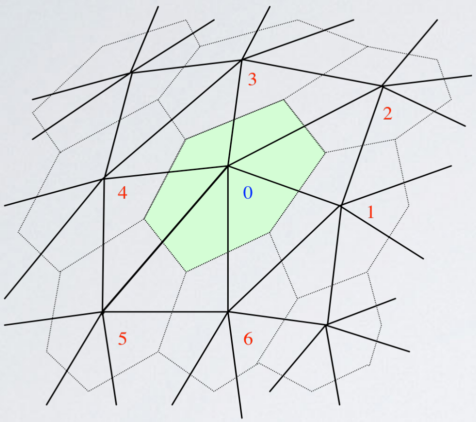
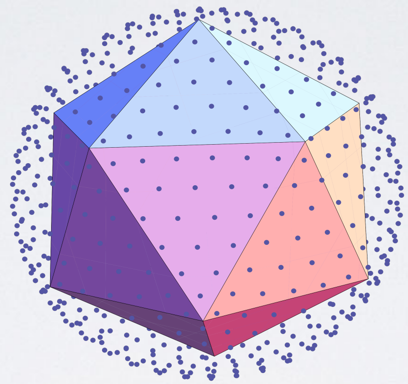
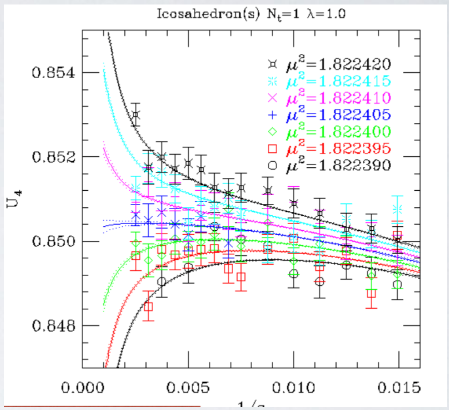
STATUS: LATTICE FIELD THEORY ON SPHERES AND CYLINDERS





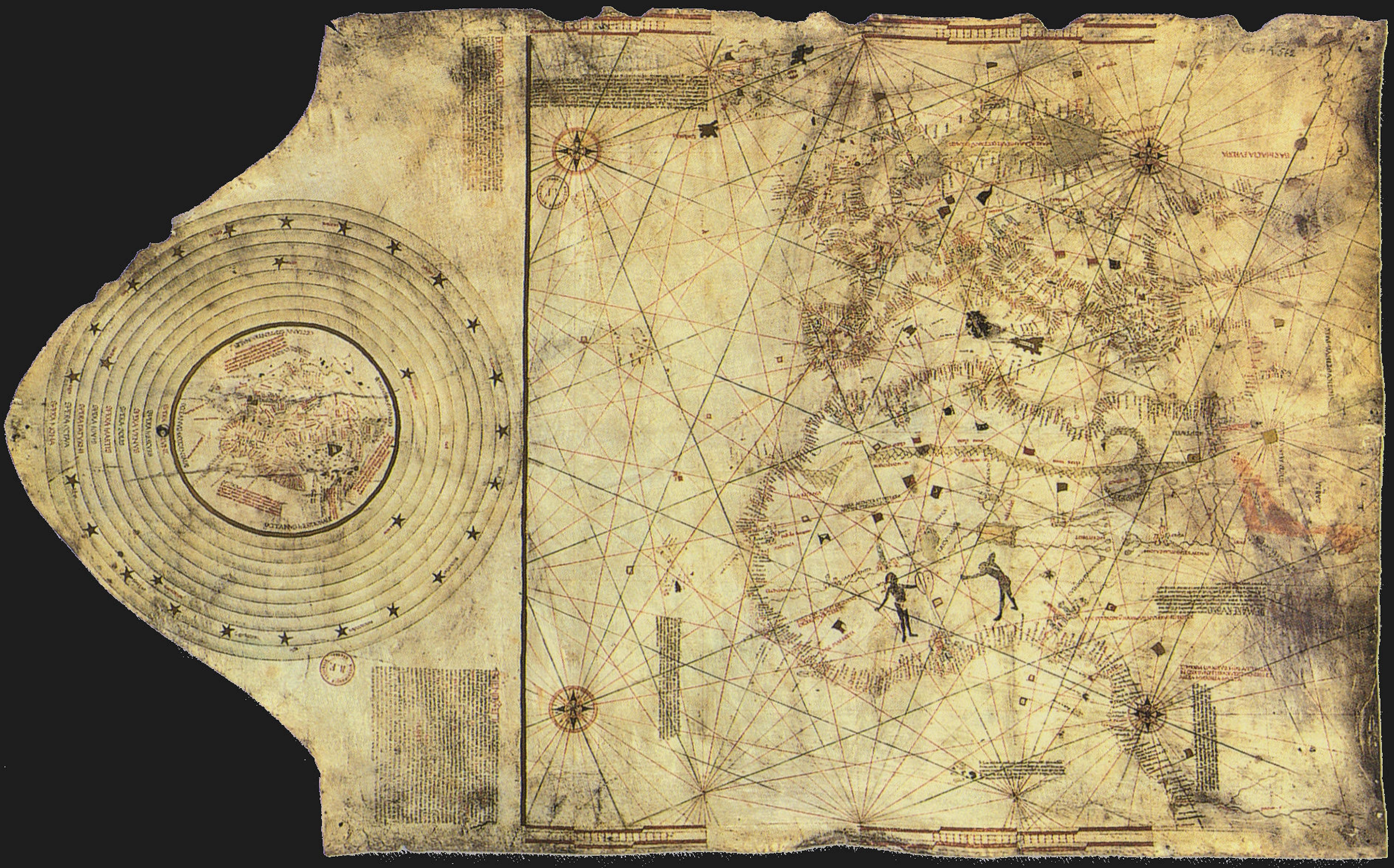


REGGE MANIFOLD + FINITE ELEMENT FIELDS + QUANTUM C=1/2 CFT

Rich Brower, Boston University at QCDNA X 2017

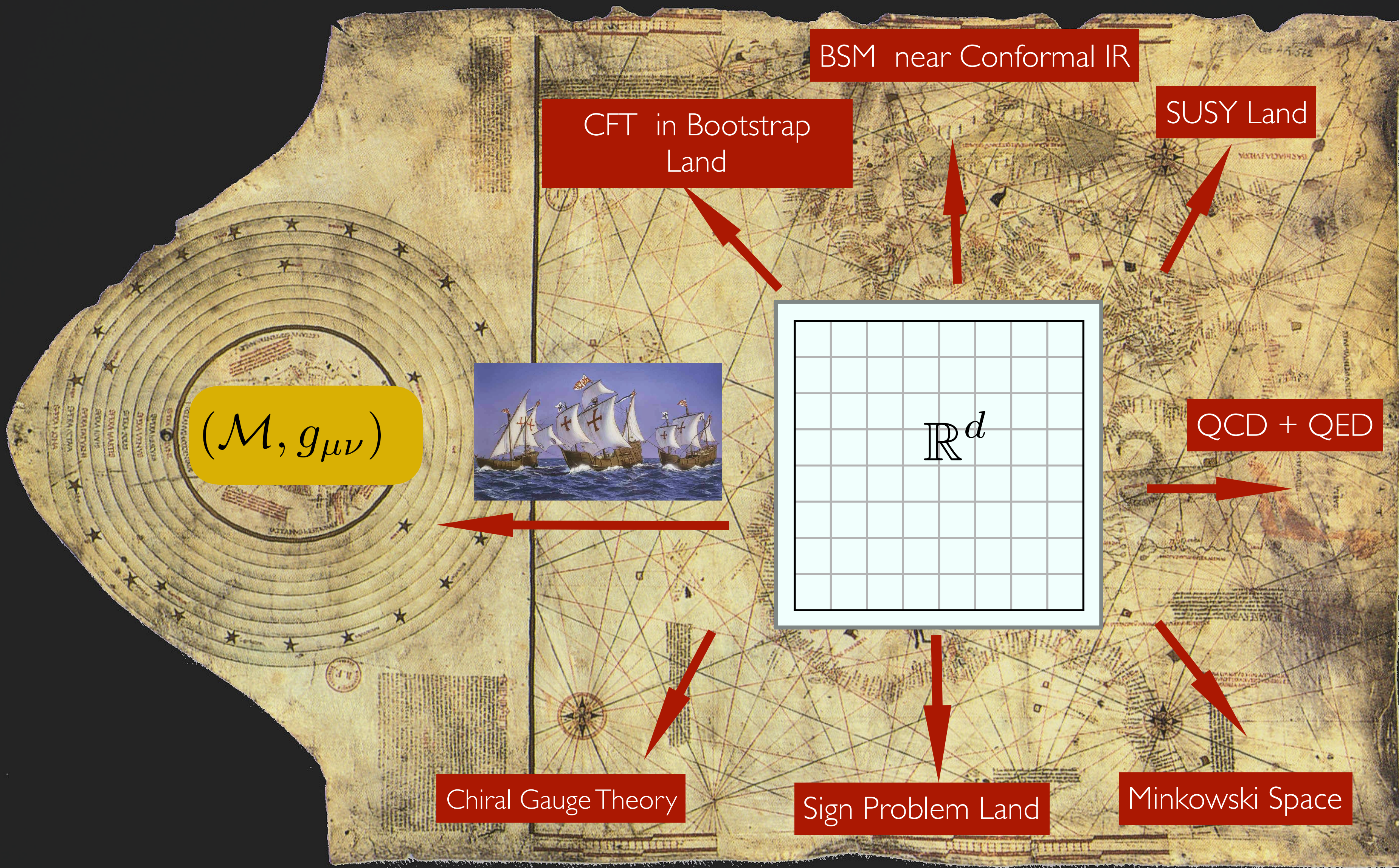
with G. Fleming, A. Gasbarro, T. Raben, C-I Tan, E. Weinberg

LATTICE THEORY BEYOND FLAT EARTH APPROXIMATION



Christopher Colombus map. c.1490.

WILD FRONTIERS OF STRONGLY INTERACTING FIELD THEORY



Christopher Colombus map c.1490.

LATTICE FIELD THEORY ON RIEMANN MANIFOLD

1. SIMPLICIAL GEOMETRY : (Regge Calculus)
2. CLASSICAL ACTION: ("Finite element" approx. Hilbert space)
3. QFE: QUANTUM CT: (Simplicial Perturbation theory)

- CONVERGENCE PROOFS* (No, only heuristics)
- NUMERICAL TESTS (Yes, one and 1/2 CFT examples)

* ALL Renormalizable QFT (UV complete) ARE renormalizable on any Smooth Riemann Manifold.

CAN THE LATTICE DEFINE THE NON-PERTURBATIVE THEORY?

BUT WHY SPHERES AND CYLINDERS?

- Spheres and Cylinders are Weyl Maps* & CFT are "preserved".
- Sphere: For CFT, no finite volume approx. & define: "c-theorems"
- Cylinders: Radial Quant^* Bndry of global AdS (H = Dilatations)

$\mathbb{R}^d \to \mathbb{S}^d$

$$ds^2_{flat} = \sum_{\mu=1}^{d} dx^\mu dx^\mu = e^{\sigma(x)} d\Omega_d^2 \xrightarrow{Weyl} d\Omega_d^2 .$$

$\mathbb{R}^{d+1} \to \mathbb{R} \times \mathbb{S}^d$

$$ds^2_{flat} = \sum_{\mu=1}^{d+1} dx^\mu dx^\mu = e^{2t}(dt^2 + d\Omega_d^2) \xrightarrow{Weyl} (dt^2 + d\Omega_d^2) .$$

*R.C.B., G.Fleming and H.Neuberger "Lattice Radial Quantization: 3D Ising" PL B721 (2013)

My Oringinal Motivation

- Conformal Field Theories, interesting for
  - BSM composite Higgs
  - AdS/CFT weak-strong duality
  - Model building & Critical Phenomena in general

Potential Huge Advantage for CFT!

- Linear Hypercubic vs Exponential Radial Lattice

$$a < r < aL \to 1 < \log(r) < L$$

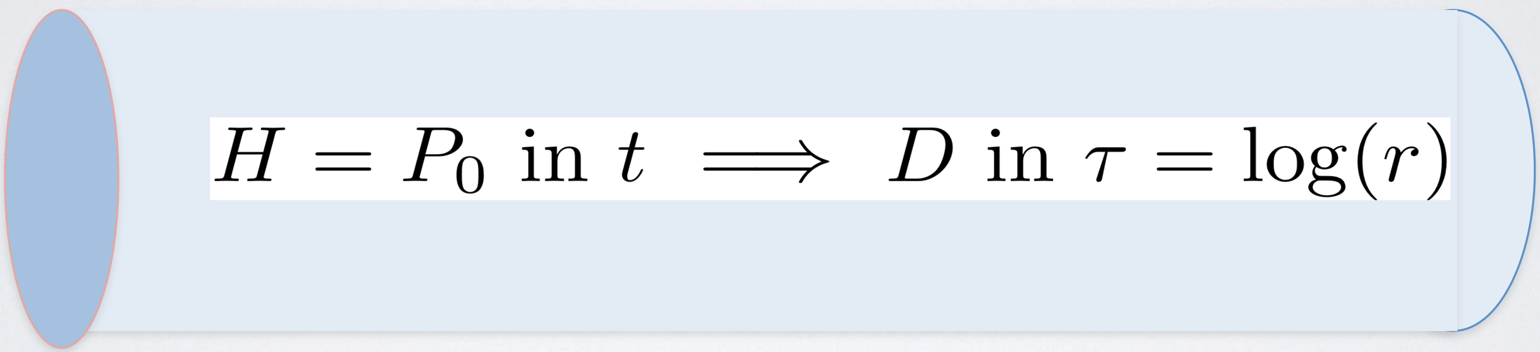
Both UV asy freedom and IR conformal on a lattice?

RADIAL QUANTIZATION: NATURAL FOR CFT

Conformal (near conformal) theories are interesting for

- *BSM composite Higgs*
- *AdS/CFT weak-strong duality*
- *Model building & Critical Phenomena in general*

$$\mathbb{R} \times \mathbb{T}^3 \qquad \text{vs} \qquad \mathbb{R} \times \mathbb{S}^3$$



$$H = P_0 \text{ in } t \implies D \text{ in } \tau = \log(r)$$

Potential advantage: Scales increases exponentially in lattice size L!

$$1 < t < aL \implies 1 < \tau = log(r) < L$$

BACK TO THE BOOTSTRAP! (CFTS : NO LOCAL LAGRANGIAN)

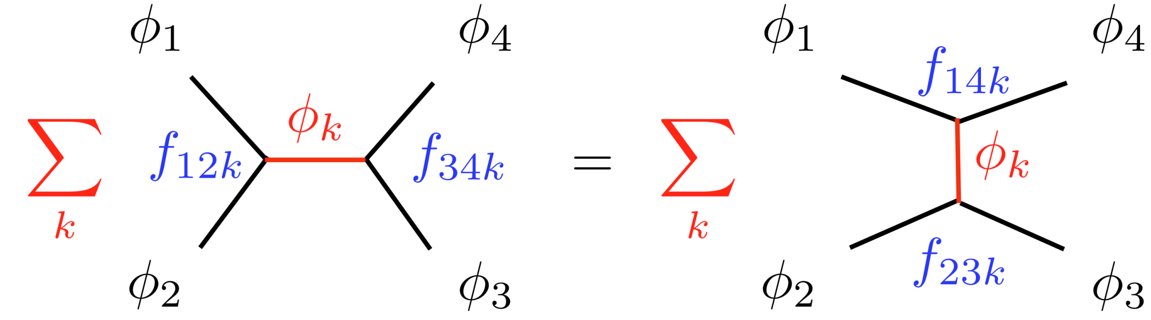
(i.e. Data: spectra + couplings to conformal blocks)

Exact 2 and 3 correlators

$$\langle \phi(x_1)\phi(x_2)\rangle = C\frac{1}{|x_1 - x_2|^{2\Delta}}$$

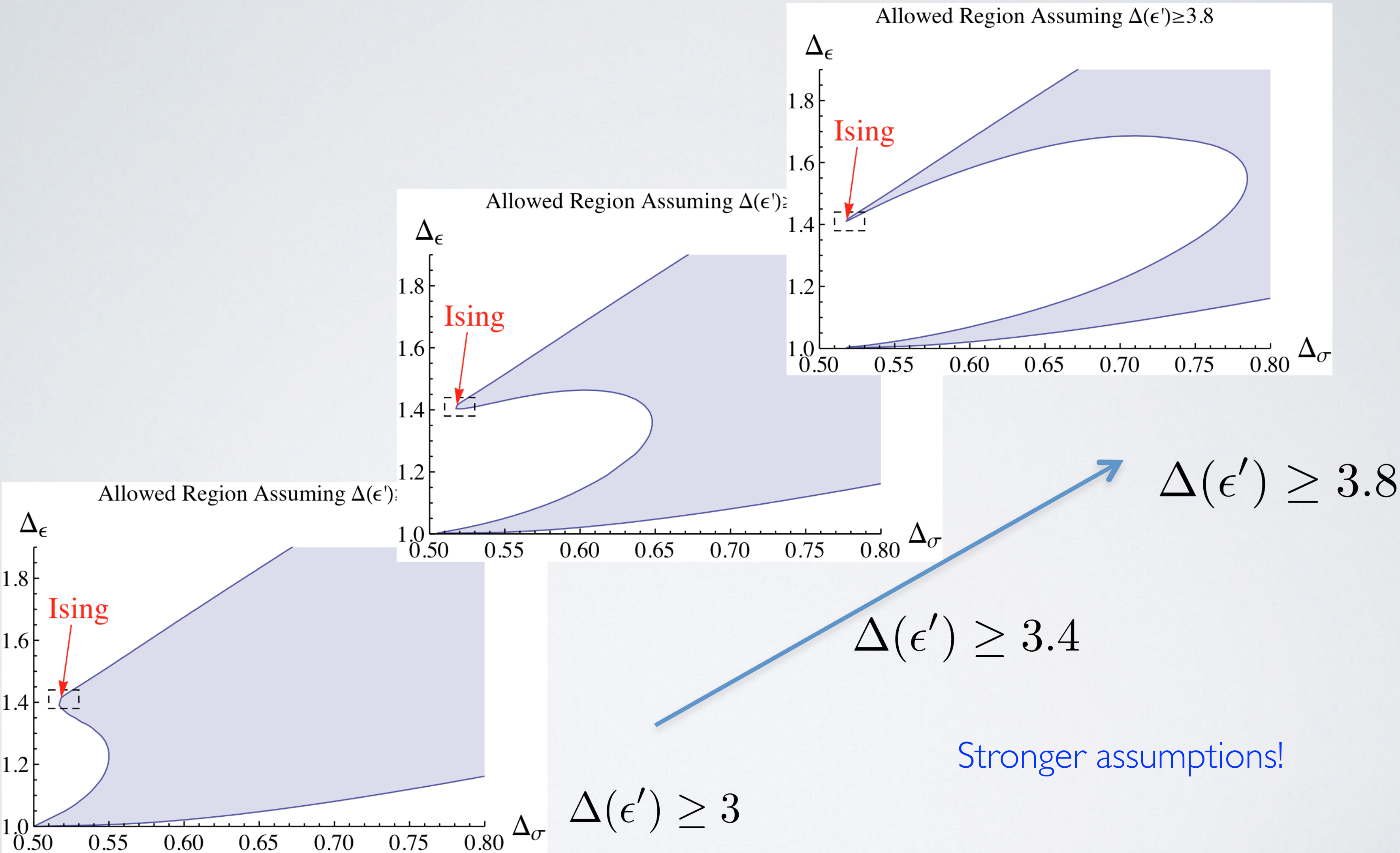
$$\mathcal{O}_i(x_1)\mathcal{O}_j(x_2) = \sum_k \frac{f_{ijk}}{|x_1 - x_2|^{\Delta_i+\Delta_j-\Delta_k}}\mathcal{O}_k(0)$$

Only "tree" diagrams!
"partial waves" exp: sum over conformal blocks



CFT Bootstrap: OPE & factorization completely fixed the theory

*INEQUALITIES FROM BOOTSTRAP**



- "Solving the 3D Ising Model with the Conformal Bootstrap" (El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin and Vichi) arXiv:1203.6064v1v [hep-th] (2012)

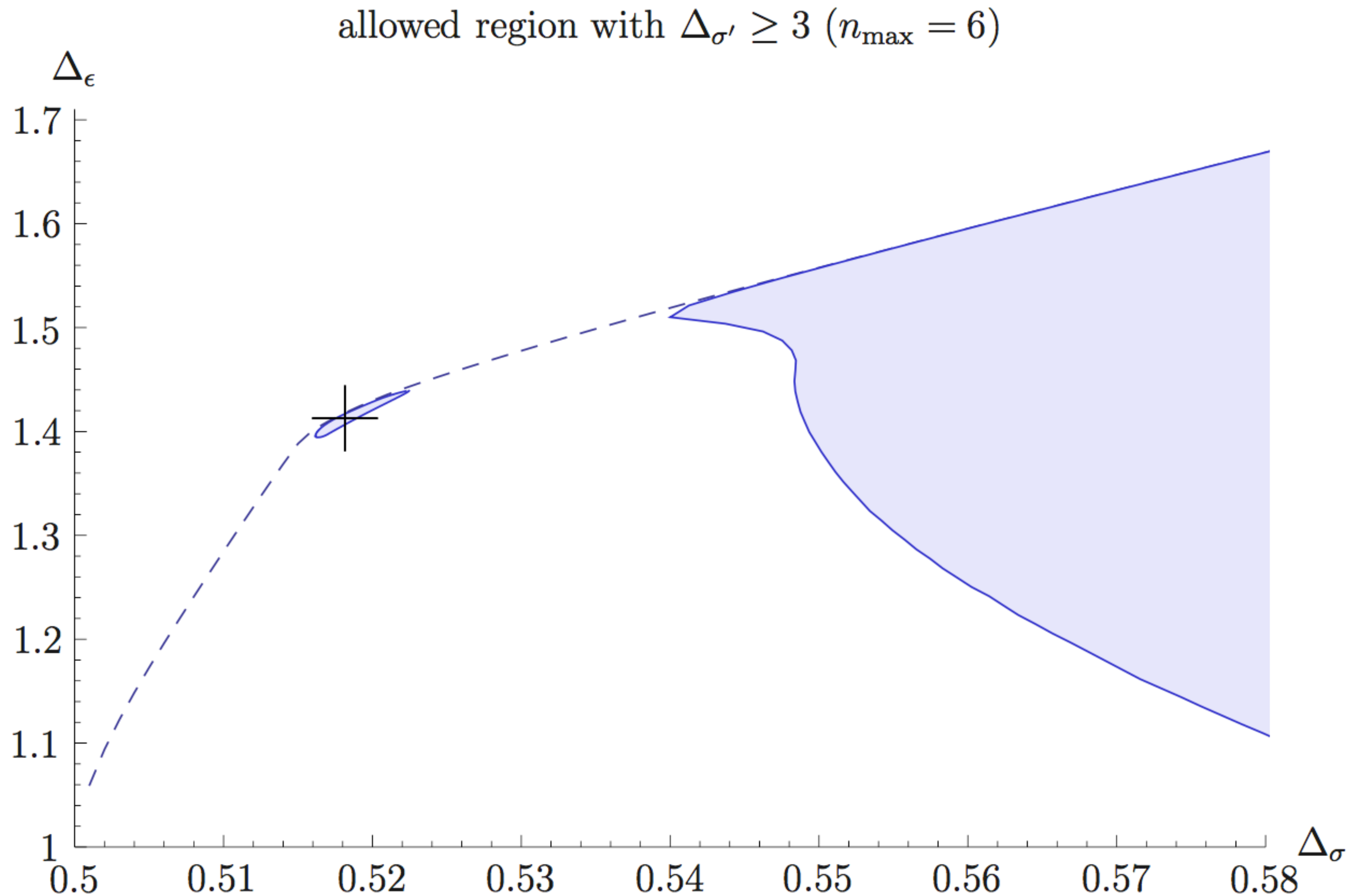
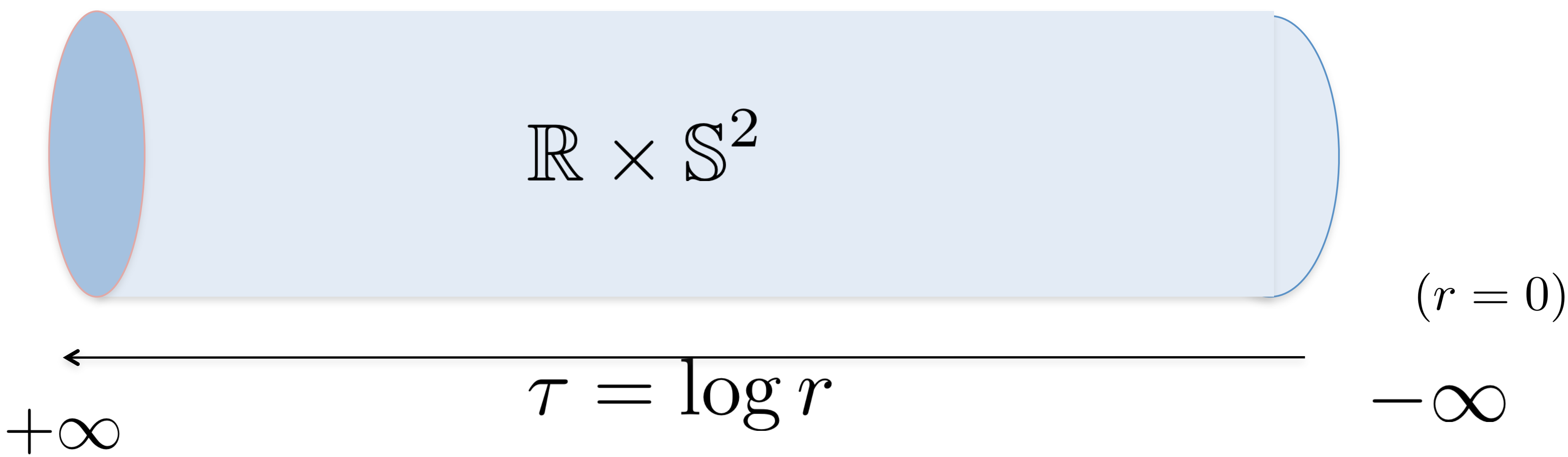


Figure 2: Allowed region of $(\Delta_\sigma, \Delta_\epsilon)$ in a $\mathbb{Z}_2$-symmetric CFT$_3$ where $\Delta_{\sigma'} \geq 3$ (only one $\mathbb{Z}_2$-odd scalar is relevant). This bound uses crossing symmetry and unitarity for $\langle\sigma\sigma\sigma\sigma\rangle$, $\langle\sigma\sigma\epsilon\epsilon\rangle$, and $\langle\epsilon\epsilon\epsilon\epsilon\rangle$, with $n_{\max} = 6$ (105-dimensional functional), $\nu_{\max} = 8$. The 3D Ising point is indicated with black crosshairs. The gap in the $\mathbb{Z}_2$-odd sector is responsible for creating a small closed region around the Ising point.

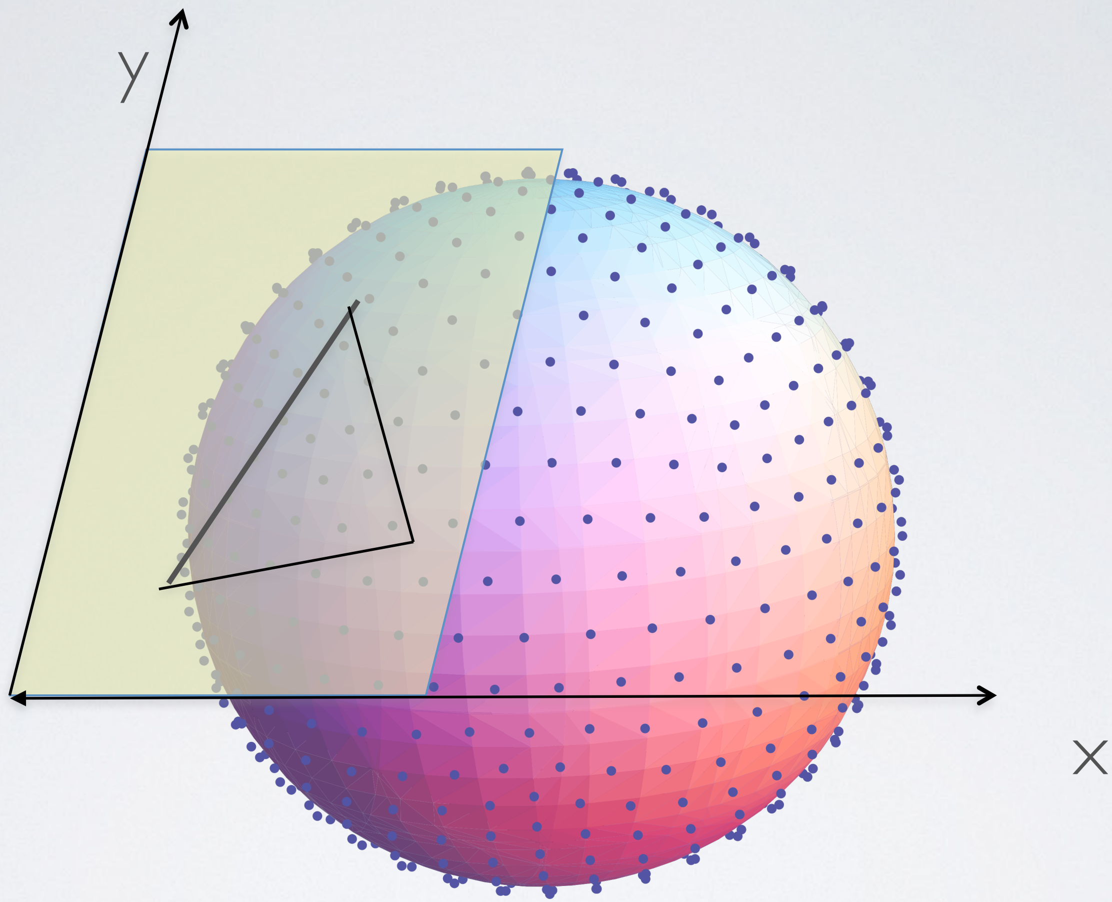
First Attempt: 3-d Ising at Wilson-Fisher FP



$$Z_{Ising} = \sum_{\sigma(x,t)=\pm 1} e^{\beta \sum_{t,\langle x,y\rangle} \sigma(t,x)\sigma(t,y) + \beta \sum_{t,x} \sigma(t+1,x)\sigma(t,x)}$$

RCB, G. Fleming, H. Neuberger Phys.Lett, B721 (2013)

FREE SCALAR AND FERMON ON S2



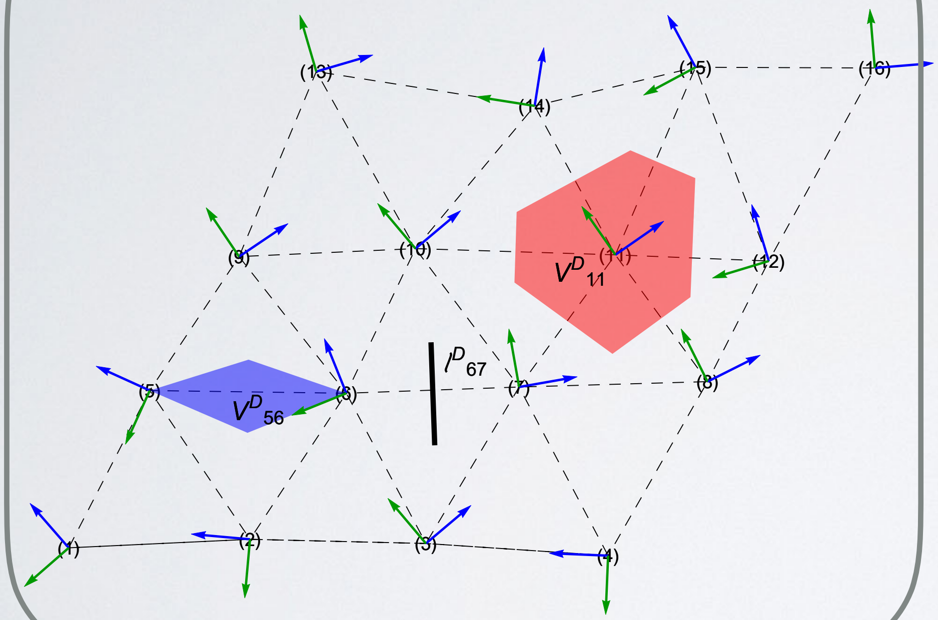
approximate spherical triangles as flat in local tangent plane

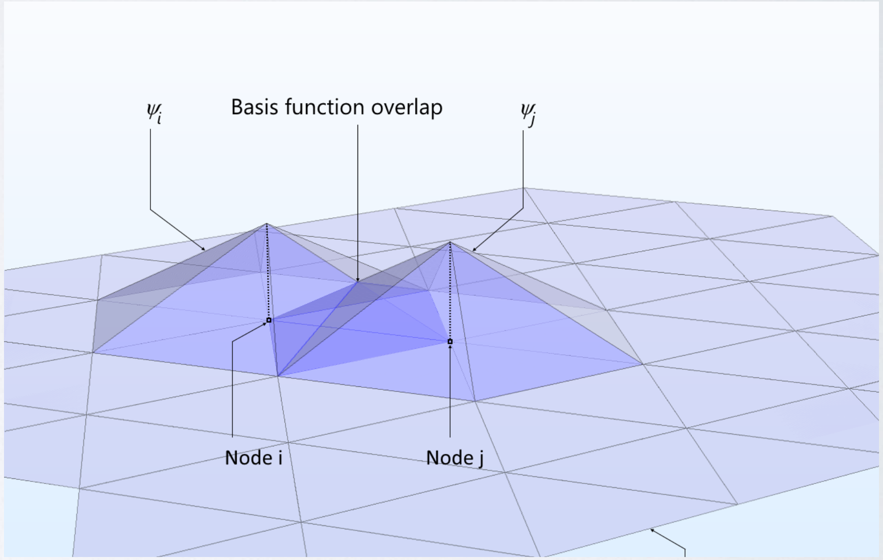
REGGE: Piecewise linear metric

$$(\mathcal{M}, g_{\mu\nu}(x)) \leftrightarrow (\mathcal{M}_\sigma, g_\sigma = \{l_{ij}\})$$

FEM: Piecewise linear fields

$$\phi(x) \leftrightarrow \phi = \sum_i \phi_i W_i(\xi)$$

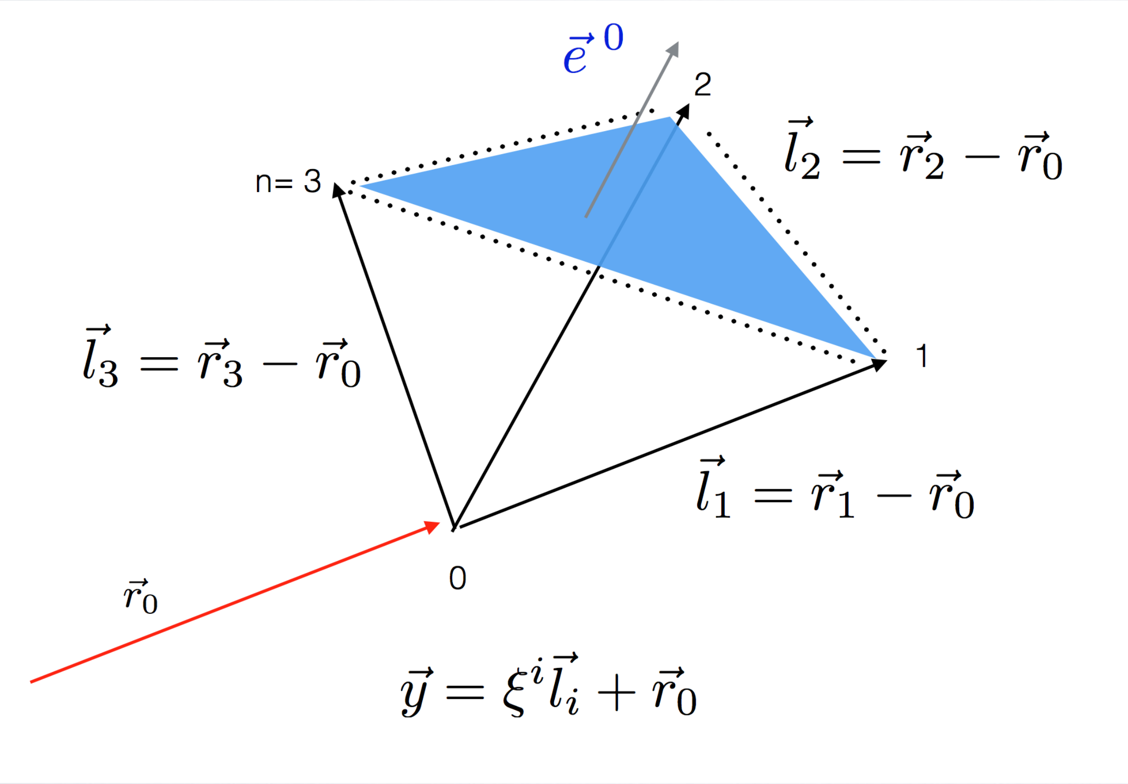




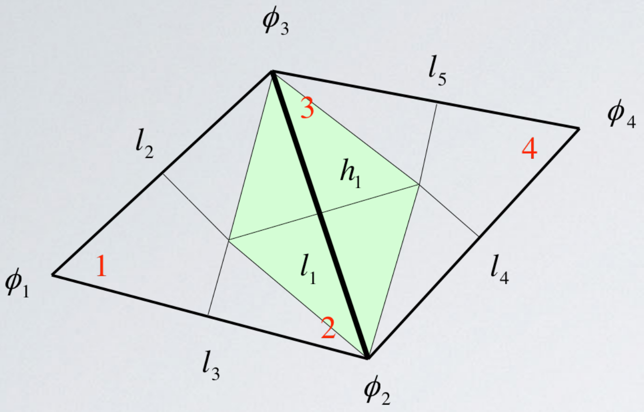
Simplicial Complex/Delaunay Dual Complex + Regge flat metric on each Simplex

Actually fancier methods: Discrete Exterior Calculus (scalar), Spin connection (Fermion), Wilson links (gauge) , etc.

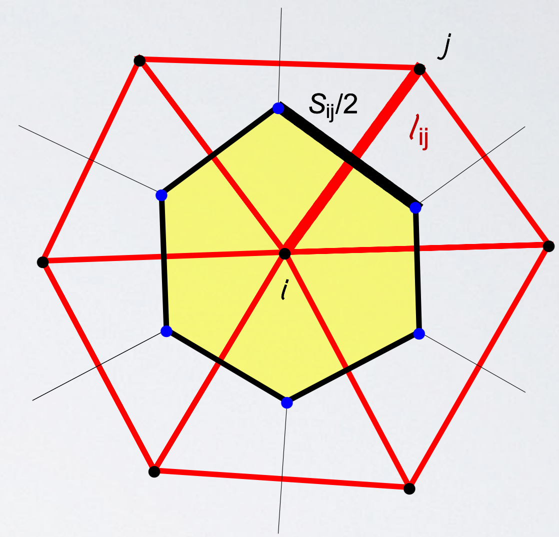
FLAT SIMPLEX IN TANGENT PLANE



REGGE CALCULUS FEM FORMULATION



LINEAR FEM/ REGGE CALCULUS *



$$*d * d\phi_i$$

$$\text{Delaunay Link Area:} \quad A_d = h_1 l_1$$

$$FEM : A_d \frac{(\phi_2 - \phi_3)^2}{l_1^2}$$

DISCRETE EXTERIOR CALCULUS
or
CHRIST FRIEBERG & LEE

* H. Hamber, S. Liu, **Feynman rules for simplicial gravity,** NP B475 (1996)

LINEAR FINITE ELEMENT APPROACH

$$S = \frac{1}{2}\int_{\mathcal{M}} d^D x\sqrt{g}\,[g^{\mu\nu}\partial_\mu\phi(x)\partial_\nu\phi(x) + m^2\phi^2(x) + \lambda\phi^4(x)]$$

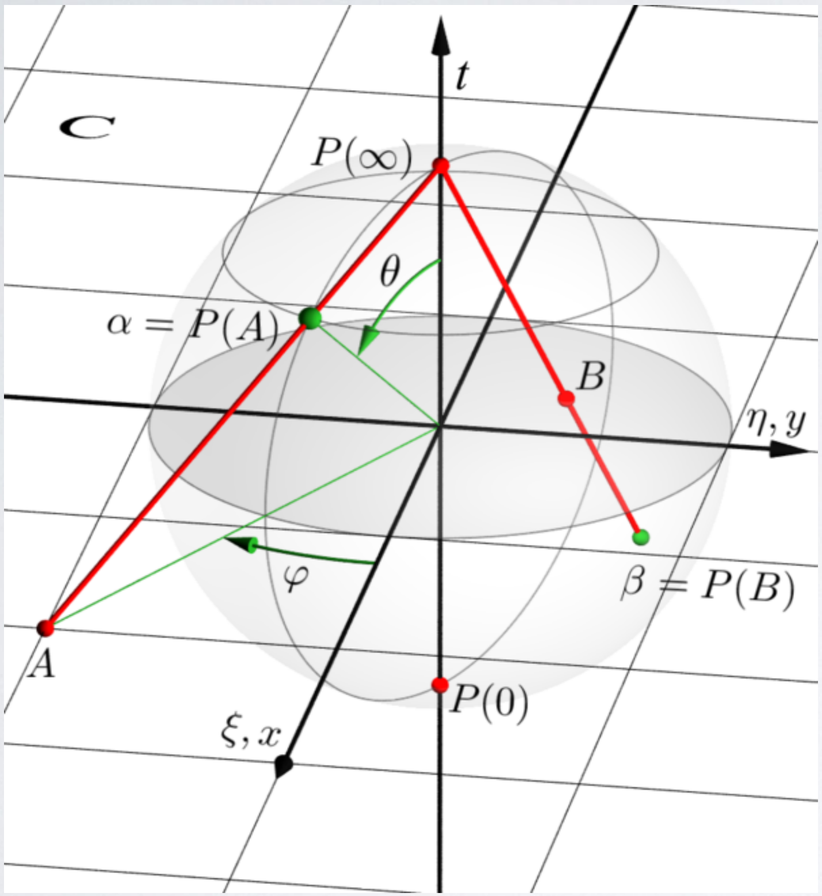
$$(y = \xi_1\vec{r}_1 + \xi_2\vec{r}_2 + \xi_3\vec{r}_3$$
$$\text{with}\xi_1 + \xi_2 + \xi_3 = 1)$$

$$\begin{aligned} I_\sigma &= \frac{1}{2}\int_\sigma d^D y[\vec{\nabla}\phi(y)\cdot\vec{\nabla}\phi(y) + m^2\phi^2(y) + \lambda\phi^4(y)] \\ &= \frac{1}{2}\int_\sigma d^D\xi\sqrt{g}\,[g^{ij}\partial_i\phi(\xi)\partial_j\phi^2(\xi) + m^2\phi^2(\xi) + \lambda\phi^4(\xi)] \end{aligned}$$

$$I_\sigma \simeq \sqrt{g_0}\left[g_0^{ij}\frac{(\phi_i - \phi_0)(\phi_j - \phi_0)}{l_{i0}\, l_{j0}} + m^2\phi_0^2 + \lambda\phi_0^4\right]$$

TEST 2D ISING/PHI 4TH ON THE RIEMANN SPHERE

Stereographic project of Complex Plane:



$$\xi = \tan(\theta/2)e^{-i\phi} = \frac{x+iy}{1+z}$$

$$|\xi| = \sqrt{\xi_1^2 + \xi_2^2} \qquad \xi = \xi_1 + i\xi_2$$

$$\vec{r} = (x, y, z) \qquad \vec{r}\cdot\vec{r} = 1$$

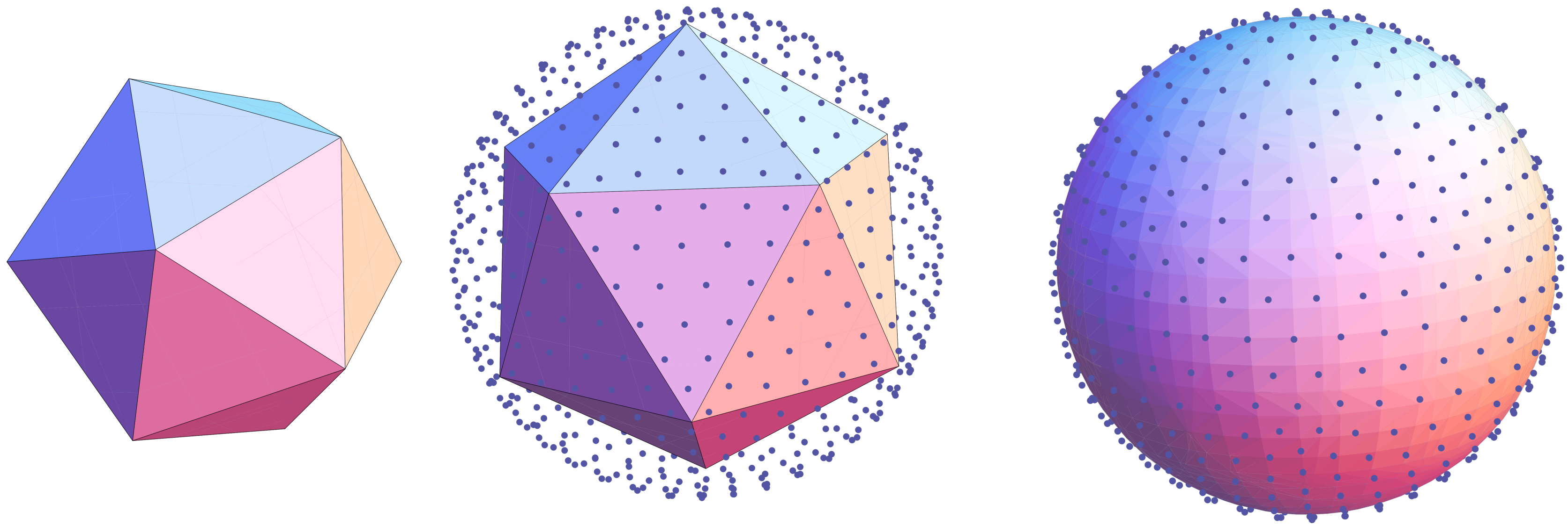
$$|\vec{r}_1 - \vec{r}_2| = 2 - 2\cos(\theta_{12})$$

Conformally Invariant
Cross Ratios are "Preserved"

$$\frac{|\xi_1 - \xi_2||\xi_3 - \xi_4|}{|\xi_1 - \xi_3||\xi_1 - \xi_4|} = \frac{|\vec{r}_1 - \vec{r}_2||\vec{r}_1 - \vec{r}_2|}{|\vec{r}_1 - \vec{r}_3||\vec{r}_1 - \vec{r}_4|}$$

Order s Refined Triangulated Icosahedron

$s = 1$ $\qquad$ $s = 8$



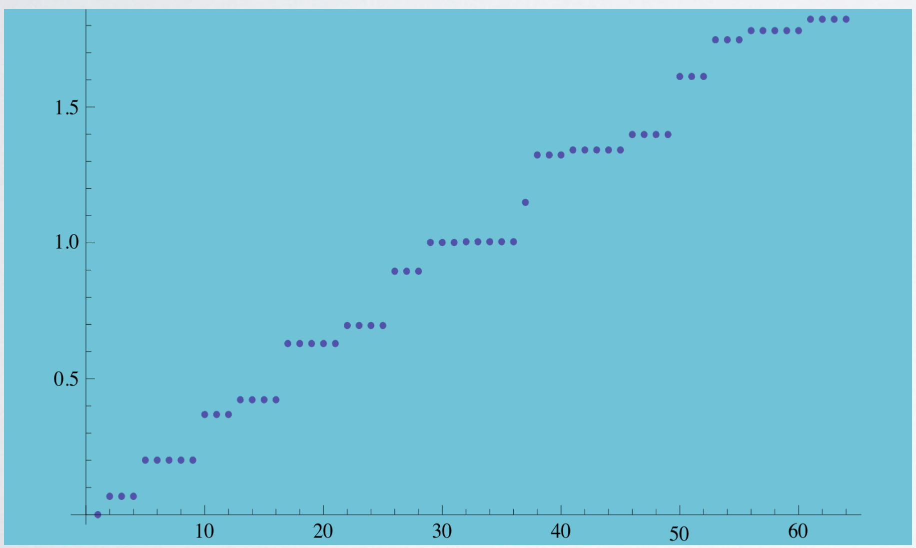
I = 0 (A),1 (T1) , 2 (H) are irreducible 120
Icosahedral subgroup of O(3)

FEM FIXES THE HUGE SPECTRAL DEFECTS OF THE LAPLACIAN ON THE SPHERE

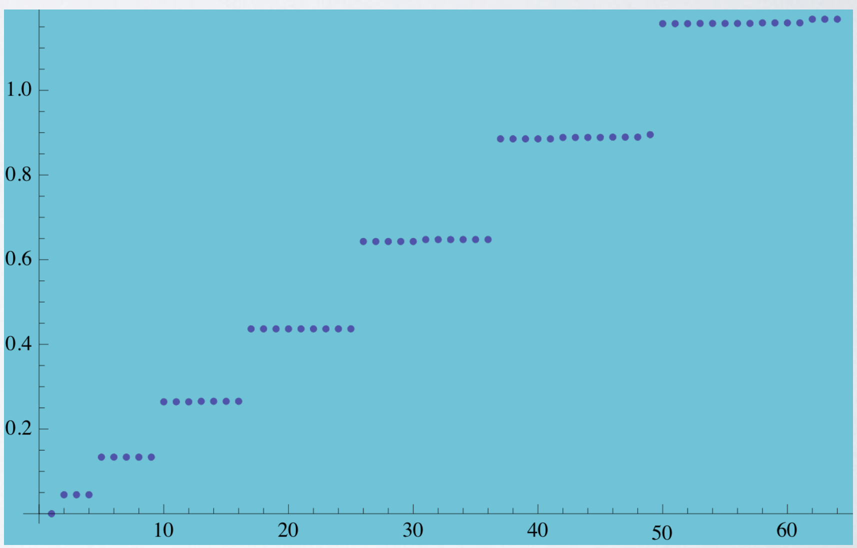
For s = 8 first (l+1)*(l +1) = 64 eigenvalues

BEFORE (K = 1)

AFTER (FEM K's)

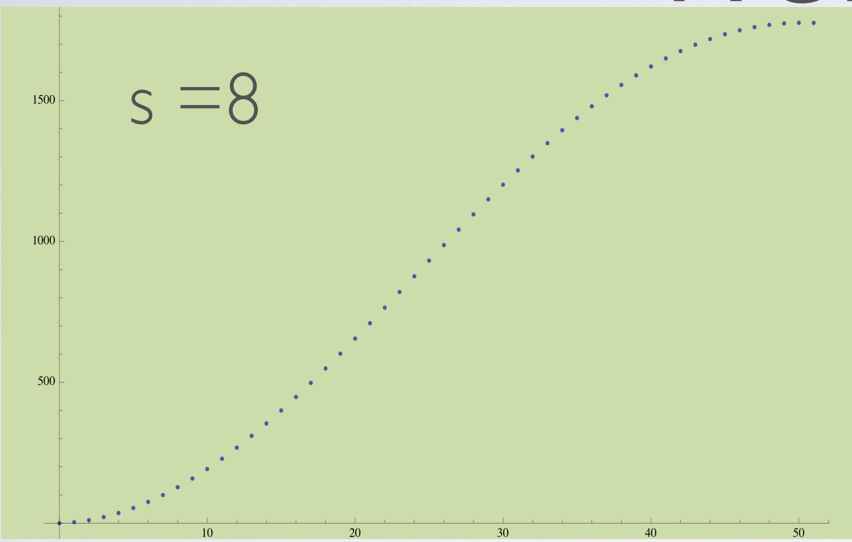


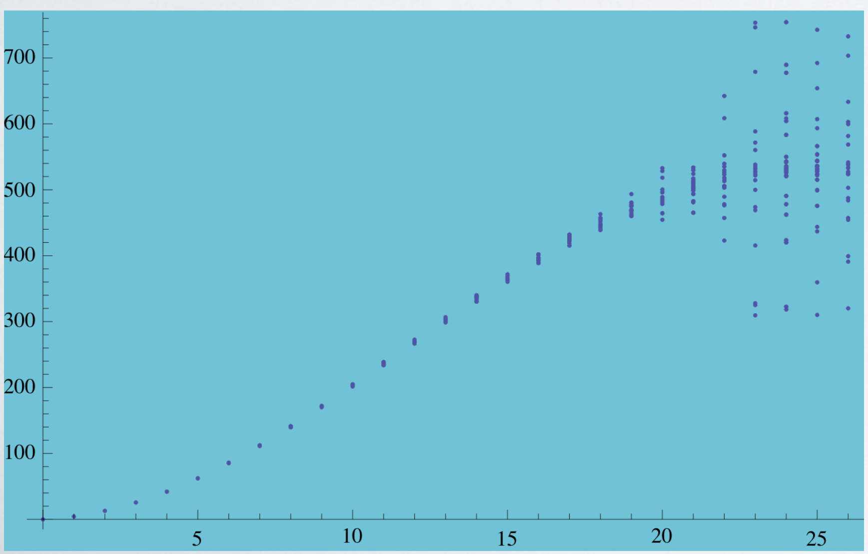
l, m

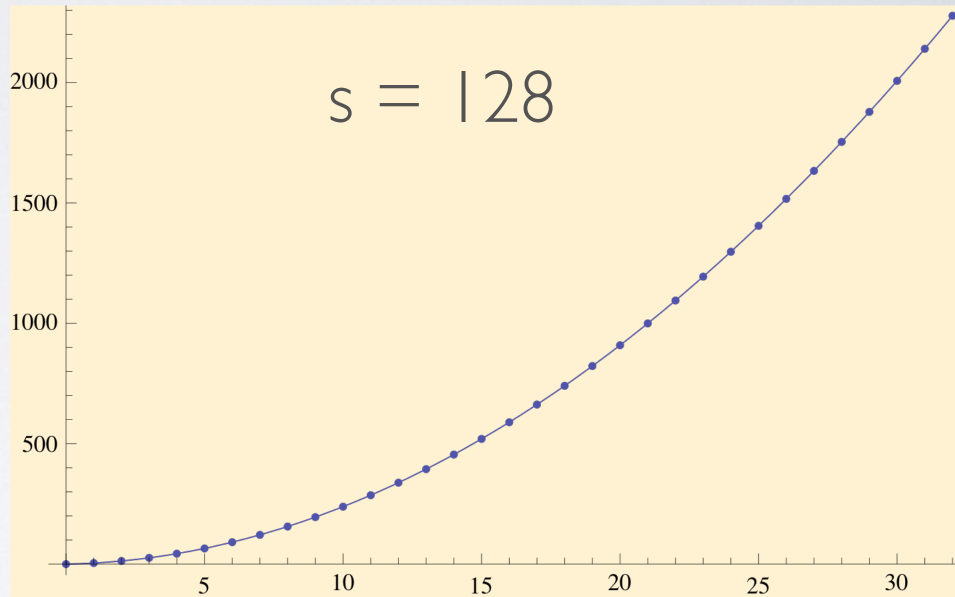


l, m

SPECTRUM OF FE LAPLACIAN ON A SPHERE







Fit

$$l + 1.00012\, l^2 - 13.428110^{-6}\, l^3 - 5.5724410^{-6}\, l^4$$

DIRAC ON SIMPLIAL MANIFOLD

$$S = \frac{1}{2}\int d^D x \sqrt{g}\bar{\psi}[\mathbf{e}^\mu(\partial_\mu - \frac{i}{4}\boldsymbol{\omega}_\mu(x)) + m]\psi(x)$$

$\mathbf{e}^\mu(x) \equiv e^\mu_a(x)\gamma^a$ Verbein & Spin connection*

$\boldsymbol{\omega}_\mu(x) \equiv \omega^{ab}_\mu(x)\sigma_{ab}$, $\sigma_{ab} = i[\gamma_a, \gamma_a]/2$

(1) New spin structure "knows" about intrinsic geometry

(2) Need to avoid simplex curvature singularities at sites.

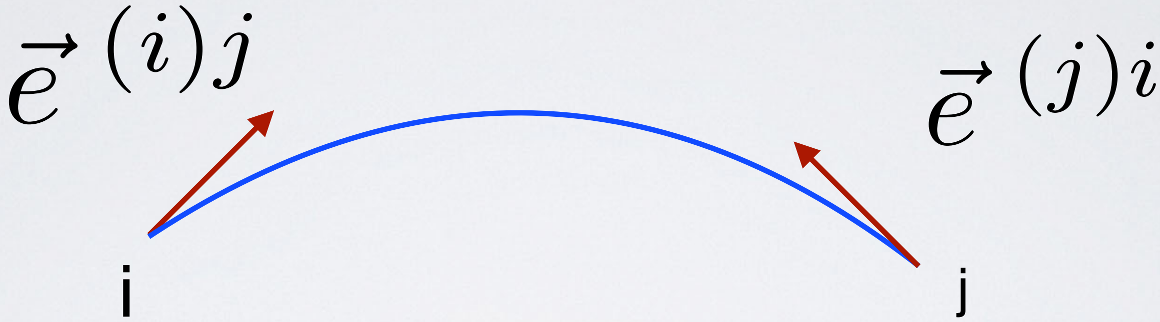
(3) Spinors rotations (Lorentz group) is double of O(D).

$$e^{i(\theta/2)\sigma_3/2} \to -1 \qquad \text{as} \quad \theta \to 2\pi$$

* Must satisfy the tetrad postulate! $\omega^{ab}_\mu = \frac{1}{2}e^{\nu[a}(e^{b]}_{\nu,\mu} - e^{b]}_{\mu,\nu} + e^{b]\sigma}e^c_\mu e_{\nu c,\sigma})$.

CONSTUCTING THE DIRAC ACTION

$$S_{naive} = \frac{1}{2} \sum_{\langle i,j \rangle} \frac{V_{ij}}{l_{ij}} [\bar{\psi}_i \vec{e}^{\,(i)j} \cdot \vec{\gamma} \Omega_{ij} \psi_j - \bar{\psi}_j \Omega_{ji} \vec{e}^{\,(i)j} \cdot \vec{\gamma} \psi_i] + \frac{1}{2} m V_i \bar{\psi}_i \psi_i$$



DO NOT USE PIECEWISE LINEAR REGGE CALCULUS MANIFOLD!

Simplicial Tetrad Hypothesis $\quad e_a^{(i)j} \gamma^a \Omega_{ij} + \Omega_{ij} e_a^{(j)i} \gamma^a = 0$

Gauge Invariance under Spin(D) transformations

$$\psi_i \to \Lambda_i \psi \quad , \quad \bar{\psi}_j \to \bar{\psi}_j \Lambda_j^\dagger \quad , \quad \mathbf{e}^{(i)j} \to \Lambda_i \mathbf{e}^{(i)j} \Lambda_i^\dagger \quad , \quad \Omega_{ij} \to \Lambda_i \Omega_{ij} \Lambda_j^\dagger$$

WILSON/CLOVER TERM

$$[\gamma_\mu(\partial_\mu - iA_\mu)]^2 = (\partial_\mu - iA_\mu)^2 - \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu} \ ,$$

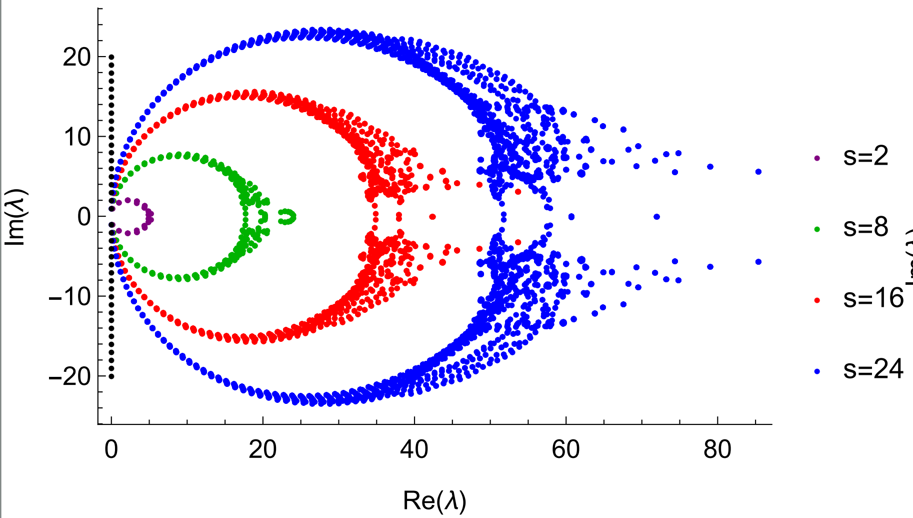
$$\Downarrow$$

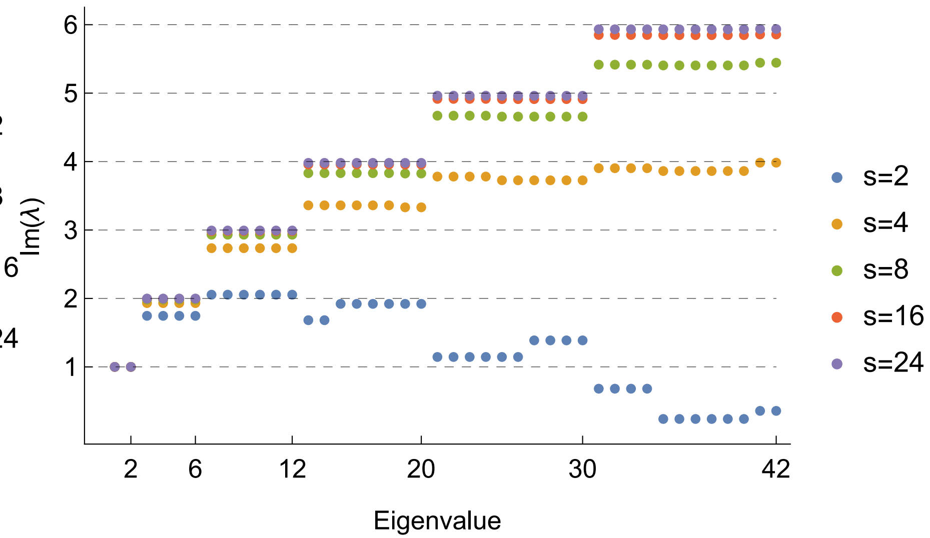
$$[\mathbf{e}^\mu_a(\partial_\mu - i\boldsymbol{\omega}_\mu)]^2 = \frac{1}{\sqrt{g}}\boldsymbol{D}_\mu\sqrt{g}g^{\mu\nu}\boldsymbol{D}_\nu - \frac{1}{2}\sigma^{ab}e^\mu_a e^\nu_b \boldsymbol{R}_{\mu\nu}$$

$$\Downarrow$$

$$S_{Wilson} = \frac{r}{2}\sum_{\langle i,j\rangle}\frac{aV_{ij}}{l^2_{ij}}(\bar{\psi}_i - \bar{\psi}_j\Omega_{ji})(\psi_i - \Omega_{ij}\psi_j)$$

2D DIRAC SPECTRA ON SPHERE





E = j + 1/2

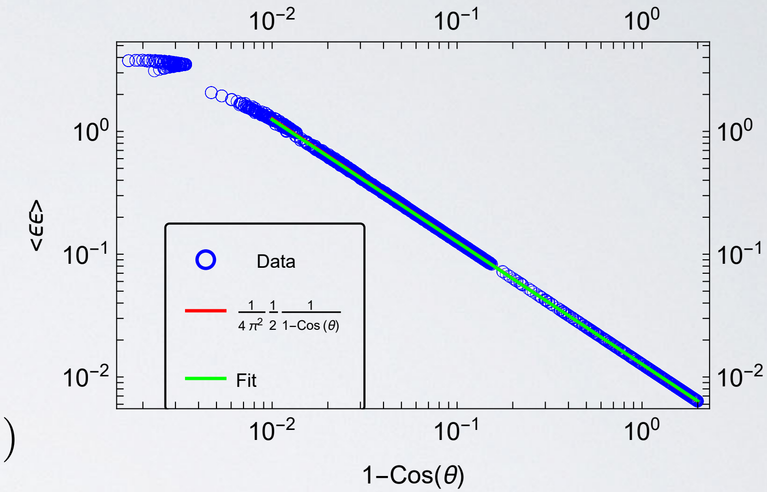
Exact is integer spacing for j = 1/2, 3/2, 5/2 … Exact degeneracy 2j + 1: No zero mode in chiral limit!.

Lattice Dirac Fermions on a Simplicial Riemannian Manifold https://arxiv.org/abs/1610.08587

Richard C. Brower, George T. Fleming, Andrew D. Gasbarro, Timothy G. Raben, Chung-I Tan, Evan S. Weinberg

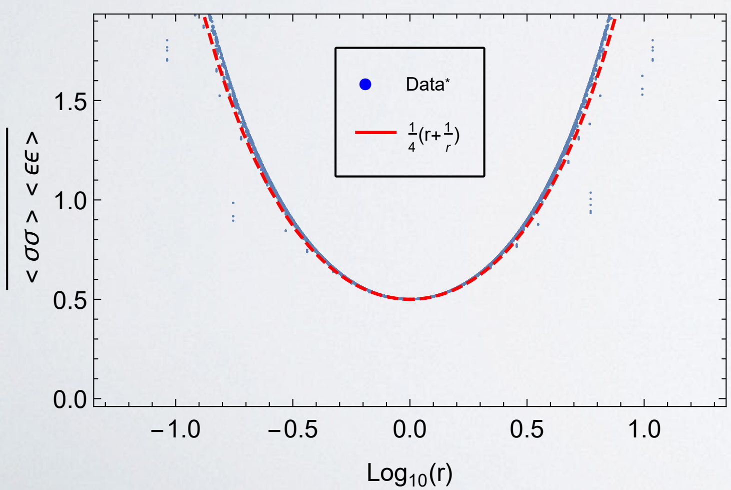
FREE MAJORANA FERMIONS ON S2

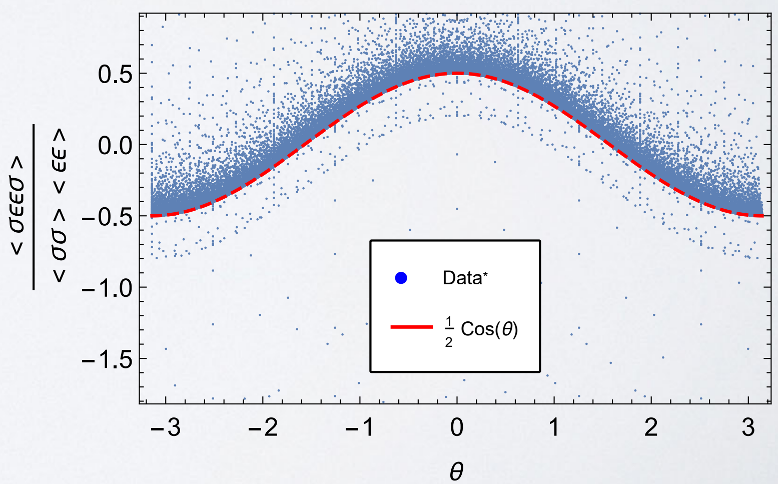
$$\langle \psi(z_1)\bar{\psi}(z_1)\bar{\psi}(z_1)\psi(z_2)\rangle = \left[\frac{1}{\partial}\right]_{z_1,z_2}\left[\frac{1}{\bar{\partial}}\right]_{z_1,z_2} = \frac{1}{4\pi^2}\frac{1}{|z_1 - z_2|^2}$$



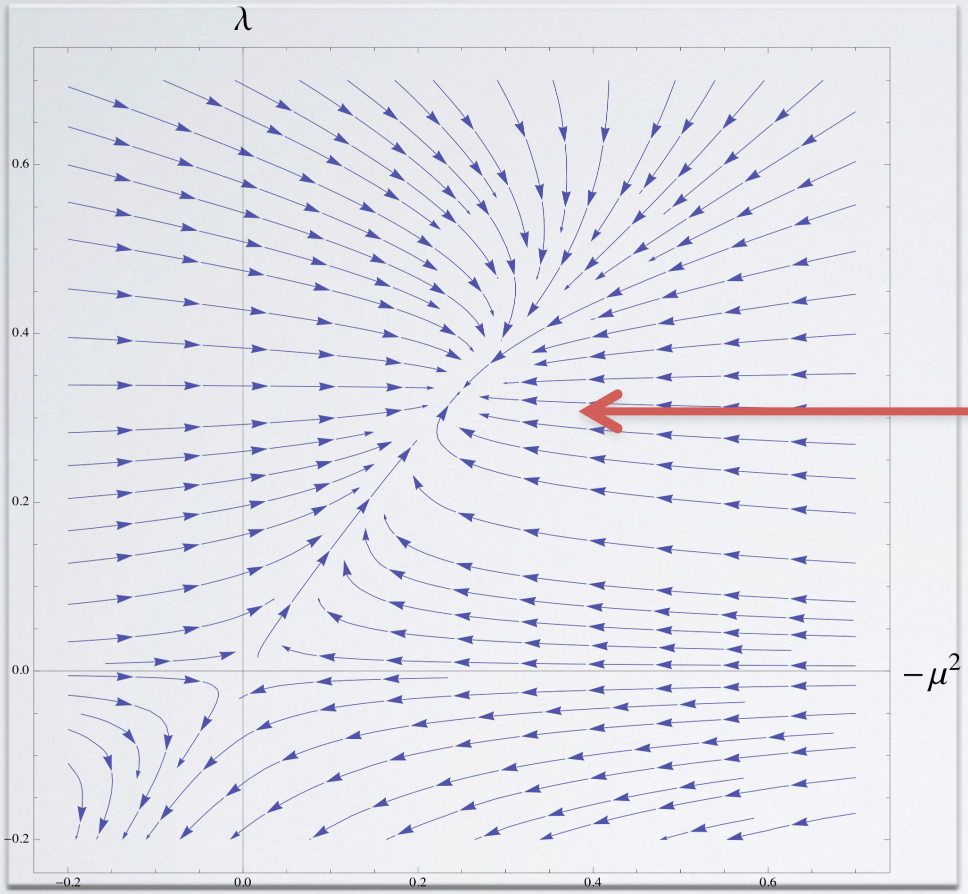
$$\frac{\langle \sigma(0)\epsilon(z_2)\epsilon(z_3)\sigma(\infty)\rangle}{\langle \epsilon(z_2)\epsilon(z_3)\rangle}$$

$$= \frac{1}{4}|\sqrt{z_1/z_2} + \sqrt{z_2/z_1}|^2 = \frac{1}{4}(r + 1/r + 2\cos\theta)$$





TEST CFT: PHI 4TH AT WILSON-FISHER FIXED POINT IN 2D & 3D.



Wilson-Fisher FP

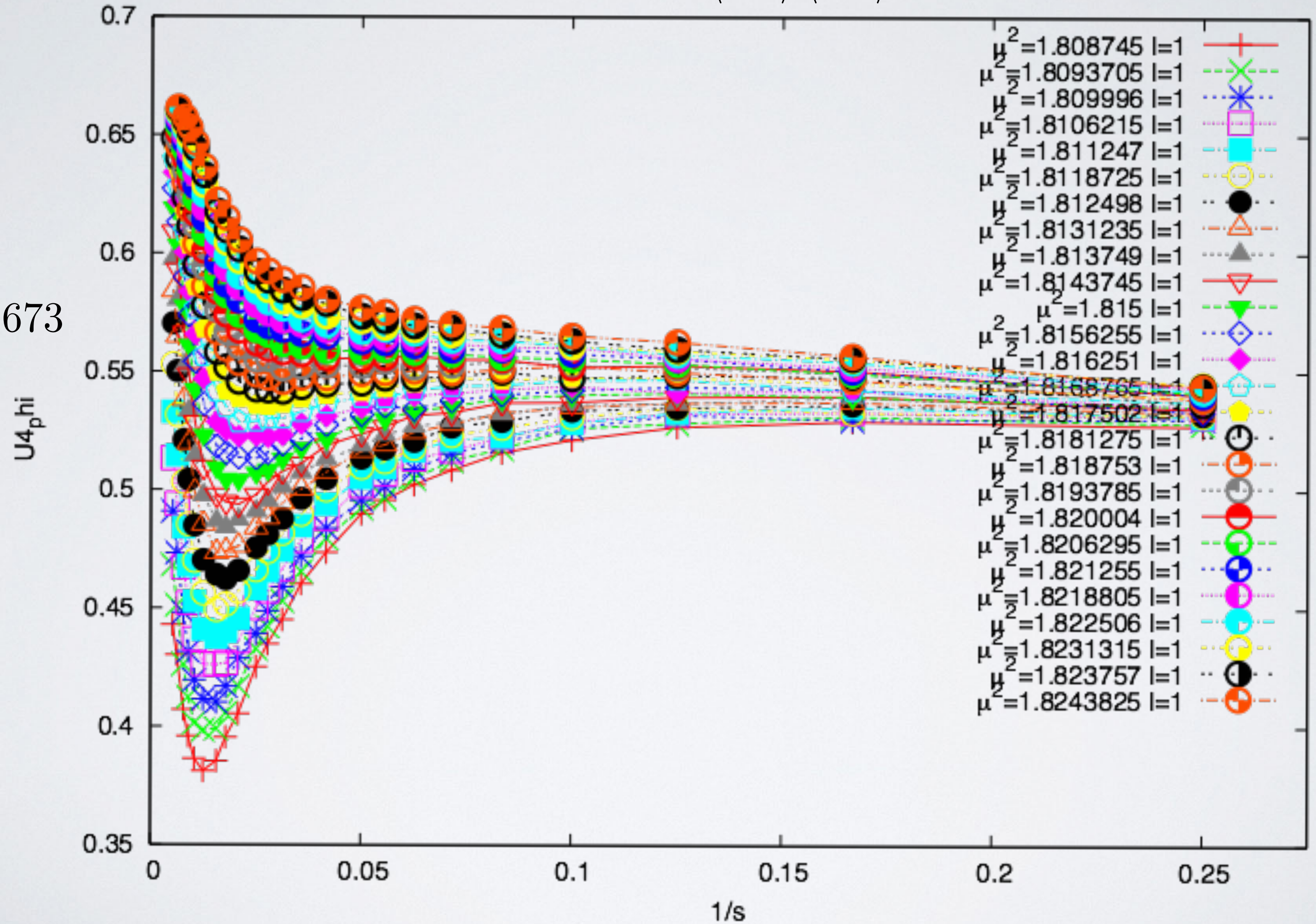
Gaussian FP

$$\mathcal{L}(x) = -\frac{1}{2}(\nabla\phi)^2 + \lambda(\phi^2 - \mu^2/2\lambda)^2$$

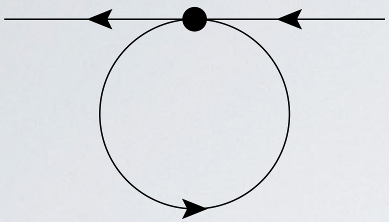
BINDER CUMULANT NEVER CONVERGES

$$U_B = 1 - \frac{\langle M^4 \rangle}{3\langle M^2 \rangle \, \langle M^2 \rangle}$$

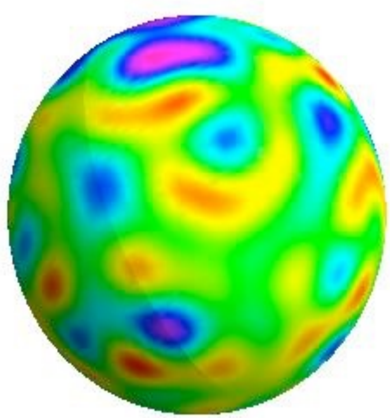
0.5673



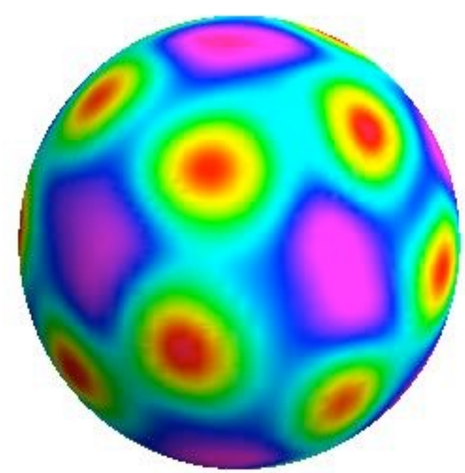
UV DIVERGENCE BREAKS ROTATIONS



$$\delta m^2 = \lambda \langle \phi(x) \phi(x) \rangle \rightarrow \frac{1}{K_{xx}}$$



one configuration



average of config.

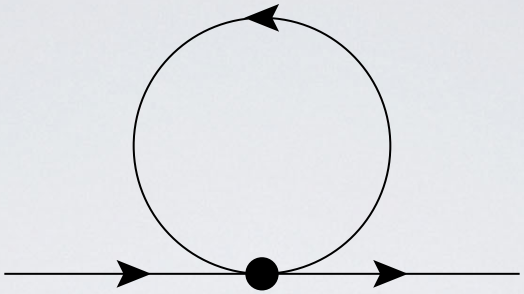
ONE LOOP COUNTER TERM

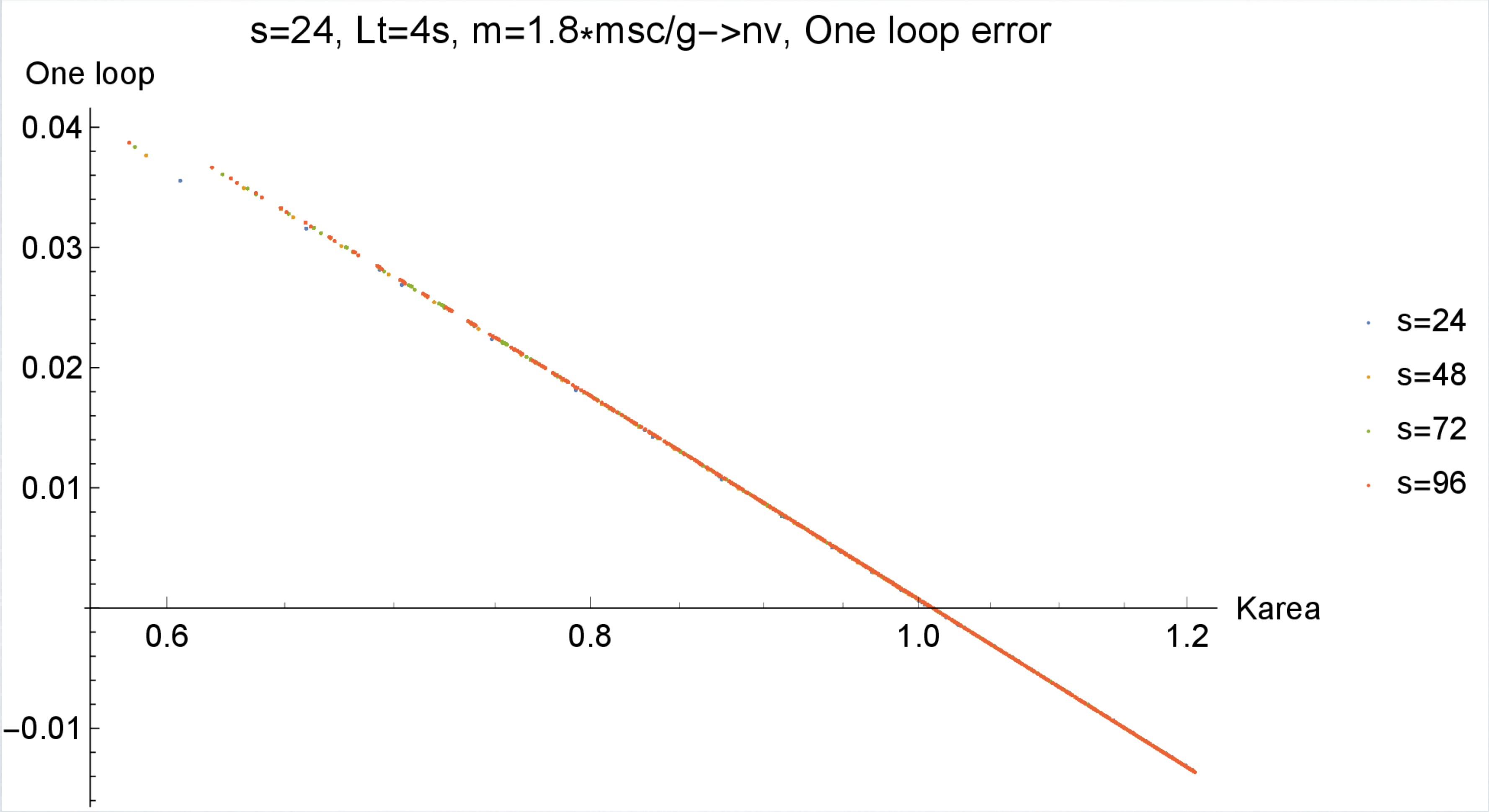
$$\Delta m_i^2 = 6\lambda \left[K^{-1}\right]_{ii} \simeq \frac{\sqrt{3}}{8\pi}\lambda \log(1/m_0^2 a_i^2) = \frac{\sqrt{3}}{8\pi}\lambda \log(N_s) + \frac{\sqrt{3}}{8\pi}\lambda \log(a^2/a_i^2)$$

Exact Continuum Divergence

Local RG Scheme Dependence

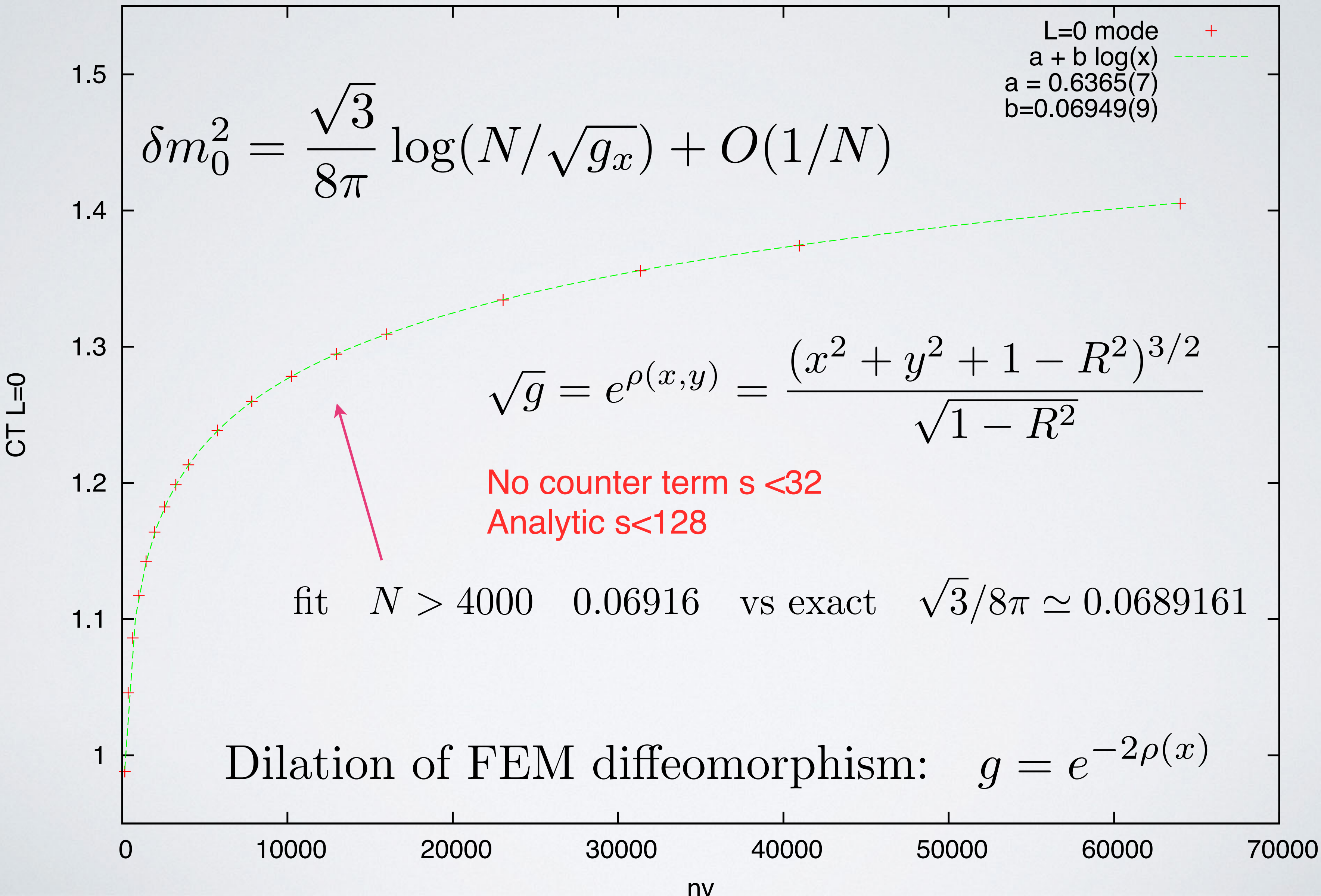
$$\delta\mu_i^2 = -6\lambda(\left[K^{-1}\right]_{ii} - \frac{1}{N_s}\sum_{j=1}^{N_s}\left[K^{-1}\right]_{jj})$$





s=24, Lt=4s, m=1.8*msc/g->nv, One loop error
One loop
0.04
0.03
0.02
0.01
-0.01
0.6
0.8
1.0
1.2
Karea
s=24
s=48
s=72
s=96

MODEL OF COUNTER TERM



EXACT C = 1/2 CFT ON 2D SPHERE

Exact Two point function

$$\langle \phi(x_1)\phi(x_2)\rangle = \frac{1}{|x_1 - x_2|^{2\Delta}} \to \frac{1}{|1 - cos\theta_{12}|^{\Delta}}$$

$$\Delta = \eta/2 = 1/8 \qquad x^2 + y^2 + z^2 = 1$$

4 pt function

$$(x_1, x_2, x_3, x_4) = (0, z, 1, \infty)$$

$$g(0, z, 1, \infty) = \frac{1}{2|z|^{1/4}|1 - z|^{1/4}}[|1 + \sqrt{1 - z}| + |1 - \sqrt{1 - z}|]$$

Critical Binder Cumulant

$$U_B^* = 1 - \frac{\langle M^4\rangle}{3\langle M^2\rangle\langle M^2\rangle} = 0.567336$$

Dual to Free Fermion

NOW BINDER CUMULANT CONVERGES

$$U_{2n}(\mu^2, \lambda, s) = U_{2n,cr} + a_{2n}(\lambda)[\mu^2 - \mu^2_{cr}]s^{1/\nu} + b_{2n}(\lambda)s^{-\omega}$$

FIT:

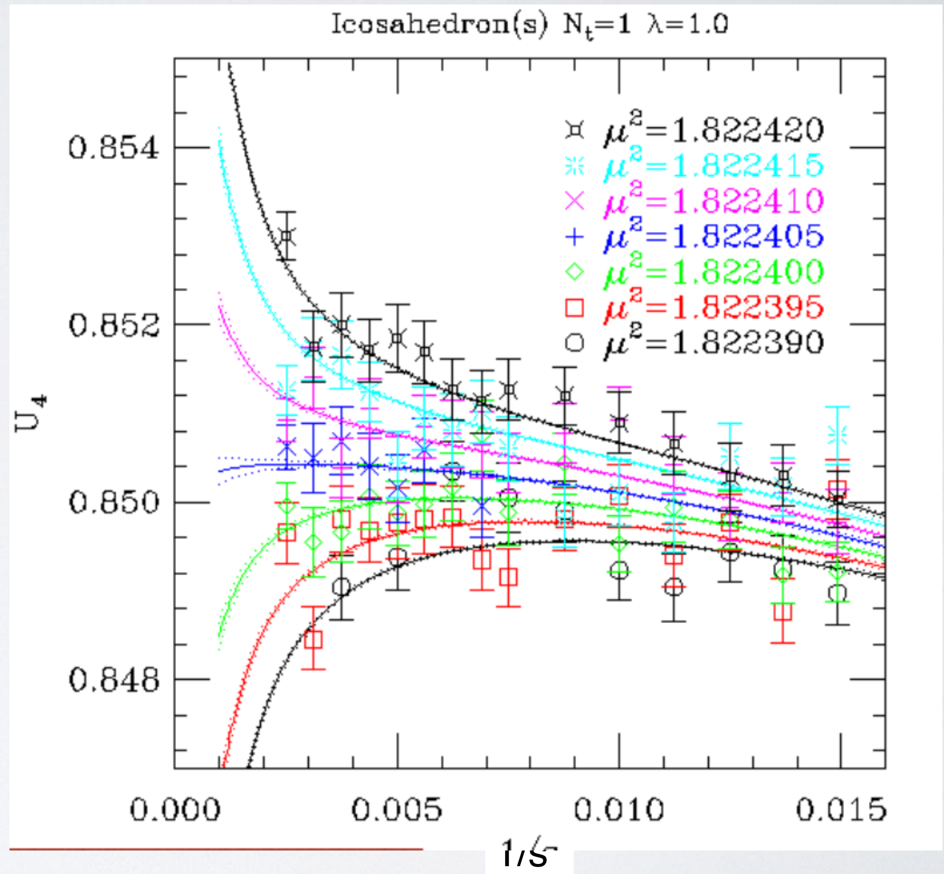
$$U_4 = 0.85081(10)$$

EXACT:

$$U_4^{exact} = 0.851021(5)$$

HIGHER MOMENT 2n = 4,6,8,10,12

$$U_6 = 0.77280(13)$$

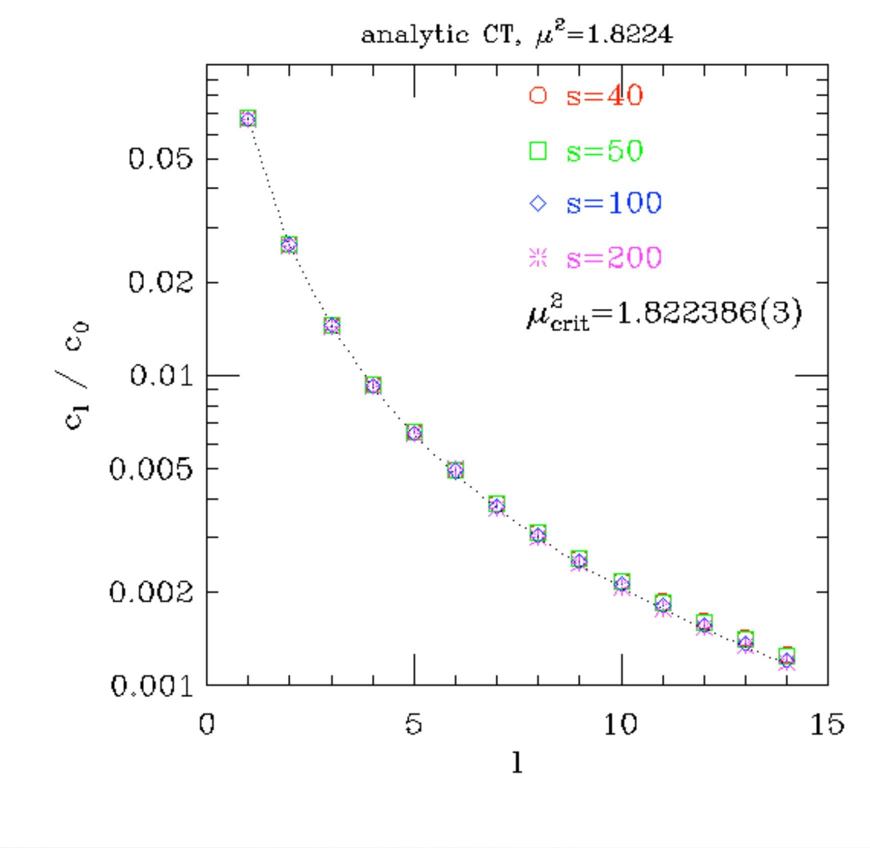


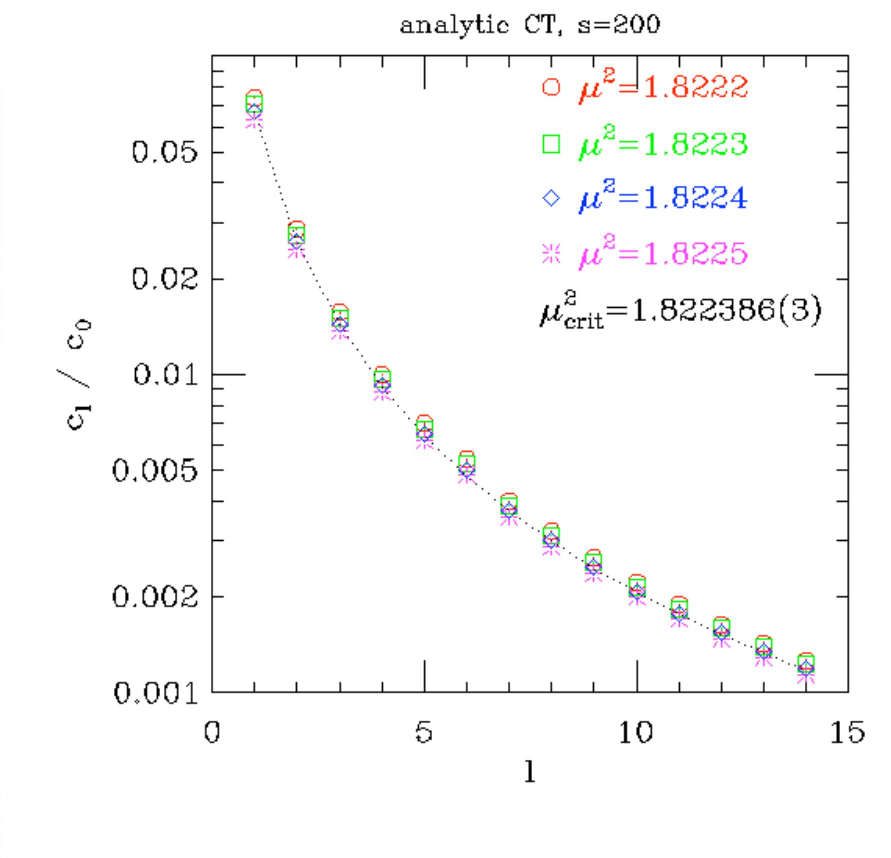
$$U_4 = \frac{3}{2}[1 - \frac{\langle M^4 \rangle}{3\langle M^2 \rangle \langle M^2 \rangle}]$$

Simultaneous fit for s up 800: E.G. 6,400,002 Sites on Sphere

$$\mu^2_{cr} = 1.82240070(34)$$

$$dof = 1701 \quad , \quad \chi^2/dof = 1.026$$





$$\int_{-1}^{1} dz \left(\frac{2}{1-z}\right)^{1/8} P_l(z)$$

$$\Delta_\sigma = \eta/2 = 1/8 \simeq 0.128$$

$$\Longrightarrow \frac{16}{7}, \frac{16}{35}, \frac{48}{161}, \frac{816}{3565}, \frac{12240}{64883}, \frac{493680}{3049501}, \frac{33456}{234577}, \frac{55760}{435643}, \frac{3602096}{30930653}, \frac{20129360}{187963199}, \frac{541373840}{5450932771} \cdots$$

Very fast cluster algorithm:
Brower,Tamayo 'Embedded Dynamics for phi 4th Theory" PRL 1989. Wolff single cluster + plus Improved Estimators etc

Using Binder Cumulants

$$
\begin{aligned}
U_4 &= \frac{3}{2}\left(1-\frac{m_4}{3\ m_2^2}\right) \\
U_6 &= \frac{15}{8}\left(1+\frac{m_6}{30\ m_2^3}-\frac{m_4}{2\ m_2^2}\right) \\
U_8 &= \frac{315}{136}\left(1-\frac{m_8}{630\ m_2^4}+\frac{2\ m_6}{45\ m_2^3}+\frac{m_4^2}{18\ m_2^4}-\frac{2\ m_4}{3\ m_2^2}\right) \\
U_{10} &= \frac{2835}{992}\left(1+\frac{m_{10}}{22680\ m_2^5}-\frac{m_8}{504\ m_2^4}-\frac{m_6\ m_4}{108 m_2^5}+\frac{m_6}{18\ m_2^3}+\frac{5\ m_4^2}{36\ m_2^4}-\frac{5\ m_4}{6\ m_2^2}\right) \\
U_{12} &= \frac{155925}{44224}\left(1-\frac{m_{12}}{1247400\ m_2^6}+\frac{m_{10}}{18900\ m_2^5}+\frac{m_8\ m_4}{2520\ m_2^6}-\frac{m_8}{420\ m_2^4}\right. \\
&\quad \left.+\frac{m_6^2}{2700 m_2^6}-\frac{m_6 m_4}{45\ m_2^5}+\frac{m_6}{15\ m_2^3}-\frac{m_4^3}{108\ m_2^6}+\frac{m_4^2}{4\ m_2^4}-\frac{m_4}{m_2^2}\right)
\end{aligned}
$$

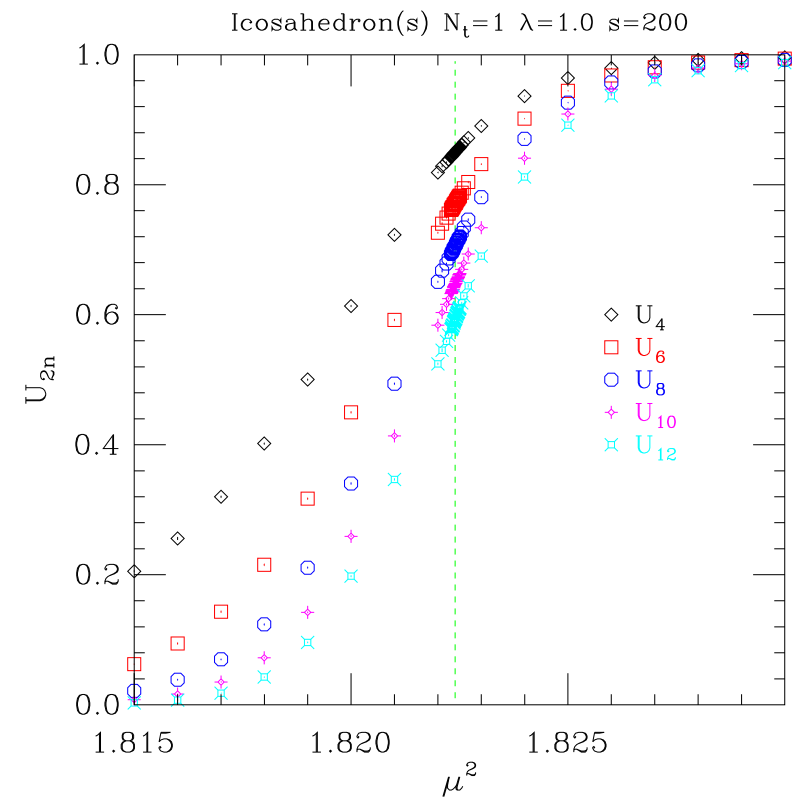
$$m_n = \langle \phi^n \rangle$$

In infinite volume

$U_{2n}=0$ in disordered phase

$U_{2n}=1$ in ordered phase

$0<U_{2n}<1$ on critical surface



- $U_{2n,cr}$ are universal quantities.
- Deng and Blöte (2003): $U_{4,cr}=0.851001$
- Higher critical cumulants computable using conformal $2n$-point functions:
  Luther and Peschel (1975)
  Dotsenko and Fateev (1984)

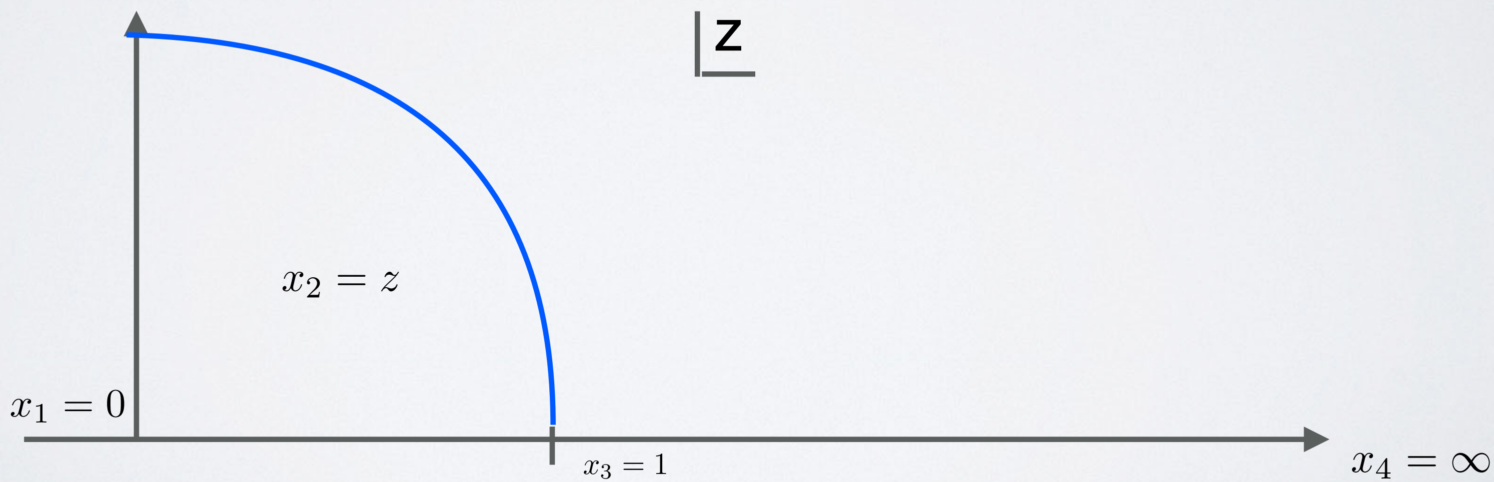
EXACT FOUR POINT FUNCTION

OPE Expansion: $\phi \times \phi = \mathbf{1} + \phi^2$ or $\sigma \times \sigma = \mathbf{1} + \epsilon$

$$g(u,v) = \frac{\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle}{\langle \phi_1 \phi_3 \rangle \, \langle \phi_2 \phi_4 \rangle}$$

$$= \frac{1}{\sqrt{2}|z|^{1/4}|1-z|^{1/4}} \left[|1 + \sqrt{1-z}| + |1 - \sqrt{1-z}| \right]$$

Crossing Sym: $|1 + \sqrt{1-z}| + |1 - \sqrt{1-z}| = \sqrt{2 + 2\sqrt{(1-z)(1-\bar{z})} + 2\sqrt{z\bar{z}}}$

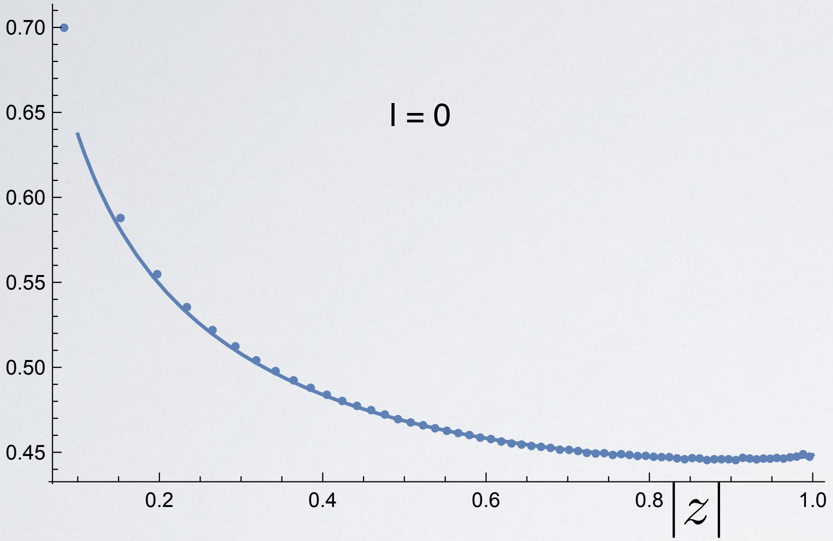


$u = \frac{r_{12}^2 r_{34}^2}{r_{13}^2 x_{24}^2}$, $v = \frac{r_{14}^2 r_{32}^2}{r_{13}^2 r_{24}^2}$ where $r_{ij}^2 = (\vec{r}_i - \vec{r}_j)^2 = 2(1 - \cos\theta_{ij})$

$u = z\bar{z}$, $v = (1-z)(1-\bar{z})$

2 TO 2 SCATTERING DATA

$g_0(|z|)$



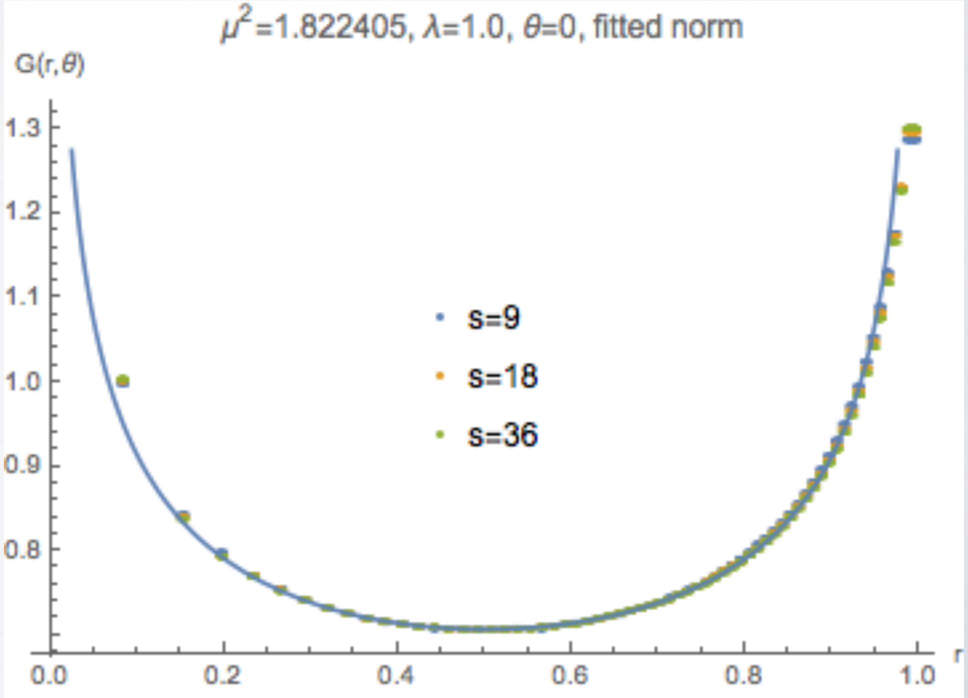
$g_l(|z|)$

ZERO PARAMETER FIT

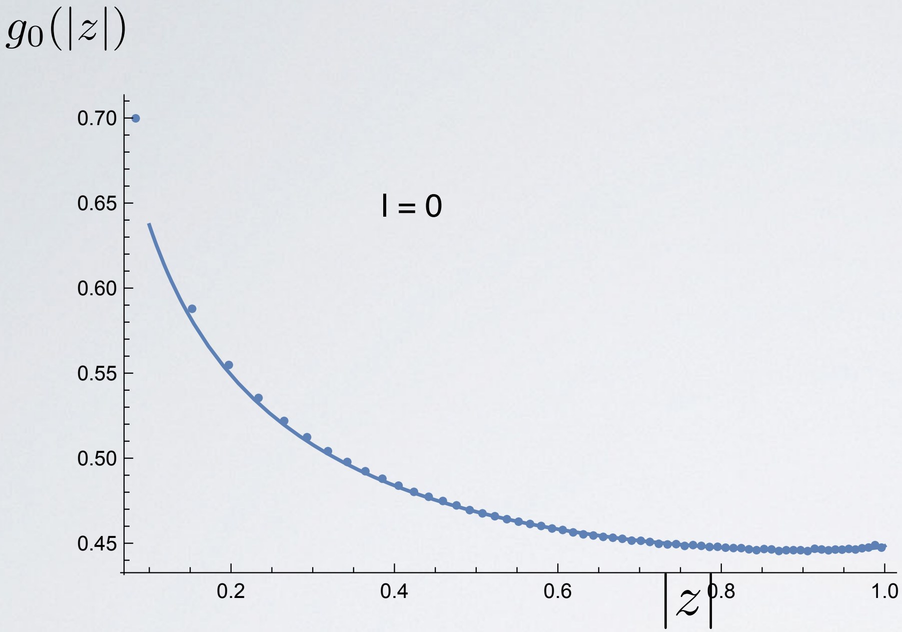
s= 10 Run for 1/2 hour

$$g(u,v) = \sum_l g_l(|z|)\cos(l\theta)$$

$$z = |z|e^{il\theta}$$



DESCENDANTS

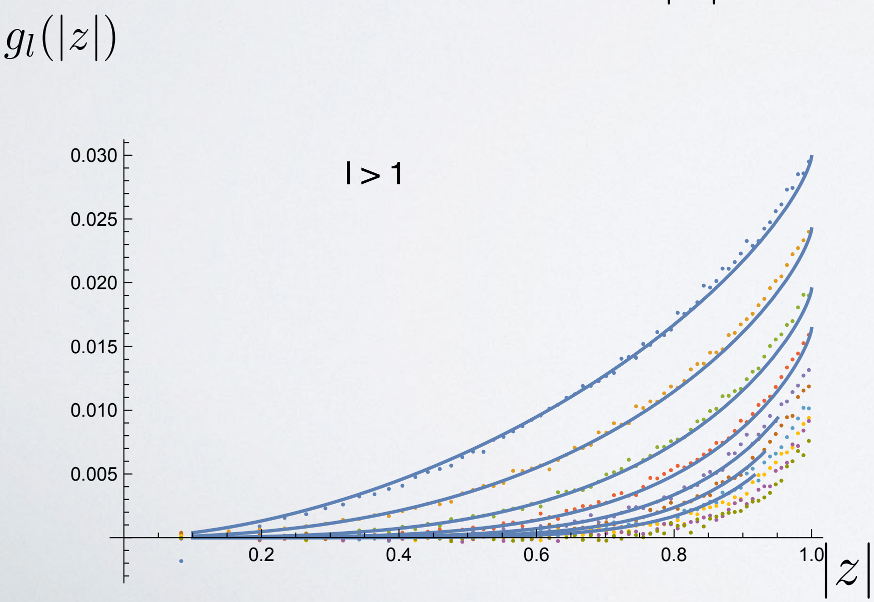


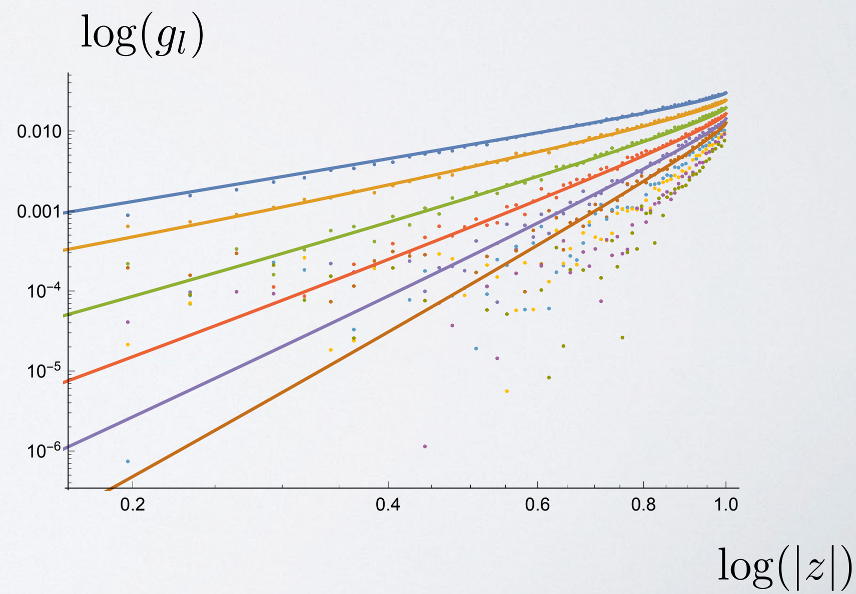
ZERO PARAMETER FIT

s= 10 Run for 1/2 hour

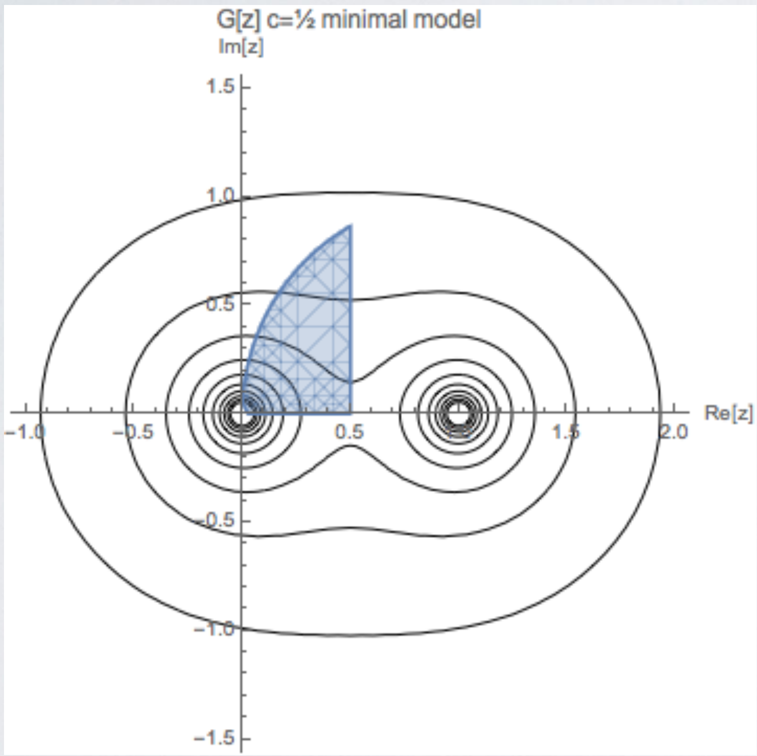
$$g(u,v) = \sum_l g_l(|z|)\cos(l\theta)$$

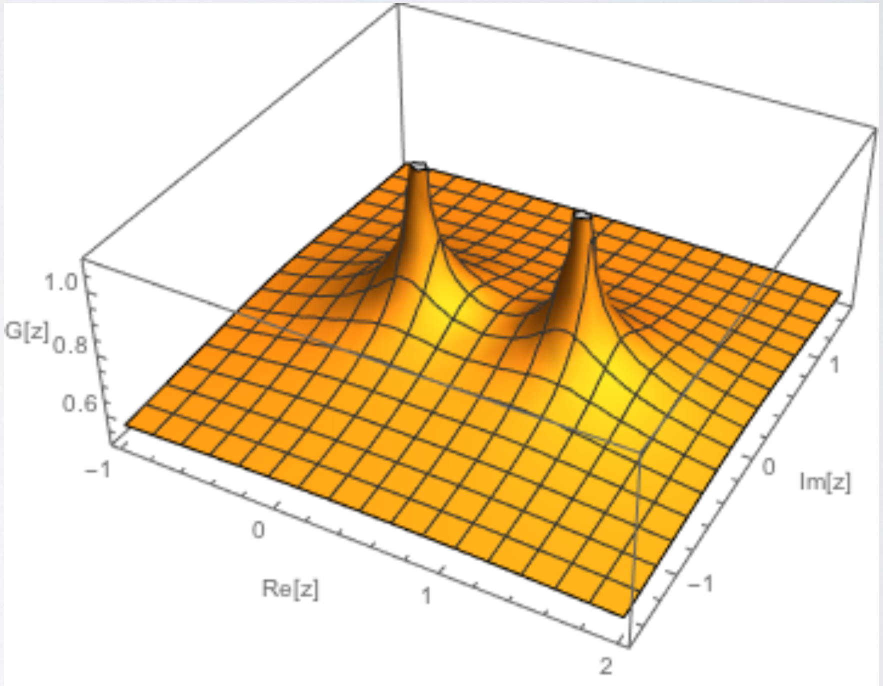
$$z = |z|e^{il\theta}$$



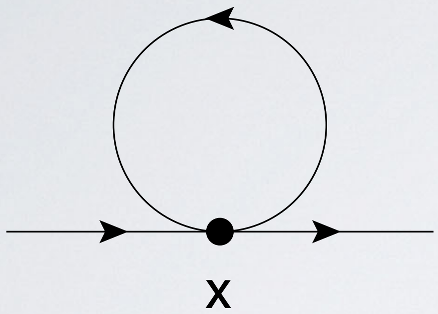


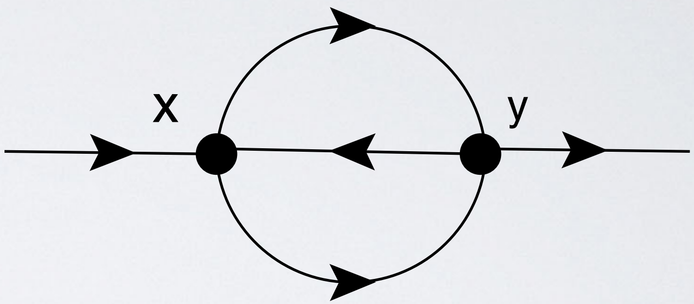
4 PT AND OPE


G[z] c=½ minimal model
Im[z]
1.5
1.0
0.5
−0.5
−1.0
−1.5
Re[z]
−1.0
−0.5
0.5
1.5
2.0


1.0
G[z]
0.8
0.6
−1
0
Re[z]
1
2
1
0
Im[z]
−1

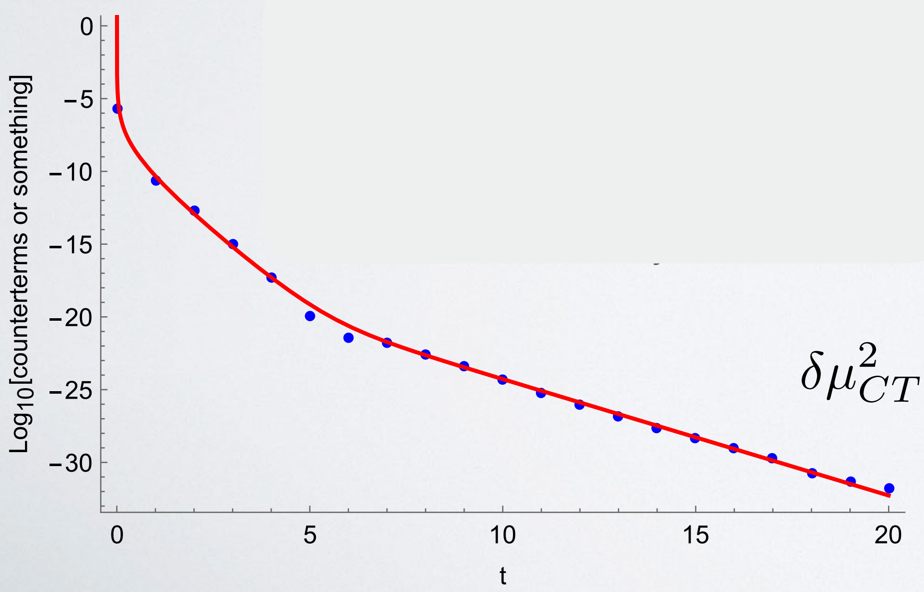
FUTURE: COUNTER TERM IN 3D





Linear Diverge —> Const shift in Lattice Variables!

Now local CT but Exponential falls |x - y| in lattice units!

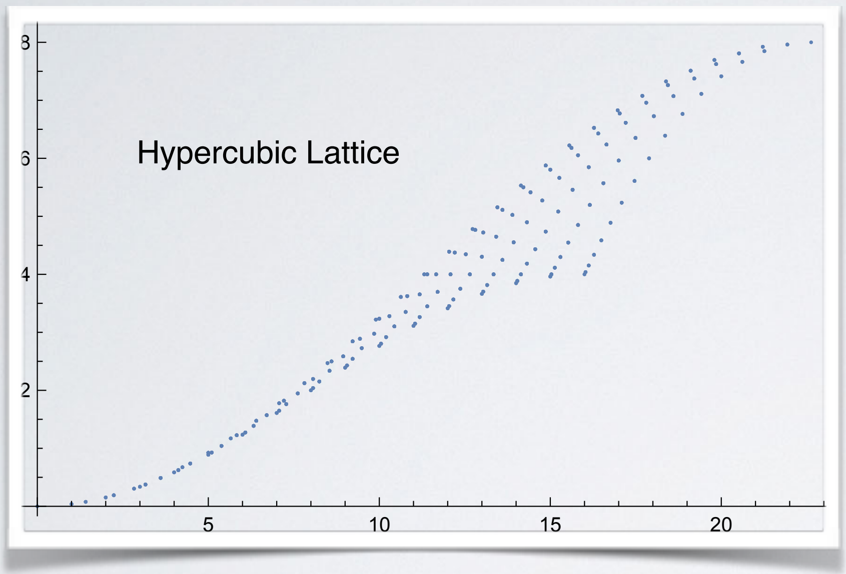


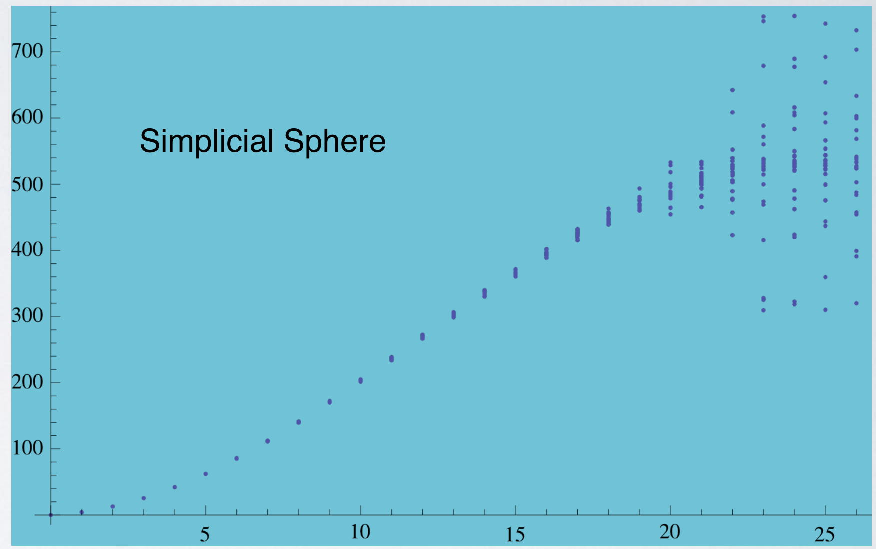
QFE Action is LODAL but not Ultra-local

$$\delta\mu^2_{CT}(x,y) \sim \lambda_0 c_x \delta_{xy} + \lambda_0^2 e^{-6|x-y|/a}$$

THE BIG PROBLEM: RESTORING ISOMETRIES FOR ON A SIMPLICIAL COMPLEX

How much help do you need from FEM ?

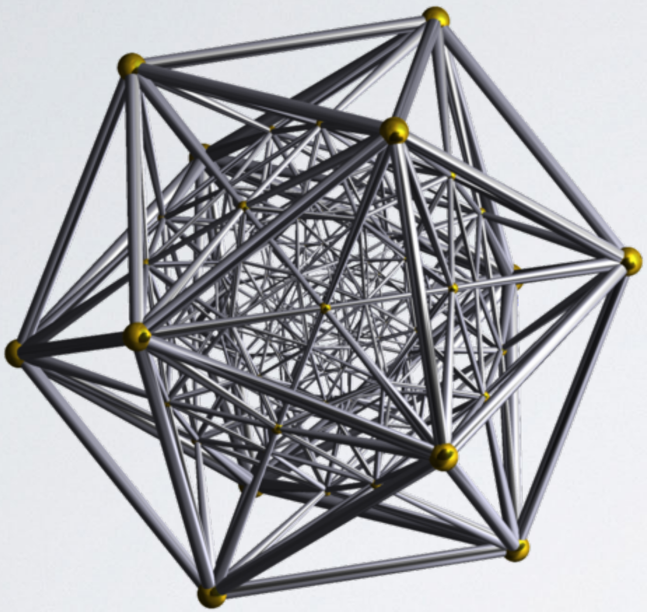


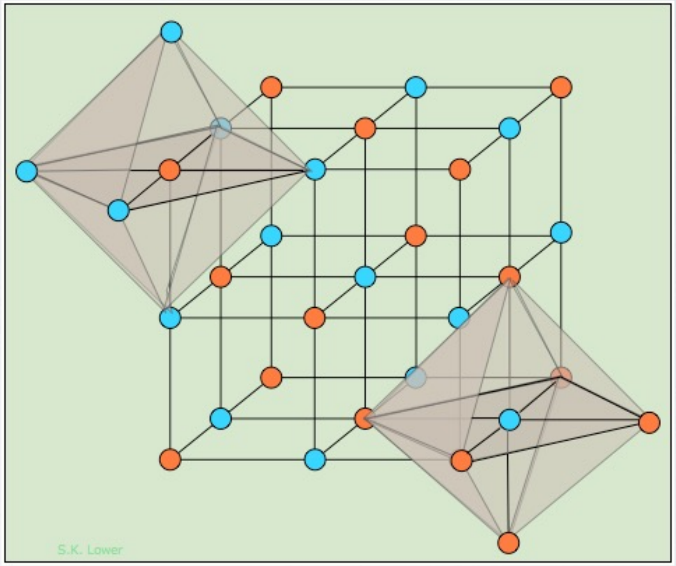


Is Renormalized perturbation theory at the UV fixed point enough to uniquely/correctly define the IR Quantum Field Theory?

DATA PARALLEL CODE AN 600 CELL ON S3

HTTPS://EN.WIKIPEDIA.ORG/WIKI/600-CELL





Aristotle's 2% Error!

$$(2\pi - 5ArcCos[1/3])/(2\pi) = 0.0204336$$

16 vertices of the form:[3] (±½, ±½, ±½, ±½),
8 vertices obtained from (0, 0, 0, ±1) by permuting coordinates.
96 vertices are obtained by taking even permutations of ½ (±φ, ±1, ±1/φ, 0).

https://en.wikipedia.org/wiki/List_of_regular_polytopes_and_compounds#Five-dimensional_regular_polytopes_and_higher

FUTURE: SEEKING FUNDING TO EXPLORE

EXTRAS

Radial Quantization

Evolution: $H = P_0 \text{ in } t \implies D \text{ in } \tau = \log(r)$

$$ds^2 = dx^\mu dx_\mu = e^{2\tau}[d\tau^2 + d\Omega^2]$$

Can drop
Weyl factor!

$$\mathbb{R}^d \to \mathbb{R} \times \mathbb{S}^{d-1}$$

$$\text{"}time\text{"} \quad \tau = log(r)\ , \quad \text{"}mass\text{"} \quad \Delta = d/2 - 1 + \eta$$

$$D \to x_\mu \partial_\mu = r\partial_r = \frac{\partial}{\partial \tau}$$

SCALING VS FULL CONFORMAL SYMMETRY

- General Field Theory with Scale invariance and Poincare Invariance
- O(d) ==> O(d,1) (Isometries of AdS space)

$$x_\mu \to \lambda x_\mu \quad , \quad x_\mu \to \frac{x_\mu}{x^2}$$

$$K_\mu : (inv \to trans \to inv)$$

$$[K_\mu, \mathcal{O}(x)] = i(x^2\partial_\mu - 2x_\mu x^\nu \partial_\nu + 2x_\mu \Delta)\mathcal{O}(x)$$

$$[D, \mathcal{O}(x)] = i(x^\mu \partial_\mu - \Delta)\mathcal{O}(x)$$

$$[D, P_\mu] = -iP_\mu \quad , \quad [D, K_\mu] = +iK_\mu \quad , \quad [K_\mu, P_\mu] = 2iD$$

EXACT CFT: POWER LAW

Conformal correlator: $\langle\phi(x_1)\phi(x_2)\rangle = C\dfrac{1}{|x_1 - x_2|^{2\Delta}}$

$$\begin{aligned} r_1^\Delta r_2^\Delta \langle\phi(\tau_1,\Omega_1)\phi(\tau_2,\Omega_2)\rangle &= C\frac{1}{[r_2/r_1 + r_1/r_2 - 2\cos(\theta_{12})]^\Delta} \\ &\simeq Ce^{-(\log(r_2) - \log(r_1)\Delta} \\ &= Ce^{-\tau\Delta} \end{aligned}$$

With $|x_1 - x_2|^2 = r_1 r_2[r_2/r_1 + r_1/r_2 - 2\cos(\theta_{12})]$

as $\tau = \log(r_2) - \log(r_1) \to \infty$