Determining the WIMP Mass from Direct Dark Matter Detection Data

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Reconstructing the velocity distribution function of WIMPs
   Deriving $f_1(v)$ from the scattering spectrum
   Reconstructing $f_1(v)$ from experimental data

Determining the WIMP mass

Summary
Deriving $f_1(v)$ from the scattering spectrum

- Differential rate for elastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = A F^2(Q) \int_{v_{\text{min}}}^{\infty} \left[ \frac{f_1(v)}{v} \right] dv$$

Here

$$v_{\text{min}} = \alpha \sqrt{Q}$$

is the minimal incoming velocity of incident WIMPs that can deposit the energy $Q$ in the detector.

$$A \equiv \frac{\rho_0 \sigma_0}{2m_\chi m_r^2} \quad \alpha \equiv \sqrt{\frac{m_N}{2m_r^2}} \quad m_r = \frac{m_\chi m_N}{m_\chi + m_N}$$

- $\rho_0$: WIMP density near the Earth
- $\sigma_0$: total cross section ignoring the form factor suppression
- $F(Q)$: elastic nuclear form factor
Deriving $f_1(v)$ from the scattering spectrum

- Normalized one-dimensional velocity distribution function
  \[
  f_1(v) = \mathcal{N} \left\{ -2Q \cdot \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] \right\}_{Q=v^2/\alpha^2} 
  \]
  \[
  \mathcal{N} = \frac{2}{\alpha} \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ \right\}^{-1}
  \]

- Moments of the velocity distribution function
  \[
  \langle v^n \rangle = \mathcal{N}(Q_{\text{thre}}) \left( \frac{\alpha^{n+1}}{2} \right) \left[ 2 Q_{\text{thre}}^{(n+1)/2} \frac{dR}{dQ} \right]_{Q=Q_{\text{thre}}} + (n + 1) I_n(Q_{\text{thre}})
  \]
  \[
  \mathcal{N}(Q_{\text{thre}}) = \frac{2}{\alpha} \left[ \frac{2 Q_{\text{thre}}^{1/2}}{F^2(Q_{\text{thre}})} \left( \frac{dR}{dQ} \right) \right]_{Q=Q_{\text{thre}}} + I_0(Q_{\text{thre}}) \right\}^{-1}
  \]
  \[
  I_n(Q_{\text{thre}}) = \int_{Q_{\text{thre}}}^{\infty} Q^{(n-1)/2} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ
  \]

[M. Drees and C. L. Shan, JCAP 0706, 011]
Reconstructing $f_1(v)$ from experimental data

- Experimental data
  \[ Q_n - \frac{b_n}{2} \leq Q_{n,i} \leq Q_n + \frac{b_n}{2} \quad i = 1, 2, \cdots, N_n, \ n = 1, 2, \cdots, B \]

- Theoretically predicted scattering spectrum

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- Ansatz: in the $n$th $Q$-bin

$$
\left( \frac{dR}{dQ} \right)_n \equiv \left( \frac{dR}{dQ} \right)_{Q \approx Q_n} = \tilde{r}_n e^{\kappa_n (Q - Q_n)} \equiv r_n e^{\kappa_n (Q - Q_{s,n})}
$$

$$
\tilde{r}_n \equiv \left( \frac{dR}{dQ} \right)_{Q = Q_n}
$$

$$
r_n \equiv \frac{N_n}{b_n}
$$

- Recoil spectrum at $Q = Q_n$

$$
\tilde{r}_n = \frac{N_n}{b_n} \left( \frac{\kappa_n}{\sinh \kappa_n} \right)
$$

$$
\kappa_n \equiv \left( \frac{b_n}{2} \right) k_n
$$

- Logarithmic slope and shifted point in the $n$th $Q$-bin

$$
\overline{Q}_n - Q_n = \frac{b_n}{2} \left( \coth \kappa_n - \frac{1}{\kappa_n} \right)
$$

$$
\overline{Q}_n = \frac{1}{N_n} \sum_{i=1}^{N_n} Q_{n,i}
$$

$$
Q_{s,n} = Q_n + \frac{1}{\kappa_n} \ln \left( \frac{\sinh \kappa_n}{\kappa_n} \right)
$$
Reconstructing $f_1(v)$ from experimental data

- Reconstructing the one-dimensional velocity distribution

$$f_{1,r}(v_s,\mu) = \mathcal{N} \left[ \frac{2Q_{s,\mu}r_\mu}{F^2(Q_{s,\mu})} \right] \left[ \frac{d}{dQ} \ln F^2(Q) \right]_{Q=Q_{s,\mu}} - k_\mu$$

$$v_{s,\mu} = \alpha \sqrt{Q_{s,\mu}}$$

$$\mathcal{N} = \frac{2}{\alpha} \left[ \sum_a \frac{1}{\sqrt{Q_a} F^2(Q_a)} \right]^{-1}$$

- Determining the moments of the velocity distribution

$$\langle v^n \rangle = \alpha^n \left[ \frac{2Q_{thre}^{1/2}r_{thre}}{F^2(Q_{thre})} + I_0 \right]^{-1} \left[ \frac{2Q_{thre}^{(n+1)/2}r_{thre}}{F^2(Q_{thre})} + (n + 1)I_n \right]$$

$$I_n = \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)}$$

$$r_{thre} = \left( \frac{dR}{dQ} \right)_{Q=Q_{thre}}$$

[M. Drees and C. L. Shan, JCAP 0706, 011]
Determining the WIMP mass

- Using two different target nuclei

\[
\langle v^n \rangle = \alpha^n_X \left[ \frac{(n+1)I_{n,X}}{I_{0,X}} \right] = \alpha^n_Y \left[ \frac{(n+1)I_{n,Y}}{I_{0,Y}} \right]
\]
Determining the WIMP mass

- Using two different target nuclei

\[ \langle v^n \rangle = \alpha^n_X \left[ \frac{(n+1)I_{n,X}}{I_{0,X}} \right] = \alpha^n_Y \left[ \frac{(n+1)I_{n,Y}}{I_{0,Y}} \right] \]

- WIMP mass

\[ m_\chi = \sqrt{m_X m_Y - m_X R_n} \]

\[ R_n = \frac{\alpha_Y}{\alpha_X} = \left( \frac{I_{n,X}}{I_{0,X}} \cdot \frac{I_{0,Y}}{I_{n,Y}} \right)^{1/n} \quad (n \neq 0, -1) \]

- 1-\(\sigma\) statistical error

\[ \sigma(m_\chi) = \frac{R_n \sqrt{m_X/m_Y} |m_X - m_Y|}{\left( R_n - \sqrt{m_X/m_Y} \right)^2} \]

\[ \times \frac{1}{|n|} \left[ \frac{\sigma^2(I_{n,X})}{I_{n,X}^2} + \frac{\sigma^2(I_{0,X})}{I_{0,X}^2} - 2\text{cov}(I_{0,X}, I_{n,X}) + \frac{I_{0,X} I_{n,X}}{I_{0,X} I_{n,X}} \right]^{1/2} \]

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Determining the WIMP mass

- 1-σ statistical error for different combinations
  
  $(1 - 200 \text{ keV}, \ n = 1, \ 25 + 25 \text{ events})$
Determining the WIMP mass

- Reproduced WIMP mass
  
  \[ (1 - 200 \text{ keV}, n = 1, ^{76}\text{Ge} + ^{28}\text{Si}, 25 + 25 \text{ events}) \]

\[ Q_{\text{max}} = 200 \text{ keV}, Q_{\text{min}} = 1 \text{ keV, } n = 1, 25 + 25 \text{ events, Ge-76 + Si-28} \]
Determining the WIMP mass

- Reproduced WIMP mass
  
  \((1 - 200 \text{ keV}, n = 1, ^{76}\text{Ge} + ^{28}\text{Si}, 250 + 250 \text{ events})\)

\[ Q_{\text{max}} = 200 \text{ keV}, Q_{\text{min}} = 1 \text{ keV}, n = 1, 250 + 250 \text{ events, Ge-76 + Si-28} \]
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- The larger the mass difference between two target nuclei, the smaller the statistical error will be.

- Our method is model-independent and needs only measured recoil energies.

- With 200 keV maximal measuring energy and 25 events from each experiment, we can already extract meaningful information about the WIMP mass.
A championship for finding new particle(s) between direct Dark Matter detection and collider experiments has been started.
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A championship for finding new particle(s) between direct Dark Matter detection and collider experiments has been started.

Thank you very much for your attention.