Constraints on mSUGRA through entropy and abundance criteria

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Abstract. We derive an expression for the entropy of a present dark matter halo described by a Navarro-Frenk-White modified model with a central core. The comparison of this entropy with the one of the halo at the freeze-out era allows us to obtain an expression for the relic abundance of neutralinos, which in turn is used to constrain the parameter space in mSUGRA models, when used with the WMAP observations. Moreover, by joning these results with the ones obtained from the usual abundance criteria, we are able to clearly discriminate validity regions among $\tan\beta$ values of the mSUGRA model, by demanding both criteria to be consistent with the 2 sigma bounds of the WMAP observations for the relic density: $0.112 < \Omega h^2 < 0.122$. We found that for $sgn\ \mu = +$, small values of $\tan\beta$ are not favored; only for $\tan\beta \sim 50$ are both criteria significantly consistent. The use of both criteria also allows us to put a lower bound on the neutralino mass, $m_\chi \geq 151$GeV.

PACS. 14.80.Ly Supersymmetric partners of known particles – 95.35.+d Dark matter – 98.62.Gq Galactic halos

1 Introduction

Supersymmetry models which have the neutralino as the lightest supersymmetric particle (LSP) and as a candidate for dark matter (DM), have several parameters that can be constrained by the bounds on the present density of DM, Ω_{CDM}, that come from several outstanding observations such as the Cosmic Microwave Background radiation (CMBR) [1], Galaxy clustering, Supernovae and Lyman α forest. One of the most recent works which combines all these data leads to: $0.112 \leq \Omega_{CDM}h^2 \leq 0.122$ [2]. In particular for mSUGRA models these constraints have been obtained using the standard approach [3,4], which is based in the Boltzmann equation considering that after the "freeze-out" era, neutralinos cease to annihilate keeping its number constant. In such an approach, the relic density of neutralinos is approximately: $\Omega_\chi \approx 1/\langle\sigma v\rangle$, where $\langle\sigma v\rangle$ is the thermally averaged cross section times the relative velocity of the LSP annihilation pair. Within the mSUGRA model five parameters (m_0, $m_{1/2}$, A_0, $\tan\beta$ and the sign of μ) are needed to specify the supersymmetric spectrum of particles and the final relic density. We will use the numerical code micrOMEGAs [5] to compute the relic density following the past scheme which will be called the "abundance criterion" (AC).

Just after "freeze-out", we can consider neutralinos then as forming a Maxwell-Boltzmann (MB) gas in thermal equilibrium with other components of the primordial cosmic structures. In the present time, such a gas is almost colisionless and either constitutes galactic halos and larger structures or it is in the process of its formation. In this context, we can conceive two equilibrium states for the neutralino gas, the decoupling (or "freeze-out") epoch and its present state as a virialized system. Computing the entropy per particle for each one of this states we can use an "entropy consistency" criterion (EC) using theoretical and empirical estimates for this entropy to obtain the relic density of neutralinos (Ω_χ).

Our objective is then to use AC and EC criteria, to obtain constraints for the parameters of the mSUGRA model by demanding that both criteria must be consistent with each other and with the observational constraints of Ω_{CDM}.

2 Abundance criterion

Relic abundance of some stable SUSY species χ is defined as $\Omega_\chi = \rho_\chi/\rho_{crit}$, where $\rho_\chi = m_\chi n_\chi$ is the relic's mass density (n_χ is the number density), ρ_{crit} is the critical density of the Universe (see [6] for a review

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on the standard method to compute the relic density). The time evolution of n_χ is given by the Boltzmann equation:

$$\frac{dn_\chi}{dt} = -3Hn_\chi - \langle\sigma v\rangle(n_\chi^2 - (n_\chi^{eq})^2) \qquad (1)$$

where H is the Hubble expansion rate, $\langle\sigma v\rangle$ is the thermally averaged cross section times the relative velocity of the LSP annihilation pair and n_χ^{eq} is the number density that species would have in thermal equilibrium. In the early Universe, the neutralinos (χ) were initially in thermal equilibrium, $n_\chi = n_\chi^{eq}$. As the Universe expanded, their typical interaction rate started to diminish an the process of annihilation froze out. Since then, the number of neutralinos in a comoving volume has remained basically constant.

There are several ways to solve equation (1), one of the more used is based on the "freeze-out" approximation (see for example [7]). However in order to have more precision, we will use the exact solution to Boltzmann equation using the public numerical code micrOMEGAs 1.3.6 [5] which calculates the relic density of the LSP in the Minimal Supersymmetric Standard Model (MSSM). We will take and mSUGRA model and its five parametersas input parameters for micrOMEGAs and use *Suspect* [8], which comes as an interface to micrOMEGAs, to calculate the supersymmetric mass spectrum.

Using micrOMEGAs, we can obtain the relic density for any region of the parameter space to discriminate regions that are consistent with the WMAP constraints in this abundance criterion.

3 Entropy consistency criterion

Since the usual MB statistics that can be formally applied to the neutralino gas at the "freeze-out" era can not be used to describe present day neutralinos subject to a long range gravitational interaction making up non-extensive systems, it is necessary to use the appropriate approach that follows from the microcanonical ensemble in the "mean field" approximation, which yields an entropy definition that is well defined for a self-gravitating gas in an intermediate state. Such an approach is valid at both the initial ("freeze-out" era, f) and final (virialized halo structures, h) states that we wish to compare. Under these conditions, the change in the entropy per particle (s) between these two states is given by [9]:

$$s^h - s^f = ln\left[\frac{n_\chi^f}{n_\chi^h}\left(\frac{x^f}{x^h}\right)^{3/2}\right] , \qquad (2)$$

where $x = m_\chi/T$, T is the temperature of the gas. A region that fits with the conditions associated with the intermediate scale is the central region of halos ($10pc^3$ within the halo core); evaluating the thermodynamical quantities at this region, using equation (2) and some extra assumptions (conservation of photon entropy), it is possible to construct a theoretical estimate for s^h that depends on the nature of neutralinos (m_χ and $\langle\sigma v\rangle$), initial conditions (given by x^f), cosmological parameters (Ω_χ, the Hubble parameter, h) and structural parameters of the virialized halo (central values for temperature and density); for details of these and the following, see section IV of [9].

An alternative estimate for s^h can be made based on empirical quantities for observed structures in the present Universe using the microcanonical entropy definition in terms of phase space volume, but restricting this volume to the actual range of velocities accessible to the central particles. That is, restricting the escape velocity up to a maximal value $v_e(0)$ which is related to the central velocity dispersion of the halo (σ_h) by an intrinsic parameter α: $v_e^2(0) \sim \alpha\sigma_h^2(0)$. In a recent work [10], we estimate the value of α using an NFW modified model with a central core, and obtain $16.4 < \alpha < 27.8$. The range of values allowed for this parameter is of the highest importance to determine the allowed region of the parameter space in the mSUGRA model as will be clear in the results presented on next section.

Equating the theoretical an empirical estimates for the entropy per particle it is obtained a relation for the relic abundance of neutralinos using the EC criterion[1]:

$$ln(\Omega_\chi h^2) = 10.853 - x^f + ln\left[\frac{(x^f\alpha)^{3/2}m_\chi}{f_g^*(x^f)}\right] \qquad (3)$$

where $f_g^*(x^f)$ is a function related to the degrees of freedom at the "freeze-out" time (see for example [7]) that will be described elsewhere [10].

Modifying the program micrOMEGAs, we obtained the value for x^f for any region of the parameter space and then Ω_χ using Eq. (3), therefore we were able to discriminate regions that are consistent with the WMAP constraints for the EC criterion.

4 Results and Conclusions

Using both the AC and EC that have been described in the preceeding sections, we can compute the total mass density of neutralinos present today and constrain the region in the mSUGRA parameter space where both criteria are fulfilled. Out of the five parameters, we will fix $\mu > 0$. Then our strategy is to explore wide regions for the values of the other four parameters, by means of a bi-dimensional analysis in the $m_0 - m_{1/2}$ plane for different fixed values of A_0 and tanβ. It is important to mention that we are not presenting an exhaustive search in all the possible regions, but we concentrated on those regions which have received more attention in the literature, see for example [3].

In Fig. (1), we present the results for tan$\beta = 10$, for three values of A_0, namely $A_0 = 1000, 0, -1000$ GeV, shown in the top, middle and bottom panels respectiveley. The yellow region (lower right corner) is

[1] This formula is a small modification to the one presented in [9]

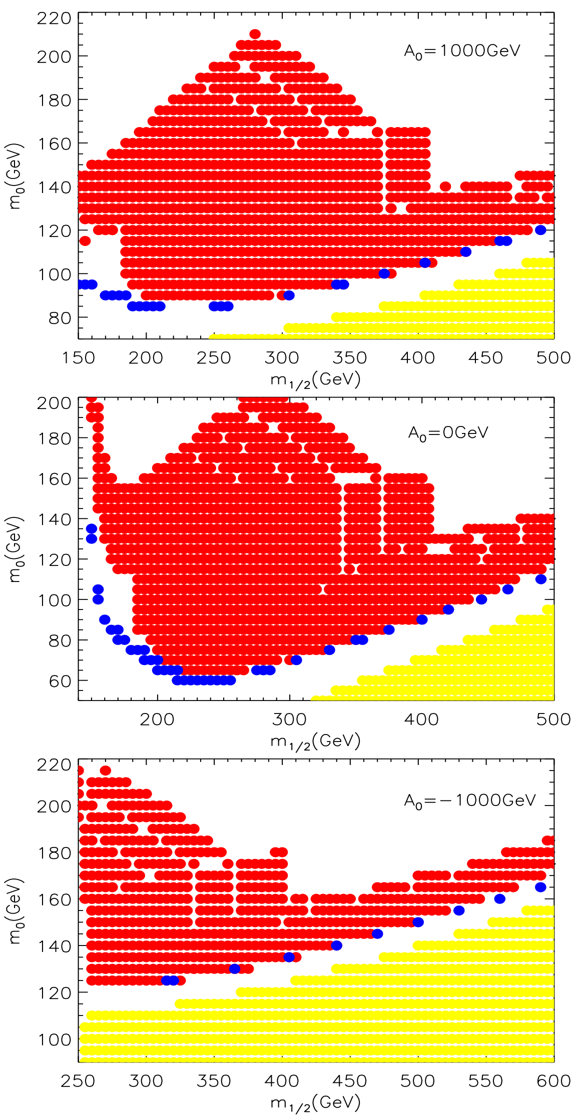

Fig. 1. Allowed regions in the parameter space for AC (lighter gray/red) and EC (darker grey/blue) for the mSUGRA model with sgnμ = +, tanβ = 10, and A_0 = 1000 GeV, top panel, A_0 = 0 GeV, middle panel, and $A_0 = -1000$ GeV, bottom panel. The figures show the so called bulk and coannihilation regions. The yellow region shows where the stau is the LSP.

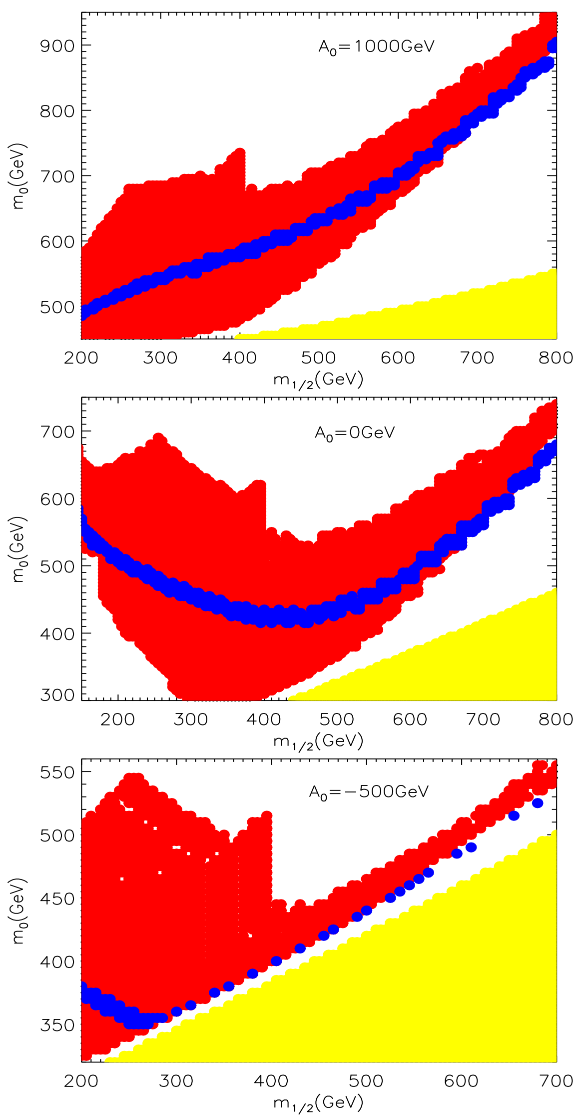

Fig. 2. The same as Fig. (1), but for tanβ = 50, and now $A_0 = -500$ GeV in the bottom panel.

where the $\tilde{\tau}$ is the LSP, the lighter and darker areas (red and blue for the online version in colours) define the allowed regions for the EC and AC respectively according to the observed DM density. The area of the EC region depends on the size of the interval of values of the parameter α, the lower and upper bounds of α determine the upper and lower boundaries of the EC region. As can be seen from the figure, the region where both criteria are fullfilled is very small, in fact, only for the highest values of α there is an intersection between both criteria. This behavior holds for all values of A_0 in the interval $[-1000, 1000]$ GeV, here we are showing only the extreme and central values.

Repeating the same procedure for larger values of tanβ, it is found that the intersection region for both criteria becomes larger, but it gets to be significant for the largest values of this parameter. This is clearly shown in Fig. (2), which is equivalent to Fig. (1), but for tanβ = 50. In this case the bottom panel is for $A_0 = -500$ GeV. It is clear from the figure that for these values of tanβ both criteria are consistent, as shown by the large intersection area for values of A_0 in the interval $[0, 1000]$ GeV. For negative values of A_0 the intersection region decreases with A_0, see the bottom panel of the figure. For even lower values of A_0 the intersection becomes insignificant.

In Figs. (3) we present the same analysis but for the Focus Point region, and for the central value $A_0 = 0$. The situation is consistent with the previous results, both criteria intersect for $\tan\beta = 50$ and there is nearly no intersection for $\tan\beta = 10$.

This analysis allows us to arrive to one of the main results of our work. The use of both criteria favours large values of tanβ.

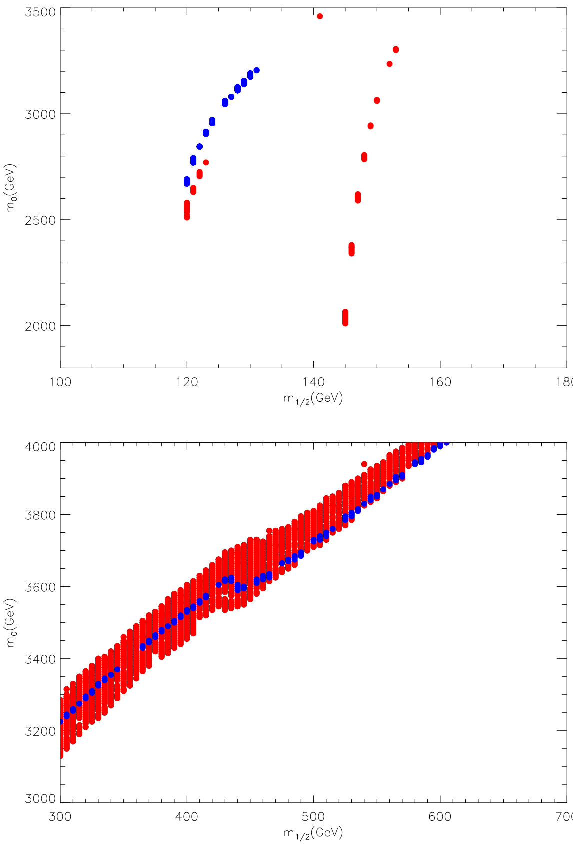

Fig. 3. Allowed regions in the parameter space for AC (lighter gray/red) and EC (darker grey/blue) in the mSUGRA model with $A_0 = 0$, sgn$\mu = +$, tan$\beta = 10$, top panel, and tan$\beta = 50$, bottom panel. The region shows the so called Focus Point region.

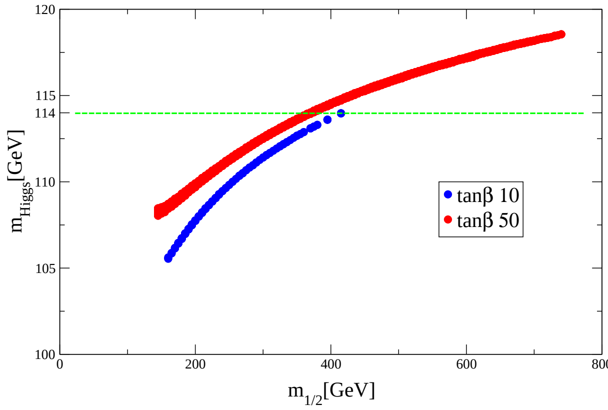

Fig. 4. Allowed values for M_{Higgs} as function of $m_{1/2}$. As can be seen from the figure, the present bound on the Higgs mass favours a large tan β.

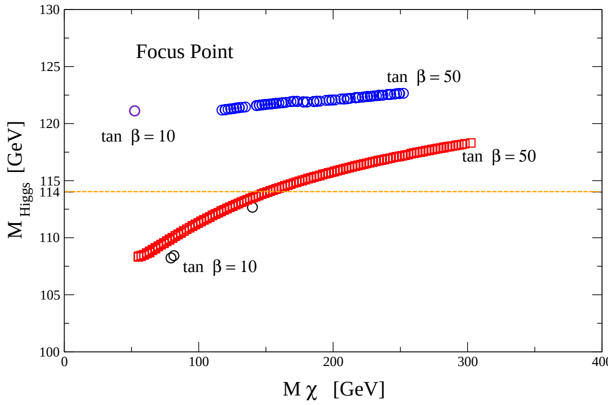

Fig. 5. The figure shows the LSP mass plotted versus the Higgs mass, points above the dashed line are the allowed values for the LSP. The points in blue correspond to the Focus Point region, the ones in red to the bulk and coannihilation regions.

In Figs.(4) and (5) we show the allowed values for the LSP and the Higgs mass after constraining the parameter space with the abundance and entropy criteria. As can be seen from Fig. (4), the current limit for the Higgs favour, combined with the AC and EC criteria, favours even more a large value of tan β. This, in turn, puts a constraint on the allowed SUSY mass spectra of the bulk and coannihilation regions: it gives an LSP of mass $m_\chi \sim 140$ GeV for tan β 10, and a lower bound for the LSP mass $m_\chi \gtrsim 150$ GeV for large tan β.

Further analysis, which is currently under way, is required to give more precise conclusions about this new method to constrain the parameter space of the mSUGRA model [10].

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