The Physics of Heavy Flavors*

Yosef Nir[1,†]

[1]*Department of Particle Physics*

Weizmann Institute of Science, Rehovot 76100, Israel

Abstract

This is a written version of a series of lectures aimed at young postdocs in experimental HEP, as well as senior students in HEP phenomenology. We begin with an overview of flavor physics and its implications for new physics. We emphasize the "new physics flavor puzzle". Then, we give three specific examples of flavor measurements and the lessons that have been (or can be) drawn from them: (i) Charm physics: lessons for supersymmetry from the upper bound on Δm_D. (ii) Bottom physics: model independent lessons on the KM mechanism and on new physics in $B^0-\overline{B}^0$ mixing from $S_{\psi K_S}$. (iii) Top physics and beyond: testing minimal flavor violation at the LHC.

---

* Lectures given at the '2007 CERN-FERMILAB Hadron Collider Physics Summer School', CERN, Geneva, Switzerland, June 6–15 2007.

[†]Electronic address: `yosef.nir@weizmann.ac.il`

I. INTRODUCTION

The Standard Model fermions appear in three generations. *Flavor physics* describes interactions that distinguish between the fermion generations.

The fermions experience two types of interactions: gauge interactions, where two fermions couple to a gauge boson, and Yukawa interactions, where two fermions couple to a scalar. Within the Standard Model, there are twelve gauge bosons, related to the gauge symmetry

$$G_{\rm SM} = SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}, \tag{1}$$

and a single Higgs scalar, related to the spontaneous symmetry breaking

$$G_{\rm SM} \to SU(3)_{\rm C} \times U(1)_{\rm EM}. \tag{2}$$

In the *interaction basis*, gauge interactions are diagonal and universal, namely described by a single gauge coupling for each factor in $G_{\rm SM}$: g_3, g_2, and g_Y. By definition, the interaction eigenstates have no gauge couplings between fermions of different generations. The Yukawa interactions are, however, quite complicated in the interaction basis. In particular, there are Yukawa couplings that involve fermions of different generations and, consequently, the interaction eigenstates do not have well-defined masses. *Flavor physics* here refers to the part of the Standard Model that depends on the Yukawa couplings.

In the *mass basis*, Yukawa interactions are diagonal (in the Standard Model, its single-Higgs extensions and even with extended Higgs sector subject to natural flavor conservation), but not universal. The mass eigenstates have, by definition, well-defined masses. The interactions related to spontaneously broken symmetries are, however, quite complicated in the mass basis. In particular, the interactions of the charged weak force carriers $W^\pm$ are not diagonal, that is, they *mix* quarks of different generations. (In extensions of the Standard Model, with left-handed $SU(2)_{\rm L}$-singlet left-handed quarks, or $SU(2)_{\rm L}$-doublet right-handed quarks, the Z-couplings can also involve mixing.) *Flavor physics* here refers to fermion masses and mixings.

Why is flavor physics interesting?

- Flavor physics and the physics of CP violation can discover new physics or probe it before it is directly observed in experiments. Here are some examples from the past:
  - The smallness of $\frac{\Gamma(K_L\to\mu^+\mu^-)}{\Gamma(K^+\to\mu^+\nu)}$ led to predicting a fourth (the charm) quark;

  - The size of Δm_K led to a successful prediction of the charm mass;
  - The size of Δm_B led to a successful prediction of the top mass;
  - The measurement of ε_K led to predicting the third generation.

- CP violation is closely related to flavor physics. Within the Standard Model, there is a single CP violating parameter, the Kobayashi-Maskawa phase $\delta_{\rm KM}$ [1]. Baryogenesis tells us, however, that there must exist new sources of CP violation. Measurements of CP violation in flavor changing processes might provide evidence for such sources.

- The fine-tuning problem of the Higgs mass, and the puzzle of the dark matter imply that there exists new physics at, or below, the TeV scale. If such new physics had a generic flavor structure, it would contribute to flavor changing neutral current (FCNC) processes orders of magnitude above the observed rates. The question of why this does not happen constitutes the *new physics flavor puzzle*.

- Most of the charged fermion flavor parameters are small and hierarchical. The Standard Model does not provide any explanation of these features. This is the *Standard Model flavor puzzle*.

  The puzzle became even deeper when neutrino masses and mixings have been measured. So far, neither smallness nor hierarchy in these parameters have been established.

In these lectures, we will not discuss that Standard Model flavor puzzle. We will, however, discuss three specific measurements that relate to the other points above:

- We show how measurements of $D^0-\overline{D}^0$ mixing allow us to explore supersymmetry and, in particular, give evidence that if there are squarks below the TeV scale, they must be quasi-degenerate.

- We explain how the measurement of the CP asymmetry in $B\to J/\psi K_S$ decays gives evidence that the KM mechanism is the dominant source of the observed CP violation, and quantitatively constrains the amount of new physics in $B^0-\overline{B}^0$ mixing.

- We present the idea of minimal flavor violation as a solution to the new physics flavor problem, and argue that the ATLAS and CMS experiments may be able to test this solution.

II. FLAVOR IN THE STANDARD MODEL

A model of elementary particles and their interactions is defined by three ingredients: (i) The symmetries of the Lagrangian; (ii) The pattern of spontaneous symmetry breaking; (iii) The representations of fermions and scalars. The Standard Model (SM) is defined as follows. (i) The gauge symmetry is $G_{\rm SM}$ of Eq. (1); (ii) Its spontaneous symmetry breaking is described by Eq. (2); (iii) There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q^I_{Li}(3,2)_{+1/6},\quad U^I_{Ri}(3,1)_{+2/3},\quad D^I_{Ri}(3,1)_{-1/3},\quad L^I_{Li}(1,2)_{-1/2},\quad E^I_{Ri}(1,1)_{-1}. \tag{3}$$

Our notations mean that, for example, left-handed quarks, Q^I_L, are triplets of $SU(3)_{\rm C}$, doublets of $SU(2)_{\rm L}$ and carry hypercharge $Y=+1/6$. The super-index I denotes interaction eigenstates. The sub-index $i=1,2,3$ is the flavor (or generation) index. There is a single scalar representation,

$$\phi(1,2)_{+1/2}=\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}. \tag{4}$$

The scalar ϕ^0 assumes a VEV,

$$\langle\phi^0\rangle=\frac{v}{\sqrt{2}}, \tag{5}$$

leading to (2).

The Standard Model Lagrangian, $\mathcal{L}_{\rm SM}$, is the most general renormalizable Lagrangian that is consistent with the gauge symmetry (1), the particle content (3,4) and the pattern of spontaneous symmetry breaking (2,5). It can be divided to three parts:

$$\mathcal{L}_{\rm SM}=\mathcal{L}_{\rm kinetic}+\mathcal{L}_{\rm Higgs}+\mathcal{L}_{\rm Yukawa}. \tag{6}$$

As concerns the kinetic terms, to maintain gauge invariance, one has to replace the derivative with a covariant derivative:

$$D^\mu=\partial^\mu+ig_sG^\mu_aL_a+igW^\mu_bT_b+ig'B^\mu Y. \tag{7}$$

Here G^μ_a are the eight gluon fields, W^μ_b the three weak interaction bosons and B^μ the single hypercharge boson. The L_a's are $SU(3)_{\rm C}$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_{\rm L}$ generators (the 2×2 Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and the Y's are the $U(1)_{\rm Y}$ charges. For example, for the left-handed quarks Q^I_L, we have

$$\mathcal{L}_{\rm kinetic}(Q_L) = i\overline{Q^I_{Li}}\gamma_\mu\left(\partial^\mu + \frac{i}{2}g_s G^\mu_a\lambda_a + \frac{i}{2}gW^\mu_b\tau_b + \frac{i}{6}g'B^\mu\right)Q^I_{Li}. \tag{8}$$

These parts of the interaction Lagrangian are always flavor-universal and CP conserving.

The Higgs potential, which describes the scalar self interactions, is given by:

$$\mathcal{L}_{\rm Higgs} = \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2. \tag{9}$$

For the Standard Model scalar sector, where there is a single doublet, this part of the Lagrangian is also CP conserving.

The quark Yukawa interactions are given by

$$-\mathcal{L}^{\rm quarks}_{\rm Yukawa} = Y^d_{ij}\overline{Q^I_{Li}}\phi D^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\tilde{\phi}U^I_{Rj} + {\rm h.c.}. \tag{10}$$

This part of the Lagrangian is, in general, flavor-dependent (that is, $Y^f \not\propto \mathbf{1}$) and CP violating.

How many independent parameters are there in $\mathcal{L}^{\rm quarks}_{\rm Yukawa}$? Each of the two Yukawa matrices Y^q ($q = u, d$) is 3×3 and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. One can think of the quark Yukawa couplings as spurions that break a global symmetry,

$$U(3)_Q \times U(3)_D \times U(3)_U \ \rightarrow\ U(1)_B. \tag{11}$$

This means that there is freedom to remove 9 real and 17 imaginary parameters (the number of parameters in three 3×3 unitary matrices minus the phase related to $U(1)_B$). We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. In the mass basis, we will be able to identify the nine real parameters as six quark masses and three mixing angles, while the single phase is $\delta_{\rm KM}$.

Upon the replacement $\mathcal{R}e(\phi^0) \rightarrow \frac{v+H^0}{\sqrt{2}}$ [see Eq. (5)], the Yukawa interactions (10) give rise to mass terms:

$$-\mathcal{L}^q_M = (M_d)_{ij}\overline{D^I_{Li}}D^I_{Rj} + (M_u)_{ij}\overline{U^I_{Li}}U^I_{Rj} + {\rm h.c.}, \tag{12}$$

where

$$M_q = \frac{v}{\sqrt{2}}Y^q, \tag{13}$$

and we decomposed the $SU(2)_{\rm L}$ quark doublets into their components:

$$Q^I_{Li} = \begin{pmatrix} U^I_{Li} \\ D^I_{Li} \end{pmatrix}. \tag{14}$$

The mass basis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices V_{qL} and V_{qR} such that

$$V_{qL} M_q V^\dagger_{qR} = M^{\rm diag}_q \quad (q = u, d), \tag{15}$$

with $M^{\rm diag}_q$ diagonal and real. The quark mass eigenstates are then identified as

$$q_{Li} = (V_{qL})_{ij} q^I_{Lj}, \quad q_{Ri} = (V_{qR})_{ij} q^I_{Rj} \quad (q = u, d). \tag{16}$$

The charged current interactions for quarks [that is the interactions of the charged $SU(2)_{\rm L}$ gauge bosons $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$], which in the interaction basis are described by (8), have a complicated form in the mass basis:

$$-\mathcal{L}^q_{W^\pm} = \frac{g}{\sqrt{2}} \overline{U_{Li}} \gamma^\mu V_{ij} D_{Lj} W^+_\mu + {\rm h.c.}. \tag{17}$$

where V is a unitary 3×3 matrix,

$$V = V_{uL} V^\dagger_{dL}, \quad (V V^\dagger = V^\dagger V = \mathbf{1}). \tag{18}$$

V is the Cabibbo-Kobayashi-Maskawa (CKM) *mixing matrix* for quarks [1, 2]. As a result of the fact that V is not diagonal, the $W^\pm$ gauge bosons couple to quark mass eigenstates of different generations. Within the Standard Model, this is the only source of *flavor changing* quark interactions.

A. The CKM matrix

The form of V is not unique:

(i) There is freedom in defining V in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, *i.e.* $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. The elements of V are written as follows:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{19}$$

(ii) There is further freedom in the phase structure of V. This means that the number of physical parameters in V is smaller than the number of parameters in a general unitary 3×3 matrix which is nine (three real angles and six phases). Let us define P_q $(q = u, d)$ to be diagonal unitary (phase) matrices. Then, if instead of using V_{qL} and V_{qR} for the rotation (16) to the mass basis we use $\tilde{V}_{qL}$ and $\tilde{V}_{qR}$, defined by $\tilde{V}_{qL} = P_q V_{qL}$ and $\tilde{V}_{qR} = P_q V_{qR}$, we still maintain a legitimate mass basis since $M_q^{\rm diag}$ remains unchanged by such transformations. However, V does change:

$$V \to P_u V P_d^*. \tag{20}$$

This freedom is fixed by demanding that V has the minimal number of phases. In the three generation case V has a single phase. (There are five phase differences between the elements of P_u and P_d and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase $\delta_{\rm KM}$ which is the single source of CP violation in the quark sector of the Standard Model [1].

The fact that V is unitary and depends on only four independent physical parameters can be made manifest by choosing a specific parametrization. The standard choice is [3]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{21}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The θ_{ij}'s are the three real mixing parameters while δ is the Kobayashi-Maskawa phase. It is known experimentally that $s_{13} \ll s_{23} \ll s_{12} \ll 1$. It is convenient to choose an approximate expression where this hierarchy is manifest. This is the Wolfenstein parametrization, where the four mixing parameters are (λ, A, ρ, η) with $\lambda = |V_{us}| = 0.23$ playing the role of an expansion parameter and η representing the CP violating phase [4, 5]:

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}. \tag{22}$$

A very useful concept is that of the *unitarity triangles.* The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \tag{23}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \tag{24}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{25}$$

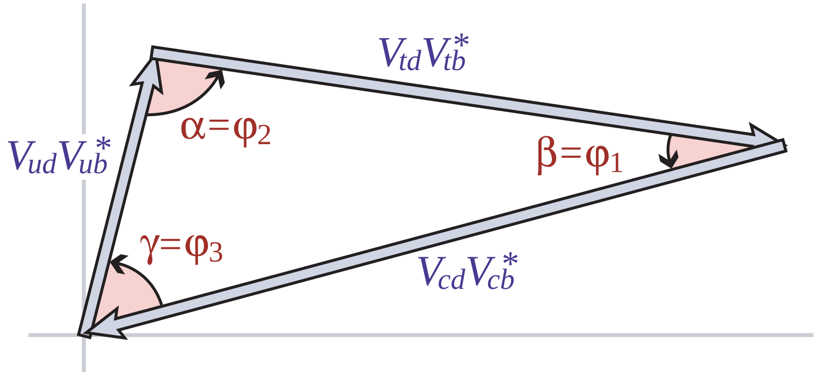


FIG. 1: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (25) only. The unitarity triangle related to Eq. (25) is depicted in Fig. 1.

The rescaled unitarity triangle is derived from (25) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ, η). The area of the rescaled unitarity triangle is $|\eta|/2$.

Depicting the rescaled unitarity triangle in the (ρ, η) plane, the lengths of the two complex sides are

$$R_u \equiv \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| = \sqrt{\rho^2+\eta^2}, \quad R_t \equiv \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right| = \sqrt{(1-\rho)^2+\eta^2}. \tag{26}$$

The three angles of the unitarity triangle are defined as follows [6, 7]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{27}$$

They are physical quantities and can be independently measured by CP asymmetries in B decays. It is also useful to define the two small angles of the unitarity triangles (24,23):

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{28}$$

III. THE NEW PHYSICS FLAVOR PUZZLE

It is clear that the Standard Model is not a complete theory of Nature:

1. It does not include gravity, and therefore it cannot be valid at energy scales above $m_{\rm Planck} \sim 10^{19}$ GeV:

2. It does not allow for neutrino masses, and therefore it cannot be valid at energy scales above $m_{\rm seesaw} \sim 10^{14}$ GeV;

3. The fine-tuning problem of the Higgs mass and the puzzle of the dark matter suggest that the scale where the SM is replaced with a more fundamental theory is actually much lower, $\Lambda_{\rm NP} \lesssim 1$ TeV.

Given that the SM is only an effective low energy theory, non-renormalizable terms must be added to $\mathcal{L}_{\rm SM}$ of Eq. (6). These are terms of dimension higher than four in the fields which, therefore, have couplings that are inversely proportional to the scale of new physics $\Lambda_{\rm NP}$. For example, the lowest dimension non-renormalizable terms are dimension five:

$$-\mathcal{L}^{\rm dim-5}_{\rm Yukawa} = \frac{Z^\nu_{ij}}{\Lambda_{\rm NP}} L^I_{Li} L^I_{Lj} \phi\phi + {\rm h.c.}. \qquad (29)$$

These are the seesaw terms, leading to neutrino masses.

As concers quark flavor physics, consider, for example, the following dimension-six, four-fermion, flavor changing operators:

$$\mathcal{L}_{\Delta F=2} = \frac{z_{sd}}{\Lambda^2_{\rm NP}} (\overline{d_L}\gamma_\mu s_L)^2 + \frac{z_{cu}}{\Lambda^2_{\rm NP}} (\overline{c_L}\gamma_\mu u_L)^2 + \frac{z_{bd}}{\Lambda^2_{\rm NP}} (\overline{d_L}\gamma_\mu b_L)^2 + \frac{z_{bs}}{\Lambda^2_{\rm NP}} (\overline{s_L}\gamma_\mu b_L)^2. \qquad (30)$$

Each of these terms contributes to the mass splitting between the corresponding two neutral mesons. For example, the term $\mathcal{L}_{\Delta B=2} \propto (\overline{d_L}\gamma_\mu b_L)^2$ contributes to Δm_B, the mass difference between the two neutral B-mesons. We use $M^B_{12} = \frac{1}{2m_B}\langle B^0|\mathcal{L}_{\Delta F=2}|\overline{B}^0\rangle$ and

$$\langle B^0|(\overline{d_{La}}\gamma^\mu b_{La})(\overline{d_{Lb}}\gamma_\mu b_{Lb})|\overline{B}^0\rangle = -\frac{1}{3} m^2_B f^2_B B_B. \qquad (31)$$

Analogous expressions hold for the other neutral mesons. This leads to $\Delta m_B/m_B \sim (z_{bd}/3)(f_B/\Lambda_{\rm NP})^2$. Experiments give:

$$\epsilon_K \sim 2.28\times 10^{-3},$$
$$\Delta m_K/m_K \sim 7.0\times 10^{-15},$$

$$\begin{aligned}\Delta m_D/m_D &\lesssim 2\times 10^{-14},\\ \Delta m_B/m_B &\sim 6.3\times 10^{-14},\\ \Delta m_{B_s}/m_{B_s} &\sim 2.2\times 10^{-13}.\end{aligned} \tag{32}$$

These measurements give then the following constraints (the bound on $\mathcal{I}m(z_{sd})$ is stronger by a factor of $(2\sqrt{2}\epsilon_K)^{-1}$ than the bound on $|z_{sd}|$):

$$\Lambda_{\rm NP} \gtrsim \begin{cases} \sqrt{\mathcal{I}m(z_{sd})}\ 2\times 10^4\ TeV & \epsilon_K \\ \sqrt{z_{sd}}\ 1\times 10^3\ TeV & \Delta m_K \\ \sqrt{z_{cu}}\ 8\times 10^2\ TeV & \Delta m_D \\ \sqrt{z_{bd}}\ 5\times 10^2\ TeV & \Delta m_B \\ \sqrt{z_{bs}}\ 2\times 10^2\ TeV & \Delta m_{B_s} \end{cases} \tag{33}$$

If the new physics has a generic flavor structure, that is $z_{ij}=\mathcal{O}(1)$, then its scale must be above 10^3-10^4 TeV. If indeed $\Lambda_{\rm NP}\gg TeV$, it means that we have misinterpreted the hints from the fine-tuning problem and the dark matter puzzle. There is, however, another way to look at these constraints:

$$\begin{aligned}\mathcal{I}m(z_{sd}) &\lesssim 5\times 10^{-9}\ (\Lambda_{\rm NP}/TeV)^2,\\ z_{sd} &\lesssim 7\times 10^{-7}\ (\Lambda_{\rm NP}/TeV)^2,\\ z_{cu} &\lesssim 2\times 10^{-6}\ (\Lambda_{\rm NP}/TeV)^2,\\ z_{bd} &\lesssim 4\times 10^{-6}\ (\Lambda_{\rm NP}/TeV)^2,\\ z_{bs} &\lesssim 3\times 10^{-5}\ (\Lambda_{\rm NP}/TeV)^2.\end{aligned} \tag{34}$$

It could be that the scale of new physics is of order TeV, but its flavor structure is far from generic.

Within the SM (a detailed derivation can be found in Appendix B of [8]), we have (using VIA for the matrix element and neglecting QCD corrections)

$$\frac{2M_{12}^B}{m_B}\approx -\frac{\alpha_2^2}{12}\frac{f_B^2}{m_W^2}S_0(x_t)(V_{tb}V_{td}^*)^2, \tag{35}$$

where $x_i=m_i^2/m_W^2$ and

$$S_0(x)=\frac{x}{(1-x)^2}\left[1-\frac{11x}{4}+\frac{x^2}{4}-\frac{3x^2\ln x}{2(1-x)}\right]. \tag{36}$$

This expression allows us to extract the weak and flavor suppression factors that apply in the SM:[1]

$$\begin{aligned}
\mathcal{I}m(z_{sd}^{\rm SM}) &\sim \alpha_2^2 y_t^2 |V_{td}V_{ts}|^2 \sim 1\times 10^{-10},\\
z_{sd}^{\rm SM} &\sim \alpha_2^2 y_c^2 |V_{cd}V_{cs}|^2 \sim 5\times 10^{-9},\\
z_{bd}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{td}V_{tb}|^2 \sim 7\times 10^{-8},\\
z_{bs}^{\rm SM} &\sim \alpha_2^2 y_t^2 |V_{ts}V_{tb}|^2 \sim 2\times 10^{-6}.
\end{aligned} \tag{37}$$

It is clear than that contributions from new physics at $\Lambda_{\rm NP} \sim 1\ TeV$ should be suppressed by factors that are comparable or smaller than the SM ones. Why does that happen? This is the new physics flavor puzzle.

The fact that the flavor structure of new physics at the TeV scale must be non-generic means that flavor measurements are a good probe of the new physics. Perhaps the best-studied example is that of supersymmetry. Here, the spectrum of the superpartners and the structure of their couplings to the SM fermions will allow us to probe the mechanism of dynamical supersymmetry breaking.

IV. LESSONS FROM $D^0 - \overline{D}^0$ MIXING

We use recent experimental information on $D^0 - \overline{D}^0$ to draw important lessons on supersymmetry. This demonstrates how flavor physics provides a significant probe of supersymmetry.

A. Supersymmetric Contributions to Neutral Meson Mixing

We consider the squark-gluino box diagram contribution to $D^0 - \overline{D}^0$ mixing amplitude that is proportional to $K^u_{2i}K^{u*}_{1i}K^u_{2j}K^{u*}_{1j}$, where K^u is the mixing matrix of the gluino couplings to a left-handed up quark and their supersymmetric squark partners. (In the language of the mass insertion approximation, we calculate here the contribution that is $\propto [(\delta^u_{LL})_{12}]^2$.) We work in the mass basis for both quarks and squarks.

[1] The SM contribution to Δm_D is dominated by long distance physics, with $z_{cu}^{\rm SM} \propto \alpha_2^2 y_s^2 (\Lambda/m_D)^2 |V_{cs}V_{us}|^2 \sim 5\times 10^{-12}$. The short distance contribution is $\propto \alpha_2^2 (y_s^4/y_c^2)|V_{cs}V_{us}|^2 \sim 5\times 10^{-13}$.

The contribution is given by

$$M_{12}^D = -i\frac{4\pi^2}{27}\alpha_s^2 m_D f_D^2 B_D \eta_{\rm QCD}\sum_{i,j}(K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*})(11\tilde{I}_{4ij} + 4\tilde{m}_g^2 I_{4ij}). \tag{38}$$

where

$$\begin{aligned}
\tilde{I}_{4ij} &\equiv \int \frac{d^4p}{(2\pi)^4}\frac{p^2}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_i^2)(p^2-\tilde{m}_j^2)} \\
&= \frac{i}{(4\pi)^2}\left[\frac{\tilde{m}_g^2}{(\tilde{m}_i^2-\tilde{m}_g^2)(\tilde{m}_j^2-\tilde{m}_g^2)}\right. \\
&\quad \left. + \frac{\tilde{m}_i^4}{(\tilde{m}_i^2-\tilde{m}_j^2)(\tilde{m}_i^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_i^2}{\tilde{m}_g^2} + \frac{\tilde{m}_j^4}{(\tilde{m}_j^2-\tilde{m}_i^2)(\tilde{m}_j^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_j^2}{\tilde{m}_g^2}\right],
\end{aligned} \tag{39}$$

$$\begin{aligned}
I_{4ij} &\equiv \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_i^2)(p^2-\tilde{m}_j^2)} \\
&= \frac{i}{(4\pi)^2}\left[\frac{1}{(\tilde{m}_i^2-\tilde{m}_g^2)(\tilde{m}_j^2-\tilde{m}_g^2)}\right. \\
&\quad \left. + \frac{\tilde{m}_i^2}{(\tilde{m}_i^2-\tilde{m}_j^2)(\tilde{m}_i^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_i^2}{\tilde{m}_g^2} + \frac{\tilde{m}_j^2}{(\tilde{m}_j^2-\tilde{m}_i^2)(\tilde{m}_j^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_j^2}{\tilde{m}_g^2}\right].
\end{aligned} \tag{40}$$

We now follow the discussion in refs. [9, 10]. To see the consequences of the super-GIM mechanism, let us expand the expression for the box integral around some value $\tilde{m}_q^2$ for the squark masses-squared:

$$\begin{aligned}
I_4(\tilde{m}_g^2,\tilde{m}_i^2,\tilde{m}_j^2) &= I_4(\tilde{m}_g^2,\tilde{m}_q^2+\delta\tilde{m}_i^2,\tilde{m}_q^2+\delta\tilde{m}_j^2) \\
&= I_4(\tilde{m}_g^2,\tilde{m}_q^2,\tilde{m}_q^2) + (\delta\tilde{m}_i^2+\delta\tilde{m}_j^2)I_5(\tilde{m}_g^2,\tilde{m}_q^2,\tilde{m}_q^2,\tilde{m}_q^2) \\
&\quad \frac{1}{2}\left[(\delta\tilde{m}_i^2)^2+(\delta\tilde{m}_j^2)^2+2(\delta\tilde{m}_i^2)(\delta\tilde{m}_j^2)\right] I_6(\tilde{m}_g^2,\tilde{m}_q^2,\tilde{m}_q^2,\tilde{m}_q^2,\tilde{m}_q^2) + \cdots,
\end{aligned} \tag{41}$$

where

$$I_n(\tilde{m}_g^2,\tilde{m}_q^2,\ldots,\tilde{m}_q^2) \equiv \int\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_q^2)^{n-2}}, \tag{42}$$

and similarly for $\tilde{I}_{4ij}$. Note that $I_n \propto (\tilde{m}_q^2)^{n-2}$ and $\tilde{I}_n \propto (\tilde{m}_q^2)^{n-3}$. Thus, using $x \equiv \tilde{m}_g^2/\tilde{m}_q^2$, it is customary to define

$$I_n \equiv \frac{i}{(4\pi)^2(\tilde{m}_q^2)^{n-2}}f_n(x), \qquad \tilde{I}_n \equiv \frac{i}{(4\pi)^2(\tilde{m}_q^2)^{n-3}}\tilde{f}_n(x). \tag{43}$$

The unitarity of the mixing matrix implies that

$$\sum_i(K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*}) = \sum_j(K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*}) = 0. \tag{44}$$

We learn that the terms that are proportional f_4, $\tilde{f}_4$, f_5 and $\tilde{f}_5$ vanish in their contribution to M_{12}. When $\delta\tilde{m}_i^2 \ll \tilde{m}_q^2$ for all i, the leading contributions to M_{12} come from f_6 and $\tilde{f}_6$. We learn that for quasi-degenerate squarks, the leading contribution is quadratic in the small mass-squared difference. The functions $f_6(x)$ and $\tilde{f}_6(x)$ are given by

$$\begin{aligned} f_6(x) &= \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(1-x)^5}, \\ \tilde{f}_6(x) &= \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(1-x)^5}. \end{aligned} \tag{45}$$

For example, with $x = 1$, $f_6(1) = -1/20$ and $\tilde{f}_6 = +1/30$; with $x = 2.33$, $f_6(2.33) = -0.015$ and $\tilde{f}_6 = +0.013$.

To further simplify things, let us consider a two generation case. Then

$$\begin{aligned} M_{12}^D &\propto 2(K_{21}^u K_{11}^{u*})^2(\delta\tilde{m}_1^2)^2 + 2(K_{22}^u K_{12}^{u*})^2(\delta\tilde{m}_2^2)^2 + (K_{21}^u K_{11}^{u*} K_{22}^u K_{12}^{u*})(\delta\tilde{m}_1^2 + \delta\tilde{m}_2^2)^2 \\ &= (K_{21}^u K_{11}^{u*})^2(\tilde{m}_2^2 - \tilde{m}_1^2)^2. \end{aligned} \tag{46}$$

We thus rewrite Eq. (38) for the case of quasi-degenerate squarks:

$$M_{12}^D = \frac{\alpha_s^2 m_D f_D^2 B_D \eta_{\rm QCD}}{108\tilde{m}_q^2}[11\tilde{f}_6(x) + 4x f_6(x)]\frac{(\Delta\tilde{m}_{21}^2)^2}{\tilde{m}_q^4}(K_{21}^u K_{11}^{u*})^2. \tag{47}$$

For example, for $x = 1.1$, $11\tilde{f}_6(x) + 4x f_6(x) = +0.14$. For $x = 2.33$, $11\tilde{f}_6(x) + 4x f_6(x) = +0.003$.

A similar expression holds for the neutral kaon system:

$$M_{12}^K = \frac{\alpha_s^2 m_K f_K^2 B_K \eta_{\rm QCD}}{108\tilde{m}_q^2}[11\tilde{f}_6(x) + 4x f_6(x)]\frac{(\Delta\tilde{m}_{21}^2)^2}{\tilde{m}_q^4}(K_{21}^d K_{11}^{d*})^2. \tag{48}$$

The mixing matrices K^u and K^d, which parametrize the gluino couplings to the left-handed quarks and their superpartners, are different from each other, but they are related through the CKM matrix:

$$K^u K^{d\dagger} = V. \tag{49}$$

The masses $\tilde{m}_q$ and $\Delta\tilde{m}_{21}^2$ are the same for the down and up sector, up to small $SU(2)_{\rm L}$ breaking effects.

B. Non-degenerate squarks at the LHC?

Eqs. (47) and (48) can be translated into our generic language:

$$\Lambda_{\rm NP} = \tilde{m}_q,$$

$$
\begin{aligned}
z_{12} &= \frac{11 f_6(x) + 4x\tilde{f}_6(x)}{18} \alpha_s^2 \left(\frac{\Delta \tilde{m}_{21}^2}{\tilde{m}_q^2}\right)^2, \\
z_{cu} &= z_{12} \sin^2 \tilde{\theta}_{12}^u, \\
z_{sd} &= z_{12} \sin^2 \tilde{\theta}_{12}^d, \qquad (50)
\end{aligned}
$$

with Eq. (49) giving

$$\sin \tilde{\theta}_{12}^u - \sin \tilde{\theta}_{12}^d \approx \sin \theta_c = 0.23. \qquad (51)$$

We now ask the following question: Is it possible that the first two generation squarks, $\tilde{Q}_{L1,2}$, are accessible to the LHC (say, $\tilde{m}_q \lesssim 1\ TeV$), and are not degenerate?

To answer this question, we use Eqs. (34). For $\Lambda_{\rm NP} \lesssim 1\ TeV$, we have $z_{cu} \lesssim 2 \times 10^{-6}$ and $z_{sd} \lesssim 7 \times 10^{-7}$. On the other hand, for non-degenerate squarks, $\Delta \tilde{m}_{21}^2/\tilde{m}_q^2 \sim 1$ and, for example, $11 f_6(1) + 4\tilde{f}_6(1) = 1/6$, we have $z_{12} = 8 \times 10^{-5}$. Then we need, simultaneously, $\sin \tilde{\theta}_{12}^u \lesssim 0.15$ and $\sin \tilde{\theta}_{12}^d \lesssim 0.09$, but this is inconsistent with Eq. (51).

There are three ways out of this situation:

1. The first two generation squarks are quasi-degenerate. The minimal level of degeneracy is $(m_2 - m_1)/(m_2 + m_1) \lesssim 0.2$. It could be the result of RGE [10].

2. The first two generation squarks are heavy. Putting $\sin \tilde{\theta}_{12}^u = 0.23$ and $\sin \tilde{\theta}_{12}^d \approx 0$, as in models of alignment [11, 12], Eq. (33) leads to

$$\tilde{m}_q \gtrsim 2\ TeV. \qquad (52)$$

3. The ratio $x = \tilde{m}_g^2/\tilde{m}_q^2$ is in a fine-tuned region of parameter space where there are accidental cancellations in $11\tilde{f}_6(x) + 4x f_6(x)$. For example, for $x = 2.33$, this combination is ~ 0.003 and the bound (52) is relaxed by a factor of 7.

Barring such accidental cancellations, the conclusion is that if squarks are within the reach of the LHC, they cannot be degenerate [13, 14].

V. LESSONS FROM $S_{\psi K_S}$

The measurement of the CP asymmetry in the $B \to J/\psi K_S$ decay and in other modes that proceed via the $b \to c\bar{c}s$ quark transition signified a new era in our understanding of CP violation. In particular, it provided the first precision test of the Kobayashi-Maskawa mechanism.

A. CP violation in neutral B decays to final CP eigenstates

We define decay amplitudes of B (which could be charged or neutral) and its CP conjugate $\overline{B}$ to a multi-particle final state f and its CP conjugate $\overline{f}$ as

$$A_f = \langle f|\mathcal{H}|B\rangle \quad , \quad \overline{A}_f = \langle f|\mathcal{H}|\overline{B}\rangle \quad , \quad A_{\overline{f}} = \langle \overline{f}|\mathcal{H}|B\rangle \quad , \quad \overline{A}_{\overline{f}} = \langle \overline{f}|\mathcal{H}|\overline{B}\rangle \ , \tag{53}$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of CP on these states introduces phases ξ_B and ξ_f according to

$$\begin{aligned} CP\ |B\rangle \ &= \ e^{+i\xi_B}\,|\overline{B}\rangle \quad , \quad CP\ |f\rangle = e^{+i\xi_f}\,|\overline{f}\rangle \ , \\ CP\ |\overline{B}\rangle \ &= \ e^{-i\xi_B}\,|B\rangle \quad , \quad CP\ |\overline{f}\rangle = e^{-i\xi_f}\,|f\rangle \ , \end{aligned} \tag{54}$$

so that $(CP\ )^2 = 1$. The phases ξ_B and ξ_f are arbitrary and unphysical because of the flavor symmetry of the strong interaction. If CP is conserved by the dynamics, $[CP\ , \mathcal{H}] = 0$, then A_f and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_B)}\, A_f \ . \tag{55}$$

A state that is initially a superposition of B^0 and $\overline{B}^0$, say

$$|\psi(0)\rangle = a(0)|B^0\rangle + b(0)|\overline{B}^0\rangle \ , \tag{56}$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\overline{B}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots \ . \tag{57}$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times t in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [15]. The simplified time evolution is determined by a 2×2 effective Hamiltonian $\mathcal{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathcal{H}$, can be written in terms of Hermitian matrices M and Γ as

$$\mathcal{H} = M - \frac{i}{2}\,\Gamma \ . \tag{58}$$

M and Γ are associated with $(B^0, \overline{B}^0) \leftrightarrow (B^0, \overline{B}^0)$ transitions via off-shell (dispersive) and on-shell (absorptive) intermediate states, respectively. Diagonal elements of M and Γ are associated with the flavor-conserving transitions $B^0 \to B^0$ and $\overline{B}^0 \to \overline{B}^0$ while off-diagonal elements are associated with flavor-changing transitions $B^0 \leftrightarrow \overline{B}^0$.

The eigenvectors of $\mathcal{H}$ have well defined masses and decay widths. We introduce complex parameters $p_{L,H}$ and $q_{L,H}$ to specify the components of the strong interaction eigenstates, B^0 and $\overline{B}^0$, in the light (B_L) and heavy (B_H) mass eigenstates:

$$|B_{L,H}\rangle = p_{L,H}|B^0\rangle \pm q_{L,H}|\overline{B}^0\rangle \tag{59}$$

with the normalization $|p_{L,H}|^2 + |q_{L,H}|^2 = 1$. If either CP or CPT is a symmetry of $\mathcal{H}$ (independently of whether T is conserved or violated) then $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, and solving the eigenvalue problem for $\mathcal{H}$ yields $p_L = p_H \equiv p$ and $q_L = q_H \equiv q$ with

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}} \, . \tag{60}$$

From now on we assume that CPT is conserved. If either CP or T is a symmetry of $\mathcal{H}$ (independently of whether CPT is conserved or violated), then M_{12} and Γ_{12} are relatively real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_B} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1 \, , \tag{61}$$

where ξ_B is the arbitrary unphysical phase introduced in Eq. (54).

The real and imaginary parts of the eigenvalues of $\mathcal{H}$ corresponding to $|B_{L,H}\rangle$ represent their masses and decay-widths, respectively. The mass difference Δm_B and the width difference $\Delta\Gamma_B$ are defined as follows:

$$\Delta m_B \equiv M_H - M_L, \quad \Delta\Gamma \equiv \Gamma_H - \Gamma_L. \tag{62}$$

Note that here Δm_B is positive by definition, while the sign of $\Delta\Gamma_B$ is to be experimentally determined. The average mass and width are given by

$$m_B \equiv \frac{M_H + M_L}{2}, \quad \Gamma_B \equiv \frac{\Gamma_H + \Gamma_L}{2}. \tag{63}$$

It is useful to define dimensionless ratios x and y:

$$x \equiv \frac{\Delta m_B}{\Gamma_B}, \quad y \equiv \frac{\Delta\Gamma_B}{2\Gamma_B}. \tag{64}$$

Solving the eigenvalue equation gives

$$(\Delta m_B)^2 - \frac{1}{4}(\Delta\Gamma_B)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \quad \Delta m_B \Delta\Gamma_B = 4\mathcal{R}e(M_{12}\Gamma_{12}^*). \tag{65}$$

All CP-violating observables in B and $\overline{B}$ decays to final states f and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of A_f, $\overline{A}_f$, $A_{\overline{f}}$ and $\overline{A}_{\overline{f}}$, together with, for neutral-meson decays only, q/p. CP violation in charged-meson decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while CP violation in neutral-meson decays is complicated by $B^0 \leftrightarrow \overline{B}^0$ oscillations and depends, additionally, on $|q/p|$ and on $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$.

For neutral D, B, and B_s mesons, $\Delta\Gamma/\Gamma \ll 1$ and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|B^0\rangle$ or $|\overline{B}^0\rangle$ after an elapsed proper time t as $|B^0{}_{\rm phys}(t)\rangle$ or $|\overline{B}^0_{\rm phys}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, we obtain

$$
\begin{aligned}
|B^0{}_{\rm phys}(t)\rangle &= g_+(t)\,|B^0\rangle - \frac{q}{p}\, g_-(t)|\overline{B}^0\rangle, \\
|\overline{B}^0_{\rm phys}(t)\rangle &= g_+(t)\,|\overline{B}^0\rangle - \frac{p}{q}\, g_-(t)|B^0\rangle\,,
\end{aligned} \tag{66}
$$

where

$$
g_\pm(t) \equiv \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right). \tag{67}
$$

One obtains the following time-dependent decay rates:

$$
\begin{aligned}
\frac{d\Gamma[B^0{}_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \left(|A_f|^2 + |(q/p)\overline{A}_f|^2\right)\cosh(y\Gamma t) + \left(|A_f|^2 - |(q/p)\overline{A}_f|^2\right)\cos(x\Gamma t) \\
&+\; 2\,\mathcal{R}e((q/p)A_f^*\overline{A}_f)\sinh(y\Gamma t) - 2\,\mathcal{I}m((q/p)A_f^*\overline{A}_f)\sin(x\Gamma t)\,, \qquad (68) \\
\frac{d\Gamma[\overline{B}^0_{\rm phys}(t)\to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} &= \left(|(p/q)A_f|^2 + |\overline{A}_f|^2\right)\cosh(y\Gamma t) - \left(|(p/q)A_f|^2 - |\overline{A}_f|^2\right)\cos(x\Gamma t) \\
&+\; 2\,\mathcal{R}e((p/q)A_f\overline{A}_f^*)\sinh(y\Gamma t) - 2\,\mathcal{I}m((p/q)A_f\overline{A}_f^*)\sin(x\Gamma t)\,, \qquad (69)
\end{aligned}
$$

where $\mathcal{N}_f$ is a common normalization factor. Decay rates to the CP-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f = \mathcal{N}_{\overline{f}}$ and the substitutions $A_f \to A_{\overline{f}}$ and $\overline{A}_f \to \overline{A}_{\overline{f}}$ in Eqs. (68,69). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $B \leftrightarrow \overline{B}$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$ are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (68,69) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of phase-space variables. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

One possible manifestation of CP-violating effects in meson decays [16] is in the interference between a decay without mixing, $B^0 \to f$, and a decay with mixing, $B^0 \to \overline{B}^0 \to f$

(such an effect occurs only in decays to final states that are common to B^0 and $\overline{B}^0$, including all CP eigenstates). It is defined by

$$\mathcal{I}m(\lambda_f) \neq 0 \,, \tag{70}$$

with

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f} \,. \tag{71}$$

This form of CP violation can be observed, for example, using the asymmetry of neutral meson decays into final CP eigenstates f_{CP}

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}\,] - d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}\,]}{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}\,] + d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}\,]} \,. \tag{72}$$

For $\Delta\Gamma = 0$ and $|q/p| = 1$ (which is a good approximation for B mesons), $\mathcal{A}_{f_{CP}}$ has a particularly simple form [17–19]:

$$\begin{aligned} \mathcal{A}_f(t) &= S_f \sin(\Delta m t) - C_f \cos(\Delta m t), \\ S_f &\equiv \frac{2\,\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2}, \quad C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \,, \end{aligned} \tag{73}$$

Consider the $B \to f$ decay amplitude A_f, and the CP conjugate process, $\overline{B} \to \overline{f}$, with decay amplitude $\overline{A}_{\overline{f}}$. There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in A_f and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model, these phases occur only in the couplings of the $W^\pm$ bosons and hence are often called "weak phases". The weak phase of any single term is convention dependent. However, the difference between the weak phases in two different terms in A_f is convention independent. A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Their origin is the possible contribution from intermediate on-shell states in the decay process. Since these phases are generated by CP-invariant interactions, they are the same in A_f and $\overline{A}_{\overline{f}}$. Usually the dominant rescattering is due to strong interactions and hence the designation "strong phases" for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The 'weak' and 'strong' phases discussed here appear in addition to the 'spurious' CP-transformation phases of Eq. (55). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any CP violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution a_i to A_f in three parts: its magnitude $|a_i|$, its weak phase ϕ_i, and its strong phase δ_i. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$\begin{aligned} A_f &= |a_1| e^{i(\delta_1+\phi_1)} + |a_2| e^{i(\delta_2+\phi_2)}, \\ \overline{A}_{\overline{f}} &= |a_1| e^{i(\delta_1-\phi_1)} + |a_2| e^{i(\delta_2-\phi_2)}. \end{aligned} \tag{74}$$

Similarly, for neutral meson decays, it is useful to write

$$M_{12} = |M_{12}| e^{i\phi_M} \quad , \quad \Gamma_{12} = |\Gamma_{12}| e^{i\phi_\Gamma} \ . \tag{75}$$

Each of the phases appearing in Eqs. (74,75) is convention dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$ and $\phi_M + \phi_1 - \overline{\phi}_1$ (where $\overline{\phi}_1$ is a weak phase contributing to $\overline{A}_f$) are physical.

In the approximations that only a single weak phase contributes to decay, $A_f = |a_f| e^{i(\delta_f + \phi_f)}$, and that $|\Gamma_{12}/M_{12}| = 0$, we obtain $|\lambda_f| = 1$ and the CP asymmetries in decays to a final CP eigenstate f [Eq. (72)] with eigenvalue $\eta_f = \pm 1$ are given by

$$\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_f)\ \sin(\Delta m t) \ \text{ with } \ \mathcal{I}m(\lambda_f) = \eta_f \sin(\phi_M + 2\phi_f). \tag{76}$$

Note that the phase so measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from $\mathcal{I}m(\lambda_f)$.

B. The CP asymmetry in $B \to \psi K_S$

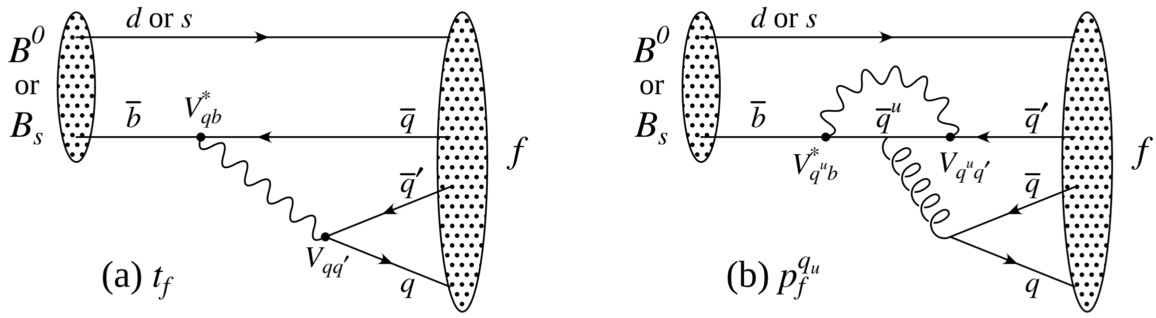
The small deviation (less than one percent) of $|q/p|$ from 1 implies that, at the present level of experimental precision, CP violation in B mixing is a negligible effect. Thus, for the purpose of analyzing CP asymmetries in hadronic B decays, we can use

$$\lambda_f = e^{-i\phi_B} (\overline{A}_f / A_f) \ , \tag{77}$$

where ϕ_B refers to the phase of M_{12} [see Eq. (75)]. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_B} = (V_{tb}^* V_{td})/(V_{tb} V_{td}^*) \ . \tag{78}$$

FIG. 2: Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s \to f$ via a $\bar{b} \to \bar{q} q \bar{q}'$ quark-level process.



Some of the most interesting decays involve final states that are common to B^0 and $\overline{B}^0$ [20–22], such as $B \to J/\psi K_S$. Here eq. (73) applies. The processes of interest proceed via quark transitions of the form $\bar{b} \to \bar{c} c \bar{s}$. There are contributions from both tree (t) and penguin (p^{q_u}, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 2) which carry different weak phases:

$$A_f = (V_{cb}^* V_{cs})\, t_f + \sum_{q_u = u,c,t} \left(V_{q_u b}^* V_{q_u s} \right) p_f^{q_u} \ . \tag{79}$$

(The distinction between tree and penguin contributions is a heuristic one, the separation by the operator that enters is more precise. For a detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, see, for example, ref. [23].) Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations:

$$A_{\psi K} = (V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u, \tag{80}$$

where $T_{\psi K} = t_{\psi K} + p_{\psi K}^c - p_{\psi K}^t$ and $P_{\psi K}^u = p_{\psi K}^u - p_{\psi K}^t$. A subtlety arises in this decay that is related to the fact that $B^0 \to J/\psi K^0$ and $\overline{B}^0 \to J/\psi \overline{K}^0$. A common final state, e.g. $J/\psi K_S$, is reached only via $K^0 - \overline{K}^0$ mixing. Consequently, the phase factor corresponding to neutral K mixing, $e^{-i\phi_K} = (V_{cd}^* V_{cs})/(V_{cd} V_{cs}^*)$, plays a role:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{(V_{cb} V_{cs}^*)\, T_{\psi K} + (V_{ub} V_{us}^*)\, P_{\psi K}^u}{(V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u} \times \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}. \tag{81}$$

For $B \to J/\psi K_S$ and other $\bar{b} \to \bar{c} c \bar{s}$ processes, we can neglect the P^u contribution to $A_{\psi K}$, in the SM, to an approximation that is better than one percent:

$$\lambda_{\psi K_S} = -e^{-2i\beta} \ \Rightarrow \ S_{\psi K_S} = \sin 2\beta, \quad C_{\psi K_S} = 0 \ , \tag{82}$$

where β is defined in Eq. (27). (Below the percent level, several effects modify this equation [24, 25].) The SM prediction for $\sin 2\beta$, based on all other constraints, is [26]

$$\sin 2\beta = 0.76 \pm 0.04. \tag{83}$$

The experimental measurements give the following ranges [27]:

$$S_{\psi K_S} = 0.68 \pm 0.03, \quad C_{\psi K_S} = 0.01 \pm 0.02 \; . \tag{84}$$

The consistency of the experimental results (84) with the SM predictions (82,83) means that the KM mechanism of CP violation has successfully passed its first precision test. For the first time, we can make the following statement based on experimental evidence:
The Kobayashi-Maskawa mechanism is the dominant source of the CP violation observed in flavor changing processes.

There are two qualifications implicit in this statement, and we now explain them in little more detail [28].

- *'Dominant'*: While $S_{\psi K}$ is measured with an accuracy of order 0.04, the accuracy of the SM prediction for $\sin 2\beta$ is only at the level of 0.1. Thus, it is quite possible that there is a new physics contribution at the level of $|M_{12}^{\rm NP}/M_{12}^{\rm SM}| \lesssim \mathcal{O}(0.1)$.

- *'Flavor changing'*: It may well happen that the KM phase, which is closely related to flavor violation through the CKM matrix, dominates meson decays while new, flavor diagonal phases (such as the two unavoidable phases in the universal version of the MSSM) dominate observables such as electric dipole moments by many orders of magnitude.

The measurement of $S_{\psi K}$ provides a significant constraint on the unitarity triangle. In the $\rho - \eta$ plane, it reads:

$$\sin 2\beta = \frac{2\eta(1-\rho)}{\eta^2 + (1-\rho)^2} = 0.68 \pm 0.03. \tag{85}$$

One can get an impression of the impact of this constraint by looking at Fig. 3, where the blue region represents $\sin 2\beta = 0.68 \pm 0.03$. An impression of the KM test can be achieved by observing that the blue region has an excellent overlap with the region allowed by all other measurements. A comparison between the constraints in the $\rho - \eta$ plane from CP conserving and CP violating processes is provided in Fig. 4. The impressive consistency between the

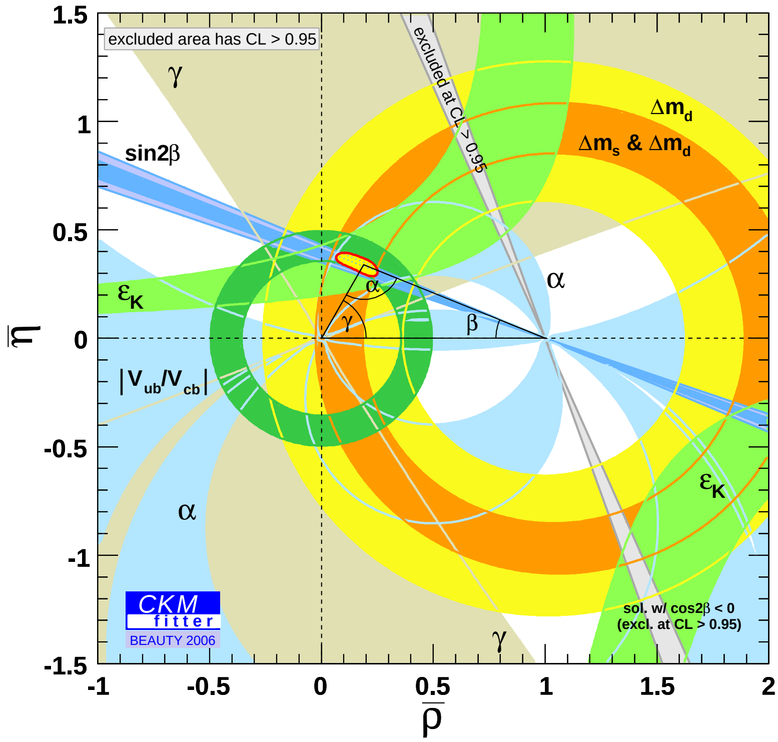


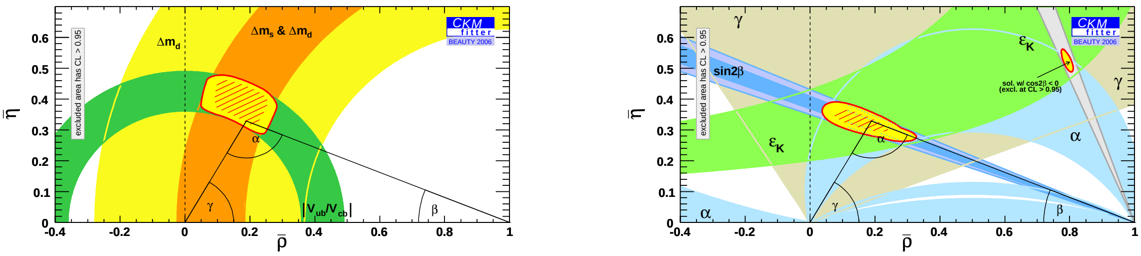
FIG. 3: Allowed region in the ρ, η plane. Superimposed are the individual constraints from charmless semileptonic B decays ($|V_{ub}/V_{cb}|$), mass differences in the B^0 (Δm_d) and B_s (Δm_s) neutral meson systems, and CP violation in $K \to \pi\pi$ (ε_K), $B \to \psi K$ ($\sin 2\beta$), $B \to \pi\pi, \rho\pi, \rho\rho$ (α), and $B \to DK$ (γ). Taken from [29].

two allowed regions is the basis for our statement that the KM mechanism has passed its first precision tests. The fact that the allowed region from the CP violating processes is more strongly constrained is related to the fact that CP is a good symmetry of the strong interactions and that, therefore, various CP violating observables – in particular $S_{\psi K}$ – can be cleanly interpreted.

The measurement of $S_{\psi K_S}$ cleanly determines the relative phase between the $B^0 - \overline{B}^0$ mixing amplitude and the $b \to c\bar{c}s$ decay amplitude ($\sin 2\beta$ in the SM). The $b \to c\bar{c}s$ decay has Standard Model tree contributions and therefore is very unlikely to be significantly affected by new physics. On the other hand, the mixing amplitude can be easily modified by new physics. We parametrize such a modification as follows:

$$r_d^2 \, e^{2i\theta_d} = \frac{M_{12}}{M_{12}^{\rm SM}}. \tag{86}$$

FIG. 4: Constraints in the $\rho - \eta$ plane from (a) CP conserving or (b) CP violating processes.



Then the following observables provide constraints on r_d^2 and $2\theta_d$:

$$
\begin{aligned}
S_{\psi K_S} &= \sin(2\beta + 2\theta_d), \\
\Delta m_B &= r_d^2 (\Delta m_B)^{\rm SM}, \\
\mathcal{A}_{\rm SL} &= -\mathcal{R}e \left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\sin 2\theta_d}{r_d^2} + \mathcal{I}m \left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\cos 2\theta_d}{r_d^2}.
\end{aligned} \tag{87}
$$

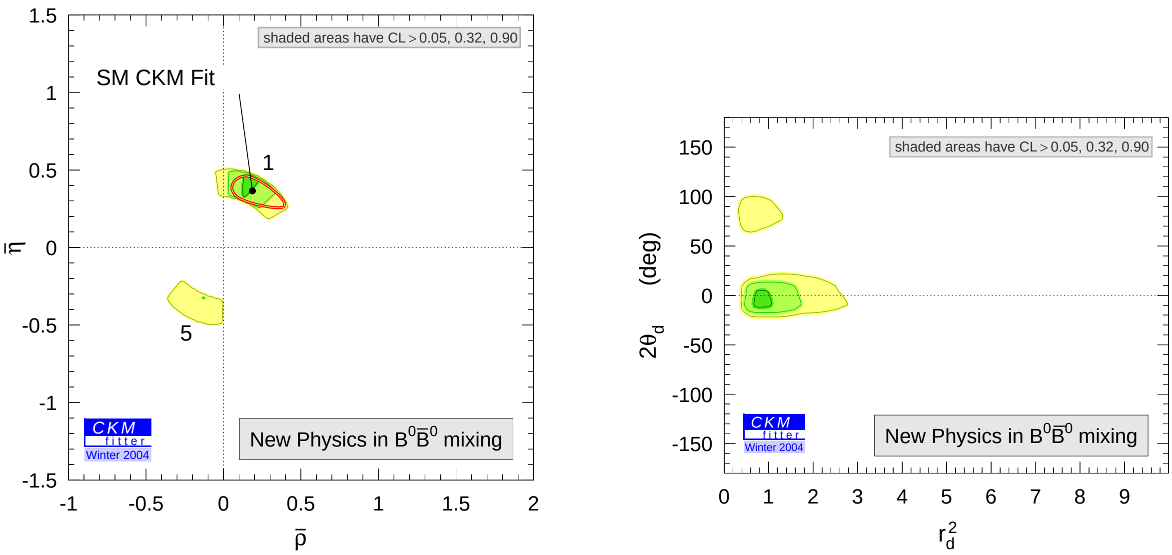
Examining whether $S_{\psi K_S}$, Δm_B and $\mathcal{A}_{\rm SL}$ fit the SM prediction, that is, whether $\theta_d \neq 0$ and/or $r_d^2 \neq 1$, we can answer the following question (see *e.g.* [30]):

Is there new physics in $B^0 - \overline{B}^0$ mixing?

Thanks to the fact that quite a few observables that are related to SM tree level processes have already been measured, we are able to refer to this question in a quantitative way. The tree level processes are insensitive to new physics and can be used to constrain ρ and η even in the presence of new physics contributions to loop processes, such as Δm_B. Among these observables we have $|V_{cb}|$ and $|V_{ub}|$ from semileptonic B decays, the phase γ from $B \to DK$ decays, and the phase α from $B \to \rho\rho$ decays (in combination with $S_{\psi K}$). One can fit these observables, and the ones in Eq. (87) to the four parameters ρ, η, r_d^2 and $2\theta_d$. The resulting constraints are shown in Fig. 5.

A long list of models that require a significant modification of the $B^0 - \overline{B}^0$ mixing amplitude are excluded. We can further conclude from Fig. 5 that a new physics contribution to the $B^0 - \overline{B}^0$ mixing amplitude at a level higher than about 30% is now disfavored.

FIG. 5: Constraints in the (a) $\rho - \eta$ plane (b) $r_d^2 - 2\theta_d$ plane, assuming that NP contributions to tree level processes are negligible [29].



VI. FLAVOR AT THE LHC

A. Top Physics

The LHC will study the physics of electroweak symmetry breaking. There are high hopes that it will not only discover the Higgs, but also shed light on the fine-tuning problem that is related to the Higgs mass.

The top quark plays a role in electroweak symmetry breaking. It poses the most severe fine tuning problem to the Higgs,

$$-\frac{3}{8\pi^2} y_t^2 \Lambda_{\rm NP}^2 = -(2\ TeV)^2 \left(\frac{\Lambda_{\rm NP}}{10\ TeV}\right)^2 . \qquad (88)$$

Therefore, very likely there exists a "top-partner" that couples to the Higgs and cancels the quadratically divergent contributions to the Higgs mass. Indeed, the LHC can, in principle, measure the mass and the production cross section and shed light on the question of the spin of the top partner [31]. Moreover, in some models – such as minimal SUGRA – it is the top quark that induces the electroweak symmetry breaking.

Given that the top quark is the only one that has an order one coupling to the Higgs, it plays a special role in various extensions of the SM. For example, in RS1 models, there is a strong enhancement of $t \to cZ$ decays (see *e.g.* [32]), and it is the only one that has a significant coupling to the Kaluza-Klein gluons (see *e.g.* [33]).

Thus, the LHC is likely to yield exciting top physics.

B. Minimal flavor violation (MFV)

A simple and rather generic principle that can guarantee that low energy flavor changing processes would deviate only very little from the SM predictions is that of *minimal flavor violation* (MFV) [34]. The basic idea can be described as follows. The gauge interactions of the SM are universal in flavor space. The only breaking of this flavor universality comes from the three Yukawa matrices, Y_U, Y_D and Y_E. If this remains true in the presence of the new physics, namely Y_U, Y_D and Y_E are the only flavor non-universal parameters, then the model belongs to the MFV class.

The Standard Model with vanishing Yukawa couplings has a large global symmetry, $U(3)^5$. In this section we concentrate only on the quarks. The non-Abelian part of the flavor symmetry for the quarks can be decomposed as follows:

$$G_{\rm f} = SU(3)_Q \otimes SU(3)_D \otimes SU(3)_U, \tag{89}$$

with the three generations of quark fields transforming as follows:

$$Q_L(3,1,1), \quad D_R(1,3,1), \quad U_R(1,1,3). \tag{90}$$

The Yukawa interactions,

$$\mathcal{L}_Y = \overline{Q_L} Y_D D_R H + \overline{Q_L} Y_U U_R H_c, \tag{91}$$

($H_c = i\tau_2 H^*$) break this symmetry. The Yukawa couplings can thus be thought of as spurions with the following transformation properties under $G_{\rm f}$:

$$Y_D \sim (3,\bar{3},1), \qquad Y_U \sim (3,1,\bar{3}). \tag{92}$$

When we say "spurions", we mean that we pretend that the Yukawa matrices are fields which transform under the flavor symmetry, and then require that all the Lagrangian terms, constructed from the SM fields, Y_D and Y_U, must be (formally) invariant under the flavor group $G_{\rm f}$. Of course, in reality, $\mathcal{L}_Y$ breaks $G_{\rm f}$ precisely because $Y_{D,U}$ are *not* fields and do not transform under the symmetry.

Treating the Yukawa couplings as $G_{\rm f}$-breaking spurions is useful to understand the flavor suppression of various flavor changing processes. Think, for example, on Δm_K, the mass difference between the two neutral K-meson mass eigenstates. It is related to an operator with the flavor structure of $(\overline{d_L} s_L)^2$. The coefficient of this operator must be $\propto y_c^4 \sin^2\theta_c$.

The idea of minimal flavor violation is relevant to extensions of the SM, and can be applied in two ways:

1. If we consider the SM as a low energy effective theory, then all higher-dimension operators, constructed from SM and Y fields, are formally invariant under $G_{\rm f}$.

2. If we consider a full high-energy theory that extends the SM, then all operators, constructed from SM, Y and the new fields, are formally invariant under $G_{\rm f}$.

If the LHC discovers new particles that couple to the SM fermions, then it will be able to test solutions to the new physics flavor puzzle such as MFV [35]. Much of its power to test such frameworks is based on identifying top and bottom quarks.

We conclude that flavor physics have taught us much about the Standard Model and its extensions. Improvements in precision flavor measurements, as well as measurements at the LHC related to new particles that couple to the SM ones, are likely to teach us much more.

---

[1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).

[3] L. Chau and W. Keung, Phys. Rev. Lett. **53**, 1802 (1984).

[4] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[5] A. J. Buras, M. E. Lautenbacher, and G. Ostermaier, Phys. Rev. D **50**, 3433 (1994) [arXiv:hep-ph/9403384].

[6] C. Dib, I. Dunietz, F. J. Gilman and Y. Nir, Phys. Rev. D **41**, 1522 (1990).

[7] J. L. Rosner, A. I. Sanda and M. P. Schmidt, EFI-88-12-CHICAGO [Presented at Workshop on High Sensitivity Beauty Physics, Batavia, IL, Nov 11-14, 1987].

[8] G. C. Branco, L. Lavoura and J. P. Silva, *CP violation,* Clarendon Press, Oxford (1999).

[9] G. Raz, Phys. Rev. D **66**, 037701 (2002) [arXiv:hep-ph/0205310].

[10] Y. Nir and G. Raz, Phys. Rev. D **66**, 035007 (2002) [arXiv:hep-ph/0206064].

[11] Y. Nir and N. Seiberg, Phys. Lett. B **309**, 337 (1993) [arXiv:hep-ph/9304307].

[12] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **420**, 468 (1994) [arXiv:hep-ph/9310320].

[13] M. Ciuchini, E. Franco, D. Guadagnoli, V. Lubicz, M. Pierini, V. Porretti and L. Silvestrini, arXiv:hep-ph/0703204.

[14] Y. Nir, JHEP, in press [arXiv:hep-ph/0703235].

[15] V. Weisskopf and E. P. Wigner, Z. Phys. **63**, 54 (1930); Z. Phys. **65**, 18 (1930). [See Appendix A of P. K. Kabir, "The CP Puzzle: Strange Decays of the Neutral Kaon", Academic Press (1968).]

[16] Y. Nir, SLAC-PUB-5874 [Lectures given at 20th Annual SLAC Summer Institute on Particle Physics (Stanford, CA, 1992)].

[17] I. Dunietz and J. L. Rosner, Phys. Rev. D **34**, 1404 (1986).

[18] Ya. I. Azimov, N. G. Uraltsev, and V. A. Khoze, Sov. J. Nucl. Phys. **45**, 878 (1987) [Yad. Fiz. **45**, 1412 (1987)].

[19] I. I. Bigi and A. I. Sanda, Nucl. Phys. B **281**, 41 (1987).

[20] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980).

[21] A. B. Carter and A. I. Sanda, Phys. Rev. D **23**, 1567 (1981).

[22] I. I. Bigi and A. I. Sanda, Nucl. Phys. B **193**, 85 (1981).

[23] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996) [arXiv:hep-ph/9512380].

[24] Y. Grossman, A. L. Kagan and Z. Ligeti, Phys. Lett. B **538**, 327 (2002) [arXiv:hep-ph/0204212].

[25] H. Boos, T. Mannel and J. Reuter, Phys. Rev. D **70**, 036006 (2004) [arXiv:hep-ph/0403085].

[26] M. Bona *et al.* [UTfit Collaboration], JHEP **0610**, 081 (2006) [arXiv:hep-ph/0606167].

[27] E. Barberio *et al.* [HFAG Group], arXiv:0704.3575 [hep-ex] and online update at http://www.slac.stanford.edu/xorg/hfag

[28] Y. Nir, Nucl. Phys. Proc. Suppl. **117**, 111 (2003) [arXiv:hep-ph/0208080].

[29] CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005), [hep-ph/0406184], updated results and plots available at: http://ckmfitter.in2p3.fr

[30] Y. Grossman, Y. Nir and M. P. Worah, Phys. Lett. B **407**, 307 (1997) [hep-ph/9704287].

[31] P. Meade and M. Reece, Phys. Rev. D **74**, 015010 (2006) [arXiv:hep-ph/0601124].

[32] P. J. Fox, Z. Ligeti, M. Papucci, G. Perez and M. D. Schwartz, arXiv:0704.1482 [hep-ph].

[33] K. Agashe, A. Belyaev, T. Krupovnickas, G. Perez and J. Virzi, arXiv:hep-ph/0612015.

[34] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002) [arXiv:hep-ph/0207036].

[35] Y. Grossman, Y. Nir, J. Thaler, T. Volansky, J. Zupan, work in writing.