
B2

High Energy Gamma Conversion Models

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This work is performed in the context of the Geant4 collaboration for improving existing physics models [1] [2] [3] *and in the context of development of physics models within the GeantV project.*

[1] Schaelicke A, Ivanchenko V, Maire M and Urban L 2008 Improved Description of Bremsstrahlung for High-Energy Electrons in Geant4, 2008 IEEE NSS Conference Record N37-1

[2] A SchÃďlicke et al., Geant4 electromagnetic physics for the LHC and other HEP applications , J. Phys: Conf. Ser. 295: 012154, 2011

[3] V N Ivanchenko et al., Geant4 Electromagnetic Physics for LHC Upgrade, Journal of Physics: Conference Series 513 (2014) 022015

Contents

- Reviewing all high-energy physics processes -in view of the increased LHC energy and even more importantly for the FCC design- is crucial ($\geq \mathcal{O}(10\text{TeV})$)
- Reviewing the pair-production model in particular, including the LPM suppression mechanism at high energies
- Also of interest to other fields

  In astroparticle physics, new generation telescopes based on pair-production with minimized scattering will need accurate models describing angular resolution. Better description of pair-production detailed kinematics is therefore needed at low-energy ($\leq \mathcal{O}(100\text{MeV})$).

Contents

1. Why and how Monte Carlo simulations are used in High Energy Experiments?
   $\longrightarrow$ From detector design (FCC)
   $\longrightarrow$ To physics analysis (LHC)
2. Using randomness for modeling a large variety of physics processes $\longrightarrow$ relying on repeated random sampling to obtain numerical results
3. Cross sections: how likely is a physics process to occur
4. Differential cross-sections: for final state generation

Different ways photons interact with matter:

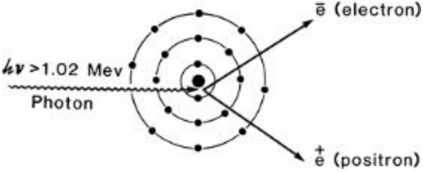
Photoelectric effect,

Compton scattering, Rayleigh scattering, Raman scattering, and

Pair-production:

coherent: e^{-}/e^{+} pair is created in the field of the nucleus

incoherent: pair is created in the field of an orbital electron



Contents

Starting point:

Bethe and Heitler (1934) [4]: **Unscreened Point Nucleus**

For photons of energy $E_\gamma > 50$ MeV, and an electron (positron) of energy $E_-(E_+)$, the differential cross-section (DCS) (in the *positron* variable) for pair-production is obtained as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_+} = \frac{4r_e^2\alpha Z^2}{E_\gamma^3}\left(E_+^2 + E_-^2 + \frac{2}{3}E_+E_-\right)\left[ln\left(\frac{2E_+E_-}{mc^2E_\gamma}\right) - \frac{1}{2}\right]$$

where α is the fine structure constant,
r_e the electron radius,

Z the atomic number

[4] H. Bethe and W. Heitler, *Proc. Roy. Soc. (London), 1934*

Starting point:

Bethe and Heitler (1934) [4]: **Unscreened Point Nucleus**

For photons of energy $E_\gamma > 50$ MeV, and an electron (positron) of energy $E_-(E_+)$, the differential cross-section (DCS) (in the *positron* variable) for pair-production is obtained as:

Corrected and extended for various effects:

- the screening of the field of the nucleus
- correction to the Born approximation
- pair-creation in the field of atomic electrons
- the LPM suppression mechanism
- Nuclear recoil

[4] H. Bethe and W. Heitler, *Proc. Roy. Soc. (London), 1934*

Bethe and Heitler (1934) [5]: **Screened Point Nucleus**

For photons of energy $E_\gamma > 50$ MeV, and an electron (positron) of energy $E_-(E_+)$, the differential cross-section (DCS) (in the *positron* variable) for pair-production is obtained as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_+} = \frac{r_e^2 \alpha Z^2}{E_\gamma^3} \times \left\{ (E_+^2 + E_-^2)\left[\phi_1(\gamma) - \frac{4}{3} lnZ\right] + \frac{2}{3}E_+E_-\left[\phi_2(\gamma) - \frac{4}{3} lnZ\right] \right\}$$

[5] H. Bethe and W. Heitler, *Proc. Roy. Soc. (London), 1934*

- $\phi_1(\gamma)$ and $\phi_2(\gamma)$ are functions accounting for the **screening effect** defined by Butcher and Messel (1960) [6] as:

$$\phi_1(\gamma) = 4\left\{\int_\delta^1 (q-\delta)^2[1-F(q)]^2 \frac{dq}{q^3}\right\} + 4 + \frac{4}{3}lnZ,$$

$$\phi_2(\gamma) = 4\int_\delta^1 \left[q^3 - 6\delta^2 q ln\left(\frac{q}{\delta}\right)\right][1-F(q)]^2\frac{dq}{q^4} + \frac{10}{3} + \frac{4}{3}lnZ,$$

$$\text{with } \gamma = \frac{100\, mc^2 E_\gamma}{Z^{1/3}E_+E_-} \text{ and } \delta = \frac{Z^{1/3}}{200}\gamma$$

q being the momentum transfer and given an atomic form factor $F(q)$ evaluated by Motz, Olsen and Koch (1964) [7].

[6] J.C. Butcher, H. Messel, *Phys. Rev.* 20 (1960) 15

[7] Motz, Olsen and Koch (1964), Formula 1A-102)

At extreme high energies $\gamma \approx 0$, the screening is essential and can be called *complete*. In that case, the values of ϕ_1 and ϕ_2 are

$$\phi_1(\gamma \approx 0) = 4ln(184),$$

$$\phi_2(\gamma \approx 0) = \phi_1(\gamma \approx 0) - \frac{2}{3}$$

It follows that the screened BH differential cross-section can be written as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_+} = \frac{4r_e^2\alpha Z^2}{E_\gamma^3}\left[\left(E_+^2 + E_-^2 + \frac{2}{3}E_+E_-\right) ln(184Z^{-1/3}) - \frac{1}{9}E_+E_-\right]$$

Davies-Bethe-Maximon (1954) [8]:

Screened Point Nucleus with Coulomb correction

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}E_+} &= \frac{r_e^2 \alpha Z^2}{E_\gamma^3} \\ &\times \left\{ (E_+^2 + E_-^2) \left[\phi_1(\gamma) - \frac{4}{3} lnZ - 4 f_c(Z) \right] \right. \\ &+ \left. \frac{2}{3} E_+ E_- \left[\phi_2(\gamma) - \frac{4}{3} lnZ - 4 f_c(Z) \right] \right\}\end{aligned}$$

[8] H. Davies, H.A. Bethe, L.C. Maximon, *Phys. Rev.* **92** 788 (1954)

- f_c : the high-energy *Coulomb distortion function* Davies et al. (1954), Maximon (1968) [9] [10]

$$f_c(Z) = (\alpha Z)^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2 + (\alpha Z)^2)}$$
$$= (\alpha Z)^2 [1/(1 + (\alpha Z)^2) + 0.20206 - 0.0369(\alpha Z)^2$$
$$+ 0.0083(\alpha Z)^4 - 0.002(\alpha Z)^6]$$

[9] H. Davies, H.A. Bethe, L.C. Maximon, *Phys. Rev.* 92 (1954) 788

[10] Maximon, *J. Res. Natl. Bur. Stand.* (1968) 79-88

In *complete screening*, where $\gamma \approx 0$ we can write:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_+} = \frac{4r_e^2\alpha Z^2}{E_\gamma^3} \times \left\{ \left(E_+^2 + E_-^2 + \frac{2}{3}E_+E_-\right)\left[ln(184Z^{-1/3}) - f_c(Z)\right] + \frac{1}{9}E_+E_- \right\}$$

Wheeler and Lamb (1939) [11]:

Screened Point Nucleus in the field of the electrons (with Atomic Excitation)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_+} = \frac{r_e^2 \alpha Z}{E_\gamma^3} \times \left\{ (E_+^2 + E_-^2)\left[\psi_1(\omega) - \frac{8}{3} lnZ\right] + \frac{2}{3} E_+ E_- \left[\psi_2(\omega) - \frac{8}{3} lnZ\right] \right\}$$

$$\text{where } \omega = \frac{100\, mc^2 E_\gamma}{Z^{2/3} E_+ E_-}$$

and $\psi_1(\omega)$ and $\psi_2(\omega)$ are inelastic correction functions

[11] Wheeler and Lamb, *Phys. Rev.* **101** 1836 (1939)

Wheeler and Lamb (1939) [11]:

Screened Point Nucleus in the field of the electrons (with Atomic Excitation)

For extreme relativistic energies, *complete screening* is considered and we obtain the following formula:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_+} = \frac{r_e^2 \alpha Z}{E_\gamma^3} \times \left\{ (E_+^2 + E_-^2)\left[29.1 - \frac{8}{3} lnZ\right] + \frac{2}{3}E_+E_-\left[28.4 - \frac{8}{3} lnZ\right] \right\}$$

[11] Wheeler and Lamb, *Phys. Rev.* **101** 1836 (1939)

Tsai (1974) [12]:

Screened + Coulomb Correction + Atomic Excitation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_+} = \frac{r_e^2\alpha}{E_\gamma}\left\{\left(\frac{4}{3}\frac{E_+^2}{E_\gamma^2}-\frac{4}{3}\frac{E_+}{E_\gamma}+1\right)\right.$$
$$\times \left[Z^2\left(\phi_1(\gamma)-\frac{4}{3}lnZ-4f_c\right)+Z\left(\psi_1(\omega)-\frac{8}{3}lnZ\right)\right]$$
$$\left.- \frac{2}{3}\frac{E_+}{E_\gamma}\left(1-\frac{E_+}{E_\gamma}\right)\left[Z^2(\phi_1(\gamma)-\phi_2(\gamma))+Z(\psi_1(\omega)-\psi_2(\omega))\right]\right\}$$

where $\phi_1(\gamma)$, $\phi_2(\gamma)$, $\psi_1(\omega)$ and $\psi_2(\omega)$ are Z-dependent functions when the Thomas Fermi model of the atom is used.

[12] Y.-S. Tsai, *Rev. of Modern Physics*, Vol. 46, **4** (1974)

Contents

For simplicity, let us consider the bremsstrahlung process:

- Ultrarelativistic electron emits a low-energy photon $\Longrightarrow q_{//}$ can be very small
- Uncertainty principle $\rightarrow$ interaction takes place over a long distance, called *formation length*
- If anything happens to the electron or photon along this distance that disturbs their coherence, the emission of the photon will be suppressed
- The *Landau-Pomeranchuk-Migdal (LPM)* effect first discussed in [13] and slightly later in [14] is the suppression due to *multiple scattering*

[13] Lev Davidovich Landau and II Pomeranchuk, *Dokl. Akad. Nauk SSSR*, volume 92, page 735, 1953.

[14] Arkady B Migdal, *Physical Review*, 103(6):1811, 1956.

- **Pair-production** differential cross-section including LPM:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\epsilon} = 4\alpha r_e^2 Z(Z+\eta(Z))\,\xi(s) \times \left\{ \left[\frac{1}{3}G(s)+\frac{2}{3}\phi(s)\right]\left[\epsilon^2+(1-\epsilon)^2\right]\left[\frac{1}{4}\phi_1-\frac{1}{3}lnZ-f_c\right] + \frac{2}{3}G(s)\,\epsilon(1-\epsilon)\left[\frac{1}{4}\phi_2-\frac{1}{3}lnZ-f_c\right]\right\}$$

with $\epsilon \equiv E_+/E_\gamma$

θ_{ms}: the mean deflection angle due to multiple scattering along the formation length and θ_r: the mean emission angle

$$s \sim \frac{\theta_r}{\theta_{ms}}$$ **LPM is effective** when $s \lesssim 1$

suppression is important when:

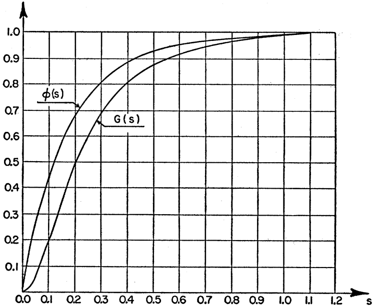
$\theta_{ms} > \theta_r \rightarrow s < 1$

$\Longrightarrow G(s) \rightarrow 0$ and $\phi(s) \rightarrow 0$

absence of suppression when:

$\theta_{ms} < \theta_r \rightarrow s > 1$

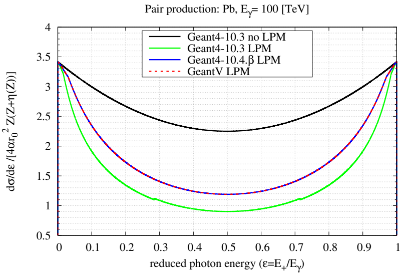
$\Longrightarrow G(s) \rightarrow 1$ and $\phi(s) \rightarrow 1$

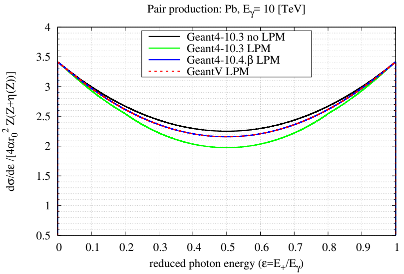


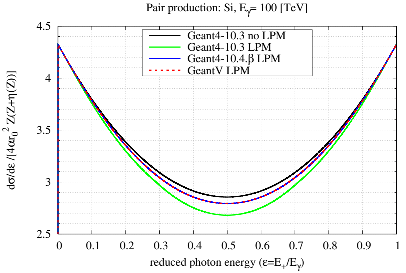
First results:

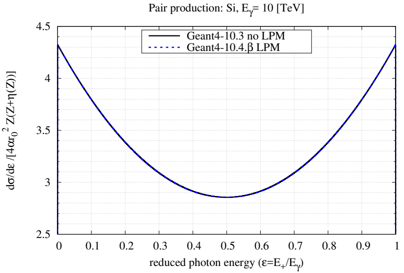
- Reviewing the pair production model including LPM suppression showed an inconsistent calculation of the LPM suppression variable and the material dependent LPM energy in the model used by *Geant4* $\leq$ *10.3*
- **An improved LPM description**, in accordance with the quantum mechanical calculations of Migdal [15] has been first implemented in a standalone code showing an improvement that would mostly affect heavy materials at FCC energies. This will be referred to as *GeantV LPM*.
- It is now integrated in Geant4. The improved Geant4 LPM will be referred to as *Geant4-10.4.*β *LPM*

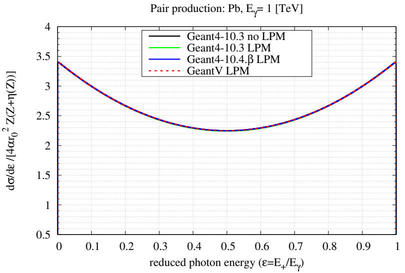
[15] Arkady B Migdal, *Physical Review*, 103(6):1811, 1956.

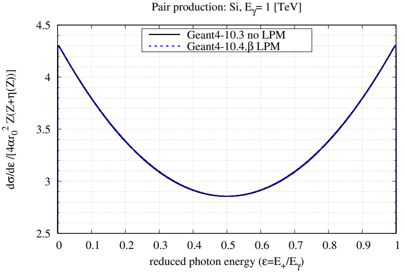

Pair production: Pb, Eγ= 100 [TeV]
Geant4-10.3 no LPM
Geant4-10.3 LPM
Geant4-10.4.β LPM
GeantV LPM
dσ/dε /[4αr0^2 Z(Z+η(Z))]
reduced photon energy (ε=E+/Eγ)
0.5 1 1.5 2 2.5 3 3.5 4
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

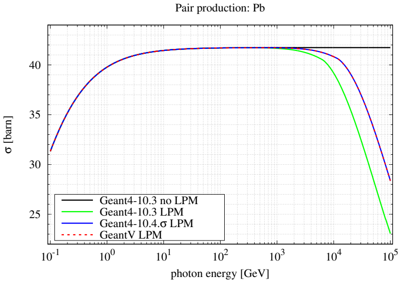

Pair production: Pb, Eγ= 10 [TeV]
Geant4-10.3 no LPM
Geant4-10.3 LPM
Geant4-10.4.β LPM
GeantV LPM
dσ/dε /[4αr0^2 Z(Z+η(Z))]
reduced photon energy (ε=E+/Eγ)
0.5 1 1.5 2 2.5 3 3.5 4
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Pair production: Si, Eγ= 100 [TeV]
Geant4-10.3 no LPM
Geant4-10.3 LPM
Geant4-10.4.β LPM
GeantV LPM
dσ/dε /[4αr0^2 Z(Z+η(Z))]
reduced photon energy (ε=E+/Eγ)
2.5 3 3.5 4 4.5
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

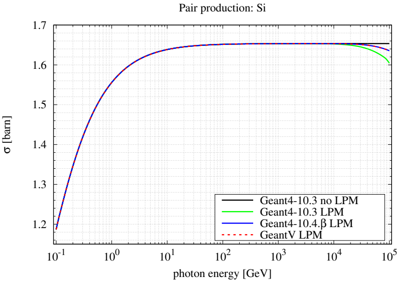

Pair production: Si, Eγ= 10 [TeV]
Geant4-10.3 no LPM
Geant4-10.4.β LPM
dσ/dε /[4αr0^2 Z(Z+η(Z))]
reduced photon energy (ε=E+/Eγ)
2.5 3 3.5 4 4.5
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1


Pair production: Pb, Eγ= 1 [TeV]
Geant4-10.3 no LPM
Geant4-10.3 LPM
Geant4-10.4.β LPM
GeantV LPM
dσ/dε /[4αr0^2 Z(Z+η(Z))]
reduced photon energy (ε=E+/Eγ)
0.5
1
1.5
2
2.5
3
3.5
4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Pair production: Si, Eγ= 1 [TeV]
Geant4-10.3 no LPM
Geant4-10.4.β LPM
dσ/dε /[4αr0^2 Z(Z+η(Z))]
reduced photon energy (ε=E+/Eγ)
2.5
3
3.5
4
4.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Pair production: Pb
σ [barn]
photon energy [GeV]
Geant4-10.3 no LPM
Geant4-10.3 LPM
Geant4-10.4.σ LPM
GeantV LPM



Pair production: Si
σ [barn]
photon energy [GeV]
Geant4-10.3 no LPM
Geant4-10.3 LPM
Geant4-10.4.β LPM
GeantV LPM

Summary:

- The LPM suppression is more important for heavier materials and more energetic gammas
- The *old* LPM in Geant4 overestimates the suppression
- For heavy materials, the *improved* LPM differs from the old one starting from few TeV gamma energy, which could be relevant for LHC
- For light materials, the improvement appears only above few tens of TeV gamma energy, relevant for FCC
- Improvement of LPM is already committed to Geant4 and just appeared in Geant4.10.03.ref03

Contents

Previous formulas apply to pair production with negligible energy transfer to the nucleus:

$$E_\gamma = E_+ + E_-$$

At extreme relativistic energies, it is necessary to account for nuclear recoil and size effects for the observation of large angle pair production. In which case the momentum conservation gives:

$$\mathbf{k} = \mathbf{p}_+ + \mathbf{p}_- + \mathbf{p_r}$$

where $\mathbf{p_r}$ is the recoil of the target (nucleus or electron)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E_+} \Longrightarrow \frac{\mathrm{d}^3\sigma}{\mathrm{d}E_+\mathrm{d}\Omega_+\mathrm{d}\Omega_-}$$

Future steps:

- Complete pair production model to be included in GeantV
- Documentation
- Investigate nuclear recoil effects for more accurate description of electromagnetic showers
- Investigate triplet production at high energies
- Extension of γ conversion to (μ_+, μ_-) and (π_+, π_-)