

# Progress on supersymmetric effects in rare K decays

$U^b$

Christopher Smith

- Outline

*A- Motivation and generalities*

*B- SUSY effects in  $K \rightarrow \pi V \bar{V}$*

*C- SUSY effects in  $K_L \rightarrow \pi^0 \ell^+ \ell^-$*

*D- Conclusion*

# Motivation and generalities

- Why rare K decays are so interesting?

$$K_L \rightarrow \pi^0 \nu \bar{\nu}, K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \pi^0 e^+ e^-, K_L \rightarrow \pi^0 \mu^+ \mu^-$$

- “*Would-be forbidden*” modes in the SM  $\rightarrow$  *New Physics can be dominant*

Flavor Changing Neutral Currents



*GIM mechanism*: probe the SM at the quantum level (loop).

- *CP-violating FCNC*: Additional suppression in the SM ( $\text{Im} \lambda_t = \text{Im}(V_{td} V_{ts}^*) \sim 10^{-4}$ )

Heaviest SM particle (top quark) gives the largest contribution

$\rightarrow$  Well-controlled perturbative regime.

- *Semi-leptonic decays*: QCD effects under excellent control (compare with  $\epsilon'/\epsilon$ )  
(FCNC and CC matrix-elements are related).

- *The only theoretically clean window on the  $\Delta S = 1$  sector*  
 $\rightarrow$  Essential input for the “inverse problem” in the LHC era.

- The  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  decays

$$B(K_L \rightarrow \pi^0 \nu \bar{\nu}) \approx \kappa^0 ( |\text{Im} \lambda_t X(x_t)|^2 ) \quad \sim \text{Br}(K_S \rightarrow \pi^0 \nu \bar{\nu})$$

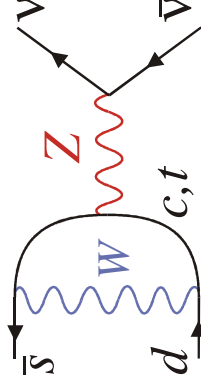
$$B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \approx \kappa^+ ( |\text{Im} \lambda_t X(x_t)|^2 + |\text{Re} \lambda_t X(x_t) + \text{Re} \lambda_c (P_c + \delta P_{u,c})|^2 )$$

1- Dimension-six:  $t$ -quark:  $X(x_t) \stackrel{\text{NLO}}{=} 1.464(41)$

$(\bar{s}d)_V (\bar{\nu}\nu)_{V-A}$

$\stackrel{\text{NNLO}}$

$c$ -quark:  $P_c \stackrel{\text{NNLO}}{=} 0.37(4) \lambda^4$

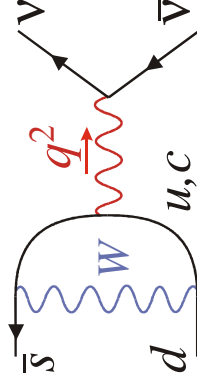


Buchalla, Buras ('93)

Buras, Gorbahn,

Haisch, Nierste ('05)

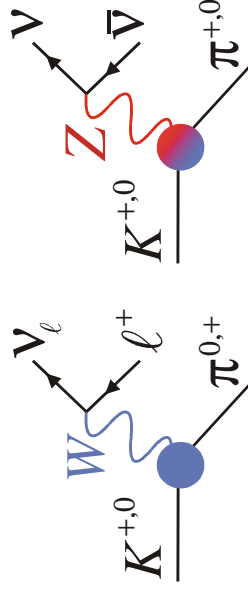
2- Higher dim.:  $\left. \begin{array}{l} \text{Dim-8 } c\text{-quark} \\ \text{LD } u\text{-quark} \end{array} \right\} \delta P_{u,c} = 0.04(2) \lambda^4$



Isidori, Mescia,

CS ('05)

3- Matrix elements:  $\left\{ \begin{array}{l} \kappa^+ = 0.5173(25) \cdot 10^{-10} \\ \kappa^0 = 2.231(13) \cdot 10^{-10} \end{array} \right.$

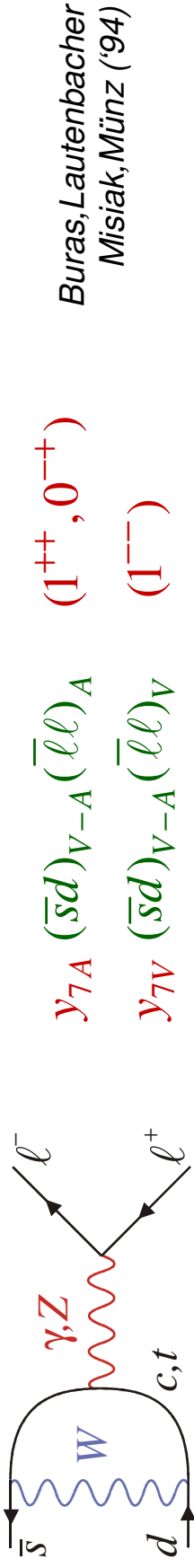


Mescia,

CS ('07)

• The  $K_L \rightarrow \pi^0 \ell^+ \ell^-$  decay

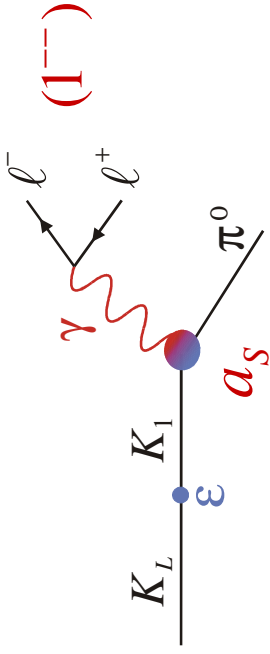
1. **Direct CPV:** Short-distance, from top & charm integrations (known at NLO):



$$y_{7A} (\bar{s}d)_{V-A} (\bar{\ell}\ell)_A \quad (1^{++}, 0^{--})$$

$$y_{7V} (\bar{s}d)_{V-A} (\bar{\ell}\ell)_V \quad (1^{--})$$

2. **Indirect CPV:** Long-distance  $\gamma$  penguin ( $\rightarrow$  ChPT) D'Ambrosio, Ecker, Isidori, Portolés ('98)

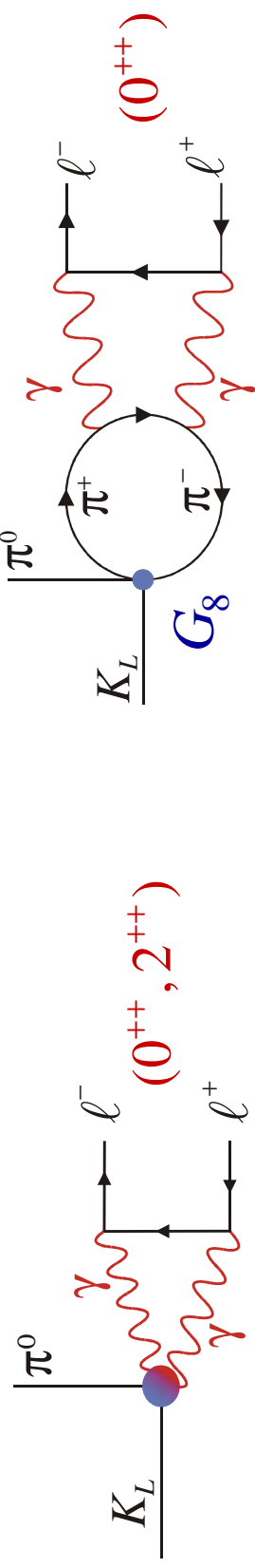


- Dominated by a counterterm, fixed using  $B(K_S \rightarrow \pi^0 \ell^+ \ell^-)$  as  $|a_S| = 1.2 \pm 0.2$ .

- Indications for constructive interference.

*Buchalla, D'Ambrosio, Isidori ('03)/de Rafael, Friot, Greynat ('04)*

3. **CP-conserving:** Long-distance  $\gamma$  penguin ( $\rightarrow$  ChPT):



Higher order corrections estimated from  $K_L \rightarrow \pi^0 \gamma \gamma$  rate and spectrum.

*Buchalla, D'Ambrosio, Isidori ('03)/ Isidori, Unterdorfer, CS ('04)*

## 4. Complete predictions

$$Br(K_L \rightarrow \pi^0 \ell^+ \ell^-) = (C_{\text{dir}}^{\ell} \kappa^2 \pm C_{\text{int}}^{\ell} |a_S| \kappa + C_{\text{ind}}^{\ell} |a_S|^2 + C_{\gamma}^{\ell}) \cdot 10^{-12}$$

SM:  $\kappa = \text{Im} \lambda_t \times 10^4 \approx 1.41$ ,  $y_{7A} \approx -0.68$ ,  $y_{7V} \approx 0.73$

$$C_{\text{dir}}^e \approx 2.3 (y_{7V}^2 + y_{7A}^2)$$

$$C_{\text{int}}^e \approx 8.1 y_{7V}$$

$$C_{\text{ind}}^e \approx 14.5, C_{\gamma}^e \approx 0$$

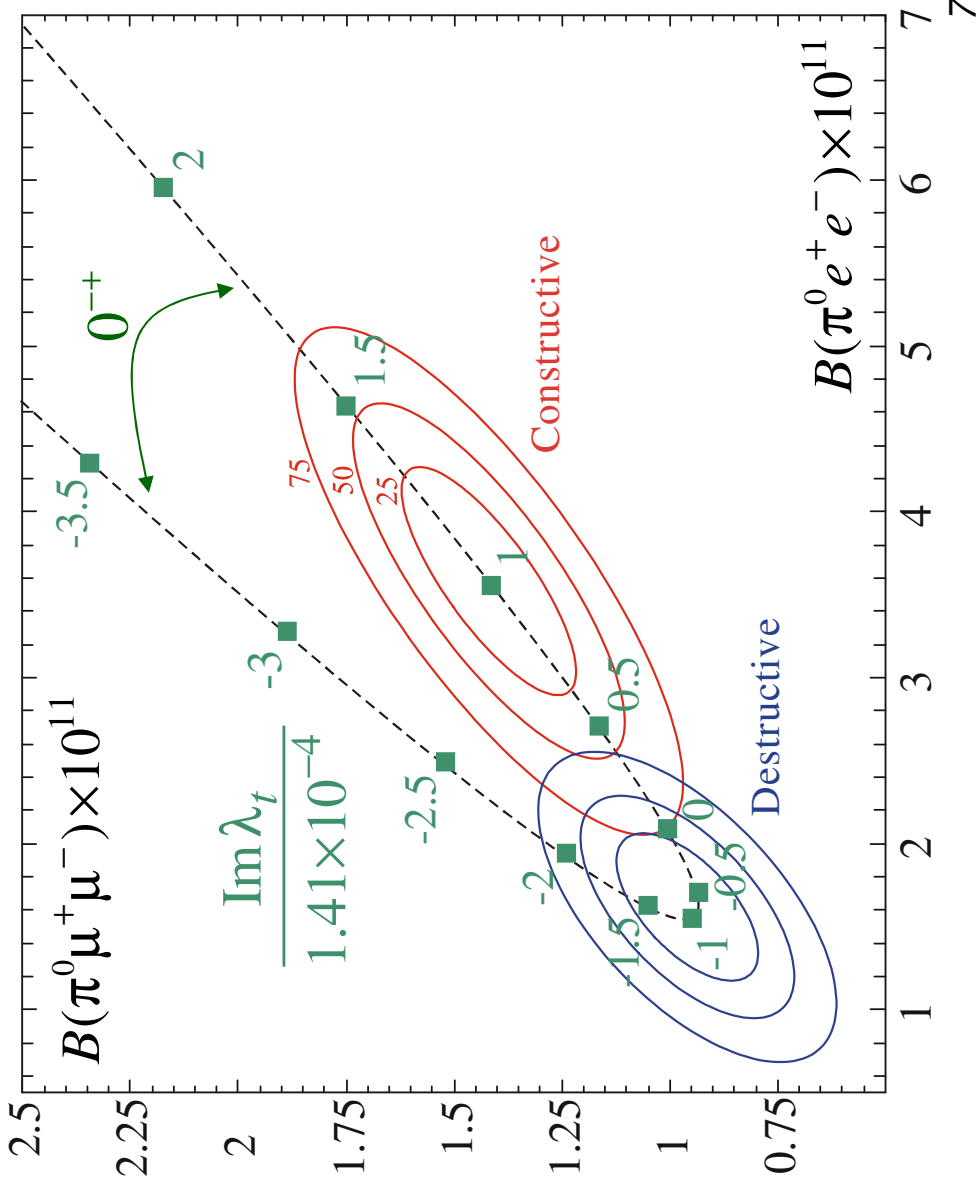
$\updownarrow$   $\frac{1}{4}$  phase-space  
suppression

$$C_{\text{dir}}^{\mu} \approx 0.55 (y_{7V}^2 + 2.33 y_{7A}^2)$$

$$C_{\text{int}}^{\mu} \approx 1.9 y_{7V}$$

$$C_{\text{ind}}^{\mu} \approx 3.4, C_{\gamma}^{\mu} \approx 5.2$$

Additional helicity-  
suppressed terms

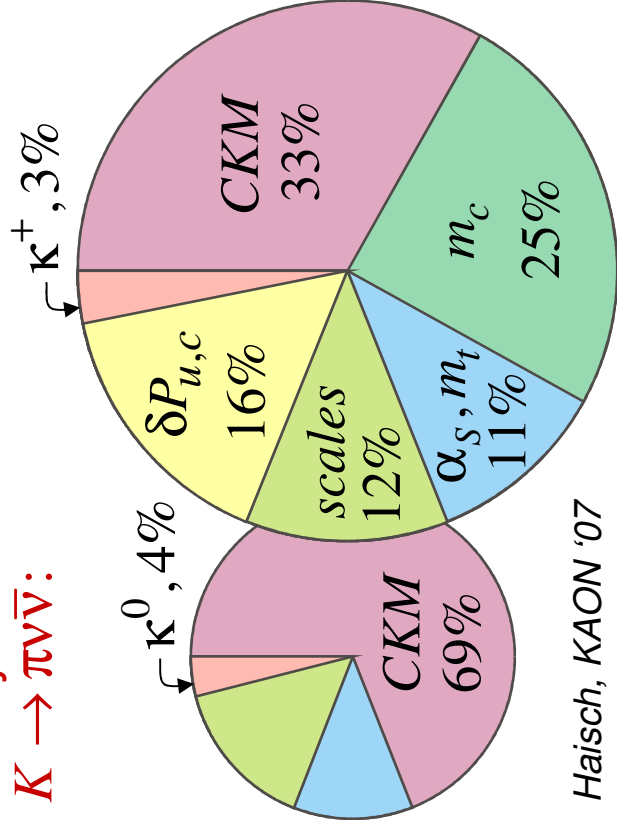


• Summary of current status in the SM:

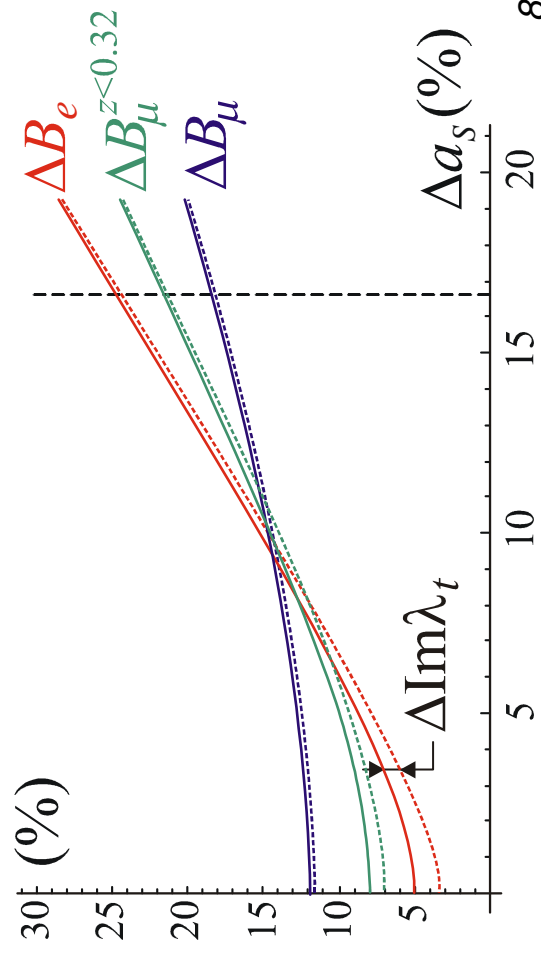
	V, A	$K^0 - \bar{K}^0$	$2^{++}$	$0^{++}$	SM ( $\times 10^{-11}$ )	Experiment
$K_L \rightarrow \pi^0 \nu \bar{\nu}$	100%	( $\approx 1\%$ )	—	—	$2.54^{+0.35}_{-0.35}$	$< 2.1 \cdot 10^{-7}$ E391a
$K_L \rightarrow \pi^0 e^+ e^-$	40%	60%	(<3%)	—	$3.54^{+0.98}_{-0.85}$	$< 2.8 \cdot 10^{-10}$ KTeV
$K_L \rightarrow \pi^0 \mu^+ \mu^-$	30%	35%	—	35%	$1.41^{+0.28}_{-0.26}$	$< 3.8 \cdot 10^{-10}$ KTeV
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	100%	—	—	—	$8.0^{+0.9}_{-0.9}$	$14.7^{+13.0}_{-8.9} \cdot 10^{-11}$ E787 E949

Theory errors for  $K \rightarrow \pi \nu \bar{\nu}$ :

Theory errors for  $K_L \rightarrow \pi^0 \ell^+ \ell^-$ :



Haisch, KAON '07

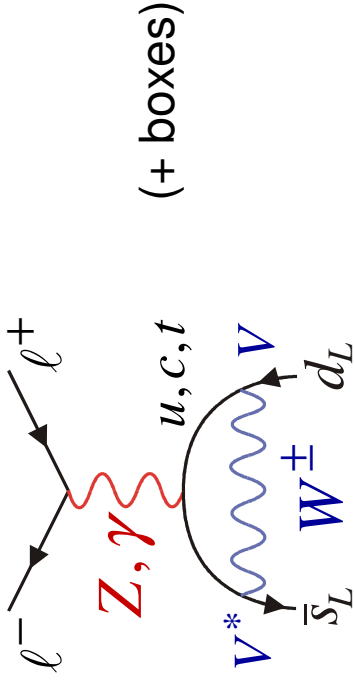




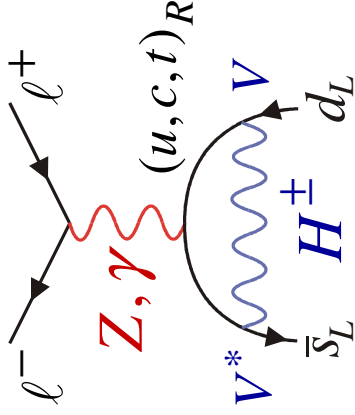
• Windows into the MSSM flavor structures:

Standard Model:

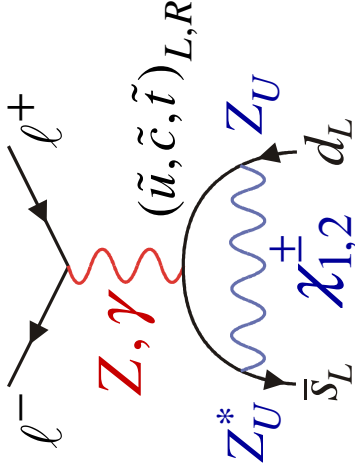
FCNC arise at one-loop,  
( $V = CKM$ )



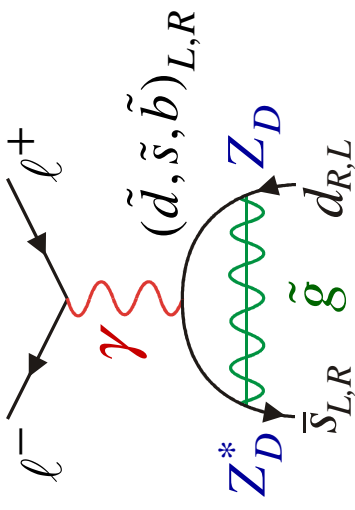
Charged Higgs:



Charginos- up-squarks:



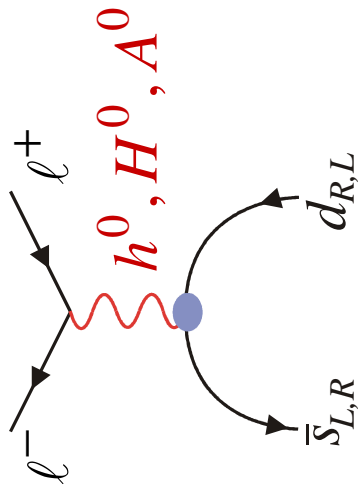
Glucinos- down-squarks:



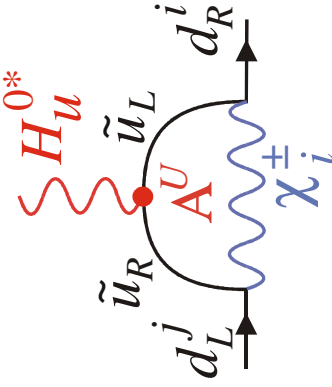
Neutral Higgses at large  $\tan \beta = v_u / v_d \approx m_t / m_b \approx 50$  :

$$\mathcal{L}_{\text{eff}} = \bar{d}_R^i Y_d^{ik} (H_d^0 + \epsilon Y_u^\dagger Y_u H_u^{0*})^{kj} d_L^j$$

$v_u$   
 $v_d$



Mismatch between mass-matrices  
and Higgs couplings at one-loop.



• The “leading order basis” : Minimal Flavor Violation

Generically, *MFV designed to suppress FCNC*, but this leaves some freedom in how it is to be defined or implemented.

Here: *MFV from a symmetry principle*:

*SM Yukawas are the only source of flavor-breaking.*

Hall, Randall ('90) / D'Ambrosio, Giudice, Isidori, Strumia ('02)

- SM has a global  $G = SU(3)^3$  flavor symmetry, broken only by  $Y_u, Y_d$ .
- In the MSSM, this symmetry also broken by the *soft-breaking terms*, therefore:

$$\left\{ \begin{aligned} \mathbf{m}_Q^2 &= m_0^2 \left( \tilde{a}_1 \mathbf{1} + \tilde{b}_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u + \tilde{b}_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + \tilde{b}_3 (\mathbf{Y}_d^\dagger \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \mathbf{Y}_d) \right), \\ \mathbf{m}_U^2 &= m_0^2 \left( \tilde{a}_2 \mathbf{1} + \tilde{b}_4 \mathbf{Y}_u \mathbf{Y}_u^\dagger \right), \mathbf{m}_D^2 = m_0^2 \left( \tilde{a}_3 \mathbf{1} + \tilde{b}_5 \mathbf{Y}_d \mathbf{Y}_d^\dagger \right), \\ \mathbf{A}^U &= a_0 \mathbf{Y}_u \left( \tilde{a}_4 \mathbf{1} + \tilde{b}_6 \mathbf{Y}_d^\dagger \mathbf{Y}_d \right), \mathbf{A}^D = a_0 \mathbf{Y}_d \left( \tilde{a}_5 \mathbf{1} + \tilde{b}_7 \mathbf{Y}_u^\dagger \mathbf{Y}_u \right) \end{aligned} \right. \quad \tilde{a}_i, \tilde{b}_i \sim \mathcal{O}(1)$$

- Introduces “minimal” departures with respect to *mSUGRA* ( $\tilde{a}_i = 1, \tilde{b}_i = 0$ )
- Approximate CCB/UFB:  $|a_{4(5)}|^2 \lesssim 3(a_1^2 + a_{2(3)}^2)$  ( $a_i \equiv m_0^2 \tilde{a}_i / a_0 \tilde{a}_i, b_i \equiv m_0^2 \tilde{b}_i / a_0 \tilde{b}_i$ )

SUSY effects in  $K \rightarrow \pi \nu \bar{\nu}$

# $K \rightarrow \pi V \bar{V}$

## 1- SUSY effects in the SM operator

$$\begin{aligned}
 H_{\text{eff}}(K \rightarrow \pi V \bar{V}) &\sim y_L^\nu (\bar{s}d)_{V-A} (\bar{V}V)_{V-A} + y_R^\nu (\bar{s}d)_{V+A} (\bar{V}V)_{V-A} \\
 &\rightarrow (y_L^\nu + y_R^\nu) (\bar{s}d)_V (\bar{V}V)_{V-A}
 \end{aligned}$$

General analysis in terms of a single complex quantity. *Buras, Romanino, Silvestrini ('98)*

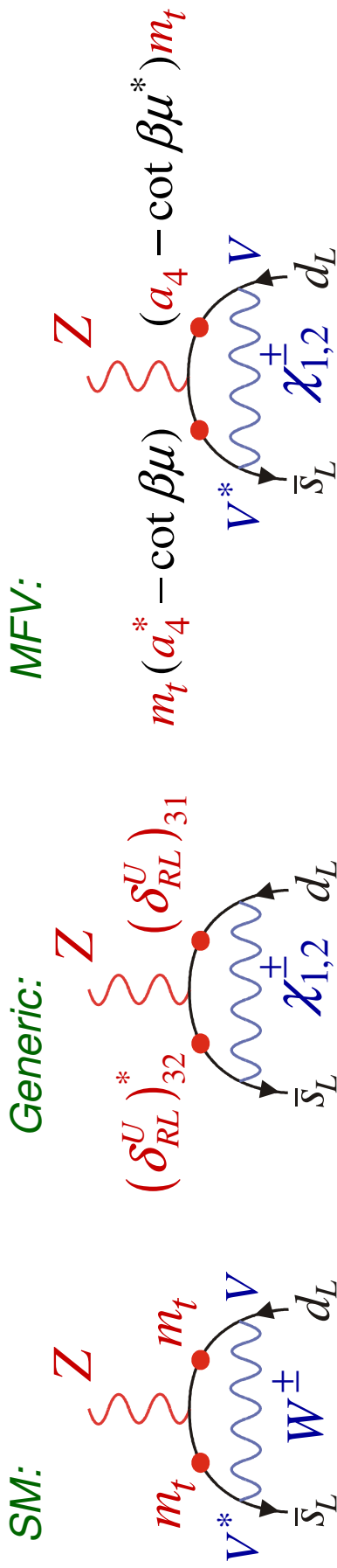
## MSSM at moderate $\tan\beta$ :

Dominant effect from *chargino penguins*, boxes smaller and constrained by  $\Delta S = 2$ .

*Nir, Worah ('98)/Buras, Romanino, Silvestrini ('98)*

LR-induced breaking of  $SU(2)_L \sim (\delta_{RL}^U)^* (\delta_{RL}^U)_{31}$ , hence very sensitive to  $A^U$  terms.

*Colangelo, Isidori ('98)*



$$\underline{K \rightarrow \pi\nu\bar{\nu}}$$

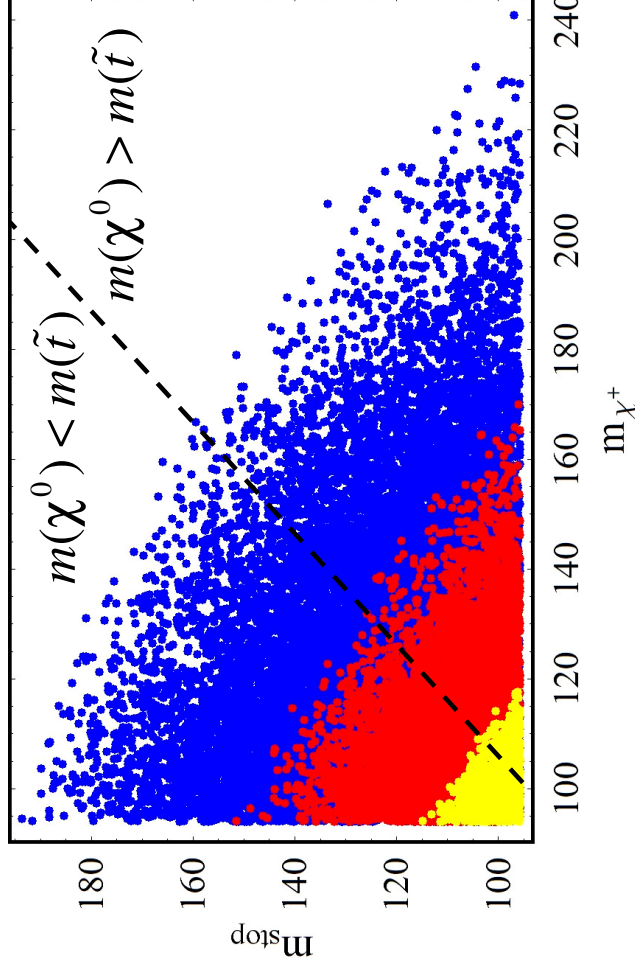
## Maximal effect under the MFV hypothesis?

Isidori, Mescia, Paradisi, Trine, CS ('06)

$$K \rightarrow \pi\nu\bar{\nu} \text{ ideal given its sensitivity to } \sim (\delta_{RL}^U)^*_{32} (\delta_{RL}^U)_{31} \sim m_t^2 V_{ts}^* V_{td} |a_4^* - \cot\beta\mu|^2.$$

Colors  $\Leftrightarrow$  enhancements of  $K_L \rightarrow \pi^0\nu\bar{\nu}$  by

**10%, 12%, 15%.**



- Enhanced only for **very light stop and chargino** ( $\sim$  higgsino).
- Small correlation with  $\Delta F = 2$
- Large correlation with  $B_{s,d} \rightarrow \mu^+\mu^-$
- **Large correlation with  $\Delta\rho$**

Buras, Gambino, Gorbahn, Jager, Silvestrini ('00)

$H^\pm$  contributions further enhance  $K \rightarrow \pi\nu\bar{\nu}$  by  $\sim 10\%$  if  $\tan\beta = 2$ ,  $m_{H^\pm} \approx 300$  GeV (effect gets smaller for larger  $\tan\beta$  and/or  $m_{H^\pm}$ ).

**SUSY masses  $> 200$  GeV &  $\tan\beta > 5$ : MFV falsified with enhancement  $\gtrsim 5\%$ .**

# $K \rightarrow \pi V \bar{V}$

Sensitivity to  $A^U$ , compared to other  $K$  &  $B$  observables?

The  $K \rightarrow \pi V \bar{V}$  modes are the best probe of the  $A^U$  terms (quadratic dependence).

Isidori, Mescia, Paradisi, Trine, CS ('06)

Scanning over  
trilinear terms:

$$|A_{13}^U|, |A_{23}^U| \leq A_0 \lambda,$$

$$A_0 = 1 \text{ TeV}$$

Phases left free.

Fixed sparticle  
masses:

$$\tan \beta = 2 - 4$$

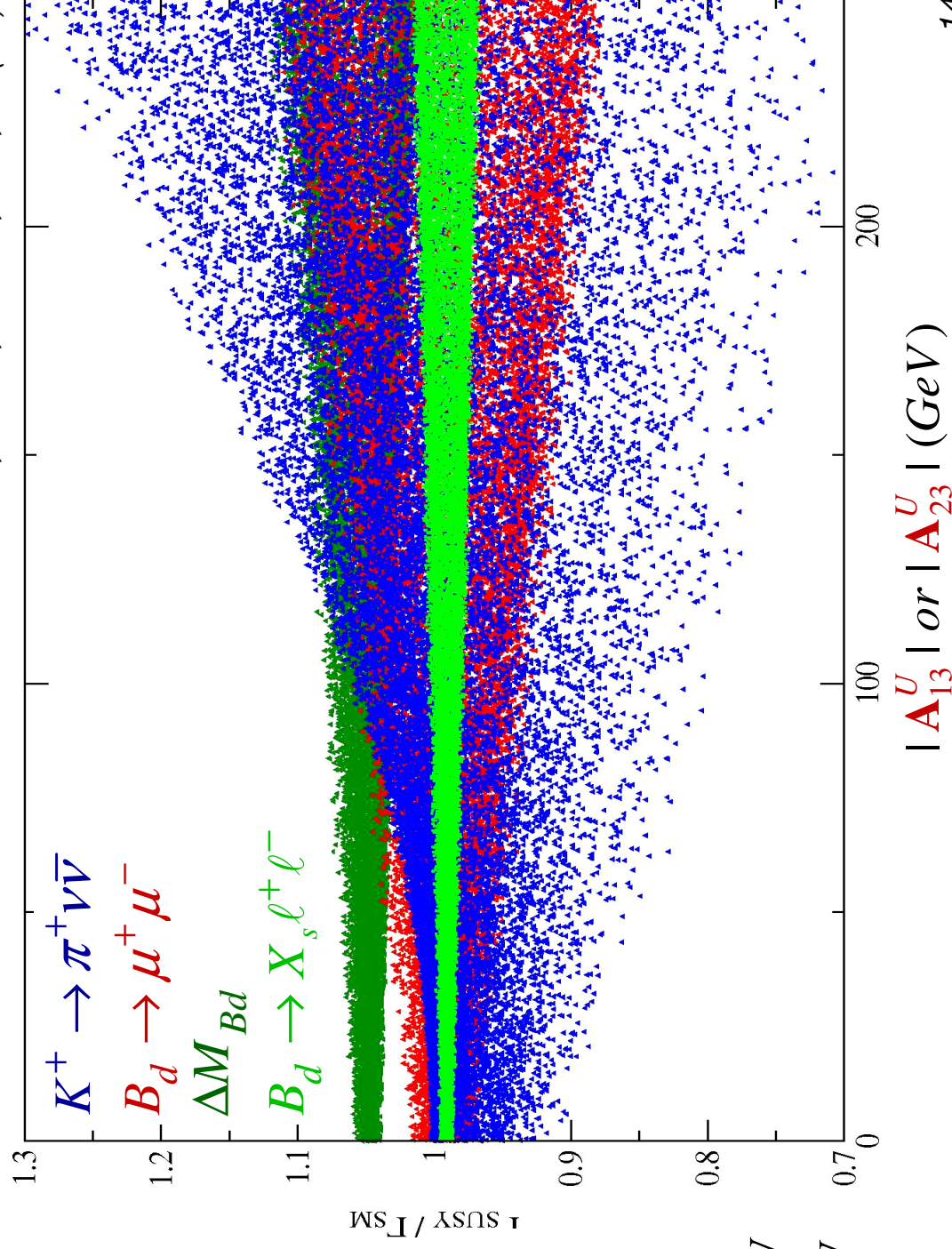
$$\mu = 500 \pm 10 \text{ GeV}$$

$$M_2 = 300 \pm 10 \text{ GeV}$$

$$m_{u_R} = 600 \pm 20 \text{ GeV}$$

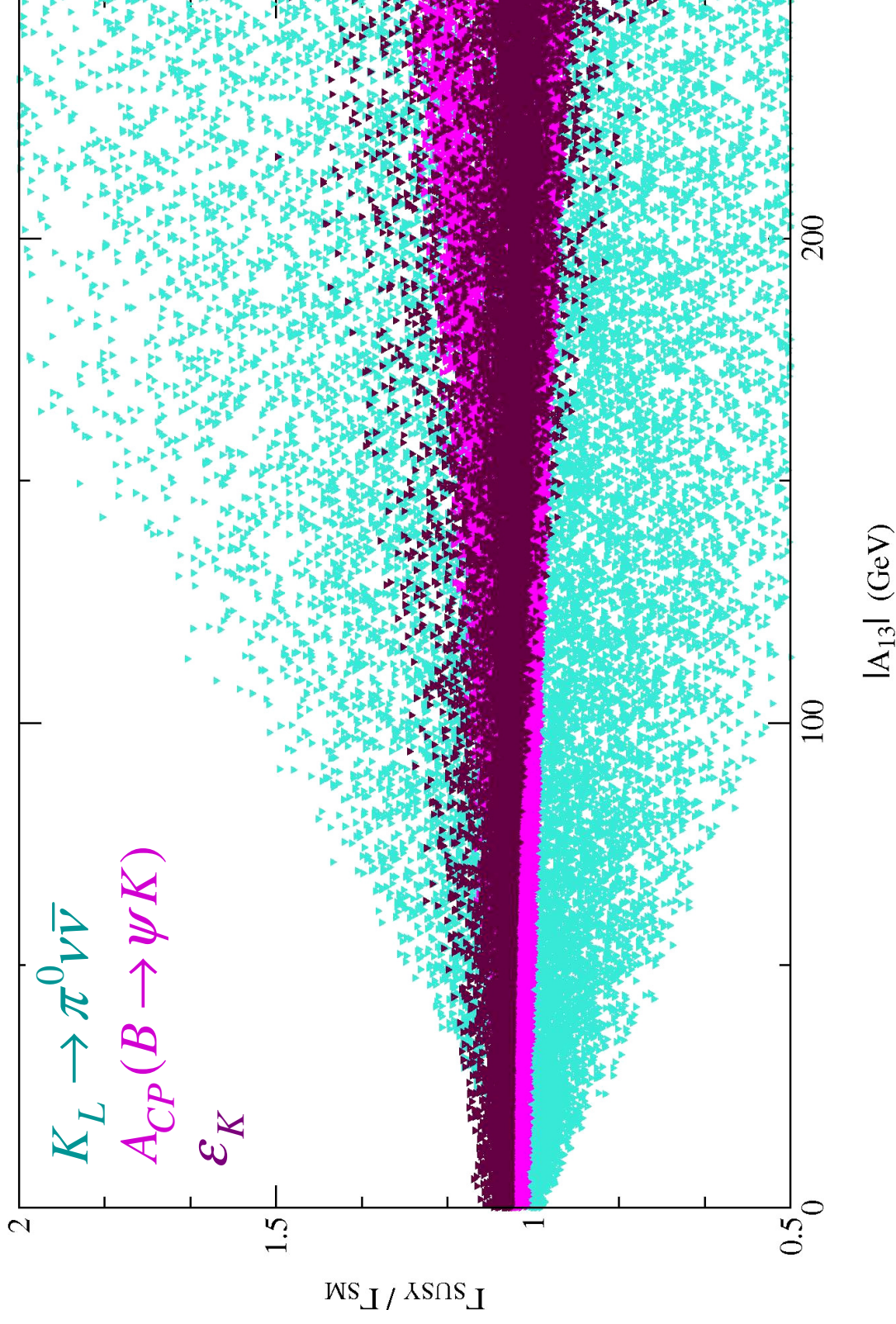
$$m_{q_L} = 800 \pm 20 \text{ GeV}$$

others : 2 TeV



Same within  $CP$ -violating  $K$  &  $B$  observables:

(Further: decoupling slower for penguins than for boxes as  $m_{\tilde{t}} \rightarrow \infty$ )



# $K \rightarrow \pi \nu \bar{\nu}$

*Is it possible to saturate the GN bound with these effects?*

The GN model-independent bound still leaves room for large effects:

$$B(K_L \rightarrow \pi^0 \nu \bar{\nu}) \leq 4.4 \times B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \approx 1.7 \cdot 10^{-9} \quad (90\% \text{ C.L.}) \quad \text{Grossman, Nir ('97)}$$

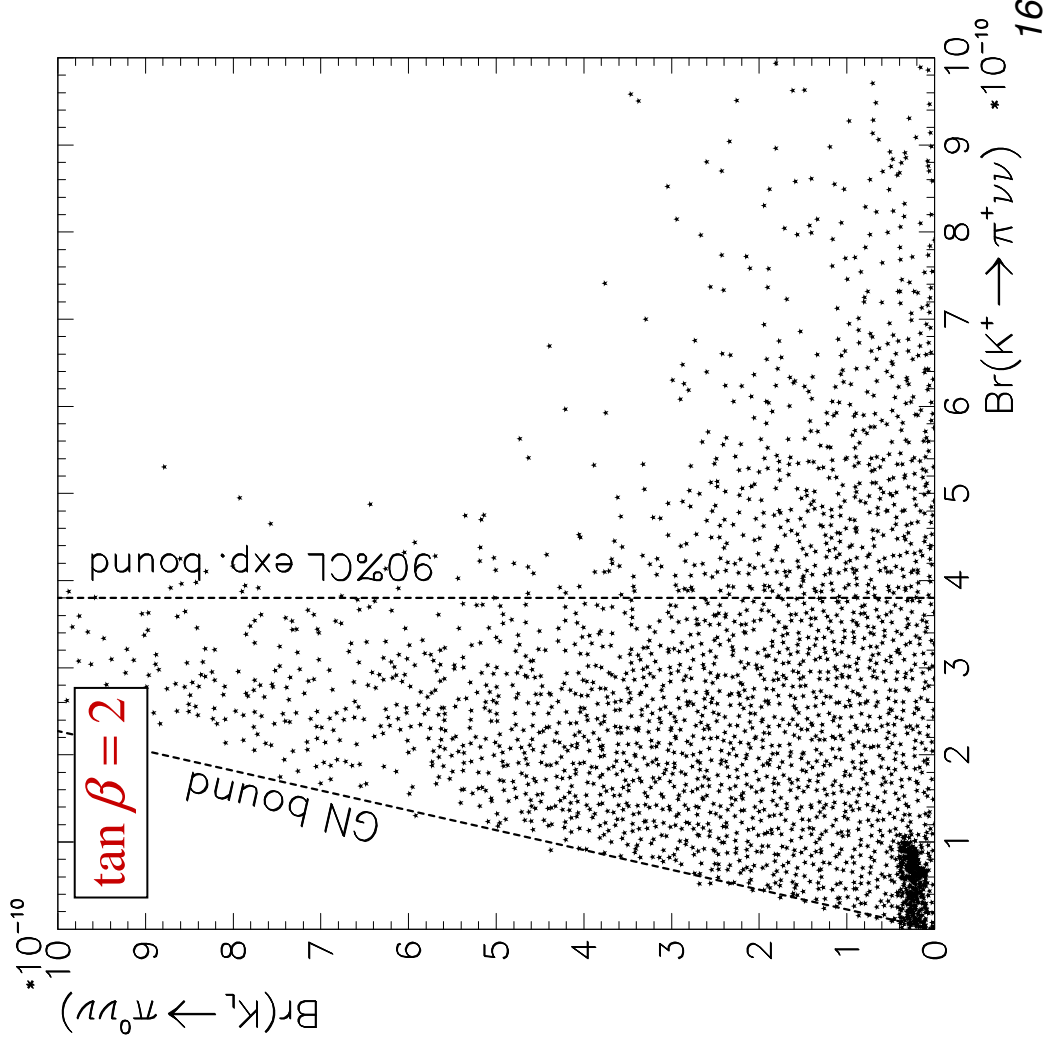
*Full scan over MSSM parameters.*

*Buras, Ewerth, Jager, Rosiek ('04)*

*Adaptive scanning (using VEGAS)  
to search for maximal effects.*

*Brein ('04)*

*Enhancement by a factor ~30 still  
allowed for the neutral mode.*





$$\underline{K \rightarrow \pi V \bar{V}}$$

## And at large $\tan\beta$ ?

- No effects from *neutral Higgs FCNC* ( $\sim$  neutrino masses).
- Effects from *charginos*:

*Within MFV*:  $\tan\beta$  not sufficient to compensate for  $m_s, m_d$  factors:

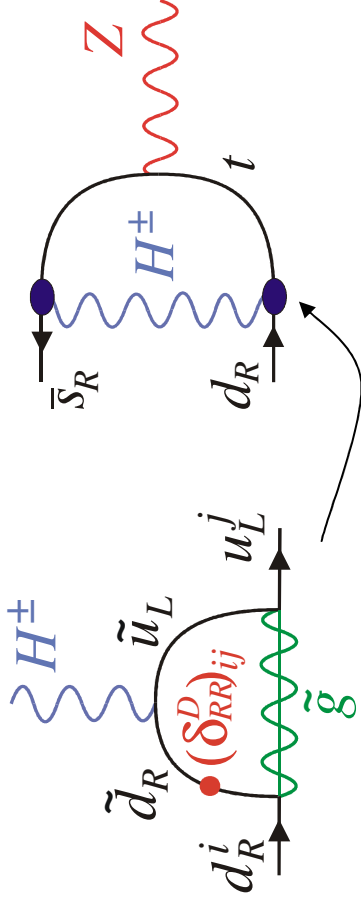
Isidori, Mescia, Paradisi, Trine, CS ('06)

$$m_t^2 V_{ts}^* V_{td} |a_4^* - \cot\beta\mu|^2 \rightarrow m_t^2 V_{ts}^* V_{td} \left( a_4^* - \cot\beta\mu + b_6^* \frac{m_s^2}{v_d^2} \right) \left( a_4 - \cot\beta\mu^* + b_6 \frac{m_d^2}{v_d^2} \right)$$

*Beyond MFV*: a priori similar as for moderate  $\tan\beta$ . Buras, Ewerth, Jager, Rosiek ('04)

- Sensitivity to higher order effects in the  $H^\pm$  penguin, though only *beyond MFV*:

Isidori & Paradisi ('06)



$$(\bar{s}_R \gamma_\mu d_R) (\bar{v}_L \gamma^\mu v_L) \sim (\tan\beta)^4$$

Slow decoupling,  $\sim x_{tH} \log(x_{tH})$ , compared to  $B_{s,d} \rightarrow \mu^+ \mu^- \sim x_{tH}^*$

## $K \rightarrow \pi \nu \bar{\nu}$

### 2- SUSY effects in new operators

Kiyo et al. ('98)/Perez ('99)

$$H_{\text{eff}}(K \rightarrow \pi \nu \bar{\nu}) \sim y_S^{\nu} (\bar{s}d)(\bar{\nu}\nu) + y_P^{\nu} (\bar{s}d)(\bar{\nu}\gamma_5\nu) \\ + y_T^{\nu} (\bar{s}\sigma_{\mu\nu}d)(\bar{\nu}\sigma^{\mu\nu}\nu) + y_{\tilde{T}}^{\nu} (\bar{s}\sigma_{\mu\nu}d)(\bar{\nu}\sigma^{\mu\nu}\gamma_5\nu)$$

Not CP-violating, but requires *active right-handed neutrinos*.

### 3- SUSY effects in new operators, different (but still invisible) final states

$$H_{\text{eff}}(K \rightarrow \pi \nu \bar{\nu}) \sim y_k (\bar{s}\Gamma_k d)(\bar{\nu}^i \Gamma_k \nu^j)$$

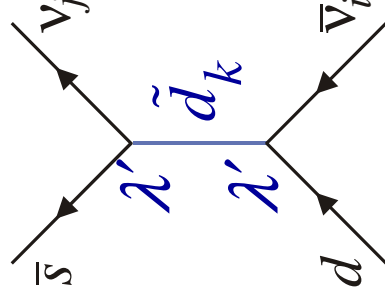
Grossman, Isidori, Murayama ('03)/  
Deandrea, Welzel, Oertel ('04)/  
Deshpande, Ghosh, He ('04)

**MSSM:** Negligible effects from boxes with LFV effects.

Can be induced by *R-parity violating couplings*:

$$W_{\Delta L=1} = \lambda^{IJK} L^I Q^J D^K + \dots$$

Scalar leptoquark tree-level exchanges:  
( $\rightarrow$  vector-current interactions)



SUSY effects in  $K_L \rightarrow \pi^0 \ell^+ \ell^-$

$$\underline{K_L \rightarrow \pi^0 \ell^+ \ell^-}$$

$K_L \rightarrow \pi^0 e^+ e^-$  and  $K_L \rightarrow \pi^0 \mu^+ \mu^-$  have very similar dynamics, but for  $m_e \neq m_\mu$

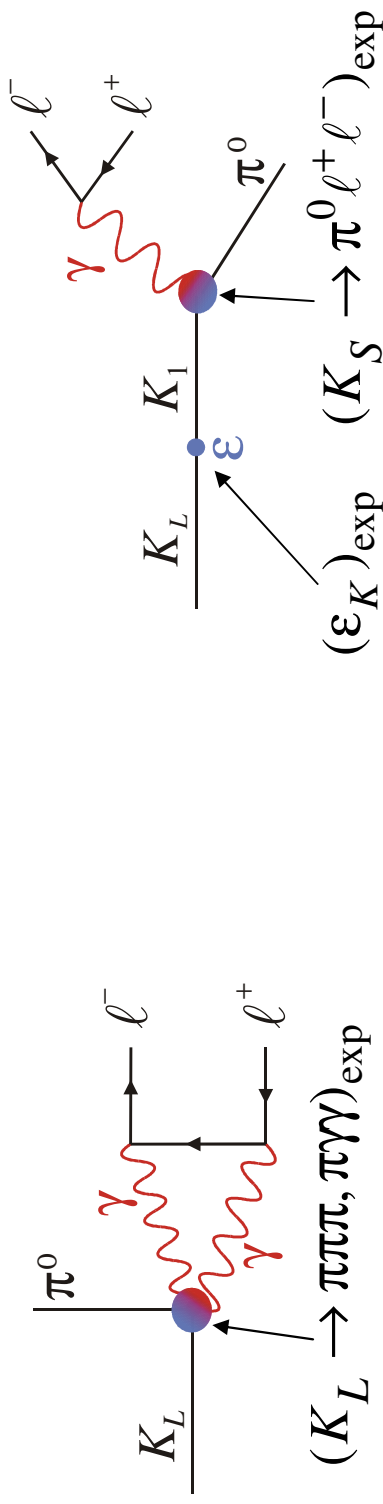
→ Sensitivity to a larger class of operators (*helicity-suppressed effects*).

Mescia, Trine, CS ('06)

### 1- SUSY effects in QCD operators

$$(\bar{s} \sigma_{\mu\nu} d) G^{\mu\nu}, (\bar{q} \Gamma q) \times (\bar{q} \Gamma q)$$

No direct impact: LD background fixed entirely from experimental data.



*CP-conserving  $\gamma$  contribution,*

Buchalla, D'Ambrosio, Isidori ('03)

Isidori, Unterdorfer, CS ('04)

*Indirect CP-violating contribution.*

D'Ambrosio, Ecker, Isidori, Portolés ('98)

At the low scale  $\mu \sim 1$  GeV, SUSY effects are in *semi-leptonic FCNC operators.*

$$\underline{K_L \rightarrow \pi^0 \ell^+ \ell^-}$$

2- SUSY effects in the SM electroweak operators

$$H_{\text{eff}}(K_L \rightarrow \pi^0 \ell^+ \ell^-) \sim y_{7V} (\bar{s}d)_{V-A} (\bar{\ell}\ell)_V + y_{7A} (\bar{s}d)_{V-A} (\bar{\ell}\ell)_A$$

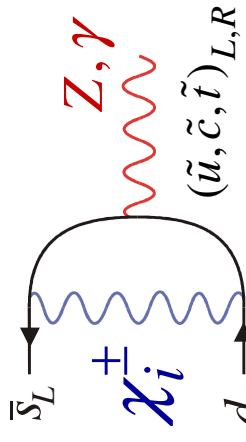
$I^{--}, CPV$        $I^{++} \& 0^{+-}, CPV$

- **Chargino penguins:** smaller but correlated with  $K \rightarrow \pi \nu \bar{\nu}$ :

Isidori, Mescia, Paradisi, Trine, CS ('06)

$$y_{7V}, y_{7A} \sim (\delta_{RL}^U)^* (\delta_{RL}^U)_{31}$$

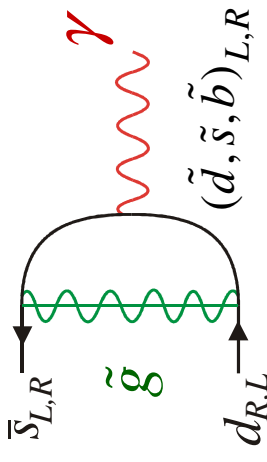
(MFV enhancement:  $\sim 7\%$  max.)



- **Gluino EMO operator,** strongly correlated with  $\epsilon'/\epsilon$ :

Buras, Colangelo, Isidori, Romanino, Silvestrini ('00)

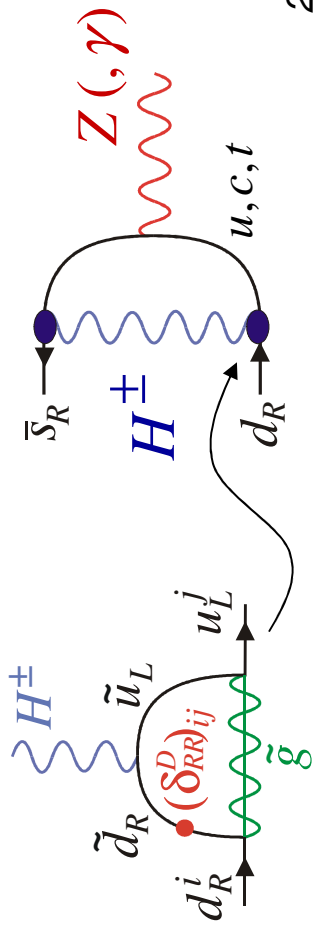
$$(\bar{s} \sigma_{\mu\nu} d) F^{\mu\nu} \rightarrow y_{7V} \sim (\delta_{RL}^D)_{12(21)}$$



- **Charged Higgs at large tan beta:**

Isidori, Paradisi ('06)

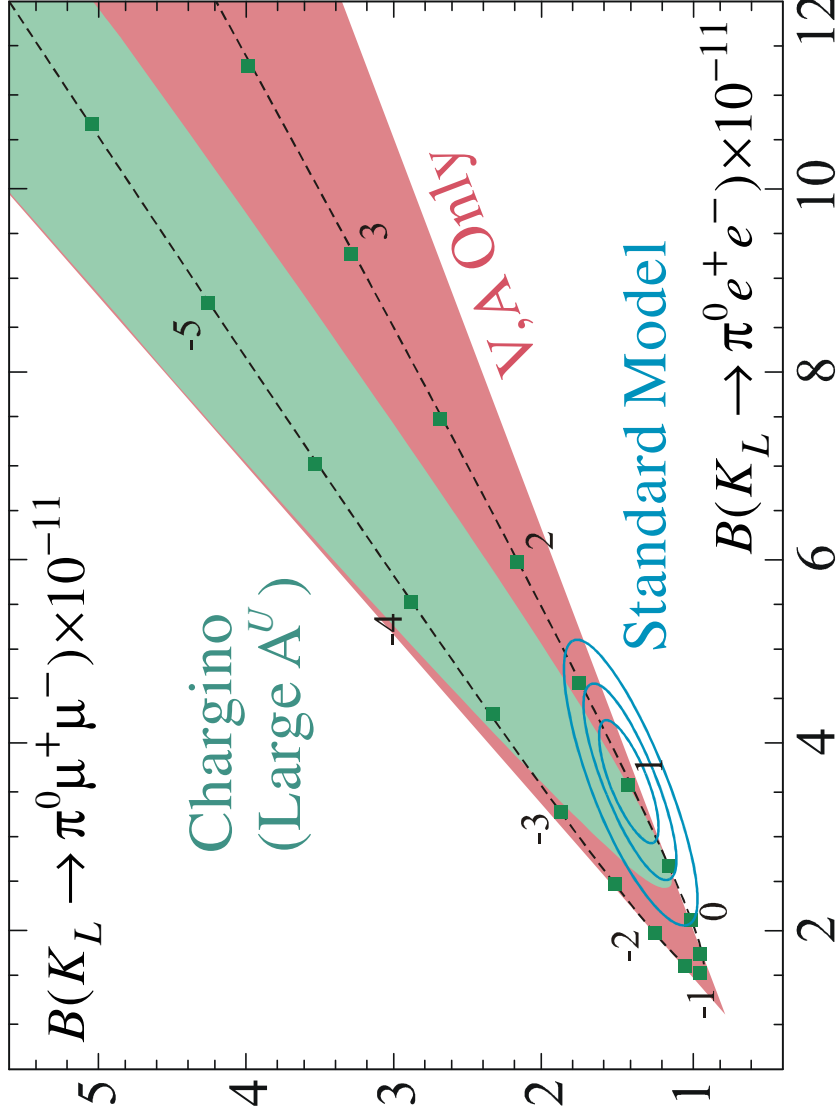
$$y_{7V}, y_{7A} \sim (\delta_{RR}^D)_{12}$$



$$\underline{K_L \rightarrow \pi^0 \ell^+ \ell^-}$$

How to disentangle  $V$  &  $A$  operators:

Specific regions in the plane signal specific correlations between  $y_{7A}$  and  $y_{7V}$ .  
 (illustrated for chargino penguins, taking unrealistically large  $A^U$  for clarity)



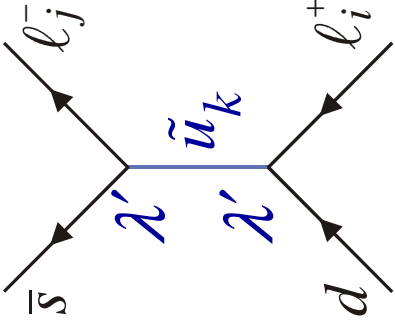
General bound if only vector/axial-vector FCNC operators are present:

$$0.1 + 0.24 B_{e^+e^-} \leq B_{\mu^+\mu^-} \leq 0.6 + 0.58 B_{e^+e^-} \quad \text{with} \quad B_{\ell^+\ell^-} \equiv B(K_L \rightarrow \pi^0 \ell^+ \ell^-) \cdot 10^{11}$$



$$\underline{K_L \rightarrow \pi^0 \ell^+ \ell^-}$$

*B- Helicity-allowed (pseudo-)scalar operators from R-parity violating couplings:*



Baring (possible) fine-tunings, must be *very suppressed* given the measured:

$$B(K_L \rightarrow e^+ e^-) = 9_{-4}^{+6} \times 10^{-12}$$

as well as bounds on  $K_L \rightarrow e^\pm \mu^\mp, \dots$

→ *No visible impact.*

#### 4- SUSY effects in the tensor/pseudotensor operators

$$H_{\text{eff}}(K_L \rightarrow \pi^0 \ell^+ \ell^-) \sim y_T (\bar{s} \sigma_{\mu\nu} d)(\bar{\ell} \sigma^{\mu\nu} \ell) + y_{\tilde{T}} (\bar{s} \sigma_{\mu\nu} d)(\bar{\ell} \sigma^{\mu\nu} \gamma_5 \ell)$$

$1^{--}, CPV$                        $1^{+-}, CPC$

Bobeth, Buras,  
Kruger, Urban ('02)

- Necessarily *helicity- and loop-suppressed* in the MSSM,
- In addition, their contributions are *phase-space suppressed*,
- *Cannot arise from R-parity violating couplings*, → *No visible impact.*
- *But:* do not contribute to  $K_L \rightarrow \ell^+ \ell^-$ .



# Conclusion

## Conclusion

Rare K decays are the *only theoretically clean window on the  $\Delta S = 1$  sector*,  
They are thus essential in the investigation of the *SUSY-breaking mechanism*.

Scenario	$K \rightarrow \pi V \bar{V}$	$K_L \rightarrow \pi^0 \ell^+ \ell^-$
MFV $\tan \beta \approx 2$	Best sensitivity, but maximum enhancement < 20-25%	Less sensitive, but precisely correlated with $K \rightarrow \pi V \bar{V}$
MFV $\tan \beta \approx 50$	Negligible effects (?)	
General $\tan \beta \approx 2$	Best probe of $\delta_{LR}^U$ (quadratic dependence)	$\delta_{LR}^U$ : correlated with $K \rightarrow \pi V \bar{V}$ $\delta_{LR}^D$ : correlated with $\varepsilon'/\varepsilon$ (but much cleaner)
General $\tan \beta \approx 50$	Good probe of $\delta_{RR}^D$ (slow decoupling as $M_H \rightarrow \infty$ )	Good probe of $\delta_{RR,LL}^D$ Correlated with $K_L \rightarrow \ell^+ \ell^-$ (but, again, much cleaner)

*If LHC finds Supersymmetry, the four modes have to be measured!*  
*The pattern of deviations with respect to the SM would become crucial.*