QCD & Monte Carlo Tools

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Topics of the lectures

1. Lecture 1: *The Monte Carlo Principle*
   - Monte Carlo as integration method
   - Hard physics simulation: Parton Level event generation
2. Lecture 2: *Dressing the Partons*
   - Hard physics simulation, cont'd: Parton Showers
3. Lecture 3: *Modelling beyond Perturbation Theory*
   - Hadronic initial states: PDFs
   - Soft physics simulation: Hadronization
   - Beyond factorization: Underlying Event
4. Lecture 4: *Higher Orders in Monte Carlos*
   - Some nomenclature: Anatomy of HO calculations
   - Merging vs. Matching

Thanks to

- the other Sherpas: T.Gleisberg, S.Höche, S.Schumann, F.Siegert, M.Schönherr, J.Winter;
- other MC authors: S.Gieseke, K.Hamilton, L.Lonnblad, F.Maltoni, M.Mangano, P.Richardson, M.Seymour, T.Sjostrand, B.Webber, ...

Simulation's paradigm

Basic strategy

Divide event into stages, separated by different scales.

- Signal/background:

  Exact matrix elements.

- QCD-Bremsstrahlung:

  Parton showers (also in initial state).

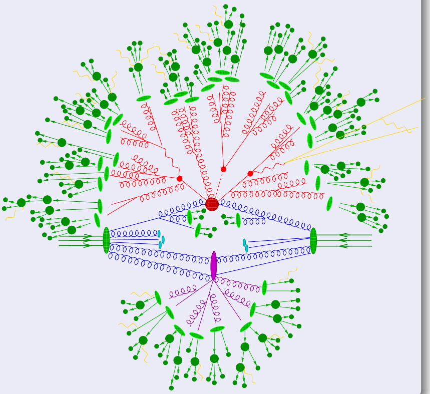
- Multiple interactions:

  Beyond factorization: Modeling.

- Hadronization:

  Non-perturbative QCD: Modeling.

Sketch of an event



Outline of today's lecture

- Why parton showers?
- Large logs in QCD radiation: Parton shower
- Including quantum effects in parton showering
- Dipole shower(s)
- Survey of existing showering tools

Motivation: Why parton showers?

Common wisdom

- Well-known: Accelerated charges radiate
- QED: Electrons (charged) emit photons
  Photons split into electron-positron pairs
- QCD: Quarks (colored) emit gluons
  Gluons split into quark pairs
- Difference: Gluons are colored (photons are not charged)
  Hence: Gluons emit gluons!
- Cascade of emissions: Parton shower

Motivation: Why parton showers?

Some more refined reasons

- Experimental definition of jets based on hadrons.
- But: Hadronization through phenomenological models (need to be tuned to data).
- Wanted: Universality of hadronization parameters (independence of hard process important).
- Link to fragmentation needed: Model softer radiation (inner jet evolution).
- Similar to PDFs (factorization) just the other way around (fragmentation functions at low scale, parton shower connects high with low scale).

Occurrence of large logarithms

$e^+e^- \to$ jets

- Differential cross section:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}x_1\mathrm{d}x_2} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

Singular for $x_{1,2}\to 1$.

- Rewrite with opening angle θ_{qg} and gluon energy fraction $x_3 = 2E_g/E_{\mathrm{c.m.}}$:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}\cos\theta_{qg}\mathrm{d}x_3} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\left[\frac{2}{\sin^2\theta_{qg}}\frac{1+(1-x_3)^2}{x_3} - x_3\right]$$

Singular for $x_3\to 0$ ("soft"), $\sin\theta_{qg}\to 0$ ("collinear").

Occurrence of large logarithms

Collinear singularities

- Use

$$\frac{2\mathrm{d}\cos\theta_{qg}}{\sin^2\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{qg}}{1+\cos\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{\bar{q}g}}{1-\cos\theta_{\bar{q}g}} \approx \frac{\mathrm{d}\theta^2_{qg}}{\theta^2_{qg}} + \frac{\mathrm{d}\theta^2_{\bar{q}g}}{\theta^2_{\bar{q}g}}$$

- Independent evolution of two jets (q and $\bar{q}$):

$$\mathrm{d}\sigma_{ee\to 3j} \approx \sigma_{ee\to 2j} \sum_{j\in\{q,\bar{q}\}} \frac{C_F\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2_{jg}}{\theta^2_{jg}} P(z)\,,$$

where $P(z) = \frac{1+(1-z)^2}{z}$ (DGLAP splitting function)

Occurrence of large logarithms

Expressing the collinear variable

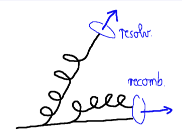
- Same form for any $t \propto \theta^2$:
- Transverse momentum $k_\perp^2 \approx z^2(1-z)^2E^2\theta^2$
- Invariant mass $q^2 \approx z(1-z)E^2\theta^2$

$$\frac{\mathrm{d}\theta^2}{\theta^2} \approx \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \approx \frac{\mathrm{d}q^2}{q^2}$$

Occurrence of large logarithms

Parton resolution

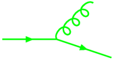
- What is a parton?
  Collinear pair/soft parton recombine!
- Introduce resolution criterion $k_\perp > Q_0$.



- Combine virtual contributions with unresolvable emissions:
  Cancels infrared divergences $\Longrightarrow$ Finite at $\mathcal{O}(\alpha_s)$

  (Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)

- Unitarity: Probabilities add up to one
  $\mathcal{P}(\text{resolved}) + \mathcal{P}(\text{unresolved}) = 1.$

 $+$  $+$  $= 1.$

Occurrence of large logarithms

The Sudakov form factor

- Diff. probability for emission between q^2 and $q^2 + \mathrm{d}q^2$:

$$\mathrm{d}\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}q^2}{q^2}\int\limits_{Q_0^2/q^2}^{1-Q_0^2/q^2} \mathrm{d}z P(z) =: \frac{\mathrm{d}q^2}{q^2}\bar{P}(q^2)\,.$$

- No-emission probability $\Delta(Q^2,\, q^2)$ between Q^2 and q^2.

Evolution equation for Δ: $-\frac{\mathrm{d}\Delta(Q^2,q^2)}{\mathrm{d}q^2} = \Delta(Q^2,\, q^2)\frac{\mathcal{P}}{\mathrm{d}q^2}$.

$$\Longrightarrow \quad \Delta(Q^2,\, q^2) = \exp\left[-\int\limits_{q^2}^{Q^2}\frac{dk^2}{k^2}\,\bar{P}(k^2)\right]\,.$$

Occurrence of large logarithms

The Sudakov form factor

- $\Delta(Q^2, q^2)$ is the Sudakov form factor.
- Remember it is given by $\Delta(Q^2, q^2) = \exp\left[-\int\limits_{q^2}^{Q^2} \frac{dk^2}{k^2} \bar{P}(k^2)\right] \approx \exp\left[-C_F \frac{\alpha_s}{2\pi} \log^2 \frac{Q^2}{Q_0^2}\right]$ for quarks.
- Use $\Delta(Q^2, Q_0^2) =: \Delta(Q^2)$.

Occurrence of large logarithms

Monte Carlo implementation

Basic idea: Sudakov form factor with probabilistic interpretation $\Longrightarrow$ lends itself to simulation.

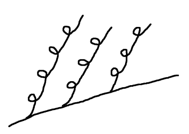
- Choose uniform random number $\#$.
- If $\# < \Delta(Q^2)$, then no branching.
- Otherwise: equate $\# = \Delta(Q^2)/\Delta(q^2)$ and solve for q^2.
- Select z according to $P(z)$.
- Remember: Freedom in interpretation of q^2 (mass, angle, transverse momentum) and z (energy, light-con fraction). No formal difference but numerical possibly large effects!

Occurrence of large logarithms

Many emissions

- Iterate emissions (jets)

Maximal result for $t_1 > t_2 > \ldots t_n$:

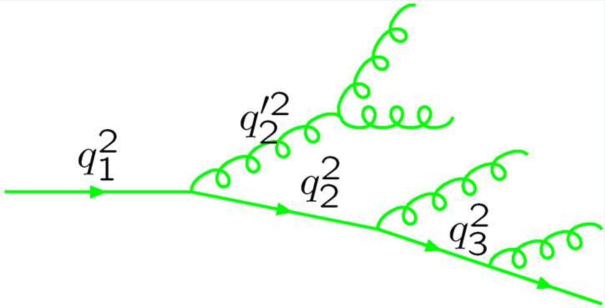


$$d\sigma \propto \sigma_0 \int\limits_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int\limits_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \ldots \int\limits_{Q_0^2}^{t_{n-1}} \frac{dt_n}{t_n} \propto \log^n \frac{Q^2}{Q_0^2}$$

- How about Q^2? Process-dependent!

Occurrence of large logarithms

Ordering the emissions : Radiation pattern



$$q_1^2 > q_2^2 > q_3^2,\; q_1^2 > {q_2'}^2$$

Occurrence of large logarithms

Final state: Forward evolution

- Basic object: Sudakov form factor

$$\Delta_{a\to bc}(t,\, t_0) = \exp\left[-\int\limits_{t_0}^{t}\frac{\mathrm{d}t'}{t'}\int \mathrm{d}z\,\frac{\alpha_s(k_\perp^2)}{2\pi}P_{a\to bc}(z)\right]$$

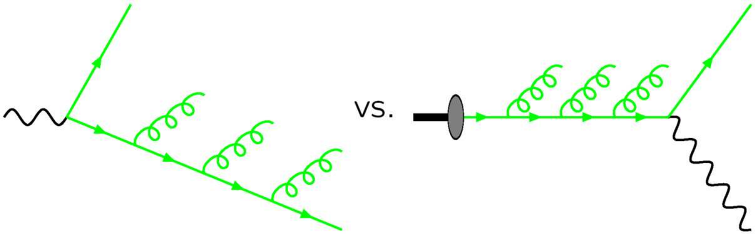
- Interpretation: Probability for a not to split into bc between t and t_0.
- Ideal for simulation. Select t and z from $1-\Delta = \#$

Initial state: Backward evolution

- In principle identical to final state, in practise different (both ends fixed)
- Use evolution equations (DGLAP-equation): Start at large Q^2 and work backwards
  Weight with PDFs at different values of x and t, "guarantee" proton

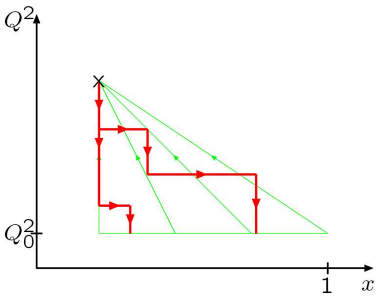
Occurrence of large logarithms

Forward vs. backward evolution: Pictorially



Occurrence of large logarithms

Use of DGLAP evolution



DGLAP evolution:

PDFs at (x, Q^2) as function of PDFs at (x_0, Q_0^2).

Backward evolution:

start from hard scattering at (x, Q^2) and work down in q^2 and up in x.

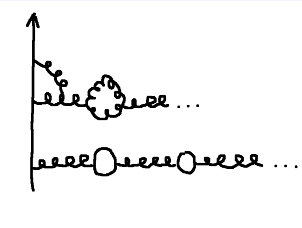
Change in algorithm:

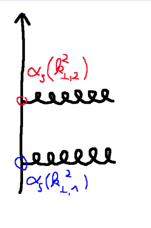
$\Delta_i(q^2) \Longrightarrow \Delta_i(q^2)/f_i(x_i, q^2)$.

Inclusion of quantum effects

Running coupling

- Effect of summing up higher orders (loops): $\alpha_s \longrightarrow \alpha_s(k_\perp^2)$



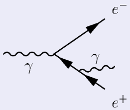


- Much faster parton proliferation, especially for small $k_\perp^2$.
- Must avoid Landau pole: $k_\perp^2 > Q_0^2 \gg \Lambda^2_{\rm QCD}$
  $\Longrightarrow Q_0^2 =$ physical parameter.

Inclusion of quantum effects

Soft logarithms : Angular ordering

- Soft limit for single emission also universal
- Problem: Soft gluons come from all over (not collinear!) Quantum interference? Still independent evolution?
- Answer: Not quite independent.
  - Assume photon into e^+e^- at θ_{ee} and photon off electron at θ
  - Energy imbalance at vertex: $k_\perp^\gamma \sim zp\theta$, hence $\Delta E \sim k_\perp^2/zp \sim zp\theta^2$.
  - Time for photon emission: $\Delta t \sim 1/\Delta E$.
  - ee-separation: $\Delta b \sim \theta_e e\Delta t > \Lambda/\theta \sim 1/(zp\theta)$
  - Thus: $\theta_{ee}/(zp\theta^2) > 1/(zp\theta) \implies \theta_{ee} > \theta$

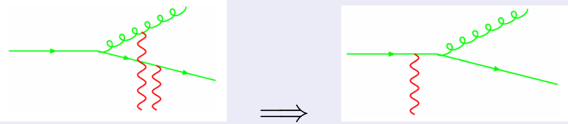


- Thus: Angular ordering takes care of soft limit.

Inclusion of quantum effects

G.Marchesini and B.R.Webber, Nucl. Phys. B **238** (1984) 1.

Soft logarithms : Angular ordering

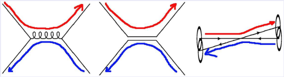


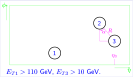
Gluons at large angle from combined color charge!

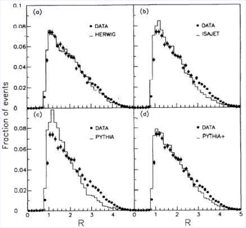
Inclusion of quantum effects

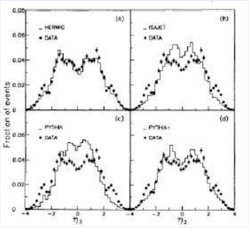
Soft logarithms : Angular ordering

Experimental manifestation:
ΔR of 2nd & 3rd jet in multi-jet events in pp-collisions









Inclusion of quantum effects

Resummed jet rates in $e^+e^- \longrightarrow$ hadrons

S.Catani *et al.* Phys. Lett. **B269** (1991) 432

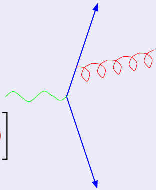
- Use Durham jet measure ($k_p erp$-type):

$$k^2_{\perp,ij} = 2\mathrm{min}(E_i^2,\, E_j^2)(1 - \cos\theta_{ij}) > Q^2_{\mathrm{jet}}\,.$$

- Remember prob. interpretation of Sudakov form factor.
- Then:

$$\mathcal{R}_2(Q_{\mathrm{jet}}) = \left[\Delta_q(E_{\mathrm{c.m.}},\, Q_{\mathrm{jet}})\right]^2$$

$$\mathcal{R}_3(Q_{\mathrm{jet}}) = 2\Delta_q(E_{\mathrm{c.m.}},\, Q_{\mathrm{jet}}) \cdot \int \mathrm{d}q \left[\alpha_s(q)\bar{P}_q(E_{\mathrm{c.m.}},\, q)\frac{\Delta_q(E_{\mathrm{c.m.}},\, Q_{\mathrm{jet}})}{\Delta_q(q,\, Q_{\mathrm{jet}})}\Delta_q(q,\, Q_{\mathrm{jet}})\Delta_g(q,\, Q_{\mathrm{jet}})\right]$$



Dipole shower(s)

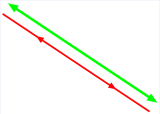
Implemented in Ariadne (L.Lonnblad, Comput. Phys. Commun. **71**, 15 (1992)).

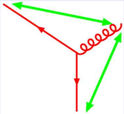
Upshot

- Expansion around soft logs, particles always on-shell

$$\mathrm{d}\sigma = \sigma_0 \frac{C_F \alpha_s(k_\perp^2)}{2\pi} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \mathrm{d}y.$$

- Always color-connected partners (recoil of emission)
  $\Longrightarrow$ emission: 1 dipole $\to$ 2 dipoles.



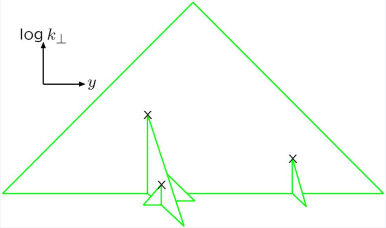


- Quantum coherence on similar grounds for angular and k_T-ordering.

Dipole shower(s)

Implemented in Ariadne (L.Lonnblad, Comput. Phys. Commun. **71**, 15 (1992)).

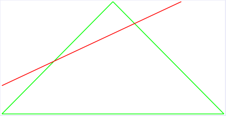
Radiation pattern



IS Radiation

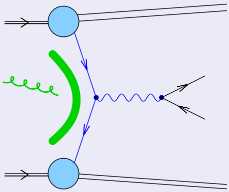
- There is none!
  Treat radiation in DIS as FS radiation between remnant & quark

  Thus, no real Dipole Shower for *pp* collisions.
- Cut FS phase space of remnants:



Dipole shower(s): further developments

Initial state dipole showers

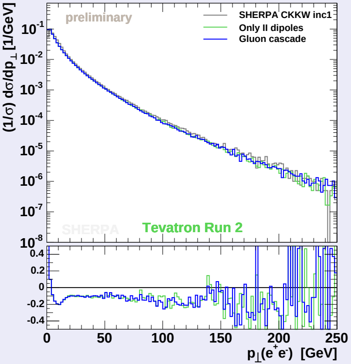


- Complete perturbative formulation.
- Dipoles and their radiation associated to **IS-IS**, **IS-FS** and **FS-FS** colour lines.
- Beam remnants kept outside evolution.
- Onshell kinematics, evolution in $k_\perp$.
- Being implemented into SHERPA by J.Winter.

Dipole shower(s): further developments

Initial state dipole showers

- Testbed: DY production.
- P_T spectrum of Z^0 boson.
- Mainly recoils vs. 1st emission:
  by construction: ME-corrected.



Dipole shower(s): further developments

DS based on Catani-Seymour splitting kernels

First discussed in: Z.Nagy and D.E.Soper, JHEP **0510** (2005) 024.

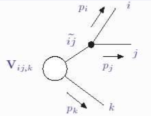
- Catani-Seymour dipole subtraction terms as universal framework for QCD NLO calculations.
- Factorization formulae for real emission process:
- Full phase space coverage & good approx. to ME.
- Currently implemented into SHERPA by S.Schumann.

Example: final-state final-state dipoles

splitting: $\tilde{p}_{ij} + \tilde{p}_k \longrightarrow p_i + p_j + p_k$

variables: $y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$, $z_i = \frac{p_i p_k}{p_i p_k + p_j p_k}$

consider $\mathrm{q}_{ij} \to \mathrm{q}_i \mathrm{g}_j$: $\langle V_{\mathrm{q}_i \mathrm{g}_j,k}(\tilde{z}_i, y_{ij,k})\rangle = C_\mathrm{F}\left\{\frac{2}{1-\tilde{z}_i+\tilde{z}_i y_{ij,k}} - (1+\tilde{z}_i)\right\}$

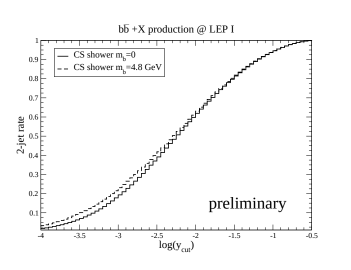


Dipole shower(s): further developments

DS based on Catani-Seymour splitting kernels

- $k_\perp$ as evolution parameter.
- Natural mass treatment: splitting variables & phase space kinematics

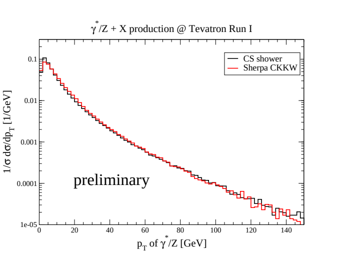
$b\bar{b} + X$ at LEP I, 2-jet rate



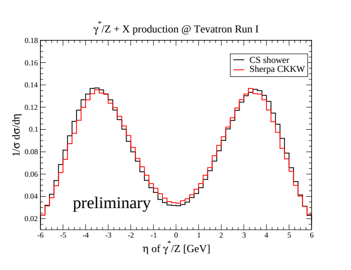
Dipole shower(s): further developments

DS based on Catani-Seymour splitting kernels

$p_\perp$ of DY pair at Tevatron I



η of DY pair at Tevatron I



Survey of existing showering tools

Tools	evolution	AO/Coherence
Ariadne	k_perp-ordered	by construction
Herwig Herwig++	angular ordering improved angular ordering	by construction by construction
Pythia	old: virtuality ordered new: $k_\perp$-ordered	by hand by construction
Sherpa	virtuality ordered (like old Pythia)	by hand

Summary of lecture 2

- Accelerated charges radiate (Bremsstrahlung);
  in QCD gluons are also charged $\Longrightarrow$ cascade of emissions.
- Probabilistic language from universal collinear or soft limits of QCD radiation.
- Factorization into individual emissions possible
  $\Longrightarrow$ nearly independent treatment (Markov chain).
- Various shower models, different levels of sophistication in realization of generic features of QCD.
- But: still need hadronization for hadron level!
- But: for jet physics soft & collinear limits maybe not good enough.
- Subject of next lectures.