Determination of α_s from hadronic τ decays

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Introduction

Quantum ChromoDynamics at 37 years is in very good shape

- perturbative QCD
  - quantum corrections to higher orders at partonic level
  - quark and gluon distribution functions
- nonperturbative QCD
  - lattice (“ab initio” calculations)
  - effective theories: ChPT, SCET, gluon resummations ...
  - new ideas for the strong coupling regime: AdS/CFT ....

QCD: The Modern View of the Strong Interactions

Berlin, 4-9 October 2009

Introduction

- the strong coupling α_s: fundamental parameter of QCD

$$\mathcal{L}_{QCD} = \sum_{n=1}^{6} \bar{\psi}_n [i\gamma^\mu(\partial_\mu - ig_s \tfrac{\lambda^a}{2} G^a_\mu) - m_n]\psi_n - \tfrac{1}{4}\sum_{a=1}^{8} G^a_{\mu\nu} G^{\mu\nu,a}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f_{abc} G^b_\mu G^c_\nu$$

$$\alpha_s = \tfrac{g_s^2}{4\pi}, \qquad a_s = \tfrac{\alpha_s}{\pi}$$

- after renormalization: $\alpha_s(\mu^2)$ becomes scale and scheme dependent
- scale dependence (RGE): $\mu^2 da_s/d\mu^2 = \beta(a_s) = -\sum_{j\geq 0} \beta_j a_s^{j+2}$

$\beta_0 = 9/4, \;\; \beta_1 = 4$ independent of RS $n_f = 3$

$\beta_2 = 10.06, \;\; \beta_3 = 47.23$ $n_f = 3$; $\overline{\text{MS}}$ scheme

$\alpha_s(\mu^2) \sim 1/\ln\mu^2$ for $\mu^2 \to \infty$; "asymptotic freedom"

Introduction

- determination of α_s: from all types of reactions that contain gluons
  - deep inelastic ep scattering, e^+e^- collisions, $p(\bar{p})$-p collisions, τ hadronic decays, Υ decays....
  - measure α_s at various scales $\Rightarrow$ evolve by RGE to a reference point
- PDG average (2008):

$$\alpha_s(M_Z^2) = 0.1176 \pm 0.0020$$

- World Average 2009 S. Bethke. arXiv:0908.1135 [hep-ph]

$$\alpha_s(M_Z^2) = 0.1184 \pm 0.0007$$

  - impressive consistency; overall error mostly theoretical

Introduction

- Ex: τ hadronic decays:

$$R_\tau = \frac{\Gamma[\tau \to \text{hadrons} + \nu_\tau]}{\Gamma[\tau \to \mu + \bar{\nu}_\mu + \nu_\tau]} = 3.640 \pm 0.010$$ LEP, CLEO

  - $R_\tau \sim S_{EW}(1 + \delta^{(0)})$
  - $\delta^{(0)} \sim d_1 \alpha_s + d_2 \alpha_s^2 + d_3 \alpha_s^3 + d_4 \alpha_s^4 + \ldots$ QCD correction

- precise determination of α_s at a low scale ($m_\tau = 1.78\,\text{GeV}$)
  - recent calculations in perturbative QCD up to fourth order

    Baikov, Chetyrkin & Kuhn 2008
  - precise experimental data (ALEPH analysis of the full LEP data)

Introduction

- Recent analyses based on two methods:
  - Contour Improved Perturbation Theory (CIPT):

    $$\alpha_s(m_\tau^2) = 0.344 \pm 0.009 \qquad \text{Davier et al 2008}$$

  - Fixed Order Perturbation Theory (FOPT):

    $$\alpha_s(m_\tau^2) = 0.320^{+0.012}_{-0.007} \qquad \text{Beneke \& Jamin 2008}$$

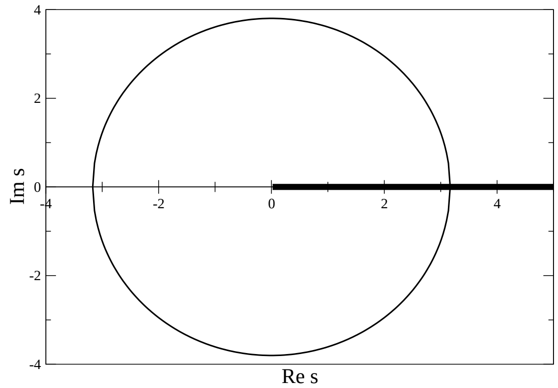
- discrepancy of 0.024 between CIPT and FOPT
- largest systematic theoretical uncertainty in $\alpha_s(m_\tau^2)$
- Aim of this talk: understand/remove this difference

Outline

- Standard CIPT and FOPT
- Higher orders in perturbative QCD
- Realistic models   Beneke & Jamin 2008
- New perturbation expansions   Caprini & Fischer 1999, 2000, 2002
- New CIPT and FOPT   Caprini & Fischer 2009
- Results and conclusions

CIPT and FOPT

- $$R_\tau \sim 1 + \delta^{(0)} = 12\pi \int_{4m_\pi^2}^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\,\Pi(s)$$
- $\Pi(s)$: polarization function $\sim i\int dx\, e^{ipx}\, \langle\Omega|\, T\{J_\mu(x)\, J_\nu(0)^\dagger\}|\Omega\rangle$
- $\Pi(s)$ analytic in the cut s-plane
- Cauchy theorem: $\Rightarrow$



$$\delta^{(0)} = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s}\, \omega(s)\, \hat{D}(s)$$

  - $\omega(s) = 1 - 2s/m_\tau^2 + 2(s/m_\tau^2)^3 - (s/m_\tau^2)^4$
  - $\hat{D}(s) = -\,s\,\frac{d}{ds}\left[\Pi(s)\right] - 1$: reduced Adler function

CIPT and FOPT

- standard perturbation series

$$\hat{D}(s) = \sum_{n\geq 1} [K_n + \kappa_n(s)] \, (a_s(\mu^2))^n, \quad n_f = 3, \; \overline{MS}$$

$$K_1 = 1, \; K_2 = 1.64, \; K_3 = 6.37, \; K_4 = 49.08, \; K_5 \sim 283$$

$$\kappa_n(s) = \sum_{k=1}^{n} \gamma_{kn} \ln^k(-s/\mu^2)$$

- choose $\mu^2 = m_\tau^2 \quad \Rightarrow$ FOPT

- choose $\mu^2 = -s \quad \Rightarrow$ CIPT (renormalization-group improved series)

$$\hat{D}(s) = \sum_{n\geq 1} K_n \, (a_s(-s))^n$$

  - expanding $\alpha_s(-s) = \sum c_k(-s/m_\tau^2) \, (\alpha_s(m_\tau^2))^k \quad \Rightarrow$ FOPT

CIPT and FOPT

- $\alpha_s(m_\tau^2)$ determined from the equation $\boxed{\delta^{(0)}_{\rm theor} = \delta^{(0)}_{\rm phen}}$

$$\boxed{\delta^{(0)}_{\rm phen} = 0.2042 \pm 0.0050}$$ Beneke & Jamin 2008

- FOPT: $\delta^{(0)}_{\rm theor}$ contains explicitly $\alpha_s(m_\tau^2)$
  - large imaginary parts of $\ln^k(-s/m_\tau^2)$ for $s = m_\tau^2 e^{i(\varphi-\pi)}$ $\Rightarrow$ poor convergence
- CIPT: $\alpha_s(-s)$ in terms of $\alpha_s(m_\tau^2)$ by solving numerically the RGE
  - avoids large logs $\Rightarrow$ in principle superior
  - but fails to describe the high-order behavior of the series

High-order behavior

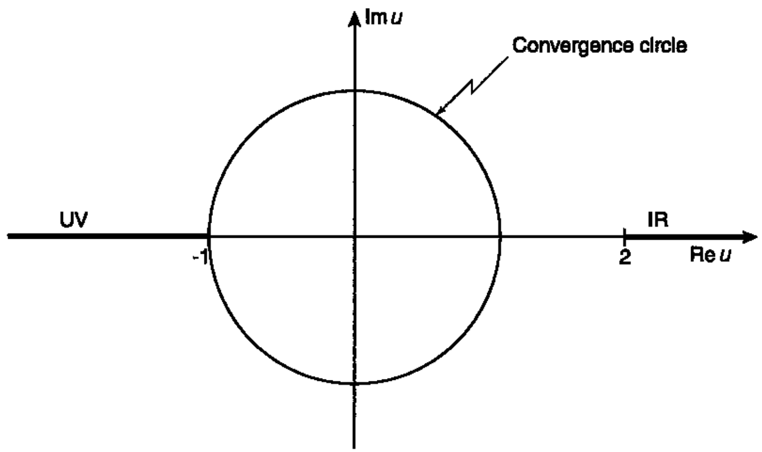
- classes of Feynman diagrams $\Rightarrow$ $K_n \sim n!$
- the perturbation series is divergent — known for QED since 1952
- the series is usually interpreted as asymptotic Dyson 1952
  - ambiguity in recovering the function from its coefficients
- A possible summation — Laplace-Borel transform

$$b_n = \frac{K_{n+1}}{\beta_0^n \, n!} \Rightarrow B(u) = \sum_{n=0}^{\infty} b_n u^n \Rightarrow \hat{D}(s) = \frac{1}{\beta_0} \int_0^{\infty} \mathrm{e}^{-u/(\beta_0 a_s(s))} B(u) \, \mathrm{d}u$$

  - if the series defining $B(u)$ has a nonzero convergence radius
  - and $B(u)$ admits an analytic continuation regular for $u > 0$
  - $\Rightarrow \hat{D}(s)$ exists and is analytic in a certain region

Borel plane in QCD

- $B(u)$ has two cuts on the real axis, for $u \leq -1$ and $u \geq 2$ (ultraviolet and infrared renormalons)



- $\sum_{n=0}^{\infty} b_n u^n$ convergences in $|u| < 1$
- near the first branch points

$$B(u) \sim \frac{r_1}{(1+u)^{\gamma_1}}, \quad B(u) \sim \frac{r_2}{(1-u/2)^{\gamma_2}}$$

- γ_1 and γ_2 known Mueller 1985, Beneke, Braun & Kivel 1997
  - Laplace-Borel integral not defined $\Rightarrow \hat{D}(s)$ not Borel summable
  - $\Rightarrow$ adopt a prescription, ex. the Principal Value

$$\hat{D}(s) = \frac{1}{\beta_0} \, \mathrm{PV} \int_0^{\infty} e^{-u/(\beta_0 a_s(s))} \, B(u) \, du$$

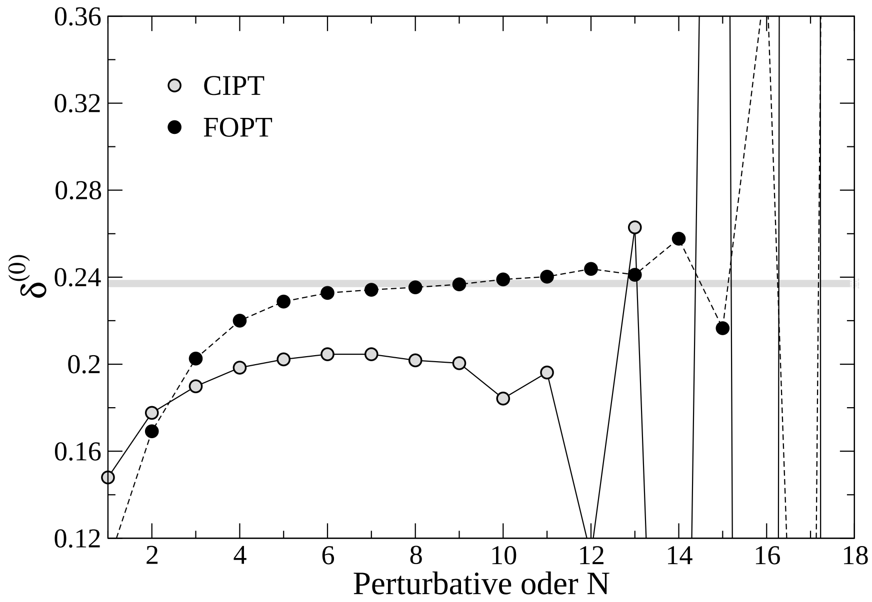
“Physical” models **Beneke & Jamin 2008**

- toy models which reproduces the low and high perturbative orders of the Adler function in massless QCD
- the “exact” Adler function is defined as:

$$\hat{D}(s) = \frac{1}{\beta_0}\,\mathrm{PV}\int_0^{\infty} e^{-u/(\beta_0 a_s(s))}\, B_{BJ}(u)\, du$$

  - $a_s(s)$ calculated exactly from RGE with β function with four Taylor terms known for QCD
  - $B_{BJ}(u)$: a parametrization consisting of UV and IR renormalons with specified branch point behaviour, multiplied by polynomials
  - parameters adjusted such as to reproduce the first 5 coefficients K_n, known from Feynman diagrams

CIPT and FOPT for the B&J models



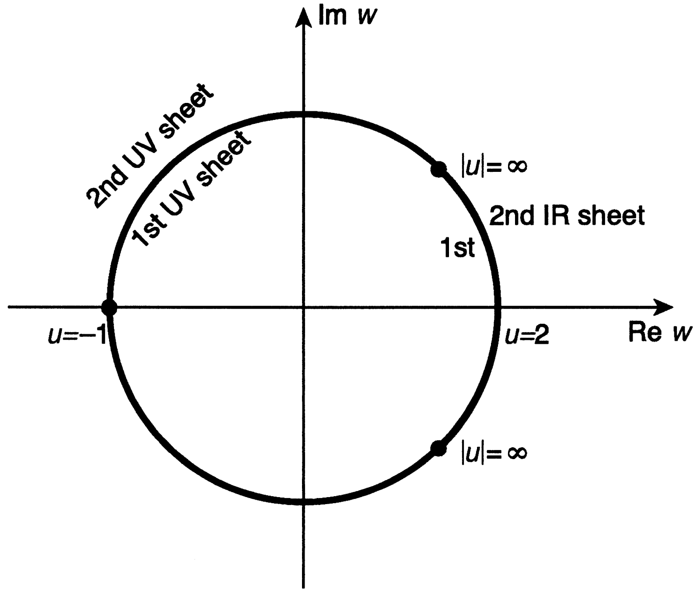
Beneke & Jamin 2008

- $\delta^{(0)}$ calculated with the standard CIPT and FOPT as a function of the order up to which the series have been summed, for $\alpha_s(m_\tau^2) = 0.34$
- horizontal band: the exact value
  - CIPT fails to approximate the true function   Beneke & Jamin 2008

Basic idea of a new expansion: analytic continuation

- the series $B(u) = \sum_{n=0}^{\infty} b_n u^n$
  - converges only in $|u| < 1$
- "Optimal" conformal mapping:

$$w(u) = \frac{\sqrt{1+u} - \sqrt{1-u/2}}{\sqrt{1+u} + \sqrt{1-u/2}}$$



  - maps the Borel cut u-plane onto $|w| < 1; \quad w(0) = 0$
- the series $B(u) = \sum_n d_n w^n$
  - converges in the whole u-plane
  - best convergence for interior points   Ciulli & Fischer 1961

New perturbative expansions

- insert $B(u) = \sum_n d_n w^n$ in the Laplace-Borel transform
- suggests the definition of the new expansion Caprini & Fischer 1999

$$\hat{D}(s) = \sum_n d_n W_n(s)$$

$$W_n(s) = \frac{1}{\beta_0} PV \int_0^\infty e^{-u/(\beta_0 a_s(s))} w^n \, du$$

- improved expansion: include the singular behaviour at the first branch-points

$$\hat{D}(s) = \sum_n c_n \mathcal{W}_n(s)$$

$$\mathcal{W}_n(s) = \frac{1}{\beta_0} PV \int_0^\infty e^{-u/(\beta_0 a_s(s))} \frac{w^n}{(1+w)^{2\gamma_1}(1-w)^{2\gamma_2}} \, du$$

Properties of the new expansion

- when reexpanded in powers of $a_s(s)$, it reproduces the coefficients K_n known from Feynman diagrams

- the expansion $W_n(s) = \sum_k \omega_{nk}(a_s(s))^k$ of each $W_n(s)$ in powers of the coupling is divergent, much like the expansion of $\hat{D}(s)$ itself

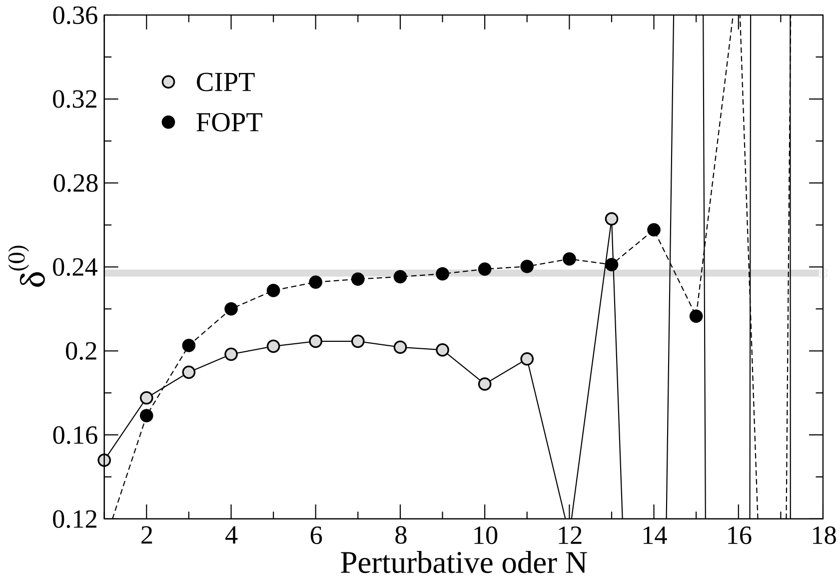
- under certain conditions, the expansion

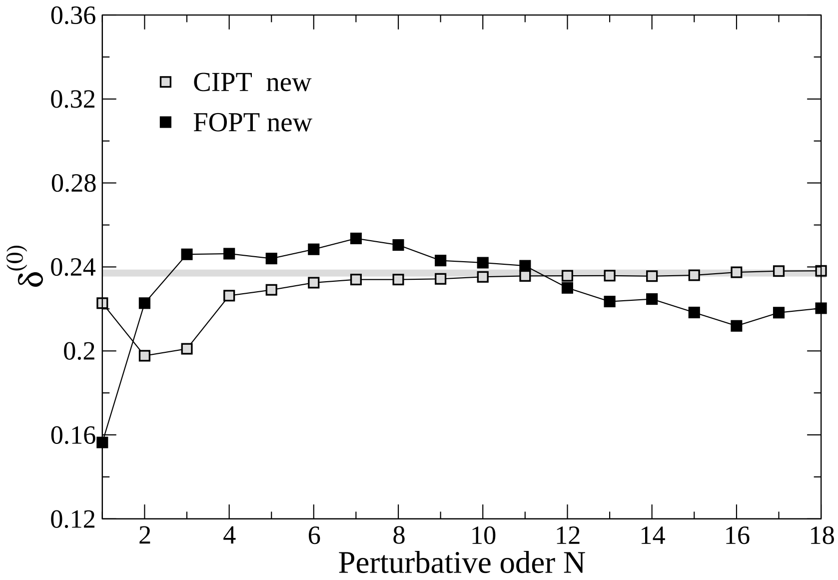
$$\hat{D}(s) = \sum_n d_n W_n(s)$$

is convergent in a domain of the energy s-plane

Caprini & Fischer 2000, 2002

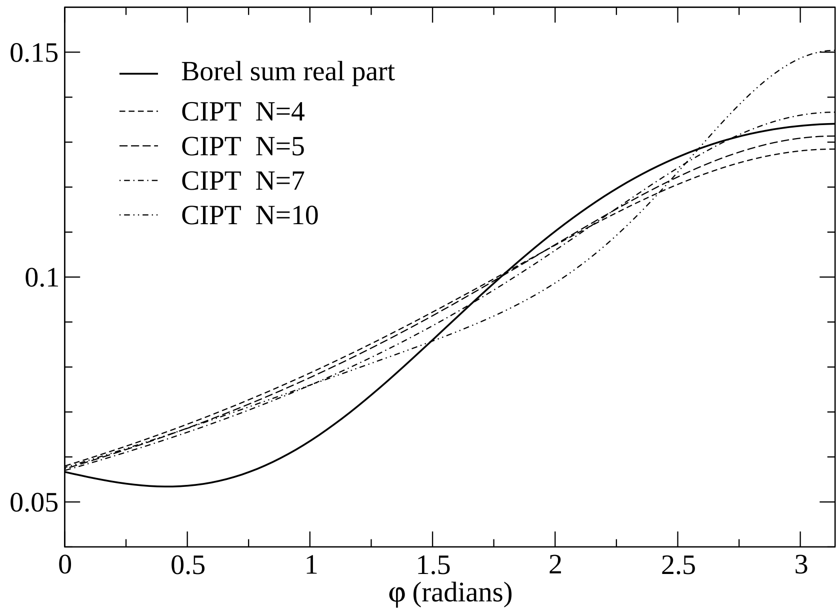
Standard and new expansions for the B&J models

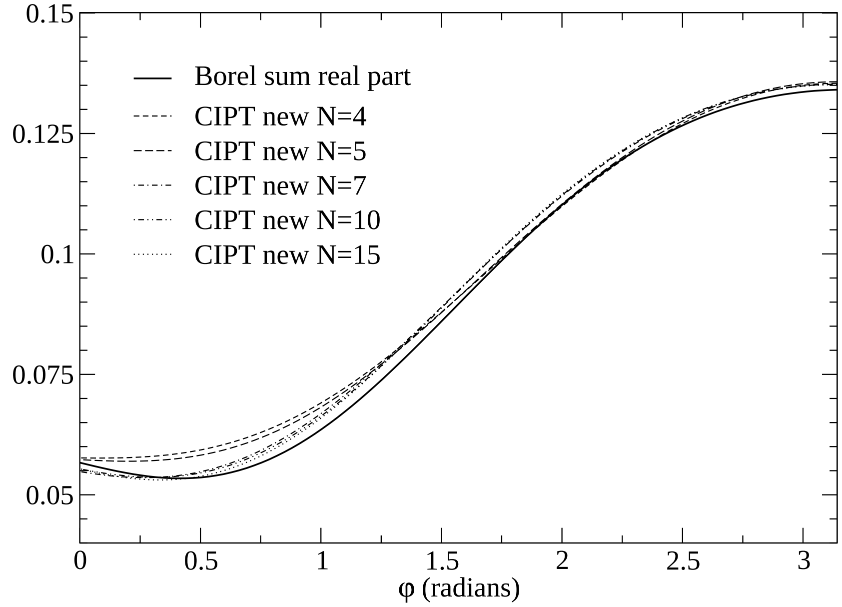




- $\delta^{(0)}$ for the standard CIPT and FOPT (left) and the new CIPT and FOPT (right), for $\alpha_s(m_\tau^2) = 0.34$
- horizontal band: the exact value

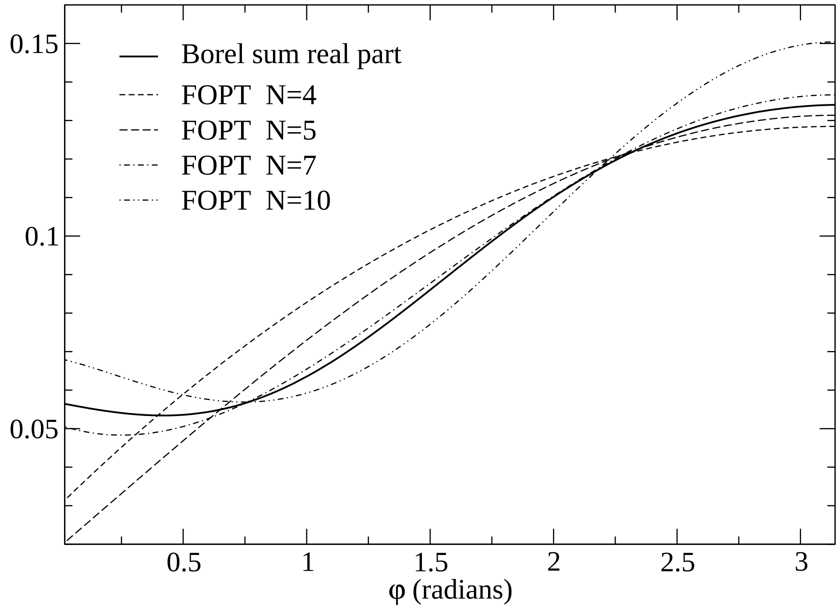
Standard and new CIPT for the B&J models

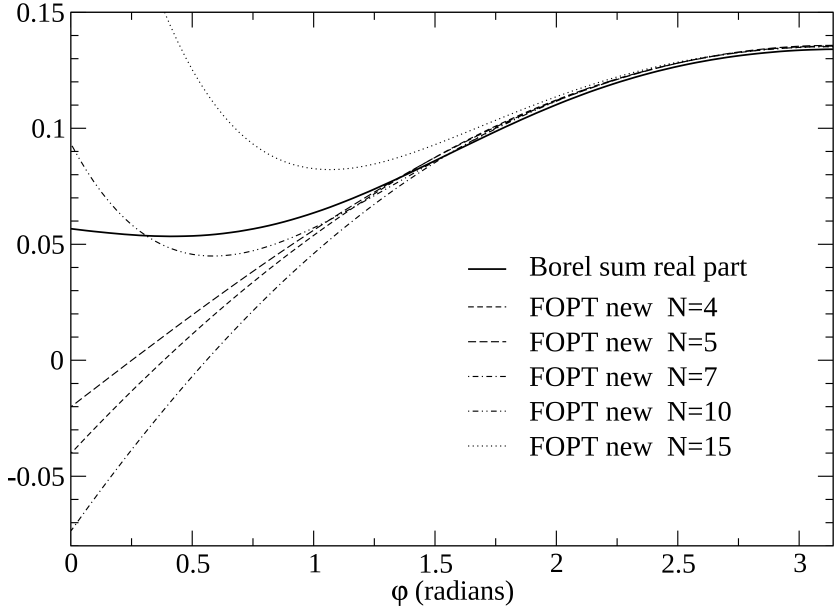




- Real part of $\hat{D}(s)$ for $s = m_\tau^2 e^{i\varphi}$, calculated with the standard CIPT (left) and the new CIPT (right)
  - very good local approximations with the new CIPT

Standard and new FOPT for the B&J models





- Real part of $\hat{D}(s)$ for $s = m_\tau^2 e^{i\varphi}$
  - standard FOPT not very good locally
  - new FOPT very good near the euclidian axis; shows the poor convergence of the expansion of $\alpha_s(s)$ on the circle

Remarks on the method

- the truncated expansion $\hat{D}(s) = \sum_{n=0}^{N} c_n \mathcal{W}_n(s)$, expanded in powers of $a_s(s)$, contains an infinite number of terms
  - $N = 3, \quad$ input K_1, K_2, K_3, K_4

    $\Rightarrow \quad K_5 = 256$ close to $K_5 = 283$
  - $N = 4, \quad$ input K_1, K_2, K_3, K_4, K_5

    $\Rightarrow \quad K_6 = 2929, \; K_7 = 1.73 \cdot 10^4$...... compared to

    $K_6 = 3275, \; K_7 = 1.88 \cdot 10^4$ in the B&J model

$\Rightarrow$ Prediction of higher order terms !

Conclusions from the physical models

- the standard expansions exhibit large oscillations for $\mathrm{N} > 10$
- the standard CIPT fails to reproduce the model at low orders
- the results of standard FOPT are not precise locally; the good values of $\delta^{(0)}$ are due to compensations of terms
- the new CIPT gives a precise description which improves by increasing the truncation order (checked up to $\mathrm{N} = 36$)
- the new FOPT gives a precise description in the regions where the effect of the imaginary logarithms is small

Determination of α_s

- Solve the equation $\boxed{\delta^{(0)}_{\text{theor}} = \delta^{(0)}_{\text{phen}}}$

  with $\delta^{(0)}_{\text{phen}} = 0.2042 \pm 0.0050$ Beneke & Jamin 2008

  - new CIPT:

    $$\alpha_s(m_\tau^2) = 0.3198 \pm 0.0042_{\text{exp}} \,{}^{+0.0099}_{-0.0076}\,{}_{\text{K}_5} \,{}^{+0.0015}_{-0.0019}\,{}_{\text{scale}}$$

  - new FOPT:

    $$\alpha_s(m_\tau^2) = 0.3113 \pm 0.0038_{\text{exp}} \pm 0.0013_{\text{K}_5} \,{}^{+0.0103}_{-0.0006}\,{}_{\text{scale}}$$

- the difference between CIPT and FOPT reduces to 0.009
- the optimal expansion: $\Rightarrow$

$$\boxed{\alpha_s(m_\tau^2) = 0.320\,^{+0.011}_{-0.009} \quad \Rightarrow \quad \alpha_s(M_Z^2) = 0.1180\,^{+0.0015}_{-0.0010}}$$

Conclusions

- QCD is a consistent theory reaching the level of precise predictions
- α_s determinations provide a solid test of QCD
- the perturbative expansion of QCD can be improved by including information about the divergent character of the series
  - clarified a theoretical discrepancy in the determination of α_s from τ decays
  - promising tool for the forthcoming calculations in perturbative QCD