LEPTONS AND THE WEAK INTERACTION - PHYSICS BEYOND THE STANDARD MODEL

W. ABDALLAH

Department of Mathematics
Faculty of Science
Cairo University

Mini-School on Experimental Tools in Particle Physics 19th April 2017

Outline

Outline

Outline

Outline

Outline

Outline

Outline

- Leptons are one of the three classes of fundamental particles in the standard model. They are spin-$\frac{1}{2}$ fermions without strong interactions. There are six known leptons, which occur in pairs called generations, written as doublets:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \tag{1}$$

- Leptons are one of the three classes of fundamental particles in the standard model. They are spin-$\frac{1}{2}$ fermions without strong interactions. There are six known leptons, which occur in pairs called generations, written as doublets:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \tag{1}$$

- In addition to the leptons, there are six corresponding antiparticles (antileptons):

$$\begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}, \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix}, \begin{pmatrix} \tau^+ \\ \bar{\nu}_\tau \end{pmatrix} \tag{2}$$

- Leptons are one of the three classes of fundamental particles in the standard model. They are spin-$\frac{1}{2}$ fermions without strong interactions. There are six known leptons, which occur in pairs called generations, written as doublets:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \tag{1}$$

- In addition to the leptons, there are six corresponding antiparticles (antileptons):

$$\begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix}, \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix}, \begin{pmatrix} \tau^+ \\ \bar{\nu}_\tau \end{pmatrix} \tag{2}$$

- The charged leptons interact via both electromagnetic and weak forces, whereas for neutral leptons only weak interactions have been observed.

- Firstly, we note that each generation of leptons has associated with it a quantum number which is called lepton numbers, defined for any state by

$$L_\ell \equiv N(\ell^-) - N(\ell^+) + N(\nu_\ell) - N(\bar{\nu}_\ell), \quad \ell = e, \mu, \tau \qquad (3)$$

- Firstly, we note that each generation of leptons has associated with it a quantum number which is called lepton numbers, defined for any state by

$$L_\ell \equiv N(\ell^-) - N(\ell^+) + N(\nu_\ell) - N(\bar{\nu}_\ell), \quad \ell = e, \mu, \tau \qquad (3)$$

- For single-particle states, $L_e = 1$ for e^- and ν_e, $L_e = -1$ for e^+ and $\bar{\nu}_e$ and $L_e = 0$ for all other particles, and so on for the other generations.

- In electromagnetic interactions, electronic lepton number conservation reduces to the conservation of $N(e^-) - N(e^+)$, since neutrinos are not involved.

- In electromagnetic interactions, electronic lepton number conservation reduces to the conservation of $N(e^-) - N(e^+)$, since neutrinos are not involved.

- This implies that electrons and positrons can only be created or annihilated in pairs.

- In electromagnetic interactions, electronic lepton number conservation reduces to the conservation of $N(e^-) - N(e^+)$, since neutrinos are not involved.

- This implies that electrons and positrons can only be created or annihilated in pairs.

- In weak interactions more general possibilities are allowed. For example, an electron could be created together with an anti-neutrino $\bar{\nu}_e$, rather than a positron.

- In electromagnetic interactions, electronic lepton number conservation reduces to the conservation of $N(e^-) - N(e^+)$, since neutrinos are not involved.

- This implies that electrons and positrons can only be created or annihilated in pairs.

- In weak interactions more general possibilities are allowed. For example, an electron could be created together with an anti-neutrino $\bar{\nu}_e$, rather than a positron.

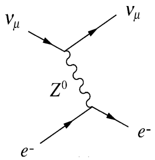
- In the Standard Model, lepton numbers are individually conserved in all known interactions.

Outline

Z^0 and $W^{\pm}$ Exchange

- Weak interactions involving only leptons are described by exchange processes in which a $W^{\pm}$ or Z^0 is emitted by one lepton and absorbed by another.
- Elastic $\nu_\mu e^-$ scattering by Z^0 exchange.

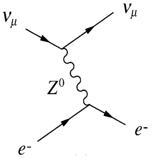
$$\nu_\mu e^- \rightarrow \nu_\mu e^-$$



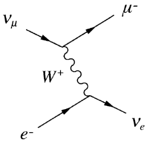
Z^0 and $W^\pm$ Exchange

- Weak interactions involving only leptons are described by exchange processes in which a $W^\pm$ or Z^0 is emitted by one lepton and absorbed by another.
- Elastic $\nu_\mu e^-$ scattering by Z^0 exchange.
- The inverse muon decay, ν_μ converts into muon by emitting a W^+ boson. The W^+ is then absorbed at the lower vertex.
- These absorption and emission processes conserve the lepton numbers and electric charge.

$\nu_\mu e^- \to \nu_\mu e^-$

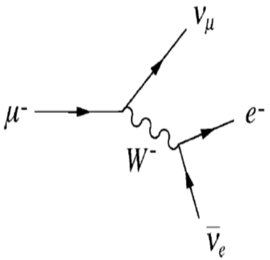


$\nu_\mu e^- \to \mu^- \nu_e$



Z^0 and $W^{\pm}$ Exchange

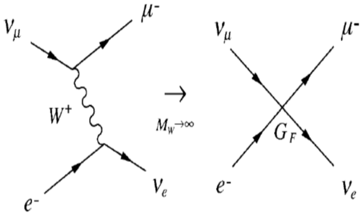
- The muon decay , the first vertex corresponds to a muon emitting a W^- boson, followed by the conversion of the W^- boson into a lepton pair in the lepton-number conserving process $W^- \to e^- \bar{\nu}_e$.



$$\mu^- \to e^- \; \bar{\nu}_e \; \nu_\mu$$

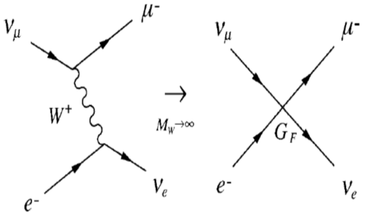
Z^0 and $W^{\pm}$ Exchange

- At low energies, the de Broglie wavelengths of all the particles involved are large compared with the range of the W-exchange interaction.



Z^0 and $W^{\pm}$ Exchange

- At low energies, the de Broglie wavelengths of all the particles involved are large compared with the range of the W-exchange interaction.



- This can then be approximated by a zero-range point interaction whose strength is characterized by the Fermi coupling constant

$$G_F = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$$

$$\frac{g_W^2}{M_W^2} = \frac{G_F}{\sqrt{2}}$$

Lepton Decay and Universality

- All known experimental data are consistent with the assumption that the interactions of the electron and its neutrino are identical with those of the muon and its associated neutrino and the tauon and its neutrino, provided the mass differences are taken into account.

Lepton Decay and Universality

- All known experimental data are consistent with the assumption that the interactions of the electron and its neutrino are identical with those of the muon and its associated neutrino and the tauon and its neutrino, provided the mass differences are taken into account.
- This fundamental assumption is called the **universality** of lepton interactions.

Lepton Decay and Universality

- All known experimental data are consistent with the assumption that the interactions of the electron and its neutrino are identical with those of the muon and its associated neutrino and the tauon and its neutrino, provided the mass differences are taken into account.
- This fundamental assumption is called the **universality** of lepton interactions.
- It was also implicit, when we assumed that the coupling strengths g_W and G_F were independent of the nature of the leptons to which the W boson coupled.

Outline

Neutrino Mixing

- The neutrino states ν_e, ν_μ and ν_τ do not have definite masses; instead they are linear combinations of three other states ν_1, ν_2 and ν_3 that do have definite masses m_1, m_2 and m_3.

Neutrino Mixing

- The neutrino states ν_e, ν_μ and ν_τ do not have definite masses; instead they are linear combinations of three other states ν_1, ν_2 and ν_3 that do have definite masses m_1, m_2 and m_3.
- For simplicity, we consider the mixing between just two of them, which we will denote ν_α and ν_β. Then,

$$\nu_\alpha = \nu_i \cos\theta_{ij} + \nu_j \sin\theta_{ij} \quad (4)$$

$$\nu_\beta = -\nu_i \sin\theta_{ij} + \nu_j \cos\theta_{ij}, \quad (5)$$

where ν_i and ν_j are the two mass eigenstates involved and θ_{ij} is a mixing angle that must be determined from experiment.

Neutrino Mixing

- The neutrino states ν_e, ν_μ and ν_τ do not have definite masses; instead they are linear combinations of three other states ν_1, ν_2 and ν_3 that do have definite masses m_1, m_2 and m_3.
- For simplicity, we consider the mixing between just two of them, which we will denote ν_α and ν_β. Then,

$$\nu_\alpha = \nu_i \cos\theta_{ij} + \nu_j \sin\theta_{ij} \quad (4)$$

$$\nu_\beta = -\nu_i \sin\theta_{ij} + \nu_j \cos\theta_{ij}, \quad (5)$$

  where ν_i and ν_j are the two mass eigenstates involved and θ_{ij} is a mixing angle that must be determined from experiment.
- If $\theta_{ij} = 0$, then $\nu_\alpha = \nu_i$, $\nu_\beta = \nu_j$ and there is no mixing.

Neutrino Oscillations

- If $\theta_{ij} \neq 0$, there is a very interesting phenomena which is called neutrino oscillations.

Neutrino Oscillations

- If $\theta_{ij} \neq 0$, there is a very interesting phenomena which is called neutrino oscillations.
- Consider a ν_α neutrino is produced with momentum $\mathbf{p}$ at time $t = 0$, the ν_i and ν_j components will have slightly different energies E_i and E_j.

Neutrino Oscillations

- If $\theta_{ij} \neq 0$, there is a very interesting phenomena which is called neutrino oscillations.
- Consider a ν_α neutrino is produced with momentum $\mathbf{p}$ at time $t = 0$, the ν_i and ν_j components will have slightly different energies E_i and E_j.
- From the quantum mechanics, we can calculate the probability of finding a ν_β state as follows

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta_{ij})\ \sin^2\left[\frac{1}{2}(E_j - E_i)t\right] \tag{6}$$

Neutrino Masses

$$E_j - E_i = (m_j^2 + |\mathbf{p}|^2)^{1/2} - (m_i^2 + |\mathbf{p}|^2)^{1/2} \simeq \frac{m_j^2 - m_i^2}{2|\mathbf{p}|} \qquad (7)$$

Therefore,

$$P(\nu_\alpha \to \nu_\beta) \simeq \sin^2(2\theta_{ij}) \ \sin^2\left[\frac{m_j^2 - m_i^2}{4|\mathbf{p}|} t\right] \qquad (8)$$

Neutrino Masses

$$E_j - E_i = (m_j^2 + |\mathbf{p}|^2)^{1/2} - (m_i^2 + |\mathbf{p}|^2)^{1/2} \simeq \frac{m_j^2 - m_i^2}{2|\mathbf{p}|} \qquad (7)$$

Therefore,

$$P(\nu_\alpha \to \nu_\beta) \simeq \sin^2(2\theta_{ij})\ \sin^2\left[\frac{m_j^2 - m_i^2}{4|\mathbf{p}|}t\right] \qquad (8)$$

So, if we define

$$\triangle(m_{ij}^2) \equiv m_i^2 - m_j^2, \qquad (9)$$

then the experiment yields the values

$$1.9 \times 10^{-3} \lesssim |\triangle(m_{32}^2)| \lesssim 3.0 \times 10^{-3}(\mathrm{eV})^2,$$
$$7.6 \times 10^{-5} \lesssim |\triangle(m_{21}^2)| \lesssim 8.6 \times 10^{-5}(\mathrm{eV})^2,$$
$$\sin^2(2\theta_{23}) \gtrsim 0.9,\ 0.32 \lesssim \tan^2(\theta_{12}) \lesssim 0.48, \sin^2(2\theta_{13}) \lesssim 0.19$$

Lepton Number Violation

- In all the above discussions, we have assumed that lepton number conservation holds and can be used to identify the neutrino flavour emitted or absorbed in any weak interaction. However, in principle, lepton number violation (LFV) can be induced in such interaction reactions by the existence of neutrino oscillations.

Lepton Number Violation

- In all the above discussions, we have assumed that lepton number conservation holds and can be used to identify the neutrino flavour emitted or absorbed in any weak interaction. However, in principle, lepton number violation (LFV) can be induced in such interaction reactions by the existence of neutrino oscillations.

Decay	Violates	B
$\mu^- \to e^- + e^+ + e^-$	L_μ, L_e	$<1.0 \times 10^{-12}$
$\mu^- \to e^- + \gamma$	L_μ, L_e	$<1.2 \times 10^{-11}$
$\tau^- \to e^- + \gamma$	L_τ, L_e	$<1.1 \times 10^{-7}$
$\tau^- \to \mu^- + \gamma$	L_τ, L_μ	$<6.8 \times 10^{-8}$
$\tau^- \to e^- + \mu^+ + \mu^-$	L_τ, L_e	$<2 \times 10^{-7}$

Why Beyond the SM?

- Many experiments have been designed to search for LFV processes, in particular, $\mu \to e\gamma$. The current upper limit is given by the MEG experiment [J. Adam et al., MEG Collaboration(2011)]

$$BR(\mu \to e\gamma) \lesssim 2.4 \times 10^{-12}.$$

It is important to note that new experiments are expected to improve this limit by three order of magnitudes and the SM result for the branching ratio of $\mu \to e\gamma$ is given by

$$BR(\mu \to e\gamma)^{\rm SM} \simeq 10^{-55}.$$

Thus, the observation of $\mu \to e\gamma$ decay will be a clear signal for physics beyond the SM.

Outline

The $B-L$ extension of the SM is based on the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

In this model:

The $B-L$ extension of the SM is based on the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

In this model:

1. Three right-handed neutrinos, N_R^i, $i = 1, 2, 3$; with $B-L$ charge $= -1$.

The $B-L$ extension of the SM is based on the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

In this model:

1. Three right-handed neutrinos, N_R^i, $i = 1, 2, 3$; with $B-L$ charge $= -1$.
2. An extra gauge boson corresponding to $B-L$ gauge symmetry, Z', and an extra SM singlet scalar, χ with $B-L$ charge $= +2$, are introduced.

[*W. Emam and S. Khalil(2007)*]

The Lagrangian of the leptonic sector is given by

$$\begin{aligned}\mathcal{L}_{B-L} &= -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + i\ \bar{\ell}_L D_\mu \gamma^\mu \ell_L + i\ \bar{e}_R D_\mu \gamma^\mu e_R + i\ \bar{N}_R D_\mu \gamma^\mu N_R \\ &+ (D^\mu\phi)^\dagger D_\mu\phi + (D^\mu\chi)^\dagger D_\mu\chi - V(\phi,\chi) \\ &- \Big(\lambda_e \bar{\ell}_L \phi e_R + \lambda_\nu \bar{\ell}_L \tilde{\phi} N_R + \lambda_N \bar{N}_R^c \chi N_R + h.c.\Big),\end{aligned}$$

$$V(\phi,\chi) = m_1^2 \phi^\dagger\phi + m_2^2 \chi^\dagger\chi + \lambda_1(\phi^\dagger\phi)^2 + \lambda_2(\chi^\dagger\chi)^2 + \lambda_3 \chi^\dagger\chi\phi^\dagger\phi.$$

Particle	ℓ_L	e_R	N_R	ϕ	χ
Y_{B-L}	-1	-1	-1	0	$+2$

After $U(1)_{B-L}$ and the Electroweak symmetry breaking, the mass matrix of the left and right-handed neutrino is

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix},$$

where

$$m_D = \frac{1}{\sqrt{2}}\lambda_\nu v, \quad M_N = \frac{1}{\sqrt{2}}\lambda_N v'.$$

The digitalization of the mass matrix leads to the light and heavy neutrinos masses, respectively:

$$\begin{aligned} m_{\nu_\ell} &\simeq -m_D M_N^{-1} m_D^T, \\ m_{\nu_H} &\simeq M_N. \end{aligned}$$

$$\Rightarrow \quad \lambda_\nu \lesssim \mathcal{O}(10^{-6})$$

Outline

The $B - L$ extension of the SM is based on the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

In this model:

The $B-L$ extension of the SM is based on the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

In this model:

1. Three right-handed neutrinos, N_R^i, $i=1,2,3$; with $B-L$ charge $=-1$.

The $B-L$ extension of the SM is based on the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

In this model:

1. Three right-handed neutrinos, N_R^i, $i = 1, 2, 3$; with $B-L$ charge $= -1$.
2. An extra gauge boson corresponding to $B-L$ gauge symmetry, Z', and an extra SM singlet scalar, χ with $B-L$ charge $= -1$, are introduced.

The $B-L$ extension of the SM is based on the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$$

In this model:

1. Three right-handed neutrinos, N_R^i, $i=1,2,3$; with $B-L$ charge $=-1$.
2. An extra gauge boson corresponding to $B-L$ gauge symmetry, Z', and an extra SM singlet scalar, χ with $B-L$ charge $=-1$, are introduced.
3. The SM singlet fermion sector includes two singlet fermions each of them has three flavors, S_j^i, $i=1,2,3$, $j=1,2$; with $B-L$ charges ∓ 2 with opposite matter parity.

[S. Khalil(2008)]

The Lagrangian of the leptonic sector is given by

$$\begin{aligned}\mathcal{L}^{ISS}_{B-L} &= -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + i\ \bar{\ell}_L D_\mu\gamma^\mu \ell_L + i\ \bar{e}_R D_\mu \gamma^\mu e_R + i\ \bar{N}_R D_\mu\gamma^\mu N_R \\ &+ i\ \bar{S}_1 D_\mu \gamma^\mu S_1 + i\ \bar{S}_2 D_\mu\gamma^\mu S_2 + (D^\mu\phi)^\dagger D_\mu\phi + (D^\mu\chi)^\dagger D_\mu \chi \\ &- V(\phi,\chi) - \Big(\lambda_e \bar{\ell}_L \phi e_R + \lambda_\nu \bar{\ell}_L \tilde{\phi} N_R + \lambda_N \bar{N}^c_R \chi S_2 + h.c.\Big) \\ &- \frac{1}{M^3}\bar{S}^c_1 \chi^{\dagger 4} S_1 - \frac{1}{M^3}\bar{S}^c_2\chi^4 S_2 .\end{aligned}$$

Particle	ℓ_L	e_R	N_R	ϕ	χ	S_1	S_2
Y_{B-L}	-1	-1	-1	0	-1	-2	$+2$

The Lagrangian of neutrino masses, in the flavor basis, is given by:

$$\mathcal{L}_m^{\nu} = \mu_s \bar{S}_2^c S_2 + (m_D \bar{\nu}_L N_R + M_N \bar{N}_R^c S_2 + h.c.),$$

where

$$\mu_s = \frac{v'^4}{4M^3} \sim 10^{-9}, \quad m_D = \frac{1}{\sqrt{2}}\lambda_\nu v, \quad M_N = \frac{1}{\sqrt{2}}\lambda_N v'.$$

Therefore, in the basis $\{\nu_L^c, N_R, S_2\}$, the 9×9 neutrino mass matrix takes the form:

$$\mathcal{M}_\nu = \left(\begin{array}{c|ccc} & \nu_L^c & N_R & S_2 \\ \hline \bar{\nu}_L & 0 & m_D & 0 \\ \bar{N}_R^c & m_D^T & 0 & M_N \\ \bar{S}_2^c & 0 & M_N^T & \mu_s \end{array} \right).$$

Diagonalizing $\mathcal{M}_\nu$ leads to the following light and heavy neutrino masses respectively:

$$\begin{aligned} m_{\nu_\ell} &= m_D M_N^{-1} \mu_s (M_N^T)^{-1} m_D^T, \\ m_{\nu_H}^2 &\simeq m_{\nu_{H'}}^2 = M_N^2 + m_D^2. \end{aligned}$$

It is now clear that

$\mu_s \ll M_N \quad \Rightarrow \quad$ light neutrino masses can be of order eV, with a TeV scale M_N.

The light neutrino mass matrix, m_{ν_ℓ}, must be diagonalized by the physical neutrino mixing matrix U_{MNS}, *i.e.*,

$$U_{MNS}^T m_{\nu_\ell} U_{MNS} = m_{\nu_\ell}^{\rm diag} \equiv {\rm diag}\{m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}\}.$$

Thus, the Dirac neutrino mass matrix, m_D, can be defined as :

$$m_D = U_{MNS}\,\sqrt{m_{\nu_\ell}^{\mathrm{diag}}}\,R\,\sqrt{\mu_S^{-1}}\,M_N,$$

where R is an arbitrary orthogonal matrix.

It is clear that this expression is a generalization to the expression of m_D in type I seesaw,

$$m_D = U_{MNS}\,\sqrt{m_{\nu_\ell}^{\mathrm{diag}}}\,R\,\sqrt{M_N}$$

Accordingly, the matrix V that diagonalizes the 9×9 neutrino mass matrix $\mathcal{M}_\nu$, *i.e.*, $V^T\mathcal{M}_\nu V = \mathcal{M}_\nu^{\rm diag}$, is given by

$$V = \begin{pmatrix} N_{3\times 3} & V_{3\times 6} \\ & \\ V_{3\times 6}^T & V_{6\times 6} \end{pmatrix},$$

where the matrix $V_{3\times 6}$ is defined as

$$V_{3\times 6} = \left(\mathbf{0}_{3\times 3}, F\right) V_{6\times 6}, \qquad F = m_D M_N^{-1}.$$

It is worth noting that here the $N_{3\times 3}$ is not unitary. It can be related to the standard U_{MNS} as

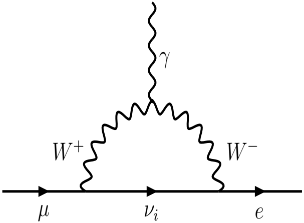
$$N_{3\times 3} \simeq \left(1 - \frac{1}{2} F F^T\right) U_{MNS} \, .$$

Thus the non-unitarity effects are measured by $\frac{1}{2} F F^T$.

The present experimental bounds on the elements of the non-unitarity parameters imply lower bounds on the right-handed neutrino masses M_N.

Outline

We perform the calculation of $\mu \to e\gamma$ due to the exchange of heavy neutrinos and W-gauge boson inside the loop:



Using the dominant decay mode of $\Gamma(\mu \to e\nu\bar{\nu})$, the branching ratio is given by

$$BR(\mu \to e\gamma) = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} = \frac{3\alpha}{64\pi}\left|\sum_{i=1}^{9} V_{\mu i}^{*} V_{ei} f(r_i)\right|^2,$$

where $\alpha = e^2/4\pi \simeq 1/137$, $r_i = (m_{\nu_i}/M_W)^2$ and $f(r_i) \simeq 10/3$.

From the MEG experiment upper bound, we have the constraint

$$\left| \sum_{i=1}^{9} V_{\mu i}^{*} V_{ei} f(r_i) \right|^2 < 0.000149.$$

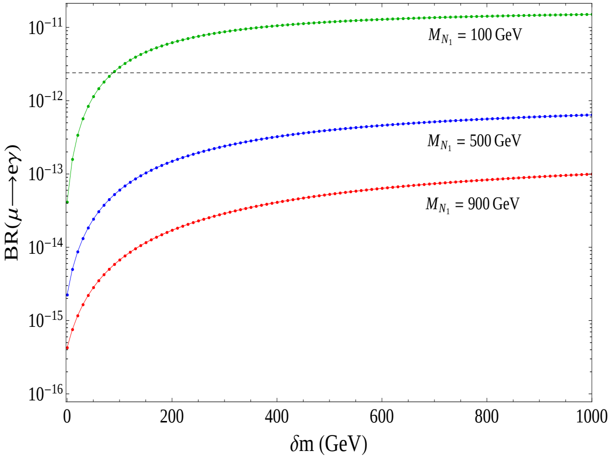
Thus, form the unitarty of V and the behavior of $f(r)$, one finds

$$\left| \sum_{i=1}^{3} V_{\mu i}^{*} V_{ei} \right| < 0.0000636,$$

which implies that $(FF^T)_{21,12} \lesssim 10^{-4}$. This bound can be easily satisfied, due to the constraints imposed on the off-diagonal elements of the non-unitary U_{MNS} mixing matrix.

[*W. Abdallah, A. Awad, S. Khalil and H. Okada(2012)*]

$BR(\mu \to e\gamma)$ versus $\delta m = M_{N_2} - M_{N_1}$, for $M_{N_1} = 100, 500, 900$ GeV from up to down, respectively.



Thank you