QCD & Monte Carlo Tools

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Topics of the lectures

1. **Lecture 1: The Monte Carlo Principle**
   - Monte Carlo as integration method
   - Hard physics simulation: Parton Level event generation

2. **Lecture 2: Dressing the Partons**
   - Hard physics simulation, cont'd: Parton Showers

3. **Lecture 3: Modelling beyond Perturbation Theory**
   - Hadronic initial states: PDFs
   - Soft physics simulation: Hadronization
   - Beyond factorization: Underlying Event

4. **Lecture 4: Higher Orders in Monte Carlos**
   - Some nomenclature: Anatomy of HO calculations
   - Merging vs. Matching

Thanks to
- the other Sherpas: T.Gleisberg, S.Höche, S.Schumann, F.Siegert, M.Schönherr, J.Winter;
Simulation’s paradigm

Basic strategy

Divide event into stages, separated by different scales.

- **Signal/background:**
  
  Exact matrix elements.

- **QCD-Bremsstrahlung:**
  
  Parton showers (also in initial state).

- **Multiple interactions:**
  
  Beyond factorization: Modeling.

- **Hadronization:**
  
  Non-perturbative QCD: Modeling.
Outline of today’s lecture

- Nomenclature: Definition of higher orders.
- ME corrections
- MC@NLO
- ME/PS merging
Nomenclature

Specifying higher-order corrections: $\gamma^* \rightarrow \text{hadrons}$

- In general: $N^n\text{LO} \leftrightarrow \mathcal{O}(\alpha_s^n)$
- But: only for inclusive quantities
  (e.g.: total xsecs like $\gamma^* \rightarrow \text{hadrons}$).

Counter-example: thrust distribution

- In general, distributions are HO.
- Distinguish real & virtual emissions:
  Real emissions $\rightarrow$ mainly distributions,
  virtual emissions $\rightarrow$ mainly normalisation.
Anatomy of HO calculations: Virtual and real corrections

**LO:** | | | | | \[ \mathcal{O}(\alpha_s) \]

**NLO:** \[ | \frac{\alpha_s}{\pi} \mathcal{O}(\alpha_s) + 2 \cdot \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) \]

NLO corrections: \( \mathcal{O}(\alpha_s) \)

- Virtual corrections = extra loops
- Real corrections = extra legs

- UV-divergencies in virtual graphs → renormalisation
- But also: IR-divergencies in real & virtual contributions

Must cancel each other, non-trivial to see:

\( N \) vs. \( N + 1 \) particle FS, divergence in PS vs. loop
Nomenclature

Cancelling the IR divergencies: Subtraction method

- **Total NLO xsec:**
  \[
  \sigma_{\text{NLO}} = \sigma_{\text{Born}} + \int d^D k |M|^2_V + \int d^4 k |M|^2_R
  \]

- **IR div. in real piece → regularise:**
  \[
  \int d^4 k |M|^2_R \rightarrow \int d^D k |M|^2_R
  \]

- **Construct subtraction term with same IR structure:**
  \[
  \int d^D k (|M|^2_R - |M|^2_S) = \int d^4 k |M|^2_{RS} = \text{finite.}
  \]

  **Possible:**
  \[
  \int d^D k |M|^2_S = \sigma_{\text{Born}} \int d^D k |\tilde{S}|^2, \text{ universal } |\tilde{S}|^2.
  \]

- \[
  \int d^D k |M|^2_V + \sigma_{\text{Born}} \int d^D k |\tilde{S}|^2 = \text{finite (analytical)}
  \]
Nomenclature

State-of-the-art NLO calculations: General strategy

- Construct Born $+$ 1st order terms
- Subtraction term: Born term $\times$ (analytical) divergencies
  
  Evaluate loop term analytically - perform cancellation

- Monte Carlo separately over subtracted real emission and virtual $+$ subtraction term

Limitations

- So far only loops with $\leq 5$ propagators under full control
  
  $\Rightarrow$ in general, only 2 $\rightarrow$ 3 processes at NLO

- Soft/collinear corners maybe still badly described
Nomenclature

Resummation: Basic idea

- Observation: Universal soft & collinear divergencies @ all orders
  Cutting them produces universal logarithms.

- Universality $\implies$ resummation of leading logs @ all orders possible.
  Improves behaviour in soft/collinear regions of phase space.
  Example: Thrust distribution.

- Nomenclature: LL, NLL, NNLL, . . .
  Limitation due to mixing with finite pieces @ some $N^n$LL.

- Leading logs also in parton shower ($=\text{resummation!!}$)
Orders in ME and PS

**ME vs. PS**

- Matrix elements good for: hard, large-angle emissions; take care of interferences.
- Parton shower good for: soft, collinear emissions; resums large logarithms.
- Want to combine both! Avoid double-counting.

**$\alpha_S$ vs. Log**

- Matrix elements at LO 4jet, but also NLO 4jet.
- Parton shower resums large logarithms.
- Non-leading logarithms (NLL) resummed in PS.
Correcting the parton shower

Example: $e^+ e^- \rightarrow q \bar{q} g$

ME : $+$ $+$

PS : $+$ $+$

Graph shows the distribution of $x_1$ and $x_2$ for ME and PS, with a comparison of ME over PS.
Correcting the parton shower

Practicalities of ME-corrections

- Obviously, $ME < PS$ is not always fulfilled.
- Could enhance $PS$ expression by a (large) factor. Question: Efficiency of the approach?
- Therefore: realised in few processes only:
  Best-known: $ee \rightarrow q\bar{q}, q\bar{q} \rightarrow V, t \rightarrow bW$
Correcting the parton shower

Power shower

Can use ME corrections for “power shower”:

- In $q\bar{q} \rightarrow V$, start parton shower @ $s_{pp}$.
- Reweight first emissions on both legs with ME.
- Effect: More hard radiation through showering.

This is the evil empire of MC event generators!
Basic principles

- **Want:**
  - NLO-Normalization and first (hard) emission correct,
  - Soft emissions correctly resummed in PS.

- **Method:**
  - Modify subtraction terms for real infrared divergences,
  - use first order parton shower-expression,
  - this is process-dependent!

- In practise much more complicated.
- Implemented for DY, $W$-pairs, $gg \rightarrow H$, $Q$-pairs.
Example results: \(W\)-pairs @ Tevatron
Combining MEs & PS

F.K., JHEP 0208 (2002) 015

Basic principles

- **Want:**
  - All jet emissions correct at tree level + LL,
  - Soft emissions correctly resummed in PS

- **Method:**
  - Separate Jet-production/evolution by $Q_{jet}$ ($k_{⊥}$ algorithm).
  - Produce jets according to LO matrix elements
  - re-weight with Sudakov form factor + running $\alpha_s$ weights,
  - veto jet production in parton shower.

- **Process-independent implementation.**
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$n$-jet rates @ NLL


At NLL-Accuracy

\[ R_2(Q_{\text{jet}}) = \left[ \Delta_q(E_{\text{c.m.}}, Q_{\text{jet}}) \right]^2 \]

\[ R_3(Q_{\text{jet}}) = 2\Delta_q(E_{\text{c.m.}}, Q_{\text{jet}}) \]

\[ \cdot \int dq \left[ \alpha_s(q) \Gamma_q(E_{\text{c.m.}}, q) \frac{\Delta_q(E_{\text{c.m.}}, Q_{\text{jet}})}{\Delta_q(q, Q_{\text{jet}})} \Delta_q(q, Q_{\text{jet}}) \Delta_g(q, Q_{\text{jet}}) \right] \]

\[ W_{\text{Sud}} = \frac{\alpha_s(q)}{\alpha_s(Q_{\text{jet}})} \cdot \Delta_q(E_{\text{c.m.}}, Q_{\text{jet}}) \]

\[ \frac{\Delta_q(E_{\text{c.m.}}, Q_{\text{jet}})}{\Delta_q(q, Q_{\text{jet}})} \Delta_q(q, Q_{\text{jet}}) \Delta_g(q, Q_{\text{jet}}) \]
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Algorithm as scale-setting prescription

- Example: $p_\perp$ distribution of jets @ Tevatron
- Consider exclusive $W + 1$- and $W + 2$-jet production


Sherpa = tree-level matrix elements with $\alpha_S$ scales and Sudakov form factors.
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Vetoing the shower

\[ \mathcal{W}_{\text{Veto}} = \left\{ 1 + \int_{Q_{\text{jet}}}^{E_{c.m.}} dq \Gamma_q(E_{c.m.}, q) + \int_{Q_{\text{jet}}}^{E_{c.m.}} dq \Gamma_q(E_{c.m.}, q) \int_{Q_{\text{jet}}}^{q} dq' \Gamma_q(E_{c.m.}, q') + \cdots \right\}^2 \]

\[ = \left\{ \exp \left( \int_{Q_{\text{jet}}}^{E_{c.m.}} dq \Gamma_q(E_{c.m.}, q) \right) \right\}^2 = \Delta_q^{-2}(E_{c.m., Q_{\text{jet}}}) \]

\[ \implies \text{Cancels dependence on } Q_{\text{jet}}. \]
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Independence on $Q_{\text{jet}}$

Example: $p_\perp$ of $W$ in $p\bar{p} \rightarrow W + X$ @ Tevatron


$Q_{\text{jet}} = 10 \text{ GeV}$

$Q_{\text{jet}} = 30 \text{ GeV}$

$Q_{\text{jet}} = 50 \text{ GeV}$
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Merging issues: Dependence on scales

$p_\perp$ distribution of 1st jet @ Tevatron
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Comparison with other codes

$p_\perp$ of $W$-bosons & jets in $p\bar{p} \rightarrow W + X$ @ Tevatron

$p_\perp^W$ & $p_\perp^{1st\ jet}$

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Comparison with other codes

$p_\perp$ of $W$-bosons & jets in $p\bar{p} \rightarrow W + X$ @ Tevatron

$\frac{d^2\sigma}{dp_\perp^2}$

- Sherpa
- PYTHIA
- MC@NLO

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Comparison with data from Tevatron

$p_\perp$ of $Z$-bosons in $p\bar{p} \rightarrow Z + X$

Data from CDF, Phys. Rev. Lett. 84 (2000) 845
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Comparison with data from Tevatron

Jet rates in $p\bar{p} \rightarrow Z + X$

(D0-Note 5066)
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Comparison with data from Tevatron

Jet spectra (1st jet) in $p\bar{p} \rightarrow Z + X$ (D0-Note 5066)
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Comparison with data from Tevatron

Jet spectra (2nd jet) in $p\bar{p} \rightarrow Z + X$

(D0-Note 5066)
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Comparison with data from Tevatron

Jet spectra (3rd jet) in $p\bar{p} \rightarrow Z + X$ (D0-Note 5066)

Data / PYTHIA

Data / SHERPA

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Comparison with data from Tevatron

Azimuthal correlation \((\angle_{1,jet,2,jet})\) in \(p\bar{p} \to Z + X\) (D0-Note 5066)
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Extrapolation to LHC: Jets

$p_\perp$ of jets in inclusive $Z+\text{jets}$

- Influence of more jets.
- Displayed here: $\sigma$-sections.
- Difference in shape & $\sigma$-sec.

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Comparison with other merging algorithms: MLM

$p_\perp$ of jets in inclusive $W+\text{jets}$ at Tevatron
Combining MEs & PS

Comparison with other merging algorithms: MLM

$p_{\perp}$ of jets in inclusive $W$+jets at LHC

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QCD & Monte Carlo Tools
Summary & outlook

Summary: QCD & simulation tools

- Many interesting signals at LHC “spoiled” by QCD.
- Need to understand & describe QCD to high precision.
- Simulation tools mandatory for success of LHC (example: jets in backgrounds)
- Time to improve & validate essential tools is now!
- New methods of merging of ME& PS extremely powerful.
- Different, complementary aspects w.r.t. MC@NLO.
- Important: educated choice which tool to use!
- Important: know your Monte Carlo!
- Important: know the assumptions!