Renormalizability of noncommutative quantum field theory (NCQFT)

ADRIAN TANASĂ

Măgurele, October 2009

Plan

- Introduction and motivations

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- Noncommutative renormalization (UV/IR mixing); the Grosse-Wulkenhaar solution

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- Conclusions

Renormalizability

how to chose between all possible Lagrangians and geometries?

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what should be Ariadne's thread in this labyrinth?

we propose to use *renormalizability*

renormalizable theories - generic building blocks of physics

"Renormalization at all orders is at the heart of field theory"

C. Itzkinson and J.-B. Zuber, *"QFT"*, 1980

Noncommutativity

ORDER DOES MATTER !

noncommutativity of space-time: $[x^\mu, x^\nu] \neq 0$

H. S. Snyder, *Phys. Rev.*, '47

possible framework for the quantization of gravity
non-commutativity of space-time at (below) the Planck scale ?

(S. Doplicher *et. al.*, *Phys. Lett.* **B**, '94)

effective theories for string theory or matrix models

(A. Connes *et. al. JHEP*, '98, M. Douglas and C. Hull, *JHEP*, '98)

$\hookrightarrow$ *Seiberg-Witten map* (N. Seiberg and E. Witten, *JHEP*, '99)

relation with loop quantum gravity

(L. Freidel and E. Livine, *Phys. Rev. Lett.*, '06, E. Young *et. al.*, *J. Math. Phys.*, '09)

non-local effective interactions - *fractional quantum Hall effect*

(A. P. Polychronakos *JHEP*, '01, L. Susskind, hep-th/0101029)

Φ^4 model:

$$\mathcal{S} = \int d^4x \, [\frac{1}{2}\partial_\mu \Phi \star \partial^\mu \Phi + \frac{1}{2} m^2 \Phi \star \Phi + \frac{\lambda}{4!} \Phi \star \Phi \star \Phi \star \Phi],$$

$\star$ - Moyal product
(non-local, noncommutative, associative product)

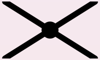
$\int d^4x \, (\Phi \star \Phi)(x) = \int d^4x \, \Phi(x) \, \Phi(x)$
(same propagation)

Implications of the use of the Moyal product in QFT

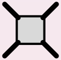
interaction term (in position space)

$$\int d^4x \Phi^{\star 4}(x) \propto \int \prod_{i=1}^{4} d^4x_i \Phi(x_i) \delta(x_1 - x_2 + x_3 - x_4)$$

$$e^{2i \sum_{1 \leq i < j \leq 4} (-1)^{i+j+1} x_i \Theta^{-1} x_j}$$



$\rightarrow$



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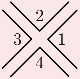
$\hookrightarrow$ non-locality

$\hookrightarrow$ restricted invariance: only under cyclic permutation

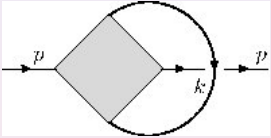


$\longrightarrow$ ribbon graphs

$\longrightarrow$ clear distinction between planar and non-planar graphs

Renormalization on the Moyal space

UV/IR mixing (S. Minwalla *et. al.*, *JHEP*, '00)



$$\lambda \int d^4k \frac{e^{ik_\mu \Theta^{\mu\nu} p_\nu}}{k^2 + m^2} \to_{|p|\to 0} \frac{1}{\theta^2 p^2}$$

same type of behavior at any order in perturbation theory

J. Magnen, V. Rivasseau and A. T., *Europhys. Lett.* '09

$\longrightarrow$ non-renormalizability!

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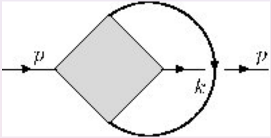

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A first solution to this problem - the Grosse-Wulkenhaar model

additional harmonic term

(H. Grosse and R. Wulkenhaar, *Comm. Math. Phys.*, '05)

$$s[\phi(x)] = \int d^4x \left(\frac{1}{2}\partial_\mu \phi \star \partial^\mu \phi + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) \star (\tilde{x}^\mu \phi) + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right),$$

$$\tilde{x}_\mu = 2(\Theta^{-1})_{\mu\nu} x^\nu .$$

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$$\tilde{x}_\mu = 2(\Theta^{-1})_{\mu\nu} x^\nu .$$

modification of the propagator - the model becomes renormalizable

- most of the techniques of QFT extend to Grosse-Wulkenhaar-like models:
  - the parametric representation

    (R. Gurău and V. Rivasseau, *Commun. Math. Phys.*, '07, A. T. and V. Rivasseau, *Commun. Math. Phys.*, '08, A. T., *J. Phys. Conf. Series*, '08, A. T., solicited by de *Modern Encyclopedia Math. Phys.*)

    (algebraic geometric properties P. Aluffi and M. Marcolli, 0807.1690[math-ph])
  - the Mellin representation

    (R. Gurău, A. Malbouisson, V. Rivasseau and A. T., *Lett. Math. Phys.*, '07)
  - dimensional regularization

    (R. Gurău and A. T., *Annales H. Poincaré*, '08)
  - the Connes-Kreimer Hopf algebra structure of renormalization

    (A. T. and F. Vignes-Tourneret, *J. Noncomm. Geom.*, '08)
- study of vacuum configurations (A. de Goursac, A. T. and J-C. Wallet, *EPJ* **C**, '08)
- gauge model propositions

  $\hookrightarrow$ vacuum state highly non-trivial

  $\longrightarrow$ perturbation theory cumbersome ...

(A. de Goursac, J-C. Wallet and R. Wulkenhaar *EPJ* **C**, '07,'08, H. Grosse and M. Wohelegant *EPJ* **C**, '07)

Translation-invariant renormalizable scalar model

(R. Gurău, J. Magen, V. Rivasseau and A. T., *Commun. Math. Phys.* '09)

the Grosse-Wulkenhaar model breaks translation-invariance !

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the complete propagator:

$$C(p,m,\theta) = \frac{1}{p^2 + a\frac{1}{\theta^2 p^2} + m^2}$$

arbitrary planar irregular 2-point function: same type of $\frac{1}{p^2}$ behavior !

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$\hookrightarrow$ other modification of the action:

$$S = \int d^4p\, [\frac{1}{2} p_\mu \phi \star p^\mu \phi + \frac{1}{2} a \frac{1}{\theta^2 p^2} \phi \star \phi + \frac{1}{2} m^2 \phi \star \phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi].$$

renormalizability at any order in perturbation theory !

NCQFT vs. CMB G. Palma and S. Patil arXiv:0906.4727 [hep-th]

Gribov-Zwanziger result D. Dudal *et. al.*, *Phys. Rev.* **D** '08

Translation-invariant NCQFT techniques

- parametric representation; relation with topologic ribbon graph polynomial

  (A. T., *J. Phys.* **A** '09, T. Krajewski, V. Rivasseau, A. T. and Z. Wang, *J. Noncomm. Geom.* (in press))

- renormalization group flow

  (J. Ben Geloun and A. T., *Lett. Math. Phys.* '08)

- commutative limit

  (J. Magnen, V. Rivasseau and A. T., *Lett. Math. Phys.* '08)

- Connes-Kreimer Hopf algebra structure of renormalization; Hochschild cohomology and combinatoric Dyson-Schwinger equations

  (A. T. and D. Kreimer, arXiv:0907.2182 [hep-th], submitted to *J. Noncomm. Geom.*)

Conclusions and perspectives

- implementation of the Polchinski flow equation method

  $\hookrightarrow$ work in progress with C. Kopper and J. Magnen

- renormalizable gauge theories ? trivial vacuum state

  (D. Blaschke *et. al., J. Phys.***A**, '08, *Europ. Phys. J.* **C**, '09, *Europ. Phys. Lett.* '09, 0908.0467 [hep-th], 0908.1743 [hep-th], L. Vilar *et. al.* 0902.2956 [hep-th])

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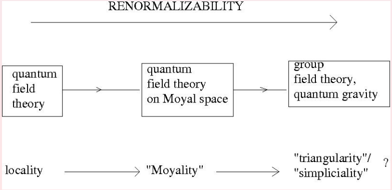
- generalization to any translation-invariant $\star$-product (*ex.:* the Wick-Voros product)

  $\hookrightarrow$ work in progress with P. Vitale

- applications of these techniques for the renormalizability study of quantum gravity models A. T., 0909.5631 [gr-qc], submitted to *Class. Quant. Grav.*



Thank you for your attention!

Glimpse of the mathematical setup

the Moyal space

The *Moyal algebra* is the linear space of smooth and rapidly decreasing functions $\mathcal{S}(\mathbb{R}^{\mathcal{D}})$ equipped with the

$$(f \star g)(x) = \int \frac{d^D k}{(2\pi)^D} d^D y \, f(x + \frac{1}{2}\Theta \cdot k) g(x+y) e^{ik\cdot y}.$$

$\star$ - Moyal product
(non-local, noncommutative, associative product)

$$[x^\mu, x^\nu]_\star = i\Theta^{\mu\nu}, \tag{2}$$

$$\Theta = \begin{pmatrix} \Theta_2 & 0 \\ 0 & \Theta_2 \end{pmatrix}, \ \ \Theta_2 = \begin{pmatrix} 0 & -\theta \\ \theta & 0 \end{pmatrix}.$$

Scales - renormalization group

definition of the RG scales:

- locus where $C^{-1}(p)$ is big
- locus where $C^{-1}(p)$ is low

$$C^{-1}_{\mathrm{comm}}(p) = p^2$$

$$C^{-1}_{GW} = p^2 + \Omega^2 x^2$$

$$C^{-1}(p) = p^2 + \frac{a}{\theta^2 p^2}$$

mixing of the UV and IR scales - key of the renormalization

BPHZ renormalization scheme
renormalization conditions

$$\Gamma^4(0,0,0,0) = -\lambda_r,\ \ G^2(0,0) = \frac{1}{m^2},\ \ \frac{\partial}{\partial p^2} G^2(p,-p)|_{p=0} = -\frac{1}{m^4}. \quad (3)$$

where Γ^4 and G^2 are the connected functions and
$0 \to p_m$ (the minimum of $p^2 + \frac{a}{\theta^2 p^2}$)

"The amount of theoretical work one has to cover before being able to solve
problems of real practical value is rather large, but this circumstance is [...] likely to become more pronounced in the theoretical physics of the future."
P.A.M. Dirac, *"The principles of Quantum Mechanics"*, 1930

Thank you for your attention

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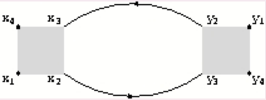
$$(f \star g)(x) = \int \frac{d^D k}{(2\pi)^D} d^D y \, f(x + \frac{1}{2}\Theta \cdot k) g(x+y) e^{ik\cdot y}$$

power counting:

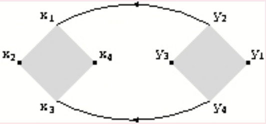
$$\omega = 4g + \frac{N-4}{2} + (B-1)$$

B - number of broken faces

improved factor in the broken faces
example:



$$n = 2,\ L = 2,\ F = 2,\ B = 1$$



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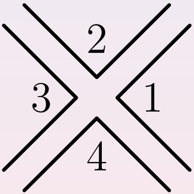
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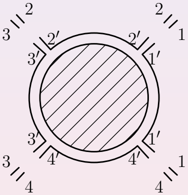
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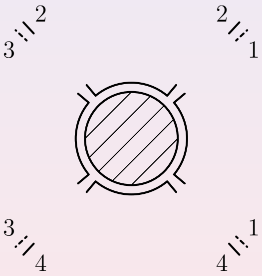

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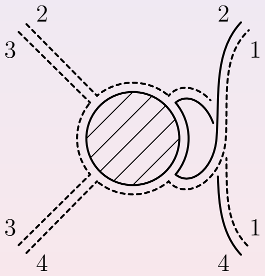

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Implementation of the Polchinski flow equation method

work in progress with J. Magnen and C. Kopper

$\hookrightarrow$ proposed regularized propagator

$$C^{\Lambda,\Lambda_0}(p) = p^2 \frac{1}{p^4 + m^2 p^2 + \frac{a}{\theta^2}} \left(e^{-\frac{p^4+m^2+\frac{a}{\theta^2}}{\Lambda_0^4}} - e^{-\frac{p^4+m^2+\frac{a}{\theta^2}}{\Lambda_0^4}} \right),$$

$$0 \leq \Lambda \leq \Lambda_0$$

$\hookrightarrow$ different type of bounds

$\hookrightarrow$ different initial conditions

$\hookrightarrow$ the Feynman amplitudes are complex $\rightarrow$ bounds on the module

$\hookrightarrow$ the bounds on the Feynman amplitudes depend on the external momenta

etc.

From Euclid to Minkowski space by Wick rotation

analytic continuation:

$$\begin{array}{rcl} p_E^0 & \longrightarrow & -\imath p^0, \\ \theta^{0i} & \longrightarrow & -\imath\theta^{0i}. \end{array}$$

the NC phase remains invariant

$\Rightarrow$ Euclidiean momenta replaced by Minkowskian momenta

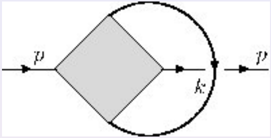
"usual" commutative propagator:

$$\frac{1}{p^2 - m^2 + \imath\varepsilon}$$

action

$$S[\phi(p)] = \int d^4p \left(\frac{1}{2}p_\mu\phi p^\mu\phi - \frac{1}{2}m^2\phi^2 + V^\star[\phi]\right).$$

J. Gomis et T. Mehen, *Nucl. Phys.* **B**, '00



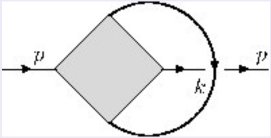
$$\int d^4k \frac{e^{\imath k_\mu \Theta^{\mu\nu} p_\nu}}{k^2 - m^2 + \imath\epsilon}$$

the propagator: $\frac{1}{k^2-m^2+\imath\epsilon} = \int_0^\infty d\alpha e^{\imath\alpha(k^2-m^2)-\varepsilon\alpha}$
allows to perform the Gaussian integration on k

$$\int_0^\infty \frac{d\alpha}{\alpha^2} e^{i\frac{\theta^2}{4\alpha}\hat{p}^2} e^{-\imath\alpha m^2} e^{-\epsilon\alpha}$$

$\hat{p}^2 = p_0^2 - p_1^2 + p_2^2 + p_3^2$
divergence:

$$\frac{\text{const}}{\theta^2\hat{p}^2}$$



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