Entropy and abundance criteria to constrain susy models with dark matter

Luis Cabral,[1], *Myriam Mondragón*,[2] Lukas Nellen,[3]
Dario Núñez,[3] Roberto Sussmann,[3] Jesús Zavala [3]

[1]CIIDET [2]IF-UNAM

[3]ICN-UNAM

SUSY 2007

Motivation

- Growing evidence for the existence of dark matter
- Neutralinos (LSP) one of the best candidates
- Little information on galactic halo thermodynamic properties
- Try to combine knowledge of astrophysics and cosmology to get an independent constraint for models with dark matter
  - Use abundance criterion
  - Define an entropy criterion
  - Combine the two, apply it to msugra
- Compatibility of abundance and entropy criteria
  $\Rightarrow$ **constraint on susy models with dark matter**

Abundance criterion

Standard approach: Boltzmann equation, after “freeze out” neutralino number is constant

$$\Omega_\chi \approx 1/\langle \sigma v \rangle,$$

where $\langle \sigma v \rangle$ is the thermally averaged cross section times the relative velocity of the LSP annihilation pair.

Exact solution using MicrOmegas, assuming most DM is LSP. Relates $\Omega_\chi h^2$ to parameters of susy model.

Belanger, Kraml, Pukhov; Belanger et al

Entropy Criterion

Another way to set a constraint equation by entropy considerations.
Consider neutralino gas in two stages of evolutions:

freeze-out era
present era

Intial and final states taken in equilibrium.

Entropy expression from microcanonical ensemble in "mean field" approximation in terms of phase space volume:

$$s = \ln \left[\frac{(2mE)^{3/2} V}{(2\pi\hbar)^3} \right],$$

where V and E are local average values of volume and energy.

Cabral-Rosetti, Hernandez, Sussman

Change in entropy between initial (s_{f}, x_{f}, n_{f}) and final states ($s^{(\mathrm{h})}$, $x^{(\mathrm{h})}$, $n^{(\mathrm{h})}$)

$$s^{(\mathrm{h})} - s_{\mathrm{f}} = \ln\left[\frac{n_{\mathrm{f}}}{n^{(\mathrm{h})}}\left(\frac{x_{\mathrm{f}}}{x^{(\mathrm{h})}}\right)^{3/2}\right].$$

where $x_{\mathrm{f}} = m_\chi/T_{\mathrm{f}}$,

m_χ is the neutralino mass and T_{f} is the temperature of the system at freeze-out
n number density of particles, s entropy

Today: centre of halos.

Rewrite with observables

Relate n_f with present day cosmological parameters like

Ω_0 and h.

Taking as an approximation:

$$n_f = n_0\,(1+z_\mathrm{f})^3$$

Entropy per particle for a photon gas at freeze-out and the one today are proportional to the cube of the temperature of the system at the corresponding epoch:

$$g_{*\mathrm{f}}\mathcal{S}_\mathrm{f} = g_{*0}\,\mathcal{S}_0\;(1+z_\mathrm{f})^3$$

g_* degrees of freedom, known function of $x = m_\chi/T$

z redshift

Gondolo

Observables

$$n_\mathrm{f} = \frac{g_{*\mathrm{f}}\left(x_\mathrm{f}\right)}{g_{*0}\left(x_0^\mathrm{CMB}\right)} \left[\frac{T_\mathrm{f}}{T_0^\mathrm{CMB}}\right]^3 = n_0 \frac{g_{*\mathrm{f}}\left(x_\mathrm{f}\right)}{g_{*0}\left(x_0^\mathrm{CMB}\right)} \left[\frac{x_0^\mathrm{CMB}}{x_\mathrm{f}}\right]^3$$

where $x_0^\mathrm{CMB} \equiv m/T_0^\mathrm{CMB} = 4{,}29 \times 10^{12}\, m/\mathrm{GeV}$, with $T_0^\mathrm{CMB} = 2{,}7\,\mathrm{K}$

At freeze-out we can consider the halo as a MB neutralino gas:

$$\rho_\mathrm{f} = m_\chi\, n_\chi \left(1 + \frac{3}{2\,x_\mathrm{f}}\right), \qquad p_\mathrm{f} = \frac{m_\chi\, n_\chi}{x_\mathrm{f}},$$

$$s_\mathrm{f} = \left[\frac{\rho + p}{n\,T}\right]_\mathrm{f} = \frac{5}{2} + x_\mathrm{f},$$

ρ density, p pressure, T temperature

Today $n_0/n_c^{(h)} = \rho_0/\rho_c^{(h)}$ and $\rho_0 = \rho_{\text{crit}}\,\Omega_0\,h^2$

Collecting results we get a theoretical expression for the entropy:

$$s_c^{(h)}\big|_{\text{th}} = \frac{5}{2} + x_f + \ln\left[\frac{g_{*f}\,(x_f)\,\left(x_0^{\text{CMB}}\right)^3}{g_{*0}\,\left(x_0^{\text{CMB}}\right)}\,\frac{h^2\,\Omega_0}{(x_f\,x_c^{(h)})^{3/2}}\,\frac{\rho_{crit}}{\rho_c^{(h)}}\right]$$

which depends on inital state x_f, observable cosmological parameters $\Omega_0,\ h$ and on generic state variables associated to the present halo structure $x_c^{(h)}$, and $\rho_c^{(h)}$.

Notice

Assumption of MB statistics does not apply to self-gravitational collision-less system.

An exactly isothermal halo is not a realistic model:
its total mass diverges
distribution function $\rightarrow$ infinite particle velocities

More realistic halo models use “energy truncated” (ET) distribution functions, with maximal “cut off” velocity.

Binney, Tremaine; Padmanabhan; Katz, Horowitz, Dekel; Katz; Magliocchetti, Pugacco, Vesperini

Empirical estimate of entropy

Take equation for entropy, restrict phase space volume to the actual range of momenta (i.e. put maximal escape velocity)
Assume a relation of the form

$$v_e^2(0) \;=\; 2\,|\Phi(0)| \;\simeq\; \alpha\,\sigma_{(h)}^2(0),$$

where $\Phi(r)$ is the newtonian gravitational potential, and α is a proportionality constant
We get empirical expression for the entropy

$$s_c^{(h)}|_{em} \;\simeq\; \ln\left[\frac{m^4\,v_e^3}{(2\pi\hbar)^3\,\rho_c^{(h)}}\right]$$
$$= \quad 89{,}17 + \ln\left[\left(\frac{m}{\mathrm{GeV}}\right)^4 \left(\frac{\alpha}{x_c^{(h)}}\right)^{3/2} \frac{\mathrm{GeV/cm^3}}{\rho_c^{(h)}}\right],$$

where we used $x_c^{(h)} = c^2/\sigma_{(h)}^2(0)$.

Entropy constraint II

Equating the theoretical and the empirical estimates for the entropy per particle we finally obtain

$$ln(\Omega_\chi h^2) = 10{,}853 - x_\mathrm{f} + ln\left[\frac{(x_\mathrm{f}\alpha)^{3/2} m_\chi g_{*0}\left(x_0^{\mathrm{CMB}}\right)}{g_{*\mathrm{f}}\left(x_\mathrm{f}\right)}\right],$$

α is the proportionality constant between the escape and dispersion velocities at the center of the halo.

Another constraint equation relating
$\Omega_\chi h^2$ and observables.

More on α

α parametrizes our ignorance of the correct mechanical-statistics treatment of non-extensive systems formed by dark matter.

Assume spherical dark matter halo with a constant density core in the center, then the dark matter density profile follows the Navarro-Frenk-White (NFW) profile, and then it has a cutt-off:

$$\rho(y) = \begin{cases} \rho_c & \text{if } y < y_c \\ \frac{\delta_0 \rho_0}{y(1+y)^2} & \text{if } y_c \leq y \leq y_v \\ 0 & \text{if } y > y_v \end{cases}$$

ρ_c is the constant central density of the core, $y = r/r_s$, $y_c = r_c/r_s$, $y_v = r/r_v$, r_s is a scale radius, r_c is the core radius and r_v is the virial radius; ρ_0, δ_0 and r_s are parameters that define the NFW profile.

For a model without core, all these parameters can be given by a series of well-established formulas

$$\delta_0 = \frac{\Delta\, c^3}{3\left[\ln(1+c) - c/(1+c)\right]},$$

$$\rho_0 = \rho_{\rm crit}\, \Omega_0\, h^2 = 253{,}8\, h^2\, \frac{M_\odot}{\rm kpc^3},$$

where $c = r_s/r_v$, ρ_{crit} is the critical density for closure in an Einstein-de Sitter Universe (central value)

Ω_0 is the ratio of the total density of the Universe today

$\Omega_0 = 1$, $\Delta \sim 100$ for a ΛCDM model

Lokas, Hoffman; Lokas Navarro, Frenk, White; Mo, Mao, White; Lokas, Mamon; Zavala et al.

NFW density profile is defined by two parameters:
a “size” parameter r_v
a concentration parameter c

$$r_v = \left(\frac{3M_v}{4\pi\Delta\rho_0}\right)^{1/3},$$

$$c_0 \approx 62{,}1\left(\frac{M_v h}{M_\odot}\right),$$

both depend on total mass contained in the halo M_v.
c_0 fit for central value of concentration, in numerical studies it has a scatter

Bullock et al

We use these eqs to describe real dark matter with only one free parameter M_v.

Model for dark matter dominated systems

From previous expressions $\rightarrow$ analytical formula for α

To compute: need values for r_v, r_s and r_c
use an observational sample of galaxies corresponding to dark matter dominated systems

$$16{,}4 \leq \alpha \leq 27{,}8$$

Conservative estimate

Apply to msugra

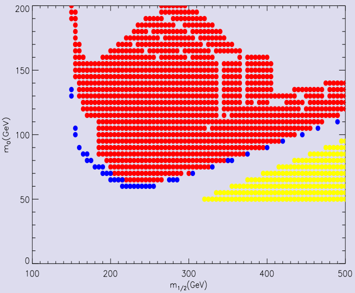
We have the AC and EC $\rightarrow$ compute relic abundance, see where they coincide

Take simple version of msugra to test the method:

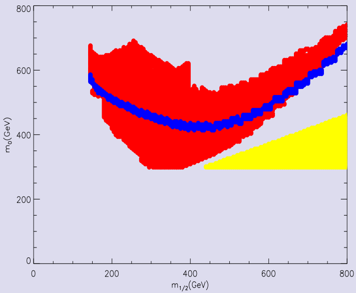
$$\begin{array}{llll} \text{fix} & A_0 = 0 & \text{and} & \text{sgn}\,\mu = + \\ \text{vary} & m_{1/2},\, m_0 & \text{and} & \tan\beta \end{array}$$

see where they are compatible

Bulk and coannihilation regions



$\tan\beta = 10$

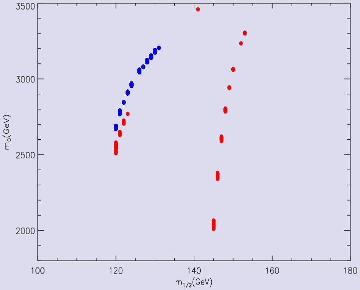


$\tan\beta = 50$.

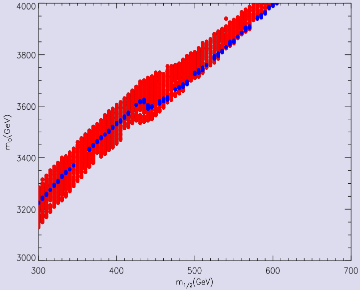
Allowed regions in the parameter space for AC (red) and EC (blue) criteria for the mSUGRA model with

$$A_0 = 0 \quad \text{and} \quad \text{sgn}\,\mu = +.$$

Focus point



$\tan\beta = 10$

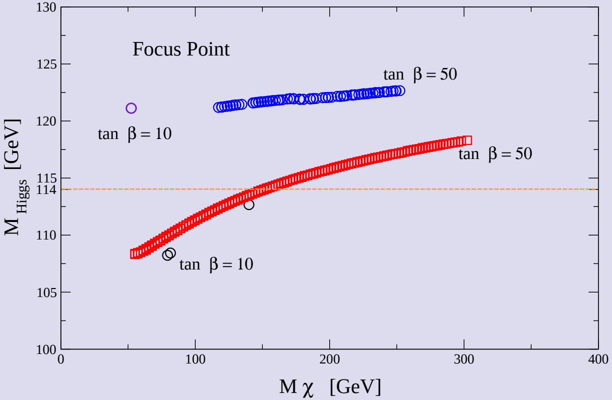


$\tan\beta = 50$.

Allowed regions in the parameter space for AC (red) and EC (blue) criteria for the mSUGRA model with

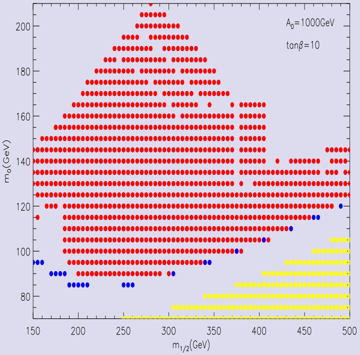
$$A_0 = 0 \quad \text{and} \quad \text{sgn}\,\mu = +.$$

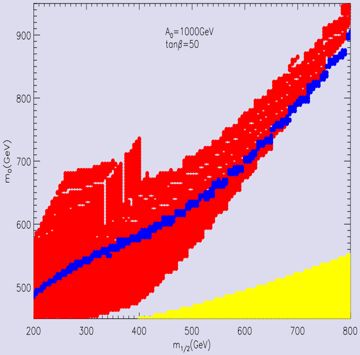
Higgs



The lightest Higgs M_{Higgs} mass vs the LSP mass m_{χ}, the dashed line indicates the present experimental limit on M_{Higgs}.

$A_0 \neq 0$





$\tan\beta = 10$ $\tan\beta = 50$.

Allowed regions in the parameter space for AC (red) and EC (blue) criteria for the mSUGRA model with

$A_0 = 1000$ GeV and sgn $\mu = +$.

Conclusions

- Through entropy considerations we get a constraint equation for Ωh^2 from cosmological/astrophysical considerations.
- By requiring the AC and EC criteria to coincide we can constrain parameter space of interesting dark matter susy models:
  example simple version of msugra

  $\Rightarrow$ large $\tan\beta$

  LSP $>$ 150 GeV
- Also, knowledge of LSP can give us feedback on astrophysical considerations to model dark matter halos
- Can be applied to any kind of dark matter
- Can be applied to any model