NLO Event Simulation for Chargino Production at the ILC


Tania Robens
in collaboration with W. Kilian, J. Reuter

RWTH Aachen

SUSY 2007, Universität Karlsruhe
1 Introduction and Motivation
- Charginos and Neutralinos in the MSSM
- Experimental accuracy and NLO results

2 Inclusion of NLO results in WHIZARD
- Implementation in WHIZARD
- Photons: fixed order vs resummation
- Results

3 Summary and Outlook
Charginos and Neutralinos in the MSSM

Chargino and Neutralino sector: Reconstruction of SUSY parameters

- Charginos $\tilde{\chi}_i^\pm$ and Neutralinos $\tilde{\chi}_i^0$: superpositions of gauge and Higgs boson superpartners
- Chargino/Neutralino sector:
  
  $\tan \beta$, $\mu$ (Higgs sector), $M_1$, $M_2$ (soft breaking terms)

  can be reconstructed from

  masses of $\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^\pm$, $\tilde{\chi}_1^0$, $2\sigma$ in the $\tilde{\chi}^\pm$ sector

  (Choi ea 98, 00, 01)

- low-scale parameters + evolution to high scales (RGEs):
  
  $\Rightarrow$ hint at SUSY breaking mechanism (Blair ea, 02)

- requires high precision in ew-scale parameter determination
Charginos and Neutralinos in the MSSM

Chargino production at the ILC

- **ILC**: future $e^+e^-$ collider, $\sqrt{s} = 500 \text{ GeV} \ (1 \text{ TeV})$
  “clean” environment, low backgrounds $\Rightarrow$ high precision

- **Charginos**: (typically) light in the MSSM
  $\Rightarrow$ easily accessible at colliders (ILC/ LHC)

- **LO production at the ILC**:

  \[
  e^+ \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^0_1 \rightarrow \tilde{\tau}^+_1 \tilde{\tau}^-_1 \nu_\tau \bar{\nu}_\tau (\rightarrow \tau^+ \tau^- \nu_\tau \bar{\nu}_\tau \tilde{\chi}^0_1 \tilde{\chi}^0_1)
  \]

  decays: typically long decay chains

  e.g. $e^+ e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^0_1 \rightarrow \tilde{\tau}^+_1 \tilde{\tau}^-_1 \nu_\tau \bar{\nu}_\tau (\rightarrow \tau^+ \tau^- \nu_\tau \bar{\nu}_\tau \tilde{\chi}^0_1 \tilde{\chi}^0_1)$
Experimental accuracy and theoretical next-to-leading-order (NLO) corrections

- experimental errors: obtained from simulation studies (LHC/ILC study, Weiglein ea, 04)
- generate “experimental data” with known SUSY input parameters
- errors: combination of statistical and systematic errors

combined $\text{LHC} + \text{ILC}$: \( \% \)

same \( \mathcal{O} \) errors from fitting routines determining SUSY parameters

**Theory:**

Full NLO SUSY corrections for \( \sigma(ee \rightarrow \tilde{\chi}\tilde{\chi}) \) at ILC: in the \% regime (Fritzsche ea 04, Öller ea 04, 05)

\( \Rightarrow \) include complete NLO contributions in analyses

Tania Robens  
NLO Event Simulation for Chargino Production at the ILC  
SUSY 2007, Universität Karlsruhe
Experimental accuracy and theoretical next-to-leading-order (NLO) corrections

- experimental errors: obtained from simulation studies (LHC/ILC study, Weiglein ea, 04)
- generate “experimental data” with known SUSY input parameters
- errors: combination of statistical and systematic errors

combined LHC + ILC: $\%_0$

same $\mathcal{O}$ errors from fitting routines determining SUSY parameters

**Theory:**

Full NLO SUSY corrections for $\sigma(ee \to \tilde{\chi}\tilde{\chi})$ at ILC:
in the $\%$ regime (Fritzsche ea 04, Öller ea 04, 05)

$\Rightarrow$ include complete NLO contributions in analyses
From $\sigma_{\text{tot}}$ to Monte Carlo event generators

MC event generators: Generate event samples (same form as experimental outcome)

- experiments: see final decay products
- need to compare with simulated event samples
- also: important irreducible background effects

(e.g. Hagiwara ea, 05, $\rightarrow$ talk by Jürgen Reuter)

$\Rightarrow$ include NLO results in Monte Carlo Generators $\leftarrow$

- MC Generator WHIZARD (W. Kilian, LC-TOOL-2001-039):
  - so far: LO Monte Carlo Event Generator for $2 \rightarrow n$ particle processes
  - includes various physical models (SM, MSSM, non-commutative geometry, little Higgs models), initial state radiation, parton shower models,...
From $\sigma_{\text{tot}}$ to Monte Carlo event generators

MC event generators: Generate event samples
(same form as experimental outcome)

- experiments: see final decay products
- need to compare with simulated event samples
- also: important irreducible background effects
  (e.g. Hagiwara ea, 05, → talk by Jürgen Reuter)

⇒ include NLO results in Monte Carlo Generators ⇐

- MC Generator WHIZARD (W. Kilian, LC-TOOL-2001-039):
  - so far: LO Monte Carlo Event Generator for $2 \rightarrow n$ particle processes
  - includes various physical models (SM, MSSM, non-commutative geometry, little Higgs models), initial state radiation, parton shower models,...
NLO cross section contributions

\[ \sigma_{\text{tot}} \text{ contributions and dependencies:} \]

- \( \sigma_{\text{born}} \)
- virtual \( \mathcal{O}(\alpha) \) corrections: \( \sigma_{\text{virt}}(\lambda) \)
- emission of soft/ hard collinear/ hard non-collinear photons:
  \[ \sigma_{\text{soft}}(\Delta E_\gamma, \lambda) + \sigma_{\text{hc}}(\Delta E_\gamma, \Delta \theta_\gamma) + \sigma_{2\rightarrow3}(\Delta E_\gamma, \Delta \theta_\gamma) \]
- higher order initial state radiation: \( \sigma_{\text{ISR}} - \sigma_{\text{ISR}}^{\mathcal{O}(\alpha)}(Q) \)

\( \lambda \): photon mass, \( \Delta E_\gamma \): soft cut, \( \Delta \theta_\gamma \): collinear angle
Including FormCalc $O(\alpha)$ results in WHIZARD

- use FeynArts / FormCalc generated code for
  
  \[ M_{\text{virt}}(\lambda) : \text{virtual corrections} \]
  \[ f_s(\Delta E_\gamma, \lambda) : \text{soft photon factor} \]
  \[ (M_{\text{born}} : \text{born contribution}) \]

- fixed order: integrate over effective matrix element:
  
  \[ |M_{\text{eff}}|^2(\Delta E_\gamma) = \left(1 + f_s(\Delta E_\gamma, \lambda)\right) |M_{\text{born}}|^2 + 2 \text{Re}(M_{\text{born}} M_{\text{virt}}^*(\lambda)) \]

  \[ \Delta E_\gamma: \text{soft photon cut, } \lambda: \text{photon mass} \]

- in practice: create library from FormCalc code, link this to WHIZARD
Photons: fixed order vs resummation

(1): Fixed $\mathcal{O}(\alpha)$ contributions

- integrate $|\mathcal{M}_{\text{eff}}|^2$ (born/ virtual/ soft photonic part)
- hard collinear photons: collinear approximation ($\mathcal{M}_{\text{born}}$)
- hard non-collinear photons: explicit $e^+e^- \to \tilde{\chi}\tilde{\chi}\gamma$ process ($\mathcal{M}_{\text{born}}^2 \to 3$)
- corresponds to analytic results in literature (Fritzsche ea/ Öller ea)
(1): Fixed $\mathcal{O}(\alpha)$ contributions

- integrate $|\mathcal{M}_{\text{eff}}|^2$ (born/ virtual/ soft photonic part)
- + hard collinear photons: collinear approximation ($\mathcal{M}_{\text{born}}$)
- + hard non-collinear photons: explicit $e e \rightarrow \tilde{\chi} \bar{\chi} \gamma$ process ($\mathcal{M}_{\text{born}}^2 \rightarrow 3$)
- corresponds to analytic results in literature
  (Fritzsche et al./ Öller et al.)

problem: too low energy cuts: $|\mathcal{M}_{\text{eff}}|^2 < 0$
⇒ use negative weights
or set $\mathcal{M}_{\text{eff}} = 0$

event generator specific problem
($\sigma_{\text{tot}} \geq 0$)

$\mathcal{M}^2$ behaviour, different cuts [GeV]
Photons: fixed order vs resummation

(2): Resumming leading logs to all orders

- idea: subtract $\mathcal{O}(\alpha)$ soft + virtual collinear contributions in $\mathcal{M}_{\text{eff}}$:

$$|\tilde{\mathcal{M}}_{\text{eff}}|^2 = (1 + f_s(\Delta E_\gamma)) |\mathcal{M}_{\text{born}}|^2 + 2 \text{Re}(\mathcal{M}_{\text{born}} \mathcal{M}_{\text{virt}}^*) - 2 f_s^{\text{ISR},\mathcal{O}(\alpha)}(\Delta E_\gamma) |\mathcal{M}_{\text{born}}|^2$$

- fold this with ISR structure function:

$$\int d\Gamma \int_0^1 dx_1 \int_0^1 dx_2 f^{\text{ISR}}(x_1) f^{\text{ISR}}(x_2) |\tilde{\mathcal{M}}_{\text{eff}}|^2(s, x_i)$$

- $f^{\text{ISR}}(x)$: Initial state radiation (Jadach, Skrzypek, Z.Phys. 1991)

$\Rightarrow$ describes collinear (real + virtual) photons in leading log accuracy $\Leftarrow$

- $f_s^{\text{ISR},\mathcal{O}(\alpha)}$: soft integrated $\mathcal{O}(\alpha)$ contribution
Photons: fixed order vs resummation

**Resumming: What do we get??**

- $O(\alpha)$: equivalent to fixed order method

⇒ got rid of

$|\mathcal{M}|^2 < 0$

effects !!

**no negative weights**

![Plot](image)

$\Delta E_\gamma = 0.5 \text{ GeV}$

- higher orders:

  higher order ISR for $|\mathcal{M}_{\text{born}}|^2$ as well as Re ($\mathcal{M}_{\text{born}} \mathcal{M}_{\text{virt}}^*$) !!!

⇒ new higher order effects ⬤

- additional possibility: also fold 2 → 3 process with ISR ("res+)")
Resumming: What do we get??

- \( O(\alpha) \): equivalent to fixed order method

⇒ got rid of

\(|\mathcal{M}|^2 < 0 \)

effects !!

no negative
weights

- higher orders:
  - higher order ISR for \(|\mathcal{M}_{\text{born}}|^2\) as well as \(\text{Re} (\mathcal{M}_{\text{born}} \mathcal{M}_{\text{virt}}^*) \) !!!!

⇒ new higher order effects ⇐

- additional possibility: also fold \(2 \rightarrow 3\) process with ISR ("res+")

\[ \Delta E_\gamma = 0.5 \text{ GeV} \]
Results: cross sections

\[ e^+ e^- \rightarrow \tilde{\chi}_1 \tilde{\chi}_1^* \]

\[ \sigma [\text{fb}] \]

agrees with results in the literature (Fritzsche ea, Öller ea)
A closer look: $\Delta E_\gamma$ dependence of $\sigma_{\text{tot}}$

- **semianalytic** (*FormCalc*): tests soft approximation, shifts: 2 - 5 % (\(\Delta E_\gamma \leq 10\) GeV)

- **fixed order result** (*WHIZARD*): same as 'sa' for \(\Delta E_\gamma \geq 3\) GeV, smaller values: \(|\mathcal{M}_{\text{eff}}|^2 \leq 0\) effects
Results

\(\Delta E_\gamma\) dependence: resummation

\[e^-e^+ \rightarrow \chi^-\chi^+\]

\[\sigma_{\text{tot}}\] [fb]

\(\sqrt{s} = 1\) TeV

SPS1a'

\(E_{\text{GeV}}:\) resummation includes higher order effects

5% difference to 'sa' for \(\Delta E_\gamma \leq 10\) GeV

\(\sigma_{\text{tot}}(\Delta E_\gamma):\)

In summary:

shift in \(\Delta E_\gamma\) leads to \(\%\)0 effects, match ILC accuracy

\(\Rightarrow\) careful choice of \(\Delta E_\gamma\), method important

“best” choice: fully resummed version with low energy cut

Tania Robens  
NLO Event Simulation for Chargino Production at the ILC

SUSY 2007, Universität Karlsruhe
Inclusion of NLO results in WHIZARD

Simulation results: angular distributions

Born, fixed order, resummation

!! more than 1 $\sigma$ deviation !! $\sqrt{n_{\text{max}}} \approx \mathcal{O}(10^2)$; nbins = 20
Angular distributions: higher orders

$e^-e^+ \rightarrow \tilde{\chi}_1^- \tilde{\chi}_1^+$

$N_{\text{res}}^+ - N_{\text{ex}}$
red: 1 standard dev from Born result

$N_{\text{res}}^+$: resummation, additionly 2 $\rightarrow$ 3 folded w ISR; most complete

also higher order contributions statistically significant
Results: higher order effects

$\sqrt{s}$ dependence of different higher order contributions

Born+: only Born folded w ISR (standard way in the literature),
fully resummed result: subtraction, also fold 2 $\to$ 3 part with ISR
difference between Born+ and fully resummed result: multiple photon emission from interaction term
Summary and Outlook

- Chargino/ neutralino sector of MSSM: high precision in SUSY parameter analysis at EW scale (\%0 at ILC)
- same size/ larger NLO corrections
- include NLO results in Monte Carlo Event generators
- resummation method for photons allows lower soft cuts/ inclusion of higher order contributions
- NLO as well as higher order contributions significant !!
- next steps: include NLO corrections to $\tilde{\chi}$ decays ($\rightarrow$ talk by K.Rolbiecki), non-factorizing contributions ( start with photonic corrections in the double-pole approximation)
- general interface to FormCalc generated matrix elements: extendable to other processes...
cut dependencies: $\Delta \theta_\gamma$

tests: collinear photon approximation

\[ e^- e^+ \rightarrow \tilde{\chi}_1^- \tilde{\chi}_1^+ \]

$\sigma_{tot}$ [fb]
$\sqrt{s} = 1$ TeV

$\sigma_{tot}$ again larger for resummation method
for higher angles: second order ISR effects between 0.05° and 0.1° ($O(\%_0)$)

Tania Robens  NLO Event Simulation for Chargino Production at the ILC  SUSY 2007, Universität Karlsruhe
Inclusion of NLO results in WHIZARD

Summary and Outlook

Appendix

photon approximations

$\eta, f_s$, hard collinear approximation, $\text{ISR}^{\mathcal{O}(\alpha)}$

- $\eta = \frac{2\alpha}{\pi} \left( \log \left( \frac{Q^2}{m_e^2} \right) - 1 \right)$ ($Q$ = scale of process)

- $f_s = -\frac{\alpha}{2\pi} \sum_{i,j = e^\pm} \int_{|k| \leq \Delta E} \frac{d^3k}{2\omega_k} \frac{(\pm) p_i p_j Q_i Q_j}{p_i k p_j k}$

  (Denner 1992)

$\omega_k = \sqrt{k^2 + \lambda^2}$, $p_i$ initial/ final state momenta, $k$: $\gamma$ momentum

- hard collinear factor ($\pm$ helicity conserving/ flipping):

  $f^+(x) = \frac{\alpha}{2\pi} \left( \frac{1 + x^2}{1 - x} \right) \left( \ln \left( \frac{s (\Delta \theta)^2}{4 m^2} \right) - 1 \right)$, $f^-(x) = \frac{\alpha}{2\pi} x$

  (Dittmaier 1993)

- $f_s^{\text{ISR}, \mathcal{O}(\alpha)} = \left[ \int_{x_0}^1 f_{\text{ISR}}(x) \, dx \right]_{\mathcal{O}(\alpha)} = \frac{\eta}{4} \left( 2 \ln(1 - x_0) + x_0 + \frac{1}{2} x_0^2 \right)$
\[ \Gamma_{ee}^{LL}(x, Q^2) = \frac{\exp(-\frac{1}{2} \eta \gamma_E + \frac{3}{8} \eta)}{\Gamma(1 + \frac{\eta}{2})} \frac{\eta}{2} (1 - x)^{\left(\frac{\eta}{2}-1\right)} \]

\[ - \frac{\eta}{4} (1 + x) + \frac{\eta^2}{16} \left( -2 (1 - x) \log(1 - x) - \frac{2 \log x}{1 - x} + \frac{3}{2} (1 + x) \log x - \frac{x}{2} \right) \]

\[ - \frac{5}{2} \left( \frac{\eta}{2} \right)^3 \left[ -\frac{1}{2} (1 + x) \left( \frac{9}{32} - \frac{\pi^2}{12} + \frac{3}{4} \log(1 - x) + \frac{1}{2} \log^2(1 - x) \right) \right] \]

\[ - \frac{1}{4} \log x \log(1 - x) + \frac{1}{16} \log^2 x - \frac{1}{4} \text{Li}_2(1 - x) \]

\[ + \frac{1}{2} \frac{1 + x^2}{1 - x} \left( -\frac{3}{8} \log x + \frac{1}{12} \log^2 x - \frac{1}{2} \log x \log(1 - x) \right) \]

\[ - \frac{1}{4} (1 - x) \left( \log(1 - x) + \frac{1}{4} \right) + \frac{1}{32} (5 - 3x) \log x \right] ; \eta = \frac{2 \alpha}{\pi} \left( \log \left( \frac{Q^2}{m_e^2} \right) - 1 \right) \]
Some NLO matrix elements
Some NLO matrix elements

$$e^+ e^- \rightarrow \tilde{\chi}_i \tilde{\chi}_j$$

**T10 G4 N64**  
**T10 G5 N65**  
**T10 G6 N66**

$$e^+ e^- \rightarrow \tilde{\chi}_i \tilde{\chi}_j$$

**T11 G2 N82**  
**T11 G3 N83**  
**T11 G4 N84**

**T10 G7 N67**  
**T10 G8 N68**  
**T10 G9 N69**

**T12 G1 N85**  
**T12 G2 N86**  
**T12 G3 N87**

**T10 G10 N70**  
**T10 G11 N71**  
**T10 G12 N72**

**T12 G4 N88**  
**T13 G1 N89**  
**T13 G2 N90**

---

Tania Robens  
NLO Event Simulation for Chargino Production at the ILC  
SUSY 2007, Universität Karlsruhe
Point SPS1a'

- mSUGRA scenario
- according to Snowmass Points (Allanach ea, 02), in agreement with cosmology data/ WMAP ($\tilde{\chi}_1^0$ as DM candidate)

light sleptons
heavy squarks
some light $\tilde{\chi}$s
all masses $< 1$ TeV