Unitarity and Global Fits in Flavour Physics

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Outline

- Introduction
- Unitarity conditions
  - Standard unitarity triangle approach
  - Novel approach
- Their comparison
- A new type of fit
- Results & Conclusion

Introduction

Fundamental Tool in Flavour Physics = KM matrix

$$U = \begin{bmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{bmatrix}$$

U supposed unitary $\Longrightarrow U \times U^* = I_3$

$*$ means Hermitian conjugate, I_3 is the 3-dim. unit matrix

$$\sum_{j=d,s,b} |U_{ij}|^2 = 1,\ i = u, c, t \quad (1)$$

Introduction

$$\sum_{j=d,s,b} U_{uj}\bar{U}_{cj} = 0, \quad \sum_{j=d,s,b} U_{uj}\bar{U}_{tj} = 0, \quad \sum_{i=u,c,t} U_{id}\bar{U}_{ib} = 0 \qquad (2)$$

Above relations generate triangles

$$U = \begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$s_{12} = \sin\theta_{12}$, $c_{12} = \cos\theta_{12}$, etc. Four independent parameters: δ, s_{ij}, ij=12,13,23

Unitarity conditions

Standard unitarity triangle

Notation $|U_{ij}| = V_{ij}$

$$s_{12} = \lambda = \frac{V_{us}}{\sqrt{V_{ud}^2 + V_{us}^2}}, \; s_{23} = A\lambda^2 = \lambda \frac{V_{cb}}{V_{us}} \tag{3}$$

$$s_{13} e^{i\delta} = \bar{U}_{ub} = A\lambda^3 (\rho + i\eta) \tag{4}$$

Unitarity conditions

Standard unitarity triangle approach

$(3) + (4) \implies$

$$\rho = \frac{s_{13}}{A\lambda^3}\cos\delta = \frac{s_{13}}{s_{12}s_{23}}\cos\delta, \eta = \frac{s_{13}}{s_{12}s_{23}}\sin\delta \qquad (5)$$

$$\tan\delta = \frac{\eta}{\rho}\, , \rho^2 + \eta^2 = \left(\frac{s_{13}}{s_{12}s_{23}}\right)^2 = \left(\frac{V_{ub}(1 - V_{ub}^2)}{V_{us}V_{cb}}\right)^2 \qquad (6)$$

(6) given by M. Schmidtler and K. R. Schubert, Z.Phys. C **53**, 347 (1992)

U matrix in Wolfenstein approx.

$$U =$$

$$\begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2-\lambda^4(1+4A^2)/8 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2+A\lambda^4(1/2-(\rho+i\eta)) & 1-A^2\lambda^4/2 \end{bmatrix}$$

Marina Artuso, Elisabeta Barberio and Sheldon Stone, PMC Physics A, **3**, 3 (2009)

KM matrix determination

a. KM matrix experimental determination $\Longrightarrow$

- SM validation

- and/or new physics beyond it

a. is not easy! two reasons:

b. theoretical:

b1. s_{ij} mathematical parameters that do not enter physical quantities
b2. U present form not rephaising invariant

Novel approach

Jarlskog's solution: use only inv. quantities, $V_{ij} = |U_{ij}|$;
C.Jarlskog, Z. Phys. C **29**, 491 (1985), Phys. Rev. D **35**, 1685 (1987)

c. experimental: V_{ij} cannot be directly measured

simplest case: leptonic decays: $P \to \ell\overline{\nu_l}$, provide numbers for $V_{ij}\, f_P$

semileptonic decays:

$H \to P\ell\overline{\nu_l}$, provide numbers for $V_{ij}|f_+(q^2)|$,

Concl.: 1. V_{ij} are natural parameters entering data

Novel approach

Concl.: 2. from such simple relations one can't find two parameters!

Real problem: Use V_{ij} but you have no unitarity!!

Consistency Problem $\equiv$ what are the necessary and sufficient conditions on the set V_{ij} to represent moduli of a unitary matrix

Problem solved: P. Diţă, J. Math. Phys. **47**, 083510 (2006)

Novel approach

$$V_{ud}^2 = c_{12}^2 c_{13}^2,\ V_{us}^2 = s_{12}^2 c_{13}^2,\ V_{ub}^2 = s_{13}^2,$$
$$V_{cb}^2 = s_{23}^2 c_{13}^2, V_{tb}^2 = c_{13}^2 c_{23}^2,$$
$$V_{cd}^2 = s_{12}^2 c_{23}^2 + s_{13}^2 s_{23}^2 c_{12}^2 + 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos\delta$$
$$V_{cs}^2 = c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos\delta$$
$$V_{td}^2 = s_{13}^2 c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 - 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos\delta$$
$$V_{ts}^2 = s_{12}^2 s_{13}^2 c_{23}^2 + c_{12}^2 s_{23}^2 + 2 s_{12} s_{13} s_{23} c_{12} c_{23} \cos\delta$$

From any four independent moduli we can compute s_{ij} and δ

Unitarity conditions $\Longrightarrow$ $s_{ij} \in (0,1)$ and $\cos\delta \in (-1,1)$

Novel approach

Example: $V_{us} = a,\ V_{ub} = b,\ V_{cb} = c$;

$$s_{13} = V_{ub} = b, \quad s_{12} = \frac{a}{\sqrt{1-b^2}}, \quad s_{23} = \frac{c}{\sqrt{1-b^2}}$$

choose V_{cd}

$$\cos\delta = \frac{(1-b^2)(V_{cd}^2(1-b^2)-a^2)+c^2(a^2+b^2(a^2+b^2-1))}{2abc\sqrt{1-a^2-b^2}\sqrt{1-b^2-c^2}}$$

+ three similar formulae

Comparison

$$\begin{bmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.001 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.001 & 0.97334 \pm 0.00023 & 0.0415^{+0.001}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{bmatrix}$$

$J = (3.05\,^{+0.19}_{-0.20}) \times 10^{-5}$

A. Ceccucci, Z. Ligeti and Y. Sakai, Phys.Lett. B, **667**,145-152 (2008)

Comparison

$r_1 = V_{ud}^2 + V_{us}^2 + V_{ub}^2 - 1$, etc.

$$\begin{array}{ll} r_1 = -4.65 \times 10^{-7} & c_1 = 1.79 \times 10^{-5} \\ r_2 = 8.37 \times 10^{-6} & c_2 = -1.22 \times 10^{-5} \\ r_3 = -3.71 \times 10^{-7} & c_3 = 1.89 \times 10^{-6} \end{array}$$

Jarlskog invariant: $J = 2A$ where A is area of any unit. tr.

Unitarity $\Longrightarrow J^2 \geq 0$

$l_1 = V_{ud}V_{us},\ l_2 = V_{cd}V_{cs},\ l_3 = V_{td}V_{ts},\ \ p = (l_1 + l_2 + l_3)/2$

$J_1^c = 2\sqrt{p(p - l_1)(p - l_2)(p - l_3)}$

Comparison

$$J_1^c = 4.55 \times 10^{-5} \quad J_2^c = 3.05 \times 10^{-5} \quad J_3^c = 1.55 \times 10^{-5}$$
$$J_1^r = 3.09 \times 10^{-5} \quad J_2^r = 2.53 \times 10^{-5} \quad J_3^r = 3.05 \times 10^{-5}$$

$$\langle J \rangle = (3.18 \pm 2.98) \times 10^{-5}$$

Using all 165 $\cos\delta$ formulae, I got

$$\langle \cos\delta \rangle = 0.468 - 0.005\, i, \quad \sigma_{\langle \cos\delta \rangle} = 0.2454 + 0.0096\, i$$

Comparison

Notation: $V_{ud} = a = \frac{97419}{10^5}$, $V_{cd} = d = \frac{2256}{10^4}$,
$V_{cb} = f = \frac{415}{10^4}$, $V_{ts} = h = \frac{405}{10^4}$

$$\begin{bmatrix} a & \sqrt{d^2+f^2-h^2} & \sqrt{1-a^2-d^2-f^2+h^2} \\ d & \sqrt{1-d^2-f^2} & f \\ \sqrt{1-a^2-d^2} & h & a^2+d^2-h^2 \end{bmatrix}$$

$V_{ub} = \sqrt{1-a^2-d^2-f^2+h^2} = \frac{\sqrt{72761}}{10^5} i \approx 2.7 \times 10^{-3}\, i$

$V_{ub}^{CLS} = (3.59 \pm 0.16) \times 10^{-3}$

$DS(V_{ud}, V_{cd}, V_{cs}, V_{ts})$ leads to $\cos\delta = 1.573$

Comparison. Another example

Case of Fourier matrix: $Vij = \frac{1}{\sqrt{3}}$ $\Longrightarrow$

Standard unitarity triangle gives

$$\rho = \frac{1}{2},\ \eta = \frac{\sqrt{3}}{2}, \tan\delta = \sqrt{3},\ \delta = \frac{\pi}{3} = 60^\circ$$

$$s_{13} = \frac{1}{\sqrt{3}},\ s_{12} = s_{23} = \frac{1}{\sqrt{2}}$$

$$V_{ud}^2 = V_{us}^2 = V_{ub}^2 = V_{cb}^2 = V_{tb}^2 = \frac{1}{3}$$

$$V_{cd}^2 = V_{ts}^2 = \frac{1}{3} + \frac{\sqrt{3}}{12},\ \ V_{cs}^2 = V_{td}^2 = \frac{1}{3} - \frac{\sqrt{3}}{12}$$

true δ value: $\delta = 90^\circ$

New Fit

Concl.: 1. $\cos\delta^{(i)} \approx \cos\delta^{(j)},\ i \neq j,\ \text{all}\ \cos\delta^{(i)} \in (-1,1)$

2. χ^2 has two components: unitarity +experiment

$$\chi_1^2 = \sum_{j=u,c,t}\left(\sum_{i=d,s,b} V_{ji}^2 - 1\right)^2 + \sum_{j=d,s,b}\left(\sum_{i=u,c,t} V_{ij}^2 - 1\right)^2 +$$

$$\sum_{i<j}(\cos\delta^{(i)} - \cos\delta^{(j)})^2, \quad -1 \leq \cos\delta^{(i)} \leq 1$$

New Fit. Experimental Data

$$\mathcal{B}(P \to l\bar{\nu}_l) = \frac{G_F^2 M_P m_l^2}{8\pi\hbar}\left(1 - \frac{m_l^2}{M_P^2}\right)^2 f_P^2 V_{kl}^2 \tau_P$$

$$\frac{d\,\Gamma(H \to P\,\ell\,\nu_\ell)}{dq^2} = \frac{G_F^2\,V_{kl}^2}{192\pi^3 M_H^3}\lambda^{3/2}(q^2)|f_+(q^2)|^2$$

$$\chi_2^2 = \sum\nolimits_i \left(\frac{d_i - \widetilde{d}_i}{\sigma_i}\right)^2, \quad \chi^2 = \chi_1^2 + \chi_2^2, \quad d_i = |f_+(q_i^2)|V_{kl}$$

Results

Param.	Centr. Val. $\pm\ \sigma$	Param.	Centr. Val. $\pm\ \sigma$
f_π	131.131 $\pm$ 1.522	$f_+^{B\pi}(0)$	214.9 $\pm$ 13.4
f_K	154.97 $\pm$ 2.17	$f_+^{D\pi}(0)$	653.2 $\pm$ 19.1
f_K/f_π	1.1818 $\pm$ 0.0042	$f_+^{DK}(0)$	751.8 $\pm$ 10.4
f_B	222.8 $\pm$ 25.0	$f_+^{DK}(0)/f_+^{D\pi}(0)$	1.171 $\pm$ 0.04
f_D	207.6 $\pm$9.8	$\mathcal{F}(1)$	957.5 $\pm$ 57.7
f_{D_s}	271.0 $\pm$ 18.0	$\mathcal{G}(1)$	1,125.3 $\pm$ 40.7
$f_+^{K\pi}(0)$	955.34 $\pm$ 9.27	λ	-1.2686 $\pm$ 0.0057

Table 1: Fit results for $f_P, \ \mathrm{and}\ f_+(0)$, units in MeV

Results

Bin	$f_+^\pi(q^2)$ $(\pi^0 e^+ \nu_e)$	$f_+^\pi(q^2)$ $(\pi^- e^+ \nu_e)$
1	0.6895 $\pm$ 0.0576	0.7072 $\pm$ 0.0355
2	0.7514 $\pm$ 0.0709	0.7735 $\pm$ 0.0400
3	0.8442 $\pm$ 0.0841	0.7956 $\pm$ 0.0488
4	0.8926 $\pm$ 0.0974	0.9812 $\pm$ 0.0532
5	1.1005 $\pm$ 0.115	1.017 $\pm$ 0.065
6	1.3083 $\pm$ 0.151	1.101 $\pm$ 0.084
7	1.5789 $\pm$ 0.221	1.635 $\pm$ 0.111
8	-	1.759 $\pm$ 0.235
9	-	2.024 $\pm$ 0.30

Table 2: $|f_+(q^2)|$ FF from $D \to \pi l \nu$ decays

Results

Bin	$f_+^K(q^2)$ $(K^- e^+ \nu_e)$	$f_+^K(q^2)$ $(\bar{K}^0 e^+ \nu_e)$
1	0.7798 ± 0.0136	0.7808 ± 0.0208
2	0.8281 ± 0.0146	0.8096 ± 0.0239
3	0.8435 ± 0.0156	0.8466 ± 0.0259
4	0.9113 ± 0.0198	0.8856 ± 0.0289
5	0.9945 ± 0.0229	0.9175 ± 0.0331
6	1.007 ± 0.027	1.024 ± 0.039
7	1.128 ± 0.033	1.149 ± 0.048
8	1.212 ± 0.042	1.090 ± 0.062
9	1.303 ± 0.066	1.233 ± 0.092
10	1.561 ± 0.165	1.476 ± 0.210

Table 3: $|f_+(q^2)|$ FF from $D \to K l \nu$ decays

Results

$$V_c = \begin{pmatrix} 0.974022 & 0.226415 & 0.0042512 \\ 0.226253 & 0.973323 & 0.0381075 \\ 0.0095692 & 0.0371307 & 0.999265 \end{pmatrix}$$

$$\sigma_{V_c} = \begin{pmatrix} 1.1 \times 10^{-6} & 1.9 \times 10^{-5} & 2.1 \times 10^{-5} \\ 3.6 \times 10^{-5} & 2.7 \times 10^{-4} & 3.1 \times 10^{-4} \\ 3.5 \times 10^{-5} & 2.9 \times 10^{-4} & 3.9 \times 10^{-4} \end{pmatrix}$$

Results

$$V_+ = \begin{pmatrix} 0.974021 & 0.226332 & 0.0074844 \\ 0.226114 & 0.973083 & 0.044512 \\ 0.0124317 & 0.043391 & 0.998891 \end{pmatrix}$$

$$\begin{aligned} \delta &= (89.96 \pm 0.36)^\circ, \quad \alpha = (64.59 \pm 0.27)^\circ, \\ \gamma &= (89.98 \pm 0.06)^\circ, \quad \beta = (25.49 \pm 0.28)^\circ \end{aligned}$$

V_c and $V_+ \Longrightarrow \quad f_B \in (123.5, 222.8)$

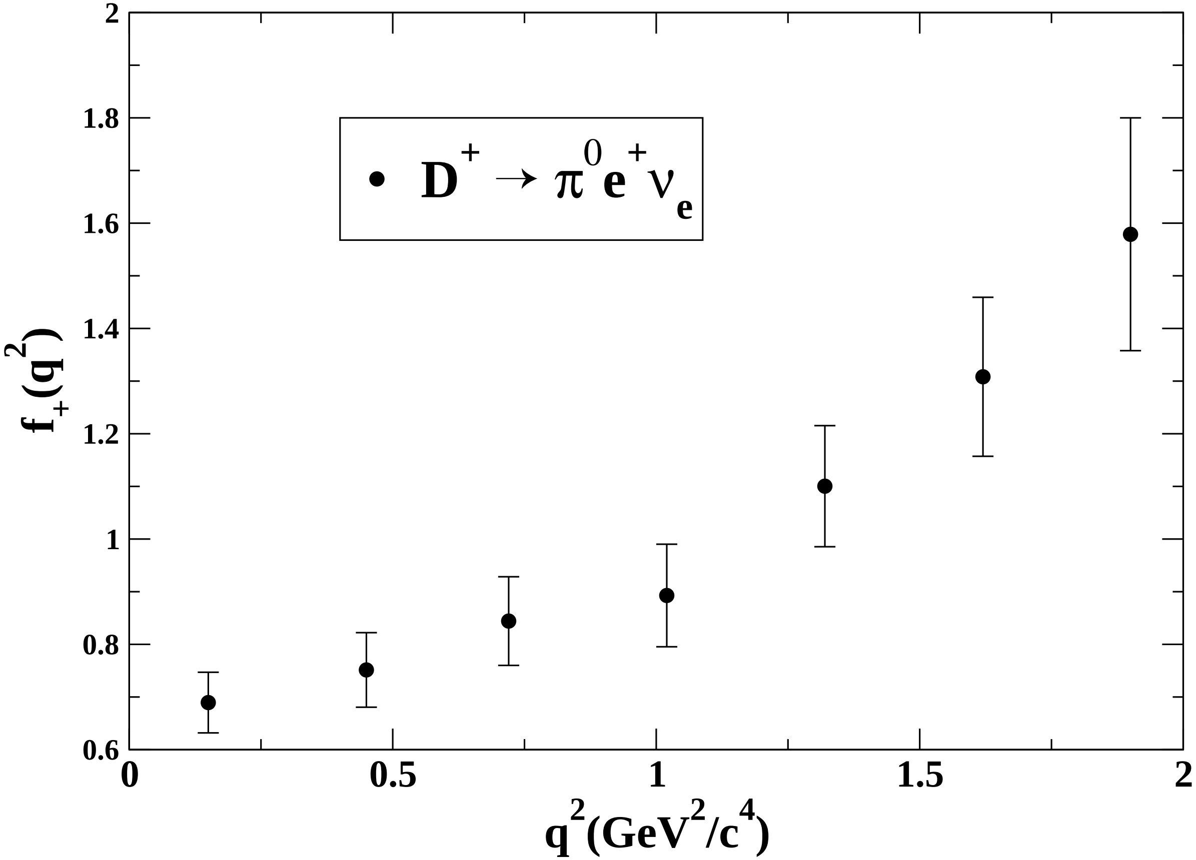
Results. Fine structure analyses

KLOE coll. F. Ambrosino et al, JHEP04, 059 (2008)
FlaviaNet Working Group arXiv:0801.1817

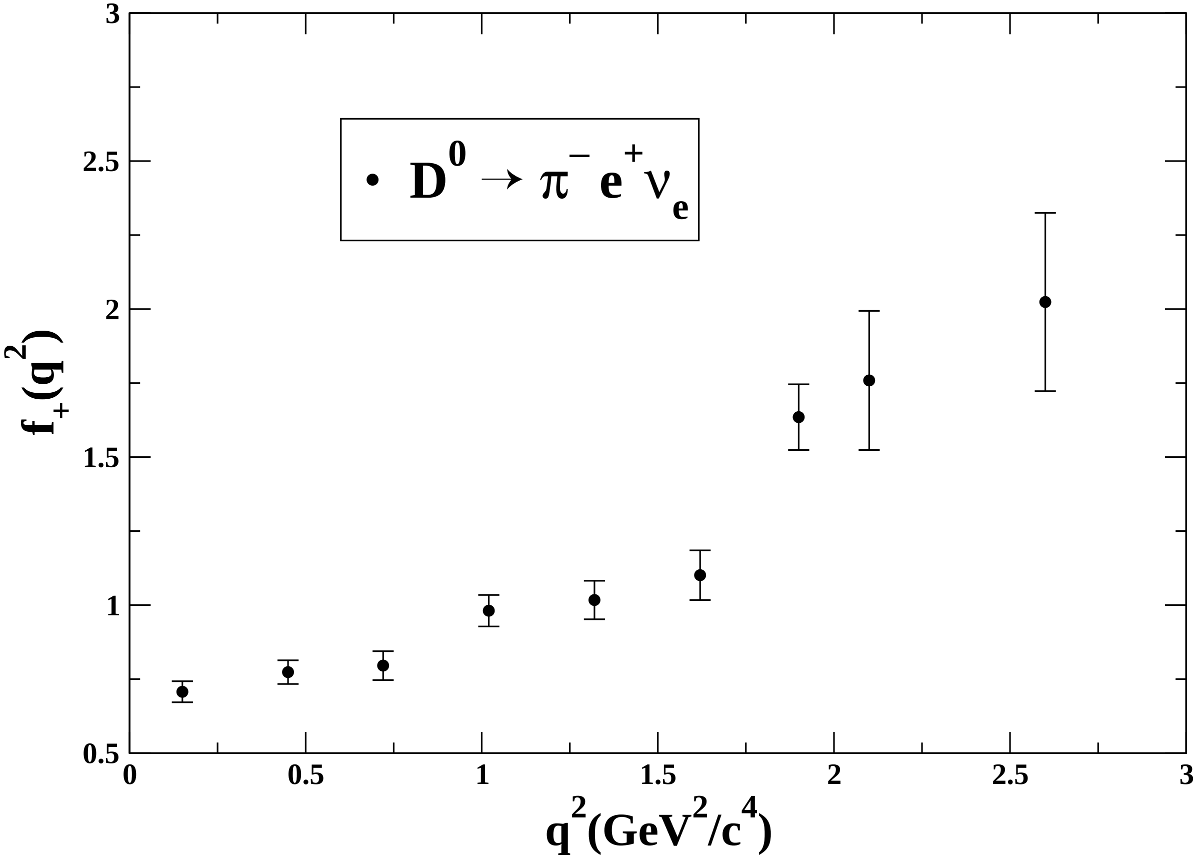
$$\begin{aligned} f_+^{\pi K}(0)_{KLOE} &= 950.38 \pm 5.56, \text{ and} \\ f_+^{\pi K}(0)_{Flavia} &= 955.06 \pm 4.31, \end{aligned}$$

From Table 1. $f_+^{\pi K}(0) = 955.34 \pm 9.27$

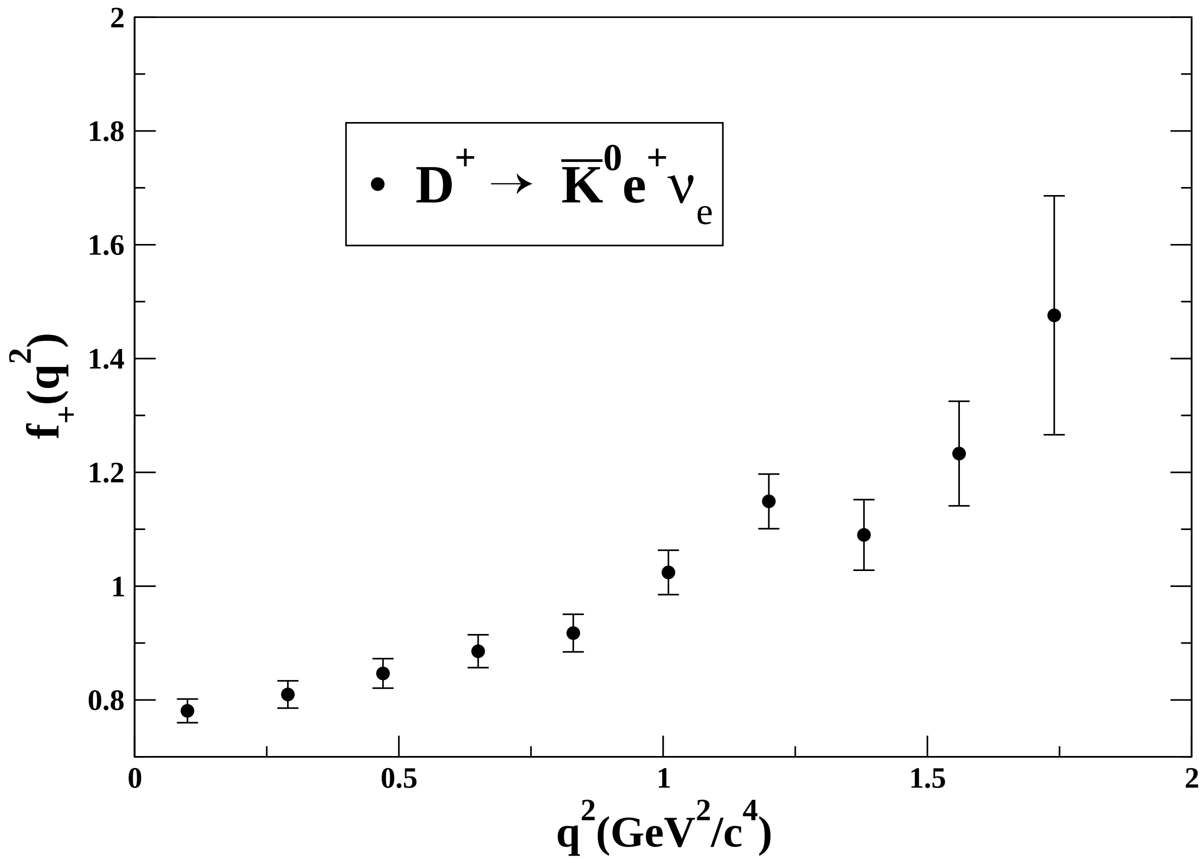
Results



Results



Results



Results