Beyond the Standard Model (II)

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• Standard Model (SM):

- currently the best model we have; it complies with experimental data.

• is there new physics "beyond" the SM ?

- SM is an effective theory, in the sense that is valid up to some scale; what is that scale? ...few TeV...?
- one possibility: Minimal Supersymmetric Standard Model (MSSM)
- new states present; superpartners of the SM states.

• Therefore, we expect to have states of masses of:

- EW scale order (from the SM: Z, W, quarks, etc....)
- TeV scale order, from the new states present in the MSSM but not in SM.

- Motivation:

- EWSB: In SM the Higgs mechanism: not dynamical.

- Hierarchy problem:

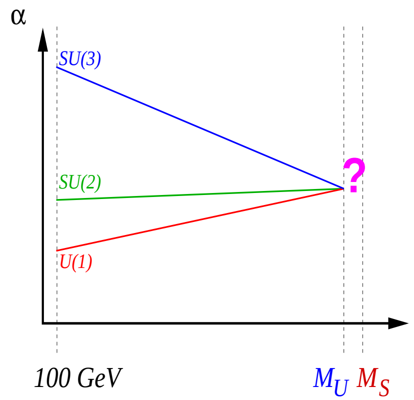
- why $M_{\rm Planck} \gg m_W, m_Z, m_H \sim 100$ GeV? M_{Planck}/m_H: unstable under quantum corrections.

- fine-tuning*

- Unification**:

- is there a single gauge group framework for all particle interactions?

- is there a "unified" gauge coupling, for all SM interactions, at some high scale?



- Gravity

- not part of the SM; would be nice to have SM, Gravity "together" in a same quantum theory.

- MSSM solves these problems: hierarchy problem, gauge unification, EWSB,...

- based on a new symmetry called Supersymmetry.

- in this talk I discuss issues related to (*) and (**).

- MSSM spectrum:

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L, \widetilde{d}_L)$	$(u_L \quad d_L)$	$(\mathbf{3},\ \mathbf{2},\ \frac{1}{6})$
$\times 3$ families	U^c	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	D^c	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \widetilde{e}_L)$	$(\nu \quad e_L)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$
$\times$families	E^c	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	$H_2 \quad (H_u)$	$(H^+_2 \quad H^0_2)$	$(\widetilde{H}^+_2 \quad \widetilde{H}^0_2)$	$(\mathbf{1},\ \mathbf{2},\ +\frac{1}{2})$
	$H_1 \quad (H_d)$	$(H^0_1 \quad H^-_1)$	$(\widetilde{H}^0_1 \quad \widetilde{H}^-_1)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$

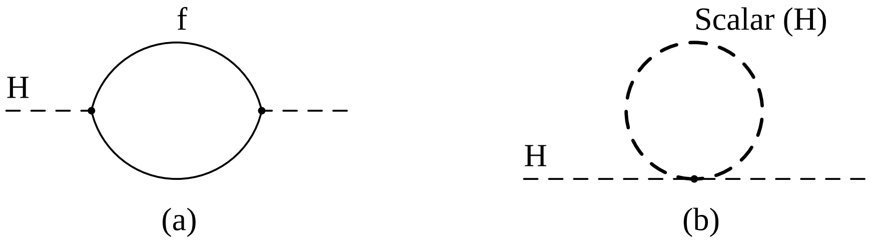
- chiral superfields: $\Phi(y,\theta) = \phi(y) + \theta\,\psi + \theta\theta\,F$; (ϕ,ψ) spartners: (slepton,lepton), (squark,quark), (higgs, higgsino)

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8},\ \mathbf{1},\ 0)$
winos, W bosons	$\widetilde{W}^\pm \quad \widetilde{W}^0$	$W^\pm \quad W^0$	$(\mathbf{1},\ \mathbf{3},\ 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\mathbf{1},\ \mathbf{1},\ 0)$

- gauge supermultiplets: $V(x,\theta,\overline{\theta}) = \theta\,\sigma^\mu\,\overline{\theta}\,V_\mu(x) + \theta\theta\,\overline{\theta}\,\overline{\lambda} + \overline{\theta\theta}\,\theta\lambda + \theta\theta\,\overline{\theta\theta}\,D(x)$. (V_μ,λ): (gauge, gaugino)
- The idea behind Supersymmetry: find a symmetry to “protect” scalars from becoming very massive $m \gg 100$ GeV.
- supermultiplets: representations of super-Poincaré algebra.

• I. Fine-tuning: SM versus MSSM:

• In the SM $v^2,\ m_h^2 \sim (coupling) \times M^2_{Planck}$.



Scalar self-energy: fermion-antifermion and scalar contributions:

$$m^2_{H,f} \sim \lambda_f^2\,\Lambda^2 + \cdots \sim \lambda_f^2\, M^2_{Planck}, \qquad m^2_{H,s} \sim \lambda_s\,\Lambda^2 + \cdots \sim \lambda_s\, M^2_{Planck},$$

$\Rightarrow \quad m_h$ and EW scale $\mathcal{O}(100\mathrm{GeV}) \leftrightarrow$ fine-tune the coupling, to all orders in perturbation theory.

• In supersymmetry (broken at $m_0 \sim 1$ TeV)

$$m_H^2 \sim m_0^2 \ln \Lambda/m_0 \sim m_0^2 \ln M_{Planck}/m_0$$

- a mild UV behaviour (logarithmic).

- In MSSM:

$$V = \tilde{m}_1^2 \, |h_1|^2 + \tilde{m}_2^2 \, |h_2|^2 - \left(m_3^2 \, h_1 \, h_2 + h.c. \right) + \frac{g_1^2 + g_2^2}{8} \left(|\, h_1 \,|^2 - |\, h_2 \,|^2 \right)^2 + \frac{1}{8} \, \delta \, |\, h_2 \,|^4$$

$$v^2 = -\frac{m^2}{\lambda}, \qquad m^2 \equiv \tilde{m}_1^2 \, \cos^2 \beta + \tilde{m}_2^2 \, \sin^2 \beta - m_3^2 \, \sin 2\beta, \qquad \lambda \equiv \frac{g^2}{8} \, \cos^2 2\beta + \frac{1}{8} \, \delta \, \sin^4 \beta$$

$v \approx 246$ GeV, $\lambda < 1$, $\tilde{m}_{1,2}, m_3 \sim 1$ TeV.

$$\tilde{m}_1^2(t) = m_0^2 + \mu_0^2 \, \sigma_8^2(t) + M_{1/2}^2 \, \sigma_1(t)$$

$$\tilde{m}_2^2(t) = \mu_0^2 \, \sigma_8^2(t) + M_{1/2}^2 \, \sigma_4(t) + A_t \, m_0 \, m_{12} \, \sigma_5(t) + m_0^2 \, \sigma_7(t) - m_0^2 \, A_t^2 \, \sigma_6(t) < 0$$

$$m_3^2(t) = \mu_0 \, M_{1/2} \, \sigma_2(t) + B_0 \, m_0 \, \mu_0 \, \sigma_8(t) + \mu_0 \, m_0 \, A_t \, \sigma_3(t)$$

- Tree level: $\sigma_{1,..6} = 0$, $\sigma_{7,8} = 1$. $m_h \leq m_Z$. However: LEPII bound: $m_h > 114.4$ GeV.
- Including quantum corrections $m_h > 114.4$ GeV, up to ~ 130 GeV. Moreover, quantum corrections trigger EWSB!
- Nevertheless, some "fine" cancellations needed to "separate" EW from susy-breaking TeV scale...

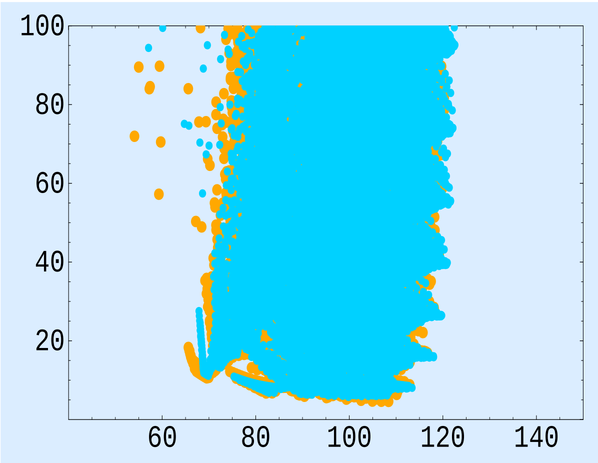
• Quantum corrections reduce the "tension" needed to separate the EW and TeV scales:

$$\delta = \frac{3\,h_t^4}{\pi^2}\left[\ln\frac{M_{\tilde t}}{m_t}+\frac{X_t}{4}+\frac{1}{32\pi^2}\left(3\,h_t^2-16g_3^2\right)\left(X_t+2\ln\frac{M_{\tilde t}}{m_t}\right)\ln\frac{M_{\tilde t}}{m_t}\right],\qquad X_t\equiv\frac{2\,(A_t\,m_0-\mu\cot\beta)^2}{M_{\tilde t}^2}\left[1-\frac{(A_t\,m_0-\mu\cot\beta)^2}{12M_{\tilde t}^2}\right].$$

• MSSM "remnant" fine-tuning: $\Delta\equiv\max\left|\Delta_{p^2}\right|_{p=\{\mu_0,m_0,A_t,B_0,M_{1/2}\}}$, $\quad \Delta_{p^2}\equiv\frac{\partial\ln v^2}{\partial\ln p^2}\sim 10$, [Barbieri, Strumia]

$$\Delta_{\mu_0^2}=\frac{-2\mu_0^2\,\sigma_8^2}{(1+\delta)\,m_Z^2}+\mathcal{O}(\cot\beta),\quad \text{similar for } \Delta^2_{m_0},\Delta_{M^2_{1/2}},\cdots\qquad \delta\sim\mathcal{O}(1)$$

Δ:



MSSM: m_h

MSSM fine-tuning $\Delta=\Delta(m_h)$

$\tan\beta\equiv\frac{v_2}{v_1}<10$, $0.1\leq\mu_0,\ m_0,\ M_{1/2}<1$ TeV.

$\Delta>30.$ $m_h>120$ GeV;

$\Delta=100\Rightarrow 1\%$ fine-tuning initial parameters

$\Rightarrow$ Quantum corrections also reduce fine-tuning Δ.

$\Delta\sim M^2_{susy}\sim M_{\tilde t^2}\sim\exp(m_h/GeV)$

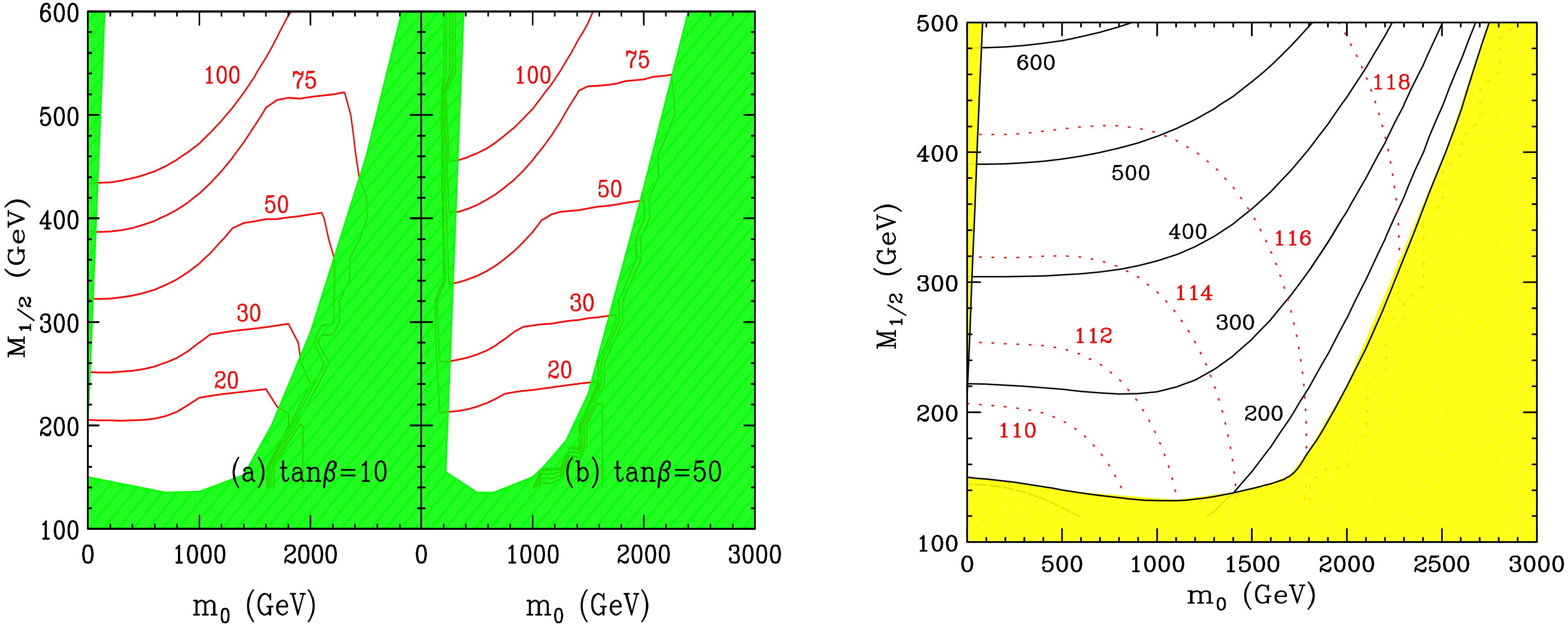


Figure 1:

$M_{1/2}$ versus m_0 with fine-tuning contours Δ (red)

$M_{1/2}$ versus m_0 for different m_h (red) and μ_0 (black)

[Moroi, Feng, Matchev] in hep-ph/9908309,9909334.

- smaller $M_{1/2}$ and smaller μ preferred.
- such values decrease fine-tuning Δ (and somewhat upset T_{eff}, see later).

- "New physics" beyond MSSM may reduce fine-tuning further.

  This is possible by incresing λ, which in MSSM is "anomalously" small.

- "New Physics'," parametrised by higher dimensional operators, could reduce Δ: [G. Ross, S. Cassel, D.G.]

- in MSSM + $d=5$ operators: $$\Delta\mathcal{L} = \frac{1}{M}\int d^2\theta\,(1+c_0\,m_0\,\theta\theta)(H_1.H_2)^2$$

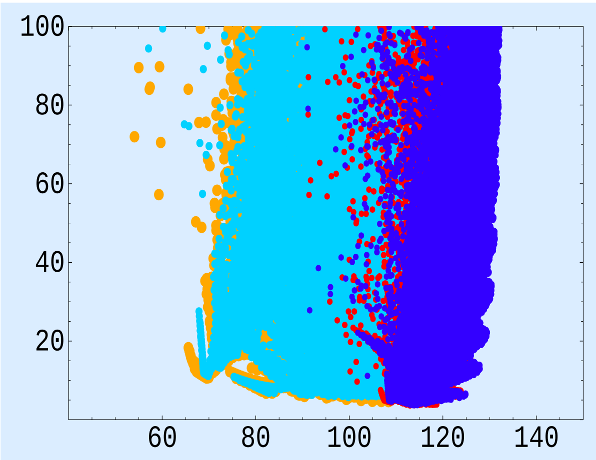
- this can be generated by: new massive gauge singlet $m \sim M$. Also $d=6$ operators ($j=1,2$)

$$\Delta\mathcal{L} = \int d^4\theta\,\mathcal{Z}_j\,(H_j^\dagger\,e^V\,H_j)^2 + \mathcal{Z}_3\,(H_1^\dagger\,e^V\,H_1)(H_2^\dagger\,e^V\,H_2) + \mathcal{Z}_{3+j}\,(H_j^\dagger\,e^V\,H_j)(H_2.H_1) + \Big[W^\alpha\,W_\alpha\,(H_2.H_1)\,\delta^2(\overline{\theta}) + h.c.\Big] + \cdots$$

$$V = V_{tree}^{MSSM} + \frac{1}{8}\delta\,|\,h_2\,|^4 + \big(|\,h_1|^2 + |\,h_2\,|^2\big)\,\big(\eta\;h_1.h_2 + h.c.\big) + \frac{1}{2}\,[\,\eta'\,(h_1.h_2)^2 + h.c.],\qquad \eta \equiv \frac{2\,\mu_0}{M},\;\; \eta' \equiv -\frac{2\,c_0\,m_0}{M}$$

$$v^2 = -\frac{m^2}{\lambda},\qquad m^2 \equiv \tilde{m}_1^2\,\cos^2\beta + \tilde{m}_2^2\,\sin^2\beta - m_3^2\,\sin 2\beta,\qquad \lambda \equiv \frac{g^2}{8}\,\cos^2 2\beta + \frac{1}{8}\,\delta\,\sin^4\beta + \eta\,\sin 2\beta + \frac{\eta'}{4}\,\sin^2 2\beta$$

Δ:



MSSM: m_h

MSSM fine-tuning reduced by "new physics" ($d=5$ op's)

$\tan\beta \equiv \frac{v_2}{v_1} < 10$, 0.1 TeV $\leq \mu_0,\, m_0,\, M_{1/2} < 1$ TeV.

$\Delta < 10$, for $114.4 \leq m_h < 130$ GeV;

$\eta = 0.07 \Rightarrow M \approx 2\,\mu_0/\eta \sim (30\,\text{to}\,65) \times \mu_0 \sim 10$ TeV.

• II. RG flow of gauge couplings in 4d N=1 susy theories, NSVZ beta function & applications to MSSM unification

Consider the Lagrangian for pure $N=1$ susy theory $SU(N)$. At scale M, this is

$$\mathcal{L}_h(V_h) = \frac{1}{16\,\kappa}\int d^2\theta\,\frac{1}{g_h^2(M)}\,\mathrm{Tr}\,\left[W^A(V_h)W_A(V_h)\right] + h.c. = \frac{1}{4g_c^2(M)}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{D^2}{2} + i\,\lambda\sigma^\mu\,\partial_\mu\,\overline{\lambda}\right] + \frac{\tilde{\theta}}{128\kappa\pi^2}\mathrm{Tr}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

This is the "holomorphic" (or Wilsonian) definition. g_h: holomorphic coupling. $V_h = V_h^a T^a$, vector superfield.

$$W_A(V_h) \;=\; -\frac{1}{4}\overline{D}\overline{D}e^{-V_h}D_A e^{V_h},\,.$$

g_h contains an Im part: the "theta" term in the action $(\tilde{\theta}F^a_{\mu\nu}\tilde{F}^{a\mu\nu})$

$$\frac{1}{g_h^2} = \frac{1}{g_c^2} + i\frac{\tilde{\theta}}{8\pi^2}$$

g_c: canonical (effective, measured) gauge coupling. Canonical formulation $\mathcal{L}_c(V_c, g_c)$:

$$\mathcal{L}_c(V_c) = \frac{1}{16}\int d^2\theta\left(\frac{1}{g_c^2} + i\frac{\tilde{\theta}}{8\pi^2}\right)\mathrm{Tr}\left[W^A(2g_cV_c)W_A(2g_cV_c)\right] + h.c.$$

If we change the cut-off from M to M', g_h changes to [Murayama, Arkani-Hamed]

$$\frac{8\pi^2}{g_h'^2} = \frac{8\pi^2}{g_h^2} + \mathcal{F}\left(\frac{8\pi^2}{g_h^2}, t \equiv \ln\frac{M}{M'}\right)$$

To preserve the holomorphicity in $g_h'^2$ of $\mathcal{L}_h(M')$, $\mathcal{F}$ must be holomorphic in g_h^2, continuous in t and $\mathcal{F}(1/g_h^2, 0) = 0$. Also

$$\mathcal{F}\left(\frac{8\pi^2}{g_h^2} + 2\pi i, t\right) = \mathcal{F}\left(\frac{8\pi^2}{g_h^2}, t\right)$$

For an infinitesimal change of scale: (M close to M'):

$$\frac{d}{dt}\left(\frac{8\pi^2}{g_h^2}\right) = \beta\left(\frac{8\pi^2}{g_h^2}\right), \qquad \beta\left(\frac{8\pi^2}{g_h^2} + 2\pi i\right) = \beta\left(\frac{8\pi^2}{g_h^2}\right)$$

β function periodic, can be Fourier expanded: $\Rightarrow \quad \beta\left(\frac{8\pi^2}{g_h^2}\right) = \sum_n b_n \, e^{-n\, 8\pi^2/g_h^2}$

$n < 0 \Rightarrow \beta = \infty$ (even in weakly coupled limit), $n > 0$ not possible (pertubatively). $\Rightarrow n = 0 \Rightarrow \beta =$ constant.

$$\Rightarrow \quad \frac{8\pi^2}{g_h'^2} = \frac{8\pi^2}{g_h^2} + b_0 \ln\frac{M'}{M}, \qquad \Rightarrow \qquad g_h \;\; \text{runs at one loop only}$$

• What does this mean for the canonical gauge coupling? In canonical normalisation

$$\mathcal{L}_c(V_c) = \frac{1}{16}\int d^2\theta \left(\frac{1}{g_c^2} + i\frac{\tilde{\theta}}{8\pi^2}\right) Tr\left[W^A(2g_cV_c)W_A(2g_cV_c)\right] + h.c.$$

- need $V_h \to 2g_cV_c$ for standard normalisation of gauge -kinetic terms: $\quad -\frac{1}{4\,g_c^2}F^a_{\mu\nu}F^{a\,\mu\nu} \to -\frac{1}{4}F_{a\,\mu\nu}F^{a\,\mu\nu}$

- in $V_h \to 2g_cV_c$, the measure of the $\int$ in partition function $\mathcal{Z}$ changes.

- Rescaling (Konishi) anomaly [Murayama, Arkani-Hamed, hep-th/9707133]

$$\mathcal{Z} = \int \mathcal{D}(V_h)\exp\mathcal{L}_h(V_h) = \int \mathcal{D}(2\,g_cV_c)\exp\mathcal{L}_c(V_c), \qquad \mathcal{D}(2g_cV_c) \neq \mathcal{D}(V_c), \qquad \mathcal{D}V_c = \mathcal{D}\lambda\,\mathcal{D}\overline{\lambda}\,\mathcal{D}A\mathcal{D}D$$

$$\mathcal{D}(2g_cV_c) \;=\; \mathcal{D}(V_c)\times\exp\left\{\frac{1}{16}\int d^4x d^2\theta\,\frac{2T(G)}{8\pi^2}\ln(2g_c)\,Tr\left[W^A(2g_cV_c)W_A(2g_cV_c)\right] + h.c.\right\}$$

where $T(G)$ is the Dynkin index for the adjoint representation for G.

Therefore

$$\mathcal{Z} = \int \mathcal{D}V_h \exp\left\{-\frac{1}{16}\int d^4x\, d^2\theta\, \frac{1}{g_h^2}\,\mathrm{Tr}\left[W^A(V_h)W_A(V_h)\right] + h.c.\right\}$$
$$= \int \mathcal{D}(2g_cV_c)\exp\left\{-\frac{1}{16}\int d^4x\, d^2\theta \frac{1}{g_h^2}\,\mathrm{Tr}\left[W^A(2g_cV_c)W_A(2g_cV_c)\right] + h.c.\right\}$$
$$= \int \mathcal{D}(V_c)\exp\left\{\frac{-1}{16}\int d^4x\, d^2\theta\,\left[\frac{1}{g_h^2} - \frac{2T(G)}{8\pi^2}\ln(2g_c)\right]\,\mathrm{Tr}\left[W^A(2g_cV_c)W_A(2g_cV_c)\right] + h.c.\right\}$$

For a canonical normalisation for the vector multiplet, the coefficient of Tr must be of form $\frac{1}{g_c^2} + i\frac{\tilde{\theta}}{8\pi^2}$

its real part defines $1/g_c^2$, so

$$\frac{1}{g_c^2} = Re\left(\frac{1}{g_h^2}\right) - \frac{2T(G)}{8\pi^2}\ln(2g_c) \qquad \text{but } g_h \text{ runs at 1-loop:} \qquad \frac{8\pi^2}{g_h'^2} = \frac{8\pi^2}{g_h^2} + b_0\ln\frac{M'}{M}$$

we find

$$\frac{1}{g_c'^2} + \frac{2T(G)}{8\pi^2}\ln(2g_c') = \frac{1}{g_c^2} + \frac{2T(G)}{8\pi^2}\ln(2g_c) + \frac{-3T(G)}{8\pi^2}\ln\frac{M}{M'}$$

⇒ for an infinitesimal change of M: "Novikov-Shifman-Vainshtein-Zakharov" β function

$$\beta(\alpha)_{NSVZ} \equiv \frac{d\,\alpha}{d(\ln\ M)} = -\frac{\alpha^2}{2\pi}\frac{3T(G)}{(1-T(G)\,\alpha/(2\pi))}, \qquad \alpha \equiv \frac{g_c^2}{(4\pi)}, \qquad T(G) = N,\ SU(N)$$

• Including matter fields:

Last eq can be generalised to include matter fields: the superpotential $\mathcal{W}$ and the matter kinetic term L_K:

• At the scale M

$$\mathcal{L}^M = \sum_i \int d^2\theta d^2\overline{\theta} \left(\Phi_i^\dagger e^{V_{h,i}} \Phi_i\right) + \left\{ \int d^2\theta \, \mathcal{W}(\Phi_1, \Phi_2, ...) + \frac{1}{16} \int d^2\theta \frac{1}{g_h^2(M)} \operatorname{Tr}\left(W^A(V_h) W_A(V_h)\right) + h.c. \right\}$$

• At the scale M'

$$\mathcal{L}^{M'} = \sum_i Z_i(M, M') \int d^2\theta \, d^2\overline{\theta} \left(\Phi_i^\dagger \, e^{V_{h,i}} \, \Phi_i\right) + \left\{ \int d^2\theta \, \mathcal{W}(\Phi_1, \Phi_2, ...) + h.c. \right\}$$
$$+ \frac{1}{16} \int d^2\theta \left[\frac{1}{g_h^2(M')} + \frac{b_o}{8\pi^2} \ln \frac{M}{M'}\right] \operatorname{Tr}\left(W^A(V_h) W_A(V_h)\right) + h.c. \Big\}$$

with notation

$$b_o \equiv -3T(G) + \sum_i T(R_i)$$

- matter kinetic term (non-holom.) receives $Z(M, M', g_c)$. For canonically normalised kinetic terms of Φ

$$\Phi = Z_i(M, M')^{-1/2} \, \Phi_i'$$

- Rescaling anomaly of a chiral multiplet, (Konishi, Shizuya) see [Murayama, Arkani-Hamed]

$$\mathcal{D}\Phi \to \mathcal{D}(Z^{-1/2}\Phi'_i) \;=\; \mathcal{D}(\Phi'_i)\times \exp\left\{-\frac{1}{16}\int d^4y\, d^2\theta\, \frac{-T_2(R_i)}{8\pi^2}\ln Z_i(M,M')\, Tr\left[W^A W_A\right] + h.c.\right\}$$

Then $\mathcal{L}'$ becomes:

$$\mathcal{L}^{M'} \;=\; \sum_i \int d^2\theta\, d^2\overline{\theta}\, \left(\Phi_i'^{\dagger}\, e^{V_{h,i}}\, \Phi'_i\right) + \left\{\int d^2\theta\, \mathcal{W}\left(Z_1^{-1/2}\Phi'_1,\; Z_2^{-1/2}\Phi'_2, \ldots\right) + \int d^2\theta\, \frac{1}{16\, g_h'^2}\,\mathrm{Tr}\left[W^A(V_h)W_A(V_h)\right] + h.c.\right\}$$

Notation:

$$\frac{1}{g_h'^2} \equiv \frac{1}{g_h^2(M)} + \frac{b_o}{8\pi^2}\ln\frac{M}{M'} - \sum_i \frac{T(R_i)}{8\pi^2}\ln Z_i(M,M'),$$

Rescale $V_h \to 2g_c V_c$ for canonical gauge kinetic term. We found this brings:

$$\frac{1}{g_c^2(M')} = \mathrm{Re}\left[\frac{1}{g_h'^2}\right] - \frac{2T(G)}{8\pi^2}\ln(2g_c(M'))$$

$$\frac{1}{g_c^2(M')} = \mathrm{Re}\frac{1}{g_h^2(M)} + \frac{b_o}{8\pi^2}\ln\frac{M}{M'} - \frac{2T(G)}{8\pi^2}\ln(2g_c(M')) - \sum_i \frac{T(R_i)}{8\pi^2}\ln Z_i(M,M')$$

$\Rightarrow$ The link of g_c^2 with holomorphic coupling g_h^2 to all orders. Useful for the link with string theory & loop corrections.

Again: $$\frac{1}{g_c^2(M')} = Re\frac{1}{g_h^2(M)} + \frac{b_o}{8\pi^2}\ln\frac{M}{M'} - \frac{2T(G)}{8\pi^2}\ln(2g_c(M')) - \sum_i \frac{T(R_i)}{8\pi^2}\ln Z_i(M,M')$$

Write it at $M' \to \Lambda$ and subtract::

$$\frac{1}{g_c^2(M')} = \frac{1}{g_c^2(\Lambda)} + \frac{b_o}{8\pi^2}\ln\frac{\Lambda}{M'} + \frac{2T(G)}{8\pi^2}\ln\frac{g_c(\Lambda)}{g_c(M')} - \sum_i \frac{T(R_i)}{8\pi^2}\ln Z_i(\Lambda,M')$$

- RG flow of canonical coupling to all orders in perturbation theory.
- Red: one loop; ($Z = 1$, $g_c(\Lambda) = g_c(M')$). Brown: 2-loop and beyond, from re-scaling anomaly cancellation

$$\beta(\alpha)_{NSVZ} \equiv \frac{d\alpha}{d(\ln M')} = -\frac{\alpha^2}{2\pi}\frac{3T(G) - \sum_i T(R_i)(1-2\gamma_i)}{(1 - T(G)\alpha/(2\pi))}, \qquad \alpha \equiv \frac{g_c^2}{4\pi}, \qquad \gamma_i \equiv -\frac{1}{2}\,\frac{d\ln Z_i}{d\Big(\ln M'\Big)},$$

- Novikov-Shifman-Vainshtein-Zakharov β function with matter fields; valid to all orders in perturbation theory.

- γ_i (anomalous dimensions) only known perturbatively, order by order ... (sum over i over matter fields in rep. R_i).

- can be generalised to $G_1 \times G_2 \times \cdots$ gauge group. Apply it to MSSM.

• Application to MSSM: Two-loop RGE with a one-loop ....work. "a": gauge group index: $SU(3) \times SU(2)_L \times U(1)_Y$

$$\beta(\alpha_a)_{NSVZ} \equiv \frac{d\alpha_a}{d(\ln\mu)} = -\frac{\alpha_a^2}{2\pi}\,\frac{3T_a(G) - \sum_\psi T_a(R_\psi)\,(1-2\gamma_\psi)}{(1 - T_a(G)\,\alpha_a/(2\pi))}$$

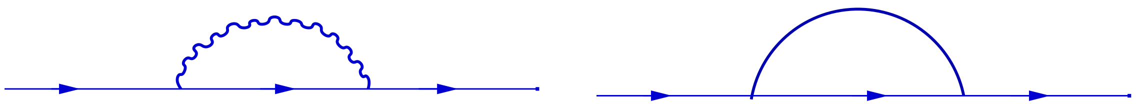
Expand the denominator of $\beta(\alpha_a)$ $\Rightarrow$ at two loop:

$$\frac{d\alpha_a}{d\ln\mu} = \frac{\alpha_a}{2\pi}\left\{-3\,T_a(G) + \sum_\psi T_a(R_\psi)\right\} - \left\{\frac{\alpha_a}{2\pi}\sum_\psi 2\,\gamma_\psi\,T_a(R_\psi) + \frac{\alpha_a^2}{4\pi^2}\,T_a(G)\,\Big(3T_a(G) - \sum_\psi T_a(R_\psi)\Big)\right\} + \mathcal{O}(\alpha^3)$$

Red: one-loop; Blue: two-loop! γ_ψ depend on gauge groups $SU(3) \times SU(2) \times U(1)$ and on Yukawa ($\mathcal{W}$).

One loop γ_ψ: gauge and Yukawa ($\mathcal{W}$)

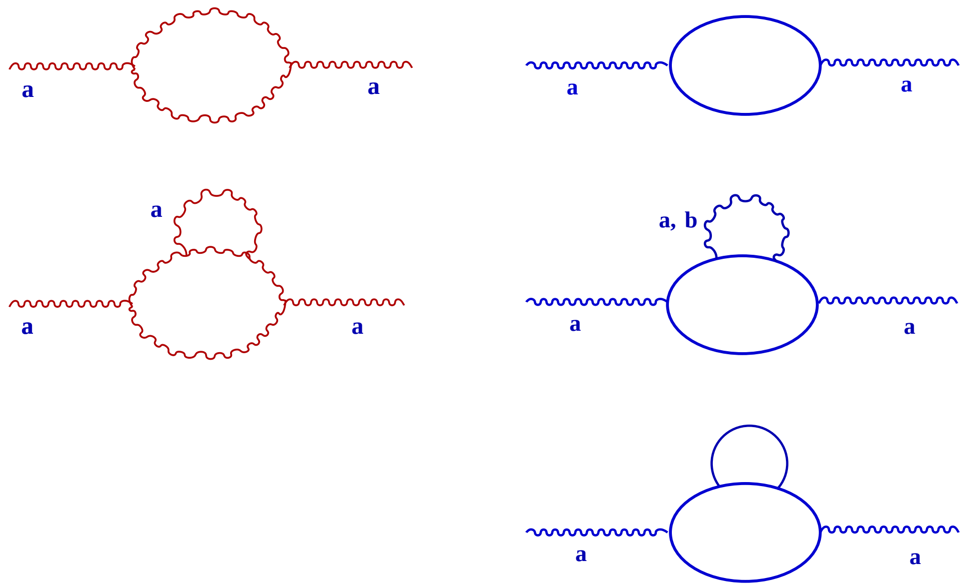
$$\gamma_\psi = -\frac{1}{2}\frac{d(\ln Z_\psi)}{d(\ln\mu)}, \qquad \gamma_\psi = -\frac{1}{4\pi}\left\{2\sum_b \alpha_b\,C_b(R_\psi) - \sum_{j=states} A_j(\psi)\,y_j\right\} + \mathcal{O}(\alpha^2)$$



• one-loop γ_ψ $\Rightarrow$ two-loop RGE for α's! RG flow of gauge couplings $\leftrightarrow$ (rescaling) anomaly cancellation.

- Diagramatic interpretation of RG flow: MSSM case. ("a": gauge greoup index)

$$\frac{1}{\alpha_a(\mu)} = -\delta_a + \frac{1}{\alpha_a(\Lambda)} + \frac{-3T(G_a)}{2\pi} \ln \frac{\Lambda}{\mu \left[\frac{\alpha_a(\Lambda)}{\alpha_a(\mu)}\right]^{1/3}} + \sum_{\psi} \frac{T(R_{\psi,a})}{8\pi^2} \ln \frac{\Lambda}{\mu \, Z_i(\Lambda, \mu)}$$



- Unification conditions:

$$g_{c,a}(\Lambda) = g_c(\Lambda) \quad \text{for any a :} \qquad SU(3), SU(2), U(1), \cdots$$

$$Z_i(\Lambda, \Lambda) = 1$$

- if $\mu = M_Z$ or $\mu < M_{susy}$: $\delta_a \neq 0$ accounts for susy breaking $\mu \rightarrow M_{susy}$

• Low-energy Supersymmetric Thresholds

$$\delta_a = \frac{1}{2\pi} \sum_\xi \beta_{a,\xi} \ln \frac{M_\xi}{M_z}, \qquad \delta_a = \delta_a^S + \delta_a^G + \delta_a^H$$

δ_a for $U(1)$, $SU(2)$ and $SU(3)$ respectively:

Squarks+sleptons:

$$\delta_1^S = \frac{1}{2\pi} \sum_{gen} \left[\frac{3}{10} \ln \frac{M_{\tilde{L}}}{M_z} + \frac{4}{5} \ln \frac{M_{\tilde{u}^c}}{M_z} + \frac{1}{5} \ln \frac{M_{\tilde{d}^c}}{M_z} + \frac{3}{5} \ln \frac{M_{\tilde{e}^c}}{M_z} + \frac{1}{10} \ln \frac{M_{\tilde{Q}_L}}{M_z} \right]$$

$$\delta_2^S = \frac{1}{2\pi} \sum_{gen} \left[\frac{1}{2} \ln \frac{M_{\tilde{L}}}{M_z} + \frac{3}{2} \ln \frac{M_{\tilde{Q}_L}}{M_z} \right]$$

$$\delta_3^S = \frac{1}{2\pi} \sum_{gen} \left[\frac{1}{2} \ln \frac{M_{\tilde{u}^c}}{M_z} + \frac{1}{2} \ln \frac{M_{\tilde{d}^c}}{M_z} + \ln \frac{M_{\tilde{Q}_L}}{M_z} \right]$$

Gauginos $U(1)$

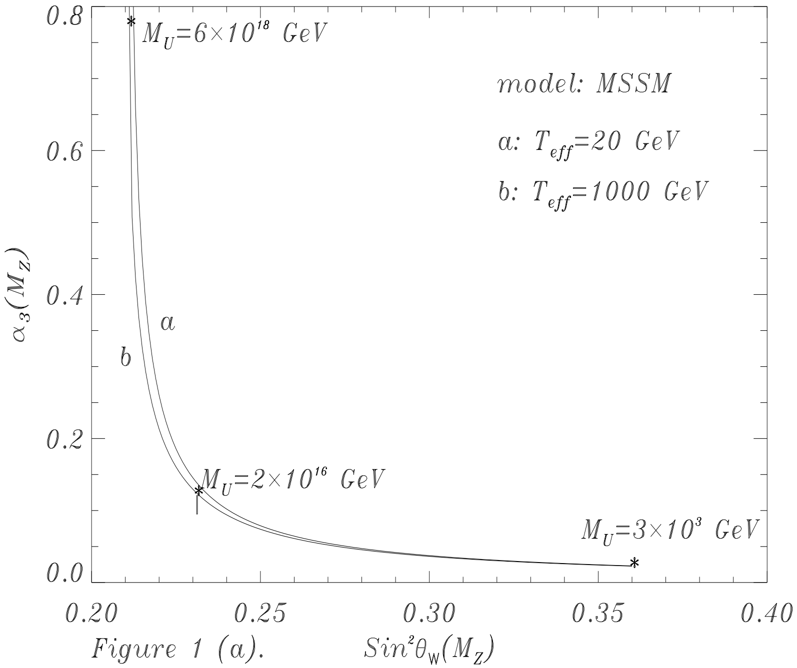
$$\delta_1^G = 0, \qquad \delta_2^G = \frac{1}{2\pi} \frac{4}{3} \ln \frac{\tilde{M}_2}{M_z}, \qquad \delta_3^G = \frac{1}{2\pi} 2 \ln \frac{\tilde{M}_3}{M_z}$$

Higgs+Higgsino:

$$\delta_1^H = \frac{1}{2\pi} \left[\frac{2}{5} \ln \frac{M_{\tilde{H}_{u,d}}}{M_z} + \frac{1}{10} \ln \frac{M_{H_2}}{M_z} \right]$$

$$\delta_2^H = \frac{1}{2\pi} \left[\frac{2}{3} \ln \frac{M_{\tilde{H}_{u,d}}}{M_z} + \frac{1}{6} \ln \frac{M_{H_2}}{M_z} \right], \qquad \delta_3^H = 0$$

• Unification at two-loop level:



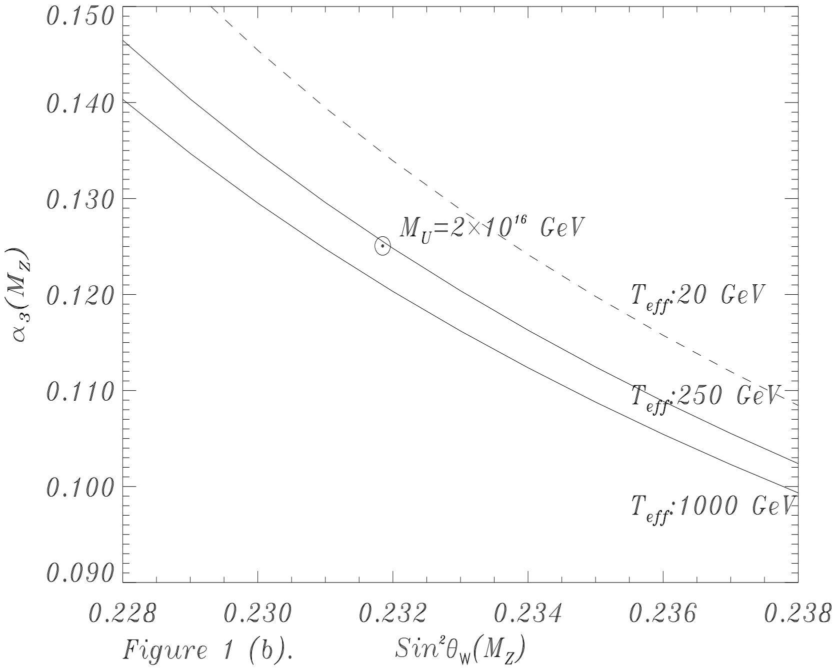


Figure 2:

$$\alpha_{1,2}(M_Z) \to \sin\theta_W, \alpha_{em}(M_Z), \quad \alpha_3^{-1}(M_Z) = \left\{\frac{15}{7}\sin^2\theta_W(M_Z) - \frac{3}{7}\right\}\alpha_{em}^{-1}(M_Z) + \frac{1}{2\pi}\frac{19}{14}\ln\frac{T_{eff}}{M_Z} + (two - loop)$$

$$T_{eff} = m_{\tilde{H}}\left(\frac{m_{\widetilde{W}}}{m_{\tilde{g}}}\right)^{\frac{28}{19}}\left(\frac{m_H}{m_{\tilde{H}}}\right)^{\frac{3}{19}}\left(\frac{m_{\widetilde{W}}}{m_{\tilde{H}}}\right)^{\frac{4}{19}}\left(\frac{m_{\tilde{l}}}{m_{\tilde{q}}}\right)^{\frac{9}{19}}, \qquad T_{eff} \simeq m_{\tilde{H}}\left(\alpha_2(M_Z)/\alpha_3(M_Z)\right)^2 \simeq |\mu|/12.$$

- M_U unification scale: $M_U \approx 2.8 \times 10^{16} \times \sqrt{4\pi\alpha_U}$ GeV.
- M_S string scale, w.heterotic: $M_S \approx 5.27 \times 10^{17} \times \sqrt{4\pi\alpha_U}$ GeV. . Larger: $m_{\tilde{W}}$, $m_{\tilde{l}}, m_{\tilde{H}}, \mu$ preferred.

• Conclusions:

- With the LHC about to start operating, this is a great time to do high energy physics!

- Supersymmetry is getting ....old, we better discover it soon!