ON THE EQUIVALENCE PRINCIPLE IN SCALAR-TENSOR THEORIES

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based on work in progress
with **Paolo Creminelli, Jerome Gleyzes, Filippo Vernizzi**

does equivalence principle hold for objects moving
on a cosmological background
in (approximately) shift symmetric modified gravity
theories?

EQUIVALENCE PRINCIPLE

- Weak Equivalence Principle (WEP): $m_{\mathrm{iner}}\ddot{X}^i = -m_{\mathrm{grav}}\partial_i\Phi$
  - the inertial and gravitational masses of a body are equal
  - "all objects fall with the same rate" independently on their internal structure
  - freely falling test particles follow geodesics of a metric
- Strong Equivalence Principle (SEP): the WEP holds also for self-gravitating bodies (e.g. neutron stars (10-20 %), black holes (100%))

★ WEP is presumed to hold in theories with universally coupled matter fields

★ SEP is a way to test theories in which the gravitational interactions are modified

- Experimental tests of SEP:
  - Nordtvedt effect
  - Dipole gravitational radiation from mixed binaries

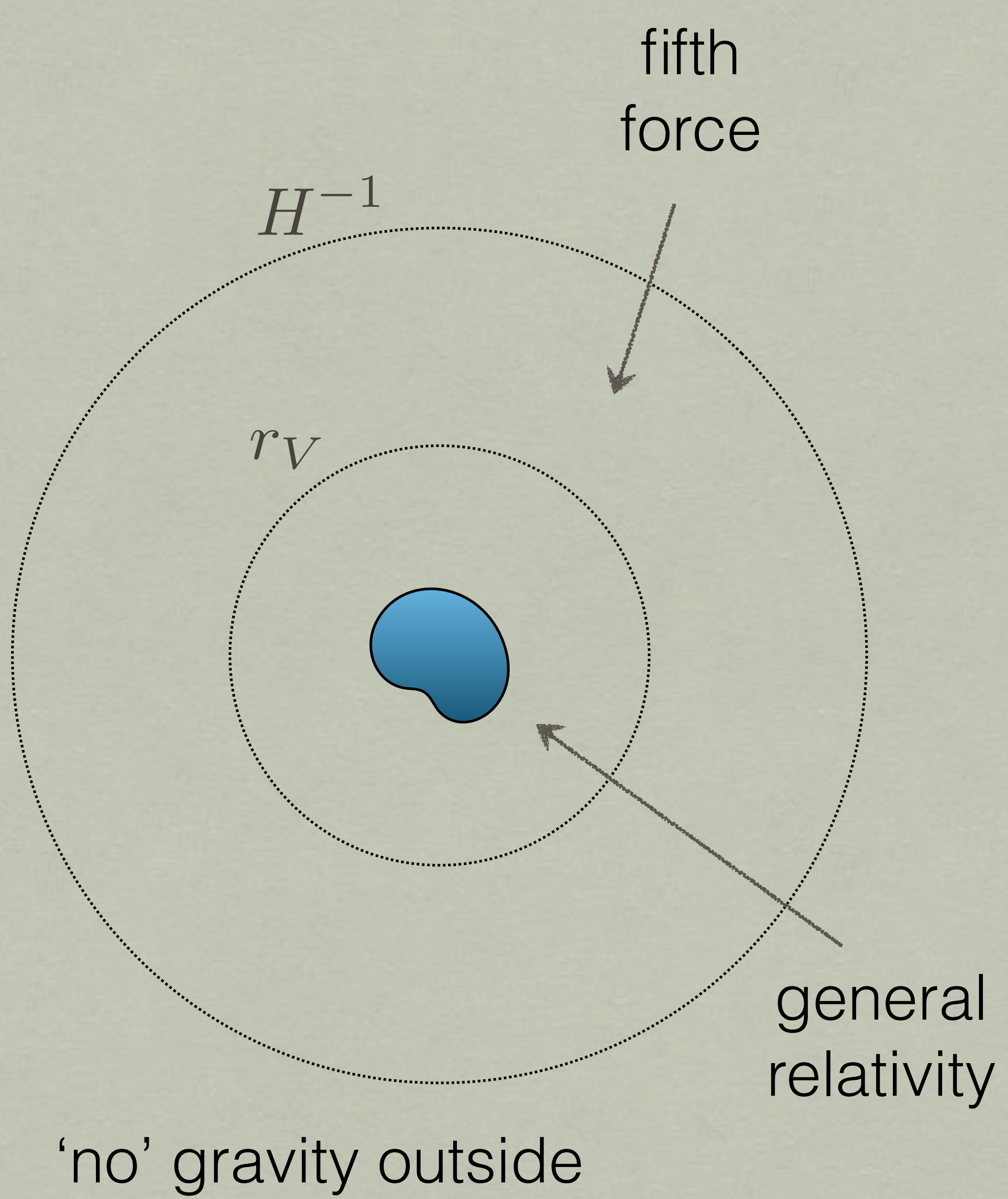
MODIFIED GRAVITY

- **Idea:** modify gravity at large scales, but keep it the same (i.e GR) at small scales
- **Examples:** galileons, massive gravity, etc.

  Nicolis, Rattazzi, Trincherini (2009)   de Rham, Gabadadze, Tolley (2010)
- **Common feature:** typically of a scalar-tensor type with an additional light scalar, π
- **Crucial:** need a screening mechanism
  - Chameleon—the scalar acquires a large mass in dense environments

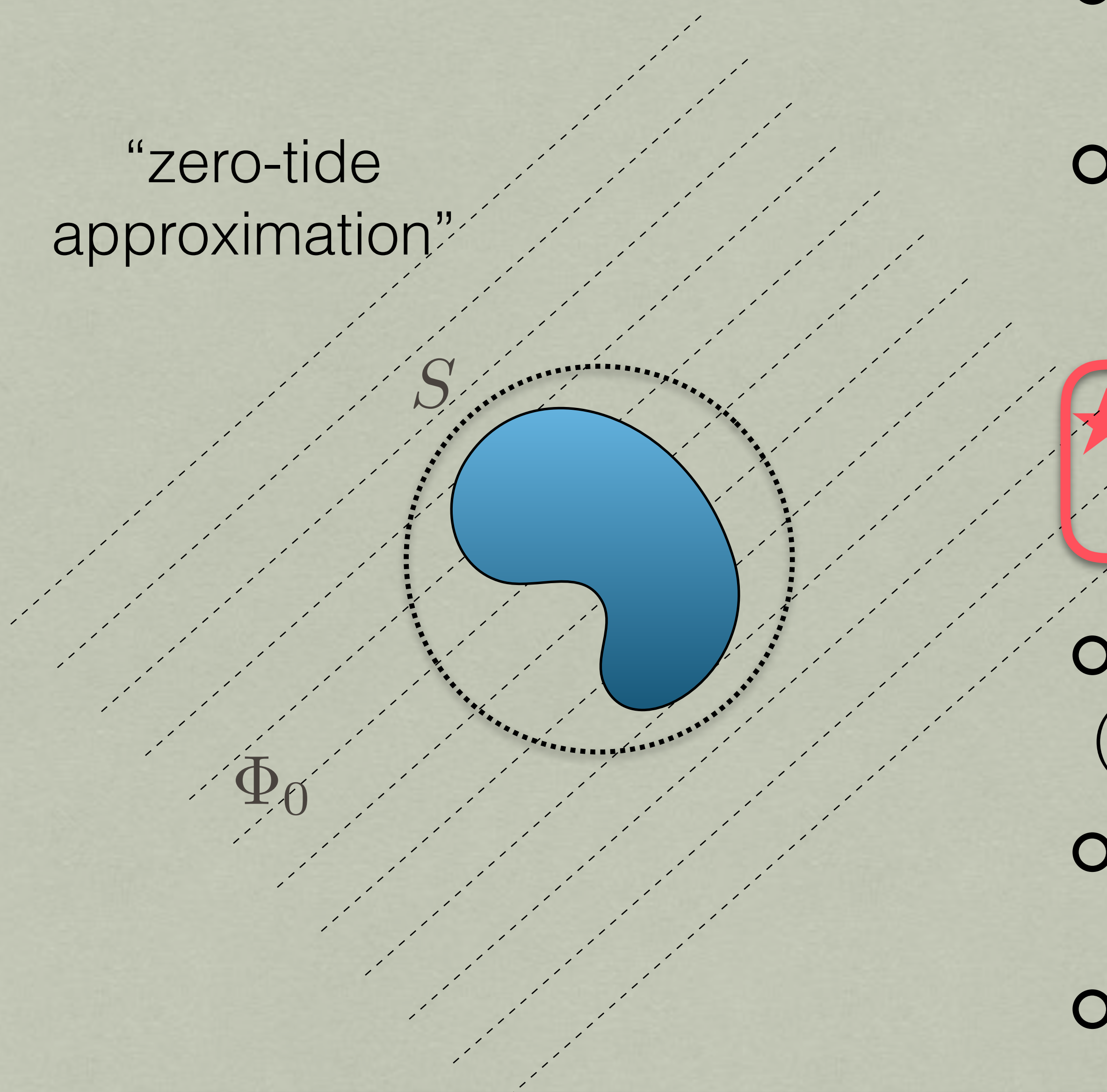
    Khoury, Weltmann (2004)
  - Vainshtein—derivative interactions $(\partial\pi)^2\Box\pi$ become strongly coupled at short scales

    Vainshtein (1972)



FRAMEWORK

Einstein, Infeld, Hoffmann (1938)
Damour (1989)



- define a pseudo-stress tensor (conserved)

$$G^{(1)}{}_{\mu}{}^{\nu} = 8\pi G_N t_{\mu}{}^{\nu}$$

- momentum of the object

$$\dot{P}_i = \partial_0 \int d^3x\, t_i{}^0 = -\oint dS_j\, t_i{}^j$$

★ **trick 1:** reduce the calculation to a surface integral—no need to know the internal structure

- ADM mass: $M = -\int d^3x\, t_0{}^0\,, \quad \dot{M} = 0$ (conserved)
- c.o.m. coordinate $X^i = -\int d^3x\, x^i t_0{}^0/M$
- acceleration: $M\ddot{X}^i = \dot{P}_i$

★ **trick 2:** background—object split $\Phi = \Phi_0 + \Phi_1(r)\,, \quad \Phi_0 = \Phi_0(0) + \partial_i\Phi_0(0)x^i$

- in GR: $\dot{P}_i = \dfrac{1}{8\pi G_N}\oint dS_j\, G^{(2)}\,i\,{}^j = -M\partial_i\Phi_0$

WEP VIOLATION

Hui, Nicolis, Stubbs (2012)

- In a modified gravity with **universal coupling** to matter fields one naively expects all objects with negligible self-gravity to move on geodesics with only post-Newtonian violations of the EP of the order $\mathcal{O}(1/c^2)$
- In theories with a **screening mechanism** this is not true in general:
  - **Chameleon screening:** $\mathcal{O}(1)$ violations of EP
    - operates in Brans-Dicke-type theories with non-linear potential

$$S = M_{\rm Pl}^2 \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2}\tilde{R} - \frac{1}{2}\tilde{\nabla}_\mu\varphi\tilde{\nabla}^\mu\varphi - V(\varphi)\right] + \int d^4x\, \mathcal{L}_m(\psi_m, \Omega^{-2}(\varphi)\tilde{g}_{\mu\nu})$$

    - screened objects have a different scalar charge: $\varphi \propto -\epsilon \dfrac{G_N M}{r}$ $(\epsilon = 0)$
    - in Jordan frame test particles move as: $M\ddot{X}^i = M\left[\dfrac{1+2\epsilon\alpha^2}{1+2\alpha^2}\right]\partial_i\Phi_0$
  - **Vainshtein screening** (operates in theories with derivative self-interactions): no $\mathcal{O}(1)$ violations of EP. **Why is that so?**

cosmological backgrounds

EFT OF DARK ENERGY I

Gubitosi, Piazza, Vernizzi (2013)

- **the guiding principle:** FRW background picks a preferred time slicing— write an EFT for the Goldstones of spontaneously broken time translations in unitary gauge

- **EFT of inflation:** unifies all single field inflationary models in one framework; to be compatible with residual symmetries write an EFT using the building blocks

  Cheung et al. (2008)

$$\delta g^{00}, \delta K^{\mu}{}_{\nu}, \delta K, \delta R$$

- **EFT of dark energy:** in late time universe matter species are present

$$S \equiv \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{m_2^4}{2} \left(\delta g^{00}\right)^2 - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 (\delta K^2 - \delta K^{\mu}_{\nu} \delta K^{\nu}_{\mu}) + \frac{\tilde{m}_4^2}{2} R\, \delta g^{00} \right] + \int d^4x \sqrt{-g}\, \mathcal{L}_m(\psi_m, g_{\mu\nu})$$

- the background evolution is determined by $c(t), \Lambda(t), f(t)$

- the mass parameters $m_i(t)$ need to be constrained by observations

EFT OF DARK ENERGY II

- covariant actions can be recovered by Stückelberg trick: $t \to t + \pi(t, \vec{x})$ and reintroducing the scalar field as

$$\frac{\phi}{M_{\rm Pl}^2} = t + \pi(t, \vec{x})\,, \quad \delta g^{00} \to 1 + \frac{X}{M_{\rm Pl}^4}\,, \quad X \equiv (\partial\phi)^2$$

- **Horndeski theories:**
  - the most general scalar-tensor theories with second order field equations
    Horndeski (1974)
  - covariant generalizations of flat space galileons
    Deffayet et al. (2011)
  - antisymmetric structure of 2nd order derivatives in the action
  - contained in our models for $m_4 = \tilde{m}_4$
- **beyond Horndeski theories:** allows for higher number of derivatives in equations of motion while still evading the 'Ostrogradski ghost', $m_4 \neq \tilde{m}_4$

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$$
\begin{aligned}
L_2^H[G_2] &\equiv G_2(\phi, X)\ ,\\
L_3^H[G_3] &\equiv G_3(\phi, X)\,\Box\phi\ ,\\
L_4^H[G_4] &\equiv G_4(\phi, X)\,{}^{(4)}R - 2G_{4X}(\phi, X)\left[(\Box\phi)^2 - (\nabla^\mu\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi)\right]\ ,\\
L_5^H[G_5] &\equiv G_5(\phi, X)\,{}^{(4)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi + \frac{1}{3}G_{5X}(\phi, X)\times\\
&\qquad \left[(\Box\phi)^3 - 3\,\Box\phi\,(\nabla^\mu\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi) + 2\,(\nabla_\mu\nabla_\nu\phi)(\nabla^\sigma\nabla^\nu\phi)(\nabla_\sigma\nabla^\mu\phi)\right]
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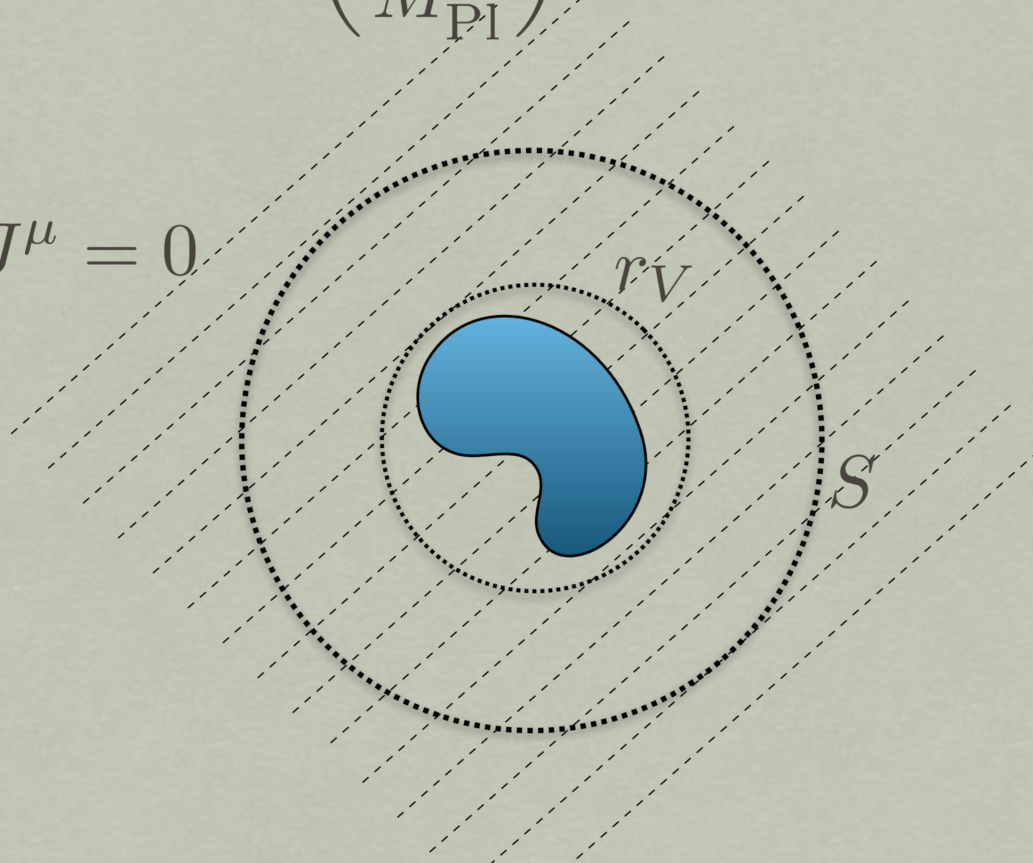
SHIFT SYMMETRY

- after covariantization the action for ϕ is 'almost' shift symmetric, except for the terms:
$$\Lambda = \Lambda\left(\frac{\phi}{M_{\rm Pl}^2}\right)\,, \quad c = c\left(\frac{\phi}{M_{\rm Pl}^2}\right)\,, \quad H = H\left(\frac{\phi}{M_{\rm Pl}^2}\right)\,.$$
- the equation of motion for the scalar
$$\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta\phi} = \frac{\partial\mathcal{L}}{\partial\phi} - \nabla_\mu J^\mu = 0$$
  - has an explicit source term
  - the time component $J^0 \neq 0$



- **in quasi-static subhorizon limit** for $k \gg H$
  - the source is always negligible
  - the non-linear contributions to the eom are dominated by $\partial_i J^i$ so that:
$$2c\Delta\pi + m_3^3\left(H\Delta\pi + \Delta\Phi\right) = \frac{m_3^3}{a^2}\partial_i\left[\partial_i\pi\Delta\pi - \partial_j\pi\partial_i\partial_j\pi\right]$$

One can solve for the asymptotic behavior of the fields!

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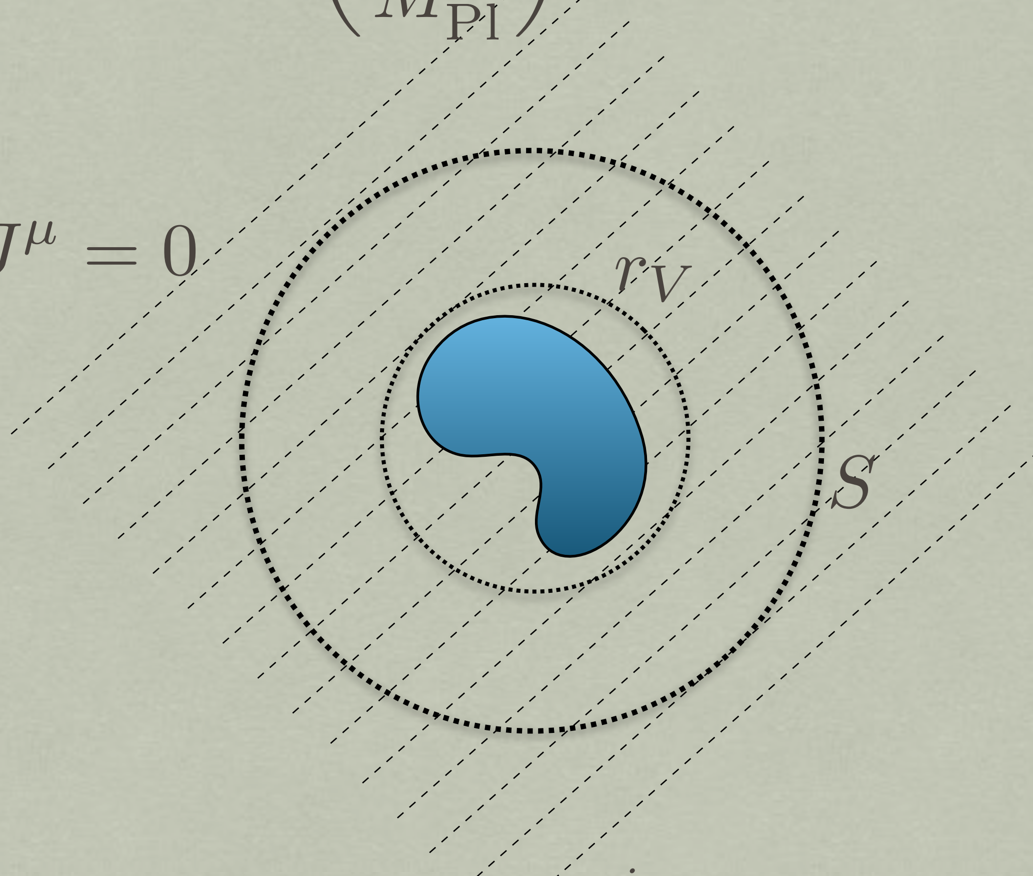

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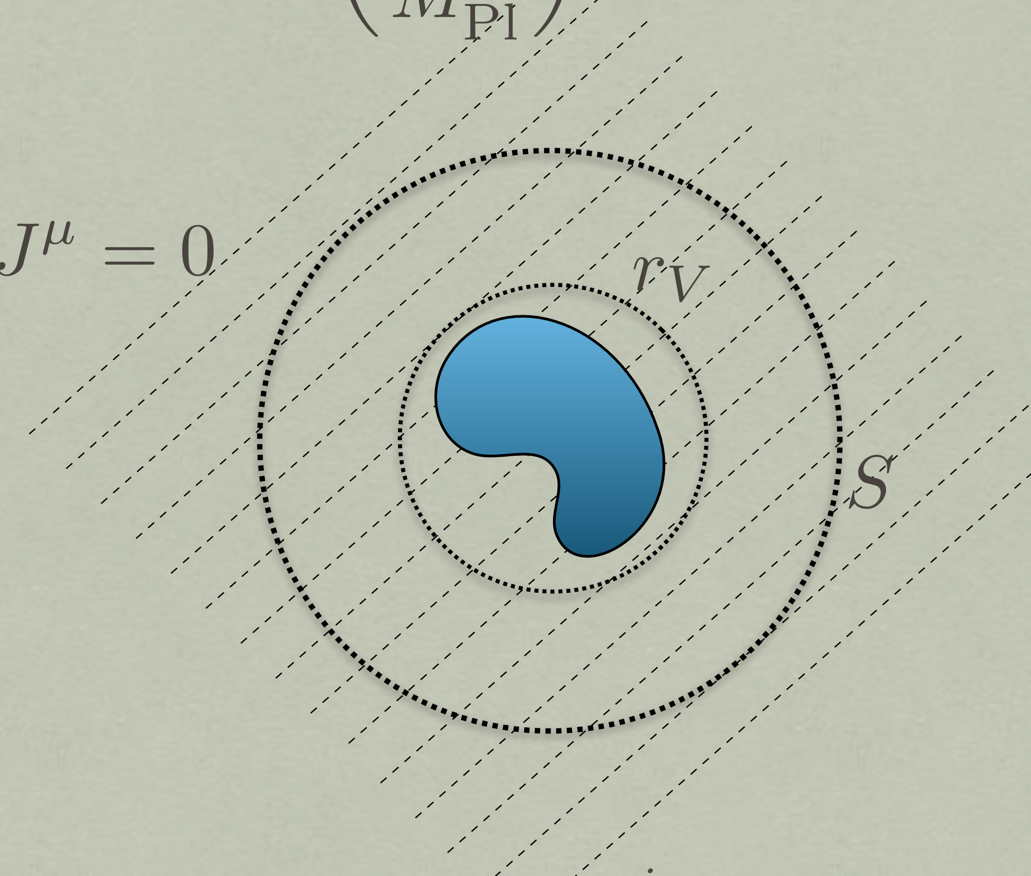

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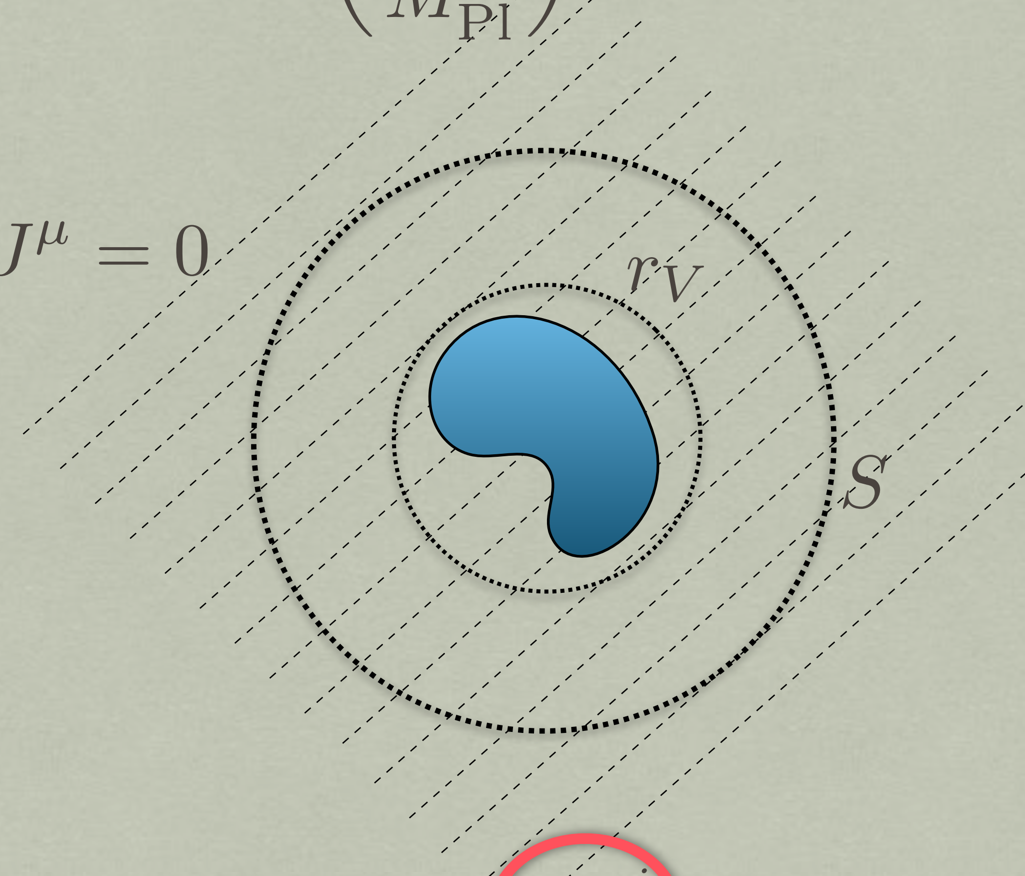

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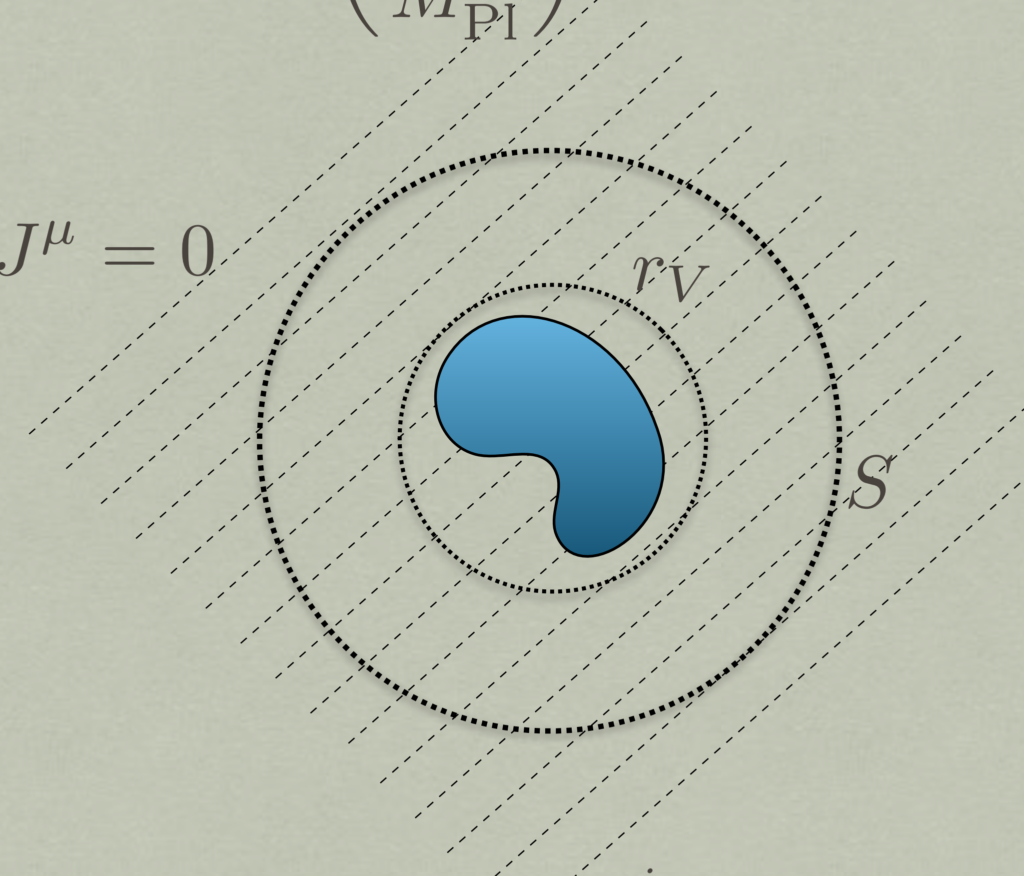

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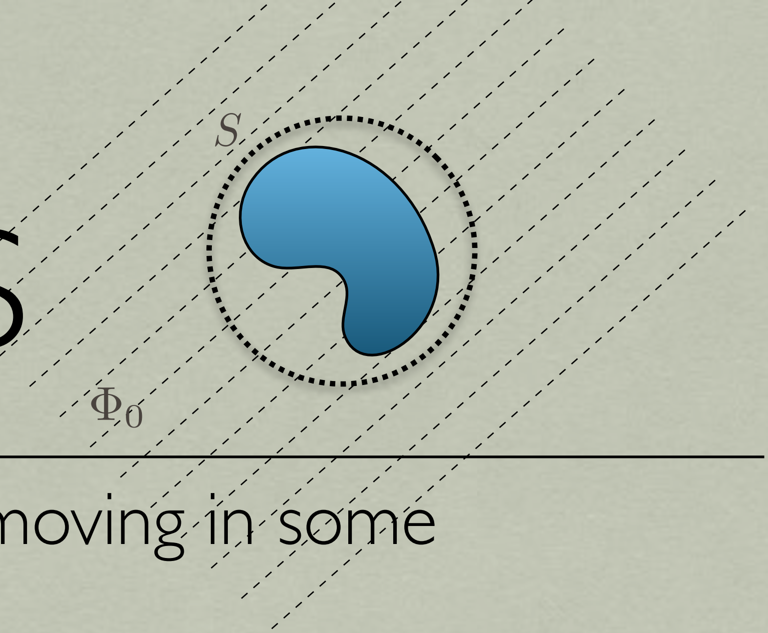
TEST PARTICLES



- **Goal:** find the gravitational force exerted on an object moving in some background gravitational field Φ_0 in FRW spacetime

$$ds^2 = -e^{2\Phi}dt^2 + a^2(t)e^{-2\Psi}\delta_{ij}dx^i dx^j$$

- define the pseudo-stress tensor as

$$M_{\rm Pl}^2 f(t) G^{\rm L}_{\mu\nu} - T^{\rm L,\pi}_{\mu\nu} = \tau_{\mu\nu}\,, \quad \tau_{\mu\nu} \equiv T^{\rm NL,\pi}_{\mu\nu} + \delta T^{\rm m}_{\mu\nu} - M_{\rm Pl}^2 f(t) G^{\rm NL}_{\mu\nu}$$

and the momentum P_i , total mass M , c.o.m. coordinate X^i wrt to it

- **in distinction from GR**:
  - $\tau_{\mu\nu}$ is not conserved, so that $\dot{M} \neq 0\,, \quad \dot{X}^i \neq P^i/M$
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- for a test particle **perturbative approximation is valid** everywhere and we find easily:

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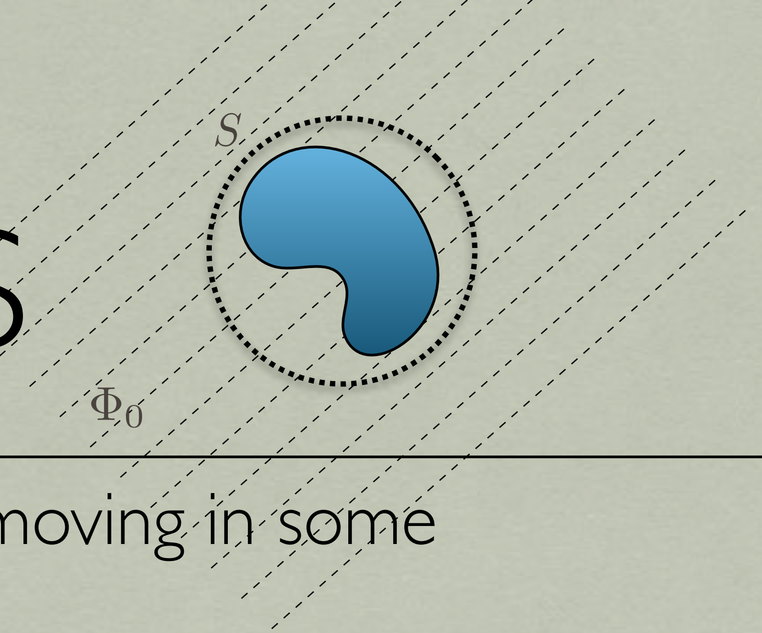

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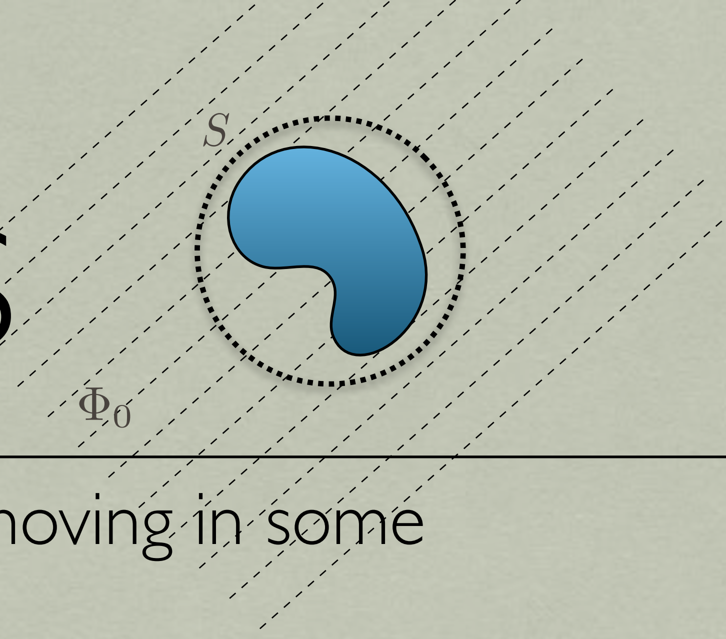

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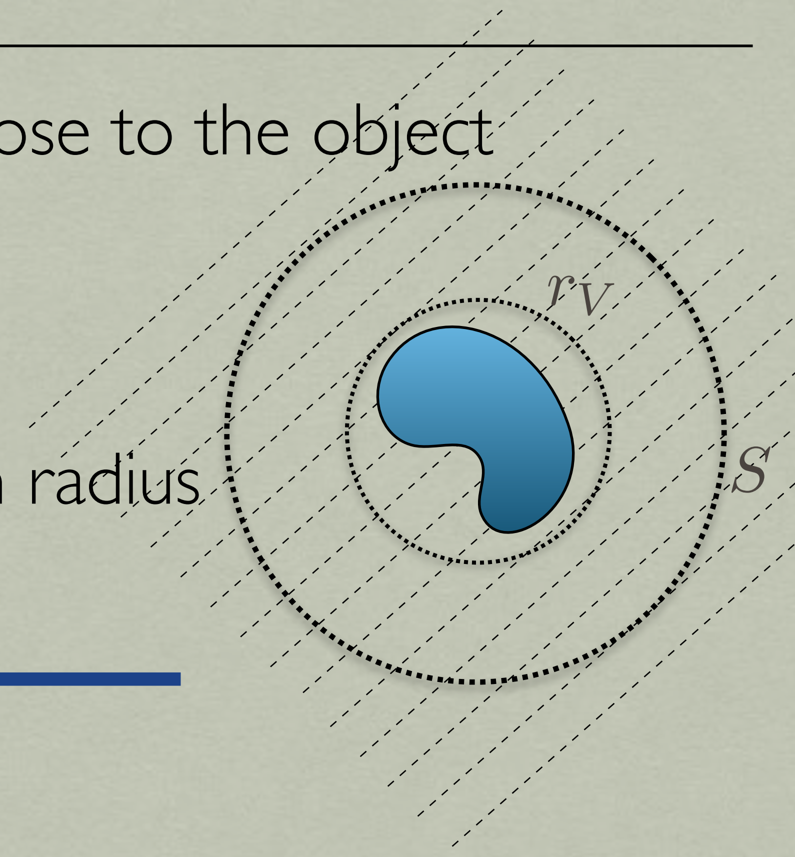
$$\ddot{X}^i + 2H\dot{X}^i = \frac{1}{a^2 M}\partial_0\left[P_i - \int d^3x\, a^3\, T^{(2,\pi)0}{}_i\right] = -\frac{1}{a^2}\partial_i\Phi_0 \quad ✓$$

(SCREENED) EXTENDED OBJECTS

- **In distinction from test particles:** screening effects close to the object must be taken into account
- this means:
  - perturbative expansion is not valid within Vainshtein radius
  - volume integrals must be taken with care



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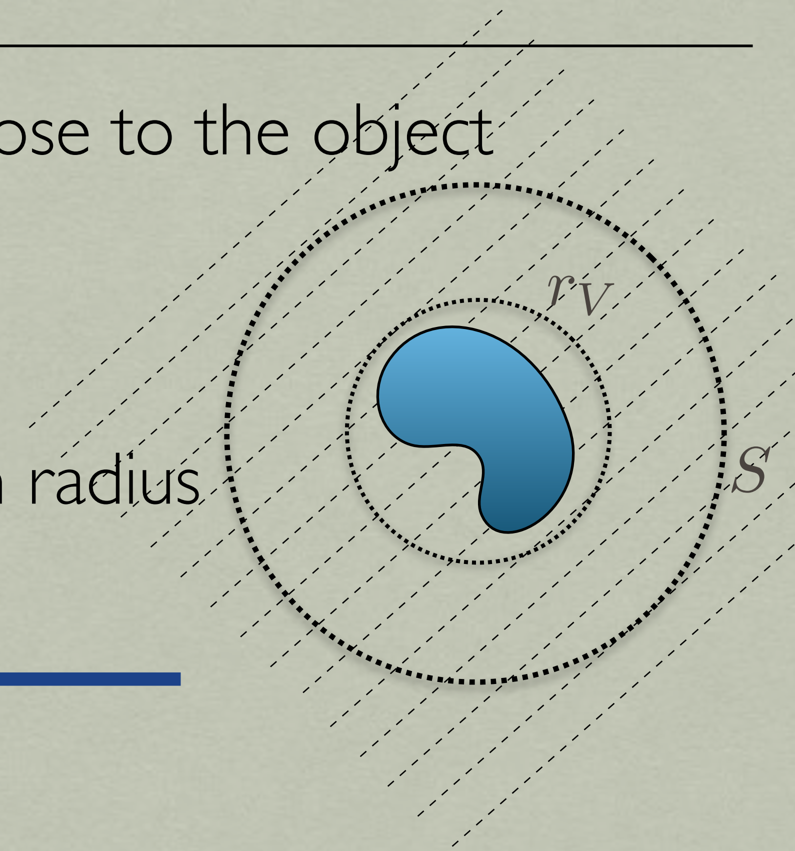

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1. asymptotic behavior depends on the total mass

$$M_0 \quad \rightarrow \quad M = -\int d^3x\, a^3\, \tau^0{}_0$$

2. it is crucial that the scalar field e.o.m. takes the form of a conserved current:

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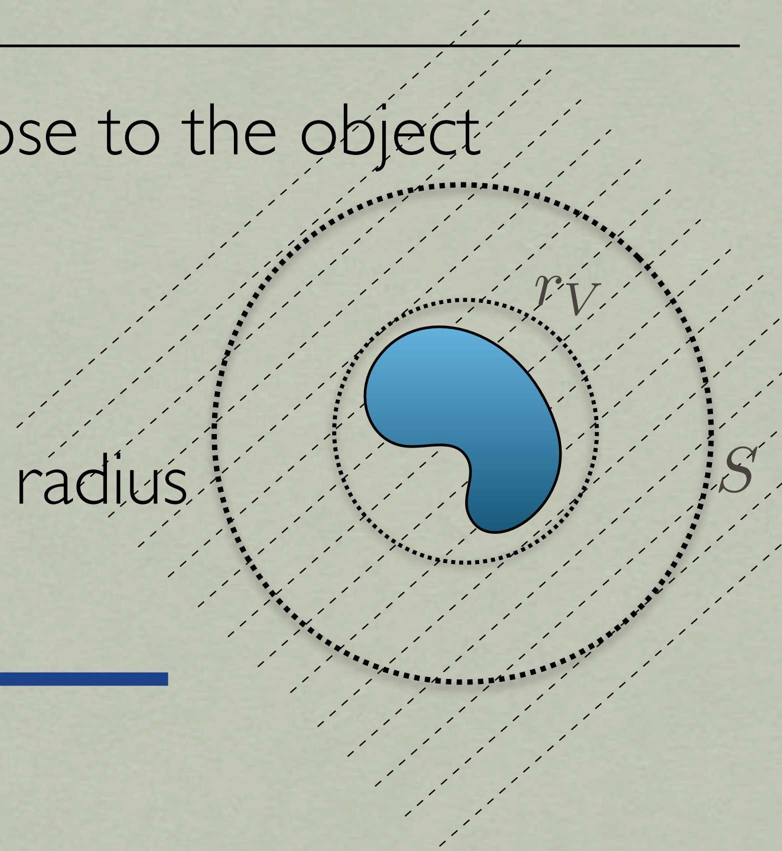

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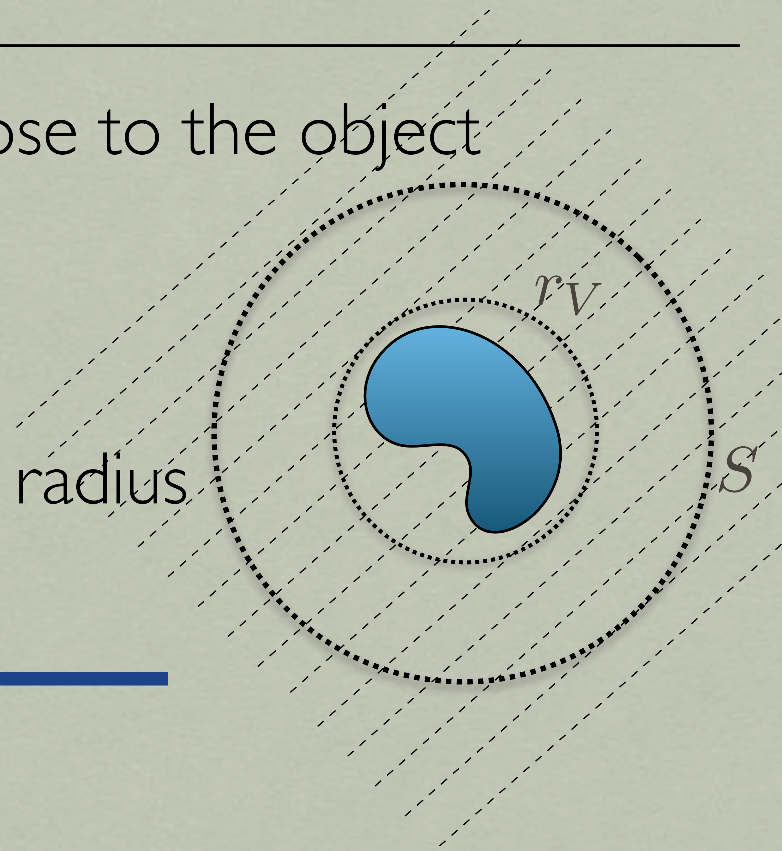

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(Tentatively) Only post-Newtonian violations of the WEP!

SUMMARY AND OUTLOOK

- In effective field theories of dark energy universally coupled to matter test particles naturally move on geodesics
- Extension to objects with finite size is non-trivial due to the strong coupling nature of the Vainshtein screening mechanism
- Shift symmetry is not exact, moreover the time component of the current is non-vanishing
- Due to the antisymmetric structure of the theories considered (i.e. of the Horndeski type) the equation of the motion of scalar takes the form of spatial divergence in the subhorizon limit, thus allowing to show that the WEP is also obeyed by screened extended objects
- OUTLOOK: more precise estimate of the order of magnitude of violation of the equivalence principle
- OUTLOOK: black holes

THANK YOU.

SEP VIOLATION

- **in GR:** a test body and a black hole of the same mass produce identical gravitational potential
- **in a scalar-tensor theory:** the scalar typically couples to matter

$$\mathcal{L}_\pi = -3M_{\rm Pl}^2(\partial\pi)^2 - 2\frac{M_{\rm Pl}^2}{m^2}(\partial\pi)^2\Box\pi + \pi T^\mu{}_\mu$$

  - for a given object this coupling sources a scalar field profile, π_0
  - for a BH there is no matter tensor—no scalar field profile—"no hair"

- **in galileon theories** the absence of scalar hair in flat space due to:

Hui, Nicolis (2012)

  - shift symmetry of the scalar self-interactions
  - staticity and spherical symmetry of the BH solution

$$\nabla_\mu J^\mu = 0 \quad \Rightarrow \quad J^r = 0 \quad \Rightarrow \quad \pi = 0$$

- for objects like neutron stars the SEP violation occurs since their 'effective test body mass' starts to depend on their internal structure $\Phi = -\frac{G_N M(\pi_0)}{r}$