QCD & Monte Carlo Tools

Frank Krauss

Institute for Particle Physics Phenomenology
Durham University

CERN, 6.-15.6.2007
Topics of the lectures

1. **Lecture 1: The Monte Carlo Principle**
   - Monte Carlo as integration method
   - Hard physics simulation: Parton Level event generation

2. **Lecture 2: Dressing the Partons**
   - Hard physics simulation, cont’d: Parton Showers

3. **Lecture 3: Modelling beyond Perturbation Theory**
   - Hadronic initial states: PDFs
   - Soft physics simulation: Hadronization
   - Beyond factorization: Underlying Event

4. **Lecture 4: Higher Orders in Monte Carlos**
   - Some nomenclature: Anatomy of HO calculations
   - Merging vs. Matching

Thanks to
- the other Sherpas: T.Gleisberg, S.Höche, S.Schumann, F.Siegert, M.Schönherr, J.Winter;
Simulation’s paradigm

Basic strategy

Divide event into stages, separated by different scales.

- **Signal/background:**
  Exact matrix elements.

- **QCD-Bremsstrahlung:**
  Parton showers (also in initial state).

- **Multiple interactions:**
  Beyond factorization: Modeling.

- **Hadronization:**
  Non-perturbative QCD: Modeling.
Outline of today’s lecture

- Why parton showers?
- Large logs in QCD radiation: Parton shower
- Including quantum effects in parton showering
- Dipole shower(s)
- Survey of existing showering tools
Motivation: Why parton showers?

Common wisdom

- Well-known: Accelerated charges radiate
- QED: Electrons (charged) emit photons
  Photons split into electron-positron pairs
- QCD: Quarks (colored) emit gluons
  Gluons split into quark pairs
- Difference: Gluons are colored (photons are not charged)
  Hence: Gluons emit gluons!
- Cascade of emissions: Parton shower
Motivation: Why parton showers?

Some more refined reasons

- Experimental definition of jets based on hadrons.
- But: Hadronization through phenomenological models (need to be tuned to data).
- Wanted: Universality of hadronization parameters (independence of hard process important).
- Link to fragmentation needed: Model softer radiation (inner jet evolution).
- Similar to PDFs (factorization) just the other way around (fragmentation functions at low scale, parton shower connects high with low scale).
Occurrence of large logarithms

\( e^+ e^- \rightarrow \text{jets} \)

- Differential cross section:

\[
\frac{d\sigma_{ee \rightarrow 3j}}{dx_1 dx_2} = \sigma_{ee \rightarrow 2j} \frac{C_F \alpha_s}{\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}
\]

**Singular for** \( x_{1,2} \rightarrow 1 \).

- Rewrite with opening angle \( \theta_{qg} \) and gluon energy fraction \( x_3 = 2E_g / E_{\text{c.m.}} \):

\[
\frac{d\sigma_{ee \rightarrow 3j}}{d \cos \theta_{qg} dx_3} = \sigma_{ee \rightarrow 2j} \frac{C_F \alpha_s}{\pi} \left[ \frac{2}{\sin^2 \theta_{qg}} \frac{1 + (1 - x_3)^2}{x_3} - x_3 \right]
\]

**Singular for** \( x_3 \rightarrow 0 \) (**“soft”**), \( \sin \theta_{qg} \rightarrow 0 \) (**“collinear”**).
Occurrence of large logarithms

Collinear singularities

- Use

\[
\frac{2 \mathrm{d} \cos \theta_{qg}}{\sin^2 \theta_{qg}} = \frac{\mathrm{d} \cos \theta_{qg}}{1 - \cos \theta_{qg}} + \frac{\mathrm{d} \cos \theta_{qg}}{1 + \cos \theta_{qg}} = \frac{\mathrm{d} \cos \theta_{\bar{q}g}}{1 - \cos \theta_{\bar{q}g}} + \frac{\mathrm{d} \cos \theta_{\bar{q}g}}{1 + \cos \theta_{\bar{q}g}} \approx \frac{\mathrm{d} \theta^2_{qg}}{\theta^2_{qg}} + \frac{\mathrm{d} \theta^2_{\bar{q}g}}{\theta^2_{\bar{q}g}}
\]

- Independent evolution of two jets \((q\text{ and } \bar{q})\):

\[
\mathrm{d}\sigma_{ee\rightarrow3j} \approx \sigma_{ee\rightarrow2j} \sum_{j\in\{q,\bar{q}\}} \frac{C_F \alpha_s}{2\pi} \frac{\mathrm{d}\theta^2_{jg}}{\theta^2_{jg}} P(z),
\]

where \(P(z) = \frac{1+(1-z)^2}{z}\) (DGLAP splitting function)
Occurrence of large logarithms

Expressing the collinear variable

- Same form for any $t \propto \theta^2$:
- Transverse momentum $k_\perp^2 \approx z^2(1 - z)^2 E^2 \theta^2$
- Invariant mass $q^2 \approx z(1 - z) E^2 \theta^2$

$$\frac{d\theta^2}{\theta^2} \approx \frac{dk_\perp^2}{k_\perp^2} \approx \frac{dq^2}{q^2}$$
**Occurrence of large logarithms**

### Parton resolution

- **What is a parton?**
  - Collinear pair/soft parton recombine!
- **Introduce resolution criterion** $k_\perp > Q_0$.
- **Combine virtual contributions with unresolvable emissions:**
  - Cancels infrared divergences $\implies$ Finite at $\mathcal{O}(\alpha_s)$
  - (Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)
- **Unitarity:** Probabilities add up to one
  - $\mathcal{P}($resolved$) + \mathcal{P}($unresolved$) = 1.$
Occurrence of large logarithms

The Sudakov form factor

- Diff. probability for emission between $q^2$ and $q^2 + dq^2$:
  \[ dP = \frac{\alpha_s}{2\pi} \frac{dq^2}{q^2} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz P(z) =: \frac{dq^2}{q^2} \bar{P}(q^2). \]

- No-emission probability $\Delta(Q^2, q^2)$ between $Q^2$ and $q^2$.

Evolution equation for $\Delta$:

\[ -\frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2) \frac{d}{dq^2}. \]

\[ \implies \Delta(Q^2, q^2) = \exp \left[ -\int_{Q^2}^{q^2} \frac{dk^2}{k^2} \bar{P}(k^2) \right]. \]
Occurrence of large logarithms

The Sudakov form factor

- $\Delta(Q^2, q^2)$ is the **Sudakov form factor**.
- Remember it is given by $\Delta(Q^2, q^2) = \exp \left[ - \int \frac{d k^2}{k^2} \bar{P}(k^2) \right] \approx \exp \left[ - C_F \frac{\alpha_s}{2\pi} \log^2 \frac{Q^2}{Q_0^2} \right]$ for quarks.
- Use $\Delta(Q^2, Q_0^2) =: \Delta(Q^2)$. 
Occurrence of large logarithms

Monte Carlo implementation

Basic idea: Sudakov form factor with probabilistic interpretation \( \implies \) lends itself to simulation.

- Choose uniform random number \( \# \).
- If \( \# < \Delta(Q^2) \), then no branching.
- Otherwise: equate \( \# = \Delta(Q^2)/\Delta(q^2) \) and solve for \( q^2 \).
- Select \( z \) according to \( P(z) \).
- Remember: Freedom in interpretation of \( q^2 \) (mass, angle, transverse momentum) and \( z \) (energy, light-con fraction). No formal difference but numerical possibly large effects!
Occurrence of large logarithms

Many emissions

- Iterate emissions (jets)

Maximal result for \( t_1 > t_2 > \ldots t_n \):

\[
d\sigma \propto \sigma_0 \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \ldots \int_{Q_0^2}^{t_{n-1}} \frac{dt_n}{t_n} \propto \log^n \frac{Q^2}{Q_0^2}
\]

- How about \( Q^2 \)? Process-dependent!
Occurrence of large logarithms

Ordering the emissions: Radiation pattern

\[ q_1^2 > q_2^2 > q_3^2, \quad q_1^2 > q_2'^2 \]
Occurrence of large logarithms

Final state: Forward evolution

- Basic object: Sudakov form factor

\[ \Delta_{a \to bc}(t, t_0) = \exp \left[ -\int_{t_0}^{t} \frac{dt'}{t'} \int dz \frac{\alpha_s(k^2_\perp)}{2\pi} P_{a \to bc}(z) \right] \]

- Interpretation: Probability for a not to split into bc between t and t_0.
- Ideal for simulation. Select t and z from 1 − Δ = #

Initial state: Backward evolution

- In principle identical to final state, in practise different (both ends fixed)
- Use evolution equations (DGLAP-equation): Start at large Q^2 and work backwards
  Weight with PDFs at different values of x and t, “guarantee” proton
Occurrence of large logarithms

Forward vs. backward evolution: Pictorially
Occurrence of large logarithms

Use of DGLAP evolution

DGLAP evolution:
PDFs at \((x, Q^2)\) as function of PDFs at \((x_0, Q_0^2)\).

Backward evolution:
start from hard scattering at \((x, Q^2)\) and work down in \(q^2\) and up in \(x\).

Change in algorithm:
\[
\Delta_i(q^2) \quad \Rightarrow \quad \Delta_i(q^2)/f_i(x_i, q^2).
\]
Inclusion of quantum effects

Running coupling

- Effect of summing up higher orders (loops): $\alpha_s \rightarrow \alpha_s(k_{\perp}^2)$

- Much faster parton proliferation, especially for small $k_{\perp}^2$.

- Must avoid Landau pole: $k_{\perp}^2 > Q_0^2 \Rightarrow \Lambda_{\text{QCD}}^2$
  $\implies Q_0^2 = \text{physical parameter.}$
Inclusion of quantum effects

Soft logarithms: Angular ordering

- Soft limit for single emission also universal
- Problem: Soft gluons come from all over (not collinear!) Quantum interference? Still independent evolution?
- Answer: Not quite independent.

  - Assume photon into $e^+ e^-$ at $\theta_{ee}$ and photon off electron at $\theta$
  - Energy imbalance at vertex: $k_\perp^\gamma \sim zp\theta$, hence $\Delta E \sim k_\perp^2 / zp \sim zp\theta^2$.
  - Time for photon emission: $\Delta t \sim 1/\Delta E$.
  - $ee$-separation: $\Delta b \sim \theta_e e\Delta t > \Lambda/\theta \sim 1/(zp\theta)$
  - Thus: $\theta_{ee}/(zp\theta^2) > 1/(zp\theta) \implies \theta_{ee} > \theta$

Thus: Angular ordering takes care of soft limit.
Inclusion of quantum effects


Soft logarithms: Angular ordering

Gluons at large angle from combined color charge!
Inclusion of quantum effects

Soft logarithms: Angular ordering

Experimental manifestation:
$\Delta R$ of 2nd & 3rd jet in multi-jet events in pp-collisions
### Inclusion of quantum effects

**Resummed jet rates in $e^+e^- \rightarrow$ hadrons**


- Use Durham jet measure ($k_p\text{erp}$-type):
  
  $$k^2_{\perp,ij} = 2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij}) > Q^2_{\text{jet}}.$$  

- Remember prob. interpretation of Sudakov form factor.

- Then:
  
  $$R_2(Q_{\text{jet}}) = [\Delta_q(E_{\text{c.m.}}, Q_{\text{jet}})]^2$$  

  $$R_3(Q_{\text{jet}}) = 2\Delta_q(E_{\text{c.m.}}, Q_{\text{jet}})$$  

  $$\cdot \int dq \left[ \alpha_s(q) \bar{P}_q(E_{\text{c.m.}}, q) \frac{\Delta_q(E_{\text{c.m.}}, Q_{\text{jet}})}{\Delta_q(q, Q_{\text{jet}})} \frac{\Delta_q(q, Q_{\text{jet}})}{\Delta_q(q, Q_{\text{jet}})} \right]$$
Dipole shower(s)


**Upshot**

- Expansion around soft logs, particles always on-shell
  \[ d\sigma = \sigma_0 \frac{C_F \alpha_s(k^2)}{2\pi} \frac{d k^2}{k^2} dy. \]

- Always color-connected partners (recoil of emission)
  \[ \implies \text{emission: 1 dipole} \rightarrow 2 \text{ dipoles}. \]

- Quantum coherence on similar grounds for angular and \( k_T \)-ordering.
Dipole shower(s)


Radiation pattern

IS Radiation

- There is none!
  Treat radiation in DIS as FS radiation between remnant & quark
  Thus, no real Dipole Shower for pp collisions.

- Cut FS phase space of remnants:
Dipole shower(s): further developments

Initial state dipole showers

- Complete perturbative formulation.
- Dipoles and their radiation associated to IS-IS, IS-FS and FS-FS colour lines.
- Beam remnants kept outside evolution.
- Onshell kinematics, evolution in $k_\perp$.
- Being implemented into SHERPA by J.Winter.
Dipole shower(s): further developments

Initial state dipole showers

- Testbed: DY production.
- $P_T$ spectrum of $Z^0$ boson.
- Mainly recoils vs. 1st emission:
  - by construction: ME-corrected.
Dipole shower(s): further developments

DS based on Catani-Seymour splitting kernels


- Catani-Seymour dipole subtraction terms as universal framework for QCD NLO calculations.
- Factorization formulae for real emission process:
  - Full phase space coverage & good approx. to ME.
- Currently implemented into SHERPA by S. Schummann.

Example: final-state final-state dipoles

splitting: \( \tilde{p}_{ij} + \tilde{p}_k \rightarrow p_i + p_j + p_k \)

variables: \( y_{ij,k} = \frac{p_ip_j}{p_ip_j + p_ip_k + p_jp_k} \), \( z_i = \frac{p_ip_k}{p_ip_k + p_ip_j + p_jp_k} \)

consider \( q_{ij} \rightarrow q_{ig_j} \): \( \langle V_{q_{ig_j,k}}(\tilde{z}_i, y_{ij,k}) \rangle = C_F \left\{ \frac{2}{1 - \tilde{z}_i \tilde{z}_j y_{ij,k}} - (1 + \tilde{z}_i) \right\} \)
Dipole shower(s): further developments

DS based on Catani-Seymour splitting kernels

- $k_{\perp}$ as evolution parameter.
- Natural mass treatment: splitting variables & phase space kinematics.

$\bar{b}b + X$ at LEP I, 2-jet rate

---

F. Krauss
IPPP
QCD & Monte Carlo Tools
Dipole shower(s): further developments

DS based on Catani-Seymour splitting kernels

$p_\perp$ of DY pair at Tevatron I

$\eta$ of DY pair at Tevatron I

$\gamma^*/Z + X$ production @ Tevatron Run I

preliminary

F. Krauss
QCD & Monte Carlo Tools
### Survey of existing showering tools

<table>
<thead>
<tr>
<th>Tools</th>
<th>evolution</th>
<th>AO/Coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ariadne</td>
<td>$k_{perp}$-ordered</td>
<td>by construction</td>
</tr>
<tr>
<td>Herwig</td>
<td>angular ordering</td>
<td>by construction</td>
</tr>
<tr>
<td>Herwig++</td>
<td>improved angular ordering</td>
<td>by construction</td>
</tr>
<tr>
<td>Pythia</td>
<td>old: virtuality ordered</td>
<td>by hand</td>
</tr>
<tr>
<td></td>
<td>new: $k_\perp$-ordered</td>
<td>by construction</td>
</tr>
<tr>
<td>Sherpa</td>
<td>virtuality ordered (like old Pythia)</td>
<td>by hand</td>
</tr>
</tbody>
</table>
Summary of lecture 2

- Accelerated charges radiate (Bremsstrahlung); in QCD gluons are also charged \(\implies\) cascade of emissions.
- Probabilistic language from universal collinear or soft limits of QCD radiation.
- Factorization into individual emissions possible \(\implies\) nearly independent treatment (Markov chain).
- Various shower models, different levels of sophistication in realization of generic features of QCD.
- But: still need hadronization for hadron level!
- But: for jet physics soft & collinear limits maybe not good enough.
- Subject of next lectures.