Entropy and abundance criteria to constrain susy models with dark matter

Luis Cabral,\textsuperscript{1} Myriam Mondragón,\textsuperscript{2} Lukas Nellen,\textsuperscript{3} Dario Núñez,\textsuperscript{3} Roberto Sussmann,\textsuperscript{3} Jesús Zavala\textsuperscript{3}

\textsuperscript{1}CIIDET \hspace{2cm} \textsuperscript{2}IF-UNAM \hspace{2cm} \textsuperscript{3}ICN-UNAM

\textbf{SUSY 2007}
Motivation

- Growing evidence for the existence of dark matter
- Neutralinos (LSP) one of the best candidates
- Little information on galactic halo thermodynamic properties
- Try to combine knowledge of astrophysics and cosmology to get an independent constraint for models with dark matter
  - Use abundance criterion
  - Define an entropy criterion
  - Combine the two, apply it to msugra
- Compatibility of abundance and entropy criteria
  \[ \Rightarrow \text{constraint on susy models with dark matter} \]
Abundance criterion

Standard approach: Boltzmann equation, after "freeze out" neutralino number is constant

\[ \Omega_\chi \approx 1/\langle \sigma v \rangle, \]

where \( \langle \sigma v \rangle \) is the thermally averaged cross section times the relative velocity of the LSP annihilation pair.

Exact solution using MicrOmegas, assuming most DM is LSP. Relates \( \Omega_\chi h^2 \) to parameters of susy model.

Belanger, Kraml, Pukhov; Belanger et al
Entropy Criterion

Another way to set a constraint equation by entropy considerations. Consider neutralino gas in two stages of evolutions:

freeze-out era
present era

Initial and final states taken in equilibrium.

Entropy expression from microcanonical ensemble in “mean field” approximation in terms of phase space volume:

\[ s = \ln \left[ \frac{(2mE)^{3/2}}{(2\pi \hbar)^3} \frac{V}{V} \right] , \]

where \( V \) and \( E \) are local average values of volume and energy.

Cabral-Rosetti, Hernandez, Sussman
Change in entropy between initial \((s_f, x_f, n_f)\) and final states \((s^{(h)}, x^{(h)}, n^{(h)})\)

\[
s^{(h)} - s_f = \ln \left[ \frac{n_f}{n^{(h)}} \left( \frac{x_f}{x^{(h)}} \right)^{3/2} \right].
\]

where \(x_f = m_\chi / T_f\),

\(m_\chi\) is the neutralino mass and \(T_f\) is the temperature of the system at freeze-out

\(n\) number density of particles, \(s\) entropy

Today: centre of halos.
Rewrite with observables

Relate $n_f$ with present day cosmological parameters like $\Omega_0$ and $h$.

Taking as an approximation:

$$n_f = n_0 (1 + z_f)^3$$

Entropy per particle for a photon gas at freeze-out and the one today are proportional to the cube of the temperature of the system at the corresponding epoch:

$$g_\ast_f S_f = g_\ast_0 S_0 (1 + z_f)^3$$

$g_\ast$ degrees of freedom, known function of $x = m_\chi / T$

$z$ redshift
Observables

\[ n_f = \frac{g_{*f}(x_f)}{g_{*0}(x_0^{\text{CMB}})} \left[ \frac{T_f}{T_0^{\text{CMB}}} \right]^3 = n_0 \frac{g_{*f}(x_f)}{g_{*0}(x_0^{\text{CMB}})} \left[ \frac{x_0^{\text{CMB}}}{x_f} \right]^3 \]

where \( x_0^{\text{CMB}} \equiv \frac{m}{T_0^{\text{CMB}}} = 4.29 \times 10^{12} \text{ m/GeV} \), with \( T_0^{\text{CMB}} = 2.7 \text{ K} \)

At freeze-out we can consider the halo as a MB neutralino gas:

\[ \rho_f = m_\chi n_\chi \left( 1 + \frac{3}{2 x_f} \right), \quad p_f = \frac{m_\chi n_\chi}{x_f}, \quad s_f = \left[ \frac{\rho + p}{n T} \right]_f = \frac{5}{2} + x_f, \]

\( \rho \) density, \( p \) pressure, \( T \) temperature
Today $n_0/n_c^{(h)} = \rho_0/\rho_c^{(h)}$ and $\rho_0 = \rho_{\text{crit}} \Omega_0 h^2$

Collecting results we get a theoretical expression for the entropy:

$$s_c^{(h)}|_{\text{th}} = \frac{5}{2} + x_f + \ln \left[ \frac{g_*(x_f)}{g_*0(x_0^{\text{CMB}})} \frac{h^2 \Omega_0}{(x_f x_c^{(h)})^{3/2}} \rho_{\text{crit}} \rho_c^{(h)} \right]$$

which depends on initial state $x_f$, observable cosmological parameters $\Omega_0$, $h$ and on generic state variables associated to the present halo structure $x_c^{(h)}$, and $\rho_c^{(h)}$. 
Assumption of MB statistics does not apply to self-gravitational collision-less system.

An exactly isothermal halo is not a realistic model: its total mass diverges. Distribution function → infinite particle velocities.

More realistic halo models use “energy truncated” (ET) distribution functions, with maximal “cut off” velocity.

Binney, Tremaine; Padmanabhan; Katz, Horowitz, Dekel; Katz; Magliocchetti, Pugacco, Vesperini
Empirical estimate of entropy

Take equation for entropy, restrict phase space volume to the actual range of momenta (i.e. put maximal escape velocity) Assume a relation of the form

\[ v_e^2(0) = 2 |\Phi(0)| \approx \alpha \sigma_{(h)}^2(0), \]

where \( \Phi(r) \) is the newtonian gravitational potential, and \( \alpha \) is a proportionality constant

We get empirical expression for the entropy

\[
\begin{align*}
S_c^{(h)}|_{em} & \approx \ln \left[ \frac{m^4 v_e^3}{(2\pi \hbar)^3 \rho_{(h)}^{(h)}} \right] \\
& = 89.17 + \ln \left[ \left( \frac{m}{\text{GeV}} \right)^4 \left( \frac{\alpha}{x_c^{(h)}} \right)^{3/2} \frac{\text{GeV/cm}^3}{\rho_{c}^{(h)}} \right],
\end{align*}
\]

where we used \( x_c^{(h)} = c^2 / \sigma_{(h)}^2(0) \).
Entropy constraint II

Equating the theoretical and the empirical estimates for the entropy per particle we finally obtain

\[ \ln(\Omega_\chi h^2) = 10.853 - x_f + \ln \left[ \frac{(x_f \alpha)^{3/2} m_\chi g^*_0 (x^\text{CMB})}{g^*_f (x_f)} \right], \]

\( \alpha \) is the proportionality constant between the escape and dispersion velocities at the center of the halo.

Another constraint equation relating \( \Omega_\chi h^2 \) and observables.
More on $\alpha$

$\alpha$ parametrizes our ignorance of the correct mechanical-statistics treatment of non-extensive systems formed by dark matter.

Assume spherical dark matter halo with a constant density core in the center, then the dark matter density profile follows the Navarro-Frenk-White (NFW) profile, and then it has a cut-off:

$$\rho(y) = \begin{cases} 
\rho_c & \text{if } y < y_c \\
\frac{\delta_0 \rho_0}{y(1+y)^2} & \text{if } y_c \leq y \leq y_v \\
0 & \text{if } y > y_v 
\end{cases}$$

$\rho_c$ is the constant central density of the core, $y = r/r_s$, $y_c = r_c/r_s$, $y_v = r/r_v$, $r_s$ is a scale radius, $r_c$ is the core radius and $r_v$ is the virial radius; $\rho_0$, $\delta_0$ and $r_s$ are parameters that define the NFW profile.
For a model without core, all these parameters can be given by a series of well-established formulas

\[ \delta_0 = \frac{\Delta c^3}{3 \left[ \ln(1 + c) - c/(1 + c) \right]}, \]

\[ \rho_0 = \rho_{\text{crit}} \Omega_0 h^2 = 253.8 \, h^2 \, \frac{M_\odot}{\text{kpc}^3}, \]

where \( c = r_s/r_v \), \( \rho_{\text{crit}} \) is the critical density for closure in an Einstein-de Sitter Universe (central value) \( \Omega_0 \) is the ratio of the total density of the Universe today \( \Omega_0 = 1, \quad \Delta \sim 100 \) for a \( \Lambda \text{CDM} \) model

Lokas, Hoffman; Lokas Navarro, Frenk, White; Mo, Mao, White; Lokas, Mamon; Zavala et al.
NFW density profile is defined by two parameters:

- a “size” parameter $r_{v}$
- a concentration parameter $c$

\[
    r_{v} = \left( \frac{3M_{v}}{4\pi \Delta \rho_{0}} \right)^{1/3},
\]

\[
    c_{0} \approx 62.1 \left( \frac{M_{v}h}{M_{\odot}} \right),
\]

both depend on total mass contained in the halo $M_{v}$. $c_{0}$ fit for central value of concentration, in numerical studies it has a scatter

Bullock et al
We use these eqs to describe real dark matter with only one free parameter $M_v$.

**Model for dark matter dominated systems**

From previous expressions $\rightarrow$ analytical formula for $\alpha$

To compute: need values for $r_v$, $r_s$ and $r_c$

use an observational sample of galaxies corresponding to dark matter dominated systems

$$16.4 \leq \alpha \leq 27.8$$

**Conservative estimate**
Apply to msugra

We have the AC and EC $\rightarrow$ compute relic abundance, see where they coincide

Take simple version of msugra to test the method:

- fix $A_0 = 0$ and $\text{sgn } \mu = +$
- vary $m_{1/2}, m_0$ and $\tan \beta$

see where they are compatible
Bulk and coannihilation regions

\[ \tan \beta = 10 \]

\[ \tan \beta = 50. \]

Allowed regions in the parameter space for AC (red) and EC (blue) criteria for the mSUGRA model with
\[ A_0 = 0 \quad \text{and} \quad \text{sgn } \mu = +. \]
Focus point

\[ \tan \beta = 10 \]

\[ \tan \beta = 50. \]

Allowed regions in the parameter space for AC (red) and EC (blue) criteria for the mSUGRA model with

\[ A_0 = 0 \quad \text{and} \quad \text{sgn} \, \mu = +. \]
The lightest Higgs $M_{\text{Higgs}}$ mass vs the LSP mass $m_{\chi}$, the dashed line indicates the present experimental limit on $M_{\text{Higgs}}$. 
$A_0 \neq 0$

\[
\tan \beta = 10 \\
\tan \beta = 50.
\]

Allowed regions in the parameter space for AC (red) and EC (blue) criteria for the mSUGRA model with

$A_0 = 1000 \text{ GeV}$ and \(\text{sgn} \mu = +\).
Conclusions

- Through entropy considerations we get a constraint equation for $\Omega h^2$ from cosmological/astrophysical considerations.
- By requiring the AC and EC criteria to coincide we can constrain parameter space of interesting dark matter susy models:
  - example simple version of msugra
    - $\Rightarrow$ large $\tan \beta$
    - $\text{LSP} > 150 \text{ GeV}$
- Also, knowledge of LSP can give us feedback on astrophysical considerations to model dark matter halos
- Can be applied to any kind of dark matter
- Can be applied to any model