Higgs Phenomenology**

**Contributed Talk**
Alexander Mück

MSSM Higgs-bosons from bottom quarks

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Table 1. The effective bottom mass and the NLO MSSM cross section $pp \rightarrow (b\bar{b})h^0/H^0/A^0 + X$ at the LHC ($\sqrt{s} = 14$ TeV) in the SPS 4 scenario including the cumulative corrections due to the different classes of corrections. See text for details on the different contributions. (Table taken from Ref. [1])

3 Results

All the results in this section are calculated in the DR scheme for $\tan \beta$. The renormalization and factorization scales are set to $\mu_R = M_{h^0}$ and $\mu_F = M_{h^0}/4$, respectively. We use the MRSTQED2004 PDF [15] which also allows the inclusion of the photon-induced partonic channels at NLO. The input-parameter scheme and the numerical input are specified in Ref. [1]. To further improve our NLO results, we use two-loop improved Higgs self-energies provided by the program package FeynHiggs [16].

Within the MSSM, we first focus on the radiative corrections and total cross sections in the SPS 4 benchmark scenario ($\tan \beta = 50$) [17] which was designed to give large cross sections for heavy Higgs bosons. At the end of this section we also show results for the other SPS points. While most of the SPS scenarios are in conflict with experimental data by now, they are still valuable because they cover typical SUSY scenarios in different regions of parameter space.

In Table 1, we show the effect of the various higher-order corrections on the effective $b$-mass. Starting from the running QCD mass at the scale of the Higgs-boson mass, the shifts from SUSY-QCD ($\Delta_b^\beta$) and the electroweak sector ($\Delta_b^{\text{weak}}$) are included. The corresponding cross sections $\sigma$ are first shown at NLO QCD. As can be seen, the QED corrections are generally very small after mass factorization. The summation of the $\tan \beta$-enhanced MSSM-QCD and MSSM-weak corrections has a significant effect on the cross sections for $H^0$ and $A^0$ production. The light Higgs boson $h^0$ is SM-like and the summation of terms $\propto \tan^2 \beta$ has thus no sizeable impact. Employing a loop-improved effective mixing angle $\alpha_{\text{eff}}$ is numerically relevant only for $h^0$ production. The cross sections in the last-but-one row of Table 1 correspond to the improved Born approximation $\sigma_{\text{IBA}}$ dressed with QCD and QED corrections.

The full MSSM cross sections, including all summations and the remaining non-universal $O(\alpha)$ and $O(\alpha)_{\text{eff}}$ corrections, are displayed in the last row of the table.

The bulk of the MSSM-QCD and -weak corrections can indeed be absorbed into the above definition of $\sigma_{\text{IBA}}$. The remaining non-universal corrections in the complete MSSM calculation turn out to be quite small, below approximately 2%.

In Fig. 1 we show the impact of the complete supersymmetric $O(\alpha)$ and $O(\alpha)_{\text{eff}}$ corrections defined relative to the improved Born approximation $\sigma_{\text{IBA}}$ for different values of the on-shell mass $M_A^\beta$. All other MSSM parameters are kept fixed at their SPS 4 values. The size of the non-universal corrections does not exceed 3% for $H^0/A^0$ production except for special model parameters where the Higgs masses are close to the production threshold for pairs of sparticles. These unphysical singularities can be removed by taking into account the finite widths of the unstable sparticles. The size of $\delta_{\text{MSSM}}$ for $h^0$ depends very sensitively on the definition of the effective mixing angle $\alpha_{\text{eff}}$ employed in $\sigma_{\text{IBA}}$. Note that in any case $h^0$ is SM-like at large $M_A^\beta$ so that $h^0$ production in bottom fusion is most likely of no phenomenological relevance.

It is important to emphasize that the non-universal MSSM corrections $\delta_{\text{MSSM}}$ at large $\tan \beta$ are quite sensitive to the choice of the $b$-mass input value within the one-loop corrections which is not fixed by the renormalization procedure. There are terms that grow as $m_b^2 \tan^2 \beta$ which are not included in $\Delta_b$. For the SPS 4 scenario the sensitivity on the $b$-mass input is shown in Fig. 2. The absolute size of the non-universal corrections varies between approximately zero and $-6\%$ for the phenomenologically relevant $H^0/A^0$ production, depending on whether a massless approximation.

![Fig. 1. Full MSSM corrections $\delta_{\text{MSSM}}$ defined relative to $\sigma_{\text{IBA}}$ as a function of the $M_A^\beta$ pole mass. All other MSSM parameters are fixed to their SPS 4 values. (Figure taken from Ref. [1])](image-url)
the effective running mass, or the pole mass is chosen as b-mass input. Although we assume that the running mass, including the corrections from $\Delta_b$ (as used for all the shown results), is a sensible choice, the sensitivity of the NLO correction to the b-mass input constitutes a theoretical uncertainty which cannot be resolved at the NLO level.

Table 2 displays the cross sections along with the non-universal corrections from the full calculation for the different SPS points. It shows that these residual corrections are small and do not exceed 2% for $H^0/A^0$ production in a wide range of MSSM parameters.

4 Conclusions

We have given a brief review on Higgs production in association with bottom quarks focussing on the basic concepts and precise predictions for Higgs production in bottom-quark fusion. The leading supersymmetric higher-order corrections can be taken into account by an appropriate definition of the couplings in an improved Born approximation. The remaining non-universal corrections are small, typically of the order of a few per cent. The theoretical uncertainty connected to the input value of the b-quark mass within the one-loop correction is emphasized.

Acknowledgments

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Table 2. Cross sections $\sigma$ and non-universal corrections $\delta$ for Higgs production in the SPS scenarios. $\delta$ is given with respect to $\sigma_{HAA}$ being dressed with NLO QCD/QED corrections.

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