Introduction to Neutrino Physics

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at
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Outline

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Identity of "Neutrino"

- Neutrinos are elementary particles.
- Neutrinos are electrically neutral.
- Neutrinos are fermions with spin-$\frac{1}{2}$.
- Neutrinos are leptons.
- Neutrinos interact only via weak interactions and gravity.
- Neutrinos exist in three flavors: electron neutrino ν_e, muon neutrino ν_μ and tau neutrino ν_τ.
- Neutrino flavors are created in weak interactions in association with the corresponding charged lepton.
- Neutrinos are tiny massive $\mathcal{O}$(eV).
- Neutrinos are only left-handed.

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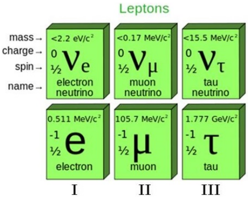
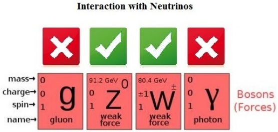
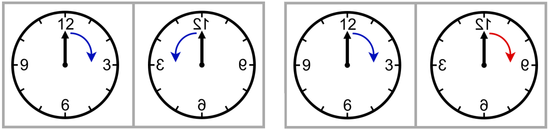




- Parity is the transformation under space reflection.
- Parity was assumed at the beginning to be a symmetry of nature.
- That's a 'mirrored' image of a natural system behaves in the same way does the 'mirror' image of that system.



P-Symmetry　　*P*-Asymmetry

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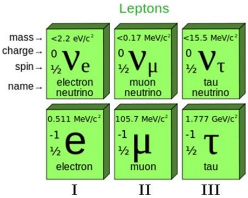

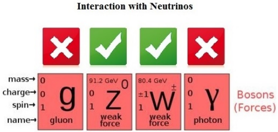

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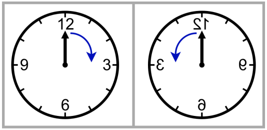

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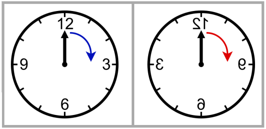

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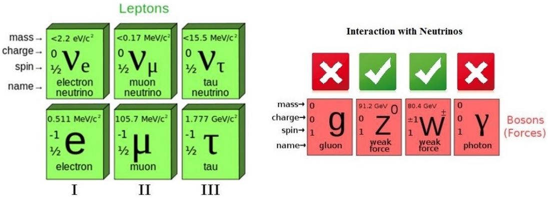

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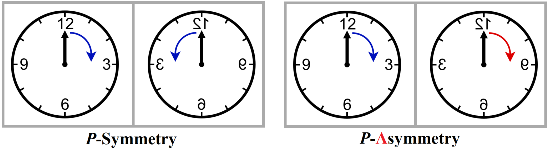

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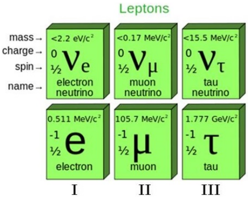

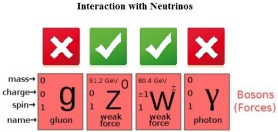

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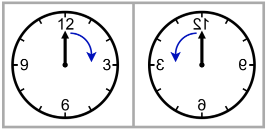

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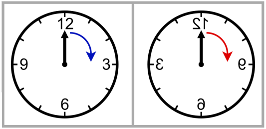

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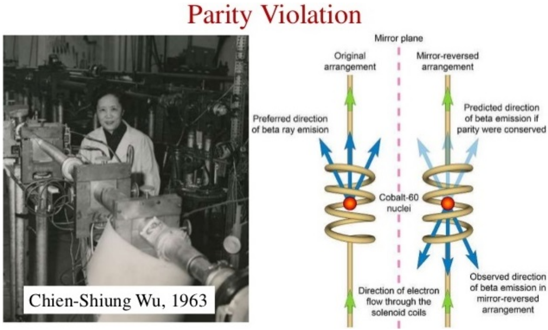
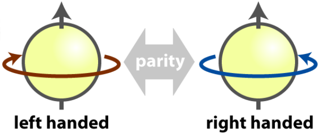
Wu Experiment & Parity Violation

- Parity is a symmetry for both the strong and electromagnetic interactions.
- In 1963, Wu found that Parity is maximally violated in the weak interactions.





- Only left-handed fermions feel the weak interactions.

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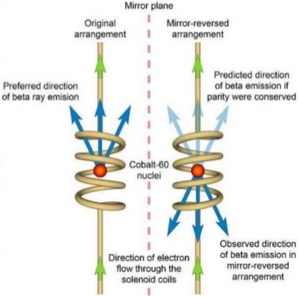

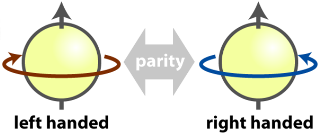

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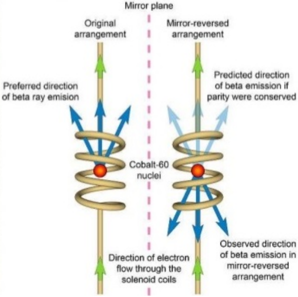

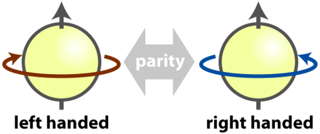

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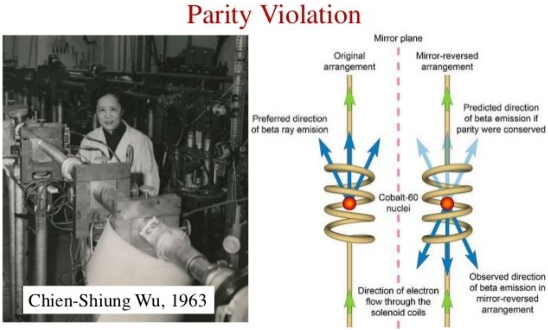

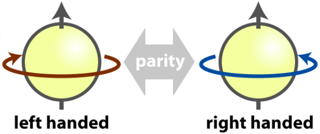

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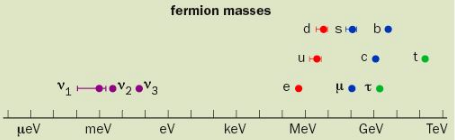
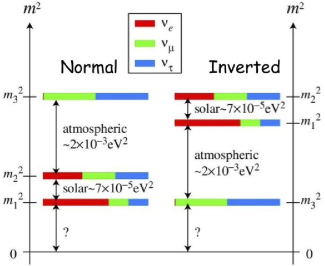
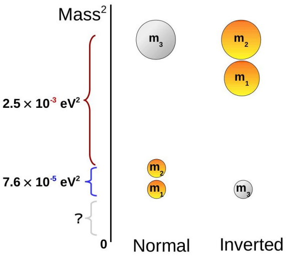
Neutrino Masses Hierarchies

- Neutrinos masses are of order of little eV's.



- Limits from solar and atmoshperic neutrino experiments propose the normal and inverted hierarchies for the neutrino masses.





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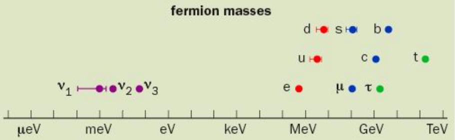

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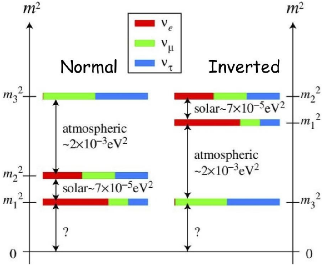

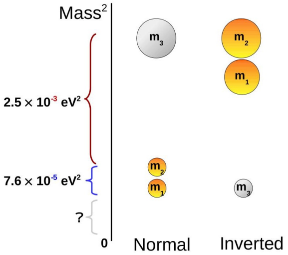

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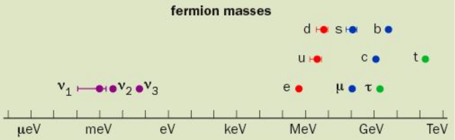

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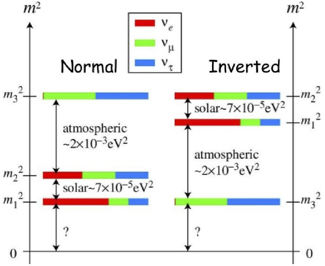

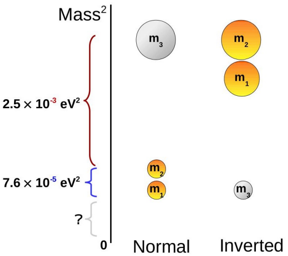

Neutrino Masses→Neutrino Mixing

- One of the new phenomena that can occur if neutrinos have nonzero masses is neutrino mixing [1].
- This is the assumption that the neutrino states ν_e, ν_μ and ν_τ that couple to electrons, muons and tauons, respectively, do not have definite masses.
- They are linear combinations of three other states ν_1, ν_2 and ν_3 that do have definite masses m_1, m_2 and m_3.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

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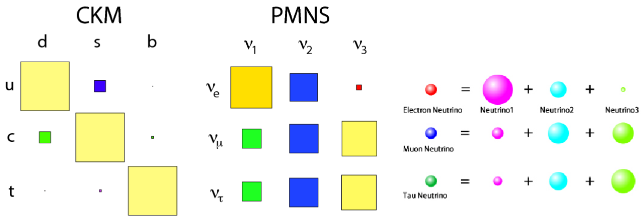
CKM & PMNS Mixing matrices

- The CKM quark mixing matrix is almost diagonal.
- The PMNS mixing matrix of neutrinos is equlibrated; i.e., all elements have approximately the same order.

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} 0.82 \pm 0.01 & 0.54 \pm 0.02 & -0.15 \pm 0.03 \\ -0.35 \pm 0.06 & 0.70 \pm 0.06 & 0.62 \pm 0.06 \\ 0.44 \pm 0.06 & -0.45 \pm 0.06 & 0.77 \pm 0.06 \end{bmatrix}$$



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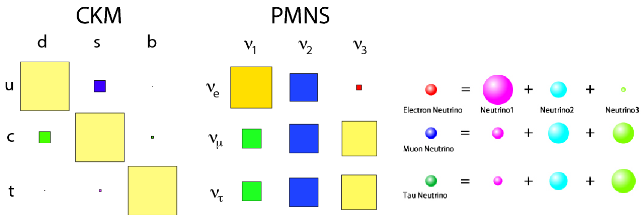

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Beats, Lagrangian

- The classical Lagrangian for forced oscillations of a particle of mass m by a force $F(t)$ in one dimension x is [2]

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2 + xF(t), \tag{1}$$

where ω is the frequency of the free oscillations.

- The *Euler-Lagrange* equation of motion of the system is

$$\ddot{x} + \omega^2 x = F(t)/m. \tag{2}$$

- The free oscillations $(F(t) \equiv 0)$ are

$$x(t) = A\cos(\omega t + \alpha), \tag{3}$$

where A is the amplitude and α is the initial phase.

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Beats, Amplitude

- We consider an oscillatory force $F(t)$ of the form

$$F(t) = f\cos(\gamma t + \beta). \tag{4}$$

- Near the resonance ($\gamma = \omega$), $\gamma = \omega + \epsilon$ and ϵ is a small quantity, the motion may be regarded as small oscillations with varaible amplitude

$$x(t) = C(t)\exp(i\omega t), \tag{5}$$

where

$$C^2 = a^2 + b^2 + 2ab\cos(\epsilon t + \beta - \alpha), \tag{6}$$

and a, b are constants.

- Clearly, the amplitude varies slowly with frequency ϵ between the limits

$$|a - b| \leq C \leq a + b. \tag{7}$$

- This phenomena is called the *"beats"*. ϵ is the beat frequency.

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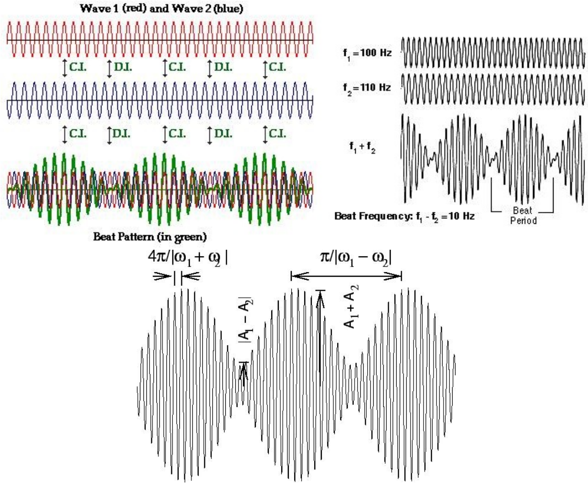
Beats, Illustration

- Click, enjoy ;) ▸ BEATS.



Two flavor Mixing Approximation

- We consider the mixing between only two of the flavor states ν_α and ν_β:

$$\nu_\alpha = \nu_i \cos\theta_{ij} + \nu_j \sin\theta_{ij}, \tag{8}$$
$$\nu_\beta = -\nu_i \sin\theta_{ij} + \nu_j \cos\theta_{ij}, \tag{9}$$

  where ν_i, ν_j are mass eigenstates involved.

- When a ν_α neutrino is produced with momentum $\vec{p}$ at time $t = 0$, the ν_i and ν_j components will have slightly different energies E_i and E_j due to their slightly different masses.
- In quantum mechanics, their associated waves will therefore have slightly different frequencies, giving rise to the '*beats*' phenomenon.

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Beats→Oscillation

- As a result, one finds that the original beam of ν_α particles develops a ν_β component whose intensity oscillates as it travels through space, while the intensity of the ν_α neutrino beam itself is correspondingly reduced.
- These are called *'neutrino oscillations'* and their existence follows from simple quantum mechanics.
- We consider now that a ν_α produced with momentum $\vec{p}$ at time $t = 0$, so that the initial state (8) is

$$|\nu_\alpha, \vec{p}\rangle = |\nu_i, \vec{p}\rangle \cos\theta_{ij} + |\nu_j, \vec{p}\rangle \sin\theta_{ij}. \tag{10}$$

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Propagation

- After time t this will become, using the equation of motion for the mass states ν_i, ν_j

$$a_i(t)|\nu_i, \vec{p}\rangle \cos\theta_{ij} + a_j(t)|\nu_j, \vec{p}\rangle \sin\theta_{ij}, \tag{11}$$

where

$$a_{i,j}(t) = \exp\left(-iE_{i,j}t\right) \tag{12}$$

are the usual oscillating time factors associated with any quantum mechanical stationary state.

- Similarly the initial state (9)

$$-a_i(t)|\nu_i, \vec{p}\rangle \sin\theta_{ij} + a_j(t)|\nu_j, \vec{p}\rangle \cos\theta_{ij}. \tag{13}$$

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- Inverting eqs. (8,9) to obtain

$$\nu_i = \nu_\alpha \cos\theta_{ij} - \nu_\beta \sin\theta_{ij}, \tag{14}$$
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- Then substitute into eq. (11) to obtain

$$|\nu_\alpha(t), \vec{p}\rangle = A(t)|\nu_\alpha(0), \vec{p}\rangle + B(t)|\nu_\beta(0), \vec{p}\rangle, \tag{16}$$

where

$$A(t) = a_i(t)\cos^2\theta_{ij} + a_j(t)\sin^2\theta_{ij}, \tag{17}$$
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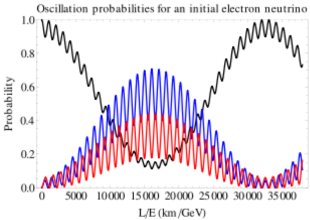
Probability

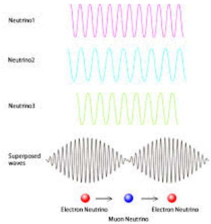
- The probability of finding a ν_β state is therefore

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta(0), \vec{p} | \nu_\alpha(t), \vec{p} \rangle$$
$$= |B(t)|^2 = \sin^2(2\theta_{ij}) \sin^2[\frac{1}{2}(E_j - E_i)t]. \quad (19)$$

- The probability of finding a ν_α state is therefore

$$P(\nu_\alpha \to \nu_\alpha) = |\langle \nu_\alpha(0), \vec{p} | \nu_\alpha(t), \vec{p} \rangle = |A(t)|^2 = 1 - |B(t)|^2. \quad (20)$$





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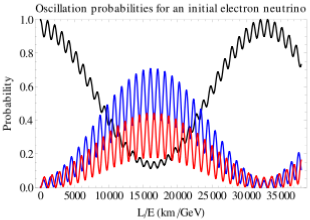

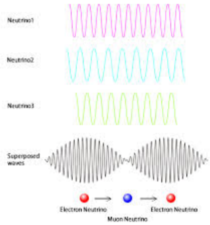

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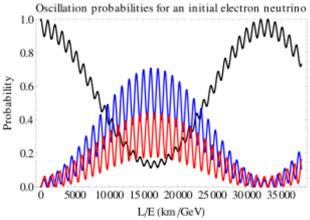

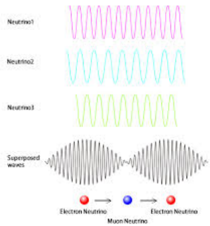

Remarks

- No oscillation if the mixing angle vanishes.
- For large energy difference $E_j - E_i$, the oscillation may be within the uncertainty time (!).
- For small mixing angles and large energy difference, the oscillation is negligible.
- These formulas assume that the neutrinos are propagating in a vacuum. This is usually a very good approximation, because of the enormous mean free paths for neutrinos to interact with matter.
- It was shown that neutrino oscillations can be enhanced when neutrinos traverse very long distances in matter.
- This is the *Mikheyev-Smirnov-Wolfenstein* (MSW) effect, due to weak interactions with matter's electrons analogous to the electromagnetic process leading to the refractive index of light in a medium.
- The MSW effect was dramatically confirmed in the 'Sudbury Neutrino Observatory (SNO)', and resolved the solar neutrino problem.

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Detection

- The time t traveled by a neutrino is determined by the distance L of the neutrino detector from the source of the neutrinos.
- Neutrino momenta are always much greater than their possible masses and they travel, to a very good approximation, at the speed of light.
- In this approximation, $t = L$,

$$E_j - E_i = (m_j^2 + p^2)^{1/2} - (m_i^2 + p^2)^{1/2} \approx \frac{m_j^2 - m_i^2}{2p}. \quad (21)$$

- Accordingly, the probability (19) may be written

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta_{ij}) \sin^2[L/L_0], \quad (22)$$

where the oscillation length

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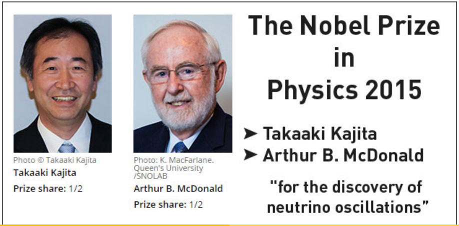
$$P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta). \quad (24)$$

- The oscillation lengths are typically of order 100 km or more.
- Oscillations can be safely neglected under normal laboratory conditions.
- The Nobel Prize in Physics 2015 for the discovery of neutrino oscillations. Click ▸ NPNO, ▸ NPNO.pdf.



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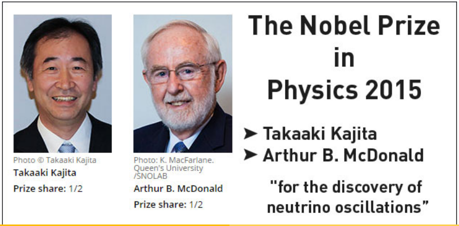

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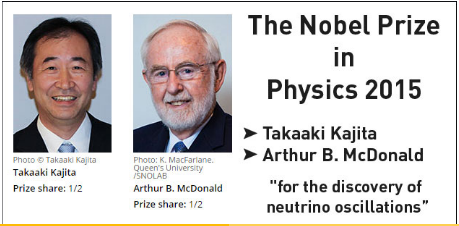

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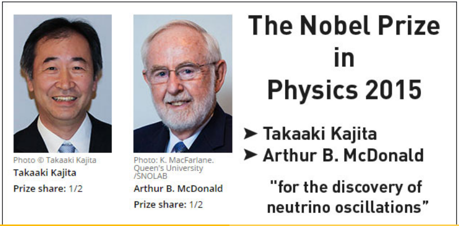

Outline

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Thanks

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