Introduction to electroweak theory and Higgs physics

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Today:

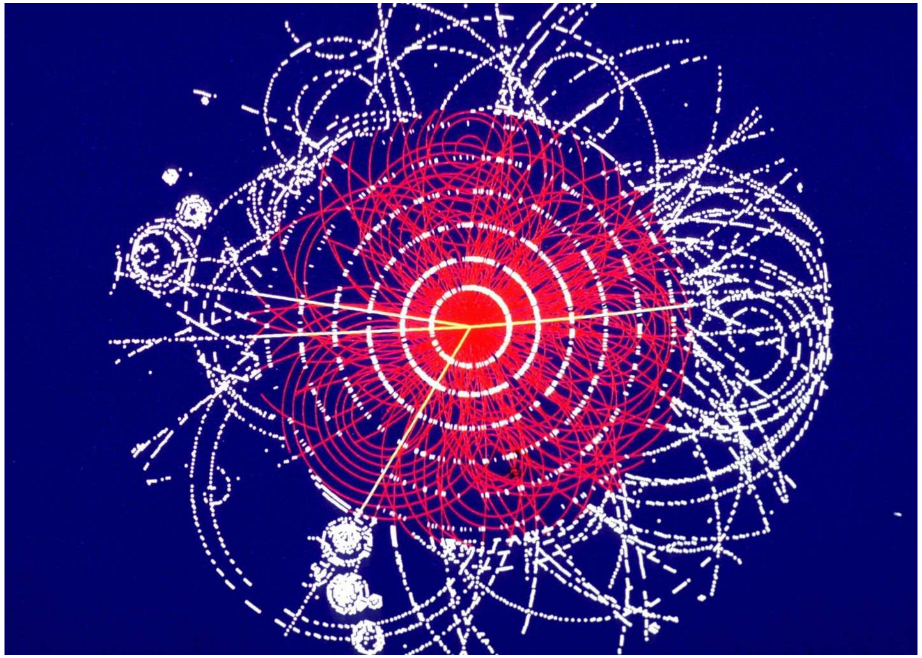
- Theoretical introduction

Tomorrow:

- Constraints on the Higgs
- Supersymmetric extension

Friday:

- Higgs boson signals at LHC



The Standard Model of particle physics (SM)

Interactions are described by gauge theory with gauge group

$$SU(3) \quad \times \quad SU(2) \times U(1)$$

Strong interactions: **QCD**

$SU(3)$ 8 massless gluons

Electroweak interactions:

$SU(2) \times U(1)$ γ massless

$W^{\pm}, Z$ massive

These gauge bosons interact with matter fields: quarks and leptons

Fermion fields of the SM and gauge quantum numbers

				$SU(3)$	$SU(2)$	$U(1)_Y$	$Q_{e.m.} = I_3 + Y$
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$	$\frac{2}{3}$ $-\frac{1}{3}$
$u_R^i =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$	$\frac{2}{3}$
$d_R^i =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$	0 -1
$e_R^i =$	e_R	μ_R	τ_R	1	1	-1	-1
$\nu_R^i =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0	0

Field theory description of the SM (and beyond)

- A quick review of non-Abelian gauge theories: many formulae but they will look familiar...
  - QED
  - Yang-Mills theories
  - electroweak interactions
- Spontaneous symmetry breaking and mass generation: the Higgs boson
- Theoretical bounds on the mass of the Higgs boson
- Experimental bounds on the mass of the Higgs boson
- Extension of the Higgs sector: two Higgs-doublet models and the MSSM

Abelian gauge theory: QED

We start with a Lagrangian

$$\mathcal{L}_0 = \bar{\psi}(x)\,(i\partial\!\!\!/ - m)\,\psi(x)$$

invariant under a GLOBAL U(1) symmetry (θ is constant)

$$\begin{aligned} \psi(x) &\rightarrow e^{iq\theta}\psi(x) \\ \partial_\mu\psi(x) &\rightarrow e^{iq\theta}\partial_\mu\psi(x) \end{aligned}$$

From Noether's theorem, there is a conserved current:

$$J_\mu(x) = q\bar{\psi}(x)\gamma_\mu\psi(x) \qquad \Longrightarrow \qquad \partial^\mu J_\mu(x) = 0$$

To gauge this theory, we promote the GLOBAL U(1) symmetry to local symmetry:

$$\begin{aligned} \psi(x) &\rightarrow e^{iq\theta(x)}\psi(x) \\ \partial_\mu\psi(x) &\rightarrow e^{iq\theta(x)}\partial_\mu\psi(x) + iqe^{iq\theta(x)}\psi(x)\partial_\mu\theta(x) \end{aligned}$$

Covariant derivative

Invent a new derivative D_μ such that

$$\psi(x) \rightarrow e^{iq\theta(x)}\psi(x) = U(x)\psi(x)$$
$$D_\mu\psi(x) \rightarrow e^{iq\theta(x)}D_\mu\psi(x) = U(x)D_\mu\psi(x)$$

i.e. both $\psi(x)$ and $D_\mu\psi(x)$ transform the same way under the U(1) local symmetry

$$D_\mu \equiv \partial_\mu + iqA_\mu$$

where A_μ transforms under the local gauge symmetry as

$$A_\mu \rightarrow A_\mu - \partial_\mu\theta(x)$$

The commutator of covariant derivatives gives the electric and magnetic fields,
i.e. the gauge invariant field strength tensor

$$F_{\mu\nu} = \frac{1}{iq}[D_\mu, D_\nu] = \frac{1}{iq}[\partial_\mu + iqA_\mu, \partial_\nu + iqA_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

QED Lagrangian

Two contributions: matter and gauge field contribution

$$\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_{gauge}$$

with

$$\begin{aligned} \mathcal{L}_\psi &= \bar{\psi}(x)\,(i\not{D} - m)\,\psi(x) \\ &= \bar{\psi}(x)\,(i\not{\partial} - m)\,\psi(x) - q\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x) \end{aligned}$$

which describes minimal coupling of the photon field $A^\mu(x)$ to the electromagnetic current $J^\mu = q\bar{\psi}\gamma^\mu\psi$, and

$$\mathcal{L}_{gauge} = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$$

$\mathcal{L}_{gauge}$ cannot contain a term proportional to $A_\mu A^\mu$ (a mass term for the photon field) since this term is not gauge invariant under

$$A_\mu \longrightarrow A_\mu - \partial_\mu\theta(x)$$

Non-Abelian (Yang-Mills) gauge theories

The starting point is a Lagrangian of free or self-interacting fields, that is symmetric under a GLOBAL symmetry

$$\mathcal{L}_\psi(\psi, \partial_\mu \psi)$$

where

$$\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} = \text{multiplet of a compact Lie group } G$$

The Lagrangian is symmetric under the transformation

$$\psi \to \psi' = U(\theta)\psi \qquad U(\theta) = \exp(igT^a\theta_a) \qquad \text{unitary matrix} \qquad UU^\dagger = U^\dagger U = 1$$

If U is unitary, the T^a are hermitian matrices, called group generators (they "generate" infinitesimal transformation around the unit element of the group)

$$U(\theta) = 1 + igT^a\theta_a + \mathcal{O}\left(\theta^2\right)$$

If U is SU(N) matrix (unitary and $\det U = 1$), then there are $N^2 - 1$ traceless, hermitian generators $T^a = \frac{\lambda^a}{2}$

Lie algebra of the generators

The generators for any representation of G satisfy the Lie Algebra relation

$$\left[T^a, T^b\right] = i f^{abc} T^c$$

where the f^{abc} are called the structure constants of the group G. The starting hypothesis is that $\mathcal{L}$ is invariant under G

$$\mathcal{L}_\psi(\psi, \partial_\mu \psi) = \mathcal{L}_\psi(\psi', \partial_\mu \psi') \qquad\qquad \psi' = U(\theta)\psi$$

Gauging the symmetry means to allow the parameters θ^a to be function of the space-time coordinates $\theta^a \to \theta^a(x)$ so that $\Longrightarrow U \to U(x)$

$$U(x) = 1 + igT^a\theta_a(x) + \mathcal{O}\left(\theta^2\right)$$

From $\partial_\mu \longrightarrow D_\mu$

We obtain a LOCAL invariant Lagrangian if we make the substitution

$$\mathcal{L}_\psi(\psi, \partial_\mu \psi) \rightarrow \mathcal{L}_\psi(\psi, D_\mu \psi) \qquad D_\mu = \partial_\mu - igA^a_\mu(x)T^a \equiv \partial_\mu - igA_\mu(x)$$

with the transformation properties

$$\begin{aligned} \psi(x) &\longrightarrow U(x)\psi(x) \\ D_\mu \psi(x) &\longrightarrow U(x)D_\mu \psi(x) = U(x)D_\mu U^{-1}(x)U(x)\psi(x) \end{aligned}$$

i.e. the covariant derivative must transform as

$$D_\mu \rightarrow U(x)D_\mu U^{-1}(x) \qquad \text{implying} \qquad A^a_\mu \rightarrow A^a_\mu + \partial_\mu \theta^a(x) + gf^{abc}A^b_\mu \theta^c + \cdots$$

We can build the kinetic term for the A^a_μ fields from

$$F_{\mu\nu} = F^a_{\mu\nu}T^a = \frac{i}{g}[D_\mu, D_\nu] \qquad \text{with} \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

which transforms homogeneously under a local gauge transformation

$$F_{\mu\nu} \rightarrow UF_{\mu\nu}U^{-1} \qquad \Longrightarrow \qquad F^a_{\mu\nu}F^{\mu\nu}_a \sim \mathrm{tr}F_{\mu\nu}F^{\mu\nu} \rightarrow \mathrm{tr}UF_{\mu\nu}U^{-1}\,UF^{\mu\nu}U^{-1} = \mathrm{tr}F_{\mu\nu}F^{\mu\nu}$$

Remarks on Yang-Mills theories

Gauge invariant Yang-Mills (YM) Lagrangian for gauge and matter fields

$$\mathcal{L}_{YM} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \mathcal{L}_\psi(\psi, D_\mu \psi) \qquad \text{with} \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

- Mass terms $A^a_\mu A^{a\mu}$ for the gauge bosons are NOT gauge invariant!
  Gauge bosons of (unbroken) YM theories are massless.
- From the $F^a_{\mu\nu} F^{a\mu\nu}$ term in the Lagrangian, we have cubic and quartic gauge boson self interactions
- gauge invariance combined with renormalizability (absence of higher powers of fields and covariant derivatives in $\mathcal{L}$) determines gauge-boson/matter couplings and gauge-boson self interactions
- if G =SU(3)$_c$ ($N = 3$) and the fermion are in triplets,

$$\psi = \begin{pmatrix} \psi_{\text{red}} \\ \psi_{\text{blue}} \\ \psi_{\text{green}} \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

  we have the QCD Lagrangian with $N^2 - 1 = 8$ gauge bosons = gluons.

Electroweak sector

From experimental facts (charged currents couple only to left-handed fermions, existence of a massless photon and a neutral Z), the gauge group is chosen as $SU(2)_L \times U(1)_Y$.

$$\psi_L \equiv \frac{1}{2}(1-\gamma_5)\psi \qquad \psi_R \equiv \frac{1}{2}(1+\gamma_5)\psi \qquad \psi = \psi_L + \psi_R$$

$$L_L \equiv \frac{1}{2}(1-\gamma_5)\begin{pmatrix} \nu_e \\ e \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad \nu_{eR} \equiv \frac{1}{2}(1+\gamma_5)\nu_e \qquad e_R \equiv \frac{1}{2}(1+\gamma_5)e$$

- $SU(2)_L$: weak isospin group. Three generators $\Longrightarrow$ three gauge bosons: W^1, W^2 and W^3.
  Generators for doublets are $T^a = \sigma^a/2$, where σ^a are the 3 Pauli matrices
  For gauge singlets (e_R, ν_R) $T^a \equiv 0$). All satisfy $\left[T^a, T^b\right] = i\epsilon^{abc}T^c$.
  The gauge coupling will be indicated with g.
- $U(1)_Y$: weak hypercharge Y. One gauge boson B with gauge coupling g'.
  One generator (charge) $Y(\psi)$, whose value depends on the fermion field

W^3 and B carry identical quantum numbers ($T_3 = 0$, $Y = 0$) $\Longrightarrow$ they will combine to produce two neutral gauge bosons: Z and γ.

Gauging the symmetry: fermion Lagrangian

Following the gauge recipe (for one generation of leptons, quarks work the same way)

$$\mathcal{L}_\psi = i\,\bar{L}_L \not{D}\, L_L + i\,\bar{\nu}_{eR} \not{D}\, \nu_{eR} + i\,\bar{e}_R \not{D}\, e_R$$

where

$$D^\mu = \partial^\mu - igW_i^\mu\, T^i - ig'Y_\psi\, B^\mu \qquad T^i = \frac{\sigma^i}{2} \quad \text{or} \quad T^i = 0, \qquad i = 1,2,3$$

$$\mathcal{L}_\psi \equiv \mathcal{L}_{kin} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

$$\begin{aligned}
\mathcal{L}_{kin} &= i\,\bar{L}_L \not{\partial}\, L_L + i\,\bar{\nu}_{eR} \not{\partial}\, \nu_{eR} + i\,\bar{e}_R \not{\partial}\, e_R \\
\mathcal{L}_{CC} &= g\,W_\mu^1\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_1}{2}\,L_L + g\,W_\mu^2\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_2}{2}\,L_L = \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{\nu}_L\,\gamma^\mu\,e_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{e}_L\,\gamma^\mu\,\nu_L \\
\mathcal{L}_{NC} &= \frac{g}{2}\,W_\mu^3\left[\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} - \bar{e}_L\,\gamma^\mu\,e_L\right] + g'\,B_\mu\Big[Y_L\left(\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} + \bar{e}_L\,\gamma^\mu\,e_L\right) \\
&\quad + Y_{\nu_{eR}}\,\bar{\nu}_{eR}\,\gamma^\mu\,\nu_{eR} + Y_{e_R}\,\bar{e}_R\,\gamma^\mu\,e_R\Big]
\end{aligned}$$

with

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

Fermion couplings fixed by renormalizability and gauge quantum numbers

				$SU(3)$	$SU(2)$	$U(1)_Y$
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$
$u_R^i =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$
$d_R^i =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
$e_R^i =$	e_R	μ_R	τ_R	1	1	-1
$\nu_R^i =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0

EW gauge-boson sector of the SM

Gauge invariance and renormalizability completely determine the kinetic terms for the gauge bosons

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu}$$

$$\begin{aligned} B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, \epsilon^{abc}\, W_{b,\mu}\, W_{c,\nu} \end{aligned}$$

The gauge symmetry does NOT allow any mass terms for $W^\pm$ and Z, i.e. forbidden are terms like

$$\mathcal{L}_{Mass} = \frac{1}{2} m_W^2 W^a_\mu W_a^\mu$$

Spontaneous symmetry breaking

Experimentally, the weak bosons are massive. We give mass to the gauge bosons through the Higgs mechanism: generate mass terms from the kinetic energy term of a scalar douplet field Φ that undergoes spontaneous symmetry breaking.

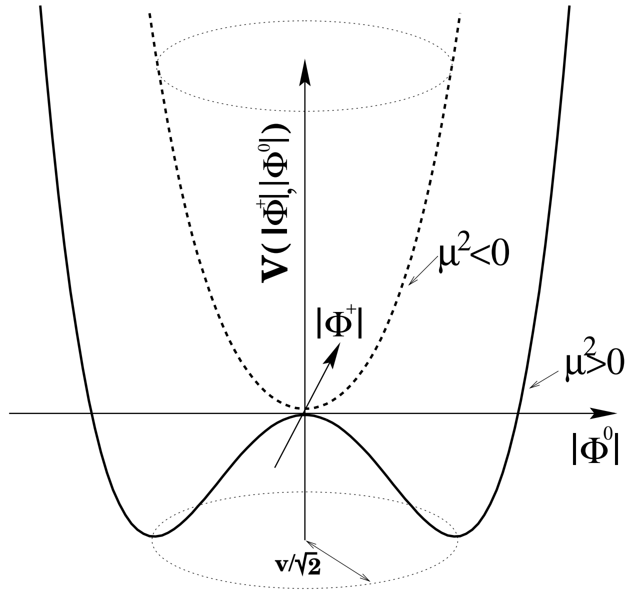
Introduce a complex scalar douplet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad Y_\Phi = \frac{1}{2}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V\left(\Phi^\dagger \Phi\right)$$

$$D^\mu = \partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig' Y_\Phi B^\mu$$

$$V\left(\Phi^\dagger\Phi\right) = V_0 - \mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2, \qquad \mu^2, \lambda > 0$$



Notice the "wrong" mass sign.

$V\left(\Phi^\dagger\Phi\right)$ is $SU(2)_L \times U(1)_Y$ symmetric.

Expanding Φ around the minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left[v + H(x) + i\chi(x)\right] \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\left[\frac{i\sigma_i \theta^i(x)}{v}\right] \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We can rotate away the fields $\theta^i(x)$ by an $SU(2)_L$ gauge transformation

$$\Phi(x) \rightarrow \Phi'(x) = U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where $U(x) = \exp\left[-\frac{i\sigma_i \theta^i(x)}{v}\right]$.

This gauge choice, called unitary gauge, is equivalent to absorbing the Goldstone modes $\theta^i(x)$.

The vacuum state can be chosen to correspond to the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Notice that only a scalar field can have a vacuum expectation value.The VEV of a fermion or vector field would break Lorentz invariance.

Consequences for the scalar field H

The scalar potential

$$V\left(\Phi^\dagger\Phi\right) = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2$$

expanded around the vacuum state

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

becomes

$$V = \frac{\lambda}{4}\left(2vH + H^2\right)^2 = \frac{1}{2}(2\lambda v^2)H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$

Consequences:

- the scalar field H gets a mass which is given by the quartic coupling λ

$$m_H^2 = 2\lambda v^2$$

- there is a term of cubic and quartic self-coupling.

Higgs kinetic terms and coupling to W, Z

$$
\begin{aligned}
D^\mu \Phi &= \left(\partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig'\frac{1}{2}B^\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2\sqrt{2}}\left[g\begin{pmatrix} W_3^\mu & W_1^\mu - iW_2^\mu \\ W_1^\mu + iW_2^\mu & -W_3^\mu \end{pmatrix} + g'B^\mu\right]\begin{pmatrix} 0 \\ v + H \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}(v+H)\begin{pmatrix} g\left(W_1^\mu - iW_2^\mu\right) \\ -gW_3^\mu + g'B^\mu \end{pmatrix}\right] \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}\left(1 + \frac{H}{v}\right)\begin{pmatrix} gvW^{\mu+} \\ -\sqrt{(g^2+g'^2)/2}\,vZ^\mu \end{pmatrix}
\end{aligned}
$$

$$
\left(D^\mu\Phi\right)^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu H \partial_\mu H + \left[\left(\frac{gv}{2}\right)^2 W^{\mu+}W_\mu^- + \frac{1}{2}\frac{\left(g^2+g'^2\right)v^2}{4}Z^\mu Z_\mu\right]\left(1+\frac{H}{v}\right)^2
$$

Consequences

- The W and Z gauge bosons have acquired masses

$$m_W^2 = \frac{g^2 v^2}{4} \qquad\qquad m_Z^2 = \frac{\left(g^2 + g'^2\right) v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

From the measured value of the Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{m_W^2} \qquad \Longrightarrow \qquad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22\,\text{GeV}$$

- the photon stays massless
- HWW and HZZ couplings from $2H/v$ term (and $HHWW$ and $HHZZ$ couplings from H^2/v^2 term)

$$\mathcal{L}_{HVV} = \frac{2m_W^2}{v} W_\mu^+ W^{-\mu} H + \frac{m_Z^2}{v} Z^\mu Z_\mu H \equiv g m_W W_\mu^+ W^{-\mu} H + \frac{1}{2}\frac{g m_Z}{\cos\theta_W} Z^\mu Z_\mu H$$

Higgs coupling proportional to mass

- tree-level HVV (V = vector boson) coupling requires VEV!
Normal scalar couplings give $\Phi^\dagger \Phi V$ or $\Phi^\dagger \Phi V V$ couplings only.

Weak mixing angle

W^3_μ and B_μ mix to produce two orthogonal mass eigenstates

$$\text{massive partner}: \quad g\,W^3_\mu - g'\,B_\mu = \sqrt{g^2+g'^2}\,Z_\mu = \sqrt{g^2+g'^2}\left(W^3_\mu \cos\theta_W - B_\mu \sin\theta_W\right)$$

$$\text{orthogonal, massless}: \quad g'\,W^3_\mu + g\,B_\mu = \sqrt{g^2+g'^2}\,A_\mu = \sqrt{g^2+g'^2}\left(W^3_\mu \sin\theta_W + B_\mu \cos\theta_W\right)$$

with mixing angle fixed by

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

Write the NC Lagrangian in terms of these mass eigenstates

$$\mathcal{L}_{NC} = \bar{\psi}\gamma_\mu\left(gT_3W_3^\mu + g'YB^\mu\right)\psi = \bar{\psi}\gamma_\mu\left(\frac{1}{\sqrt{g^2+g'^2}}(g^2T_3 - g'^2Y)Z^\mu + \frac{gg'}{\sqrt{g^2+g'^2}}(T_3+Y)A^\mu\right)\psi$$

Must identify positron charge, e, as

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_W = g'\cos\theta_W$$

and the charge of a particle, as a multiple of the positron charge, is given by the Gell-Mann–Nishijima formula: $Q = T_3 + Y$

The neutral current

It is customary to write the Z coupling to fermions in terms of the electric charge Q and the third component of isospin ($T_3 = \pm 1/2$ for left-chiral fermions, 0 for right-chiral fermions)

$$\mathcal{L}_{NC} = \bar{\psi}\gamma_\mu \left(\frac{1}{\sqrt{g^2+g'^2}}(g^2 T_3 - g'^2 Y) Z^\mu + \frac{g g'}{\sqrt{g^2+g'^2}}(T_3 + Y) A^\mu \right) \psi = e\bar{\psi}\gamma_\mu Q \psi A^\mu + \bar{\psi}\gamma_\mu Q_Z \psi Z^\mu$$

Q_Z is given by

$$Q_Z = \frac{1}{\sqrt{g^2+g'^2}}(g^2 T_3 - g'^2(Q - T_3)) = \frac{e}{\cos\theta_W \sin\theta_W}\left(T_3 - Q\sin^2\theta_W\right)$$

This procedure works for leptons and also for the quarks (see more later)

$$Q_L^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad \begin{array}{l} u_R^i = u_R, c_R, t_R \\ d_R^i = d_R, s_R, b_R \end{array}$$

Fermion mass generation

A direct mass term is not invariant under $SU(2)_L$ or $U(1)_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d \bar{Q}_L \Phi d_R - \Gamma_d \bar{d}_R \Phi^\dagger Q_L \\
&\quad -\Gamma_u \bar{Q}_L \Phi_c u_R + \text{h.c.} \\
&\quad -\Gamma_e \bar{L}_L \Phi e_R + \text{h.c.} \\
&\quad -\Gamma_\nu \bar{L}_L \Phi_c \nu_R + \text{h.c.}
\end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q, L are left-handed doublet fields and d_R, u_R, e_R, ν_R are right-handed $SU(2)$ -singlet fields.

Notice: neutrino masses can be implemented via Γ_ν term. Since $m_\nu \approx 0$ we neglect it in the following.

Fermion masses for three generations

A direct mass term is not invariant under $SU(2)_L$ or $U(1)_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij} \bar{Q}_L'^{i} \Phi d_R'^{j} - \Gamma_d^{ij*} \bar{d}_R'^{i} \Phi^\dagger Q_L'^{j} \\
&\quad -\Gamma_u^{ij} \bar{Q}_L'^{i} \Phi_c u_R'^{j} + \text{h.c.} \\
&\quad -\Gamma_e^{ij} \bar{L}_L^{i} \Phi e_R^{j} + \text{h.c.}
\end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q', u' and d' are quark fields that are generic linear combination of the mass eigenstates u and d and Γ_u, Γ_d and Γ_e are 3×3 complex matrices in generation space, spanned by the indices i and j.

$\mathcal{L}_{\text{Yukawa}}$ is Lorentz invariant, gauge invariant and renormalizable, and therefore it can (actually it must) be included in the Lagrangian.

Expanding around the vacuum state

In the unitary gauge we have

$$\bar{Q}'^{\,i}_L\, \Phi\, d'^{\,j}_R = \begin{pmatrix} \bar{u}'^{\,i}_L & \bar{d}'^{\,i}_L \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} d'^{\,j}_R = \frac{v+H}{\sqrt{2}}\, \bar{d}'^{\,i}_L\, d'^{\,j}_R$$

$$\bar{Q}'^{\,i}_L\, \Phi_c\, u'^{\,j}_R = \begin{pmatrix} \bar{u}'^{\,i}_L & \bar{d}'^{\,i}_L \end{pmatrix} \begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix} u'^{\,j}_R = \frac{v+H}{\sqrt{2}}\, \bar{u}'^{\,i}_L\, u'^{\,j}_R$$

and we obtain

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\Gamma^{ij}_d \frac{v+H}{\sqrt{2}}\, \bar{d}'^{\,i}_L\, d'^{\,j}_R - \Gamma^{ij}_u \frac{v+H}{\sqrt{2}}\, \bar{u}'^{\,i}_L\, u'^{\,j}_R - \Gamma^{ij}_e \frac{v+H}{\sqrt{2}}\, \bar{e}^{i}_L\, e^{j}_R + \text{h.c.} \\
&= -\left[M^{ij}_u\, \bar{u}'^{\,i}_L\, u'^{\,j}_R + M^{ij}_d\, \bar{d}'^{\,i}_L\, d'^{\,j}_R + M^{ij}_e\, \bar{e}^{i}_L\, e^{j}_R + \text{h.c.} \right] \left(1 + \frac{H}{v} \right)
\end{aligned}$$

with mass matrices $M^{ij} = \Gamma^{ij} \frac{v}{\sqrt{2}}$

Diagonalizing M_f

It is always possible to diagonalize M_f^{ij} $(f = u, d, e)$ with a bi-unitary transformation ($U^f_{L/R}$ must be unitary in order to preserve the form of the kinetic terms in the Lagrangian)

$$\begin{aligned} f'_{Li} &= \left(U_L^f\right)_{ij} f_{Lj} \\ f'_{Ri} &= \left(U_R^f\right)_{ij} f_{Rj} \end{aligned}$$

with U_L^f and U_R^f chosen such that

$$\left(U_L^f\right)^\dagger M_f U_R^f = \text{diagonal}$$

For example:

$$(U_L^u)^\dagger M_u U_R^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \qquad \left(U_L^d\right)^\dagger M_d U_R^d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

Mass terms

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\sum_{f',i,j} M_f^{ij}\, \bar{f}_L'^{\,i}\, f_R'^{\,j} \left(1+\frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_{f,i,j} \bar{f}_L^i \left[\left(U_L^f\right)^\dagger M_f U_R^f\right]_{ij} f_R^j \left(1+\frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_f m_f \left(\bar{f}_L f_R + \bar{f}_R f_L\right)\left(1+\frac{H}{v}\right)
\end{aligned}
$$

We succeed in producing fermion masses and we got a fermion-antifermion-Higgs coupling proportional to the fermion mass.

The Higgs Yukawa couplings are flavor diagonal: no flavor changing Higgs interactions.

Mass diagonalization and charged current interaction

The charged current interaction is given by

$$\frac{e}{\sqrt{2}\sin\theta_W}\bar{u}_L'^{\,i}\, W\!\!\!/^{+}\, d_L'^{\,i} + \text{h.c.}$$

After the mass diagonalization described previously, this term becomes

$$\frac{e}{\sqrt{2}\sin\theta_W}\bar{u}_L^{i}\left[(U_L^u)^\dagger\, U_L^d\right]_{ij} W\!\!\!/^{+} d_L^{j} + \text{h.c.}$$

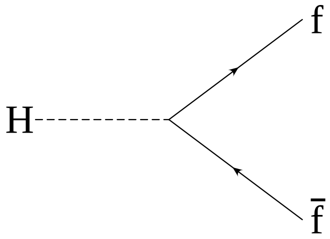
and we define the Cabibbo-Kobayashi-Maskawa matrix V_{CKM}

$$V_{CKM} = (U_L^u)^\dagger\, U_L^d$$

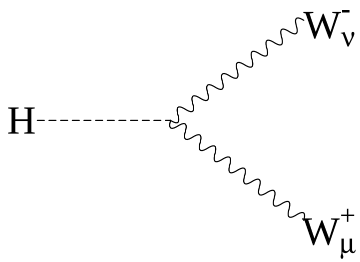
- V_{CKM} is not diagonal and then it mixes the flavors of the different quarks.

- It is a unitary matrix and the values of its entries must be determined from experiments.

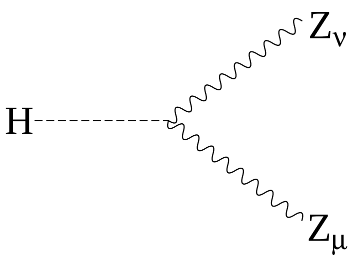
Feynman rules for Higgs couplings



$$-i\frac{m_f}{v}$$



$$ig\, m_W\, g_{\mu\nu}$$



$$i\, g\, \frac{1}{\cos\theta_W}\, m_Z\, g_{\mu\nu}$$

Within the Standard Model, the Higgs couplings are almost completely constrained. The only free parameter (not yet measured) is the Higgs mass

$$m_H^2 = 2\lambda v^2$$