Lattice QCD meets global PDF fits in the LHC era

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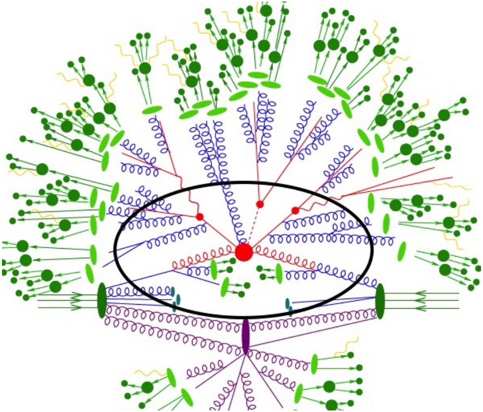
LHC processes

$$\sigma(H_1 H_2 \to X) = \sum_{i,j} \int dx_1 dx_2 \; f_{i/H_1}(x_1, \mu_F^2) f_{j/H_2}(x_2, \mu_F^2) \times \\ \times \; \hat{\sigma}_{ij \to X}(x_1 x_2 s, \mu_F^2, \mu_R^2)$$



Parton luminosities

Change of kinematic variables: $\hat{s} = x_1 x_2 s$, $y = 1/2 \log\left(x_1/x_2\right)$

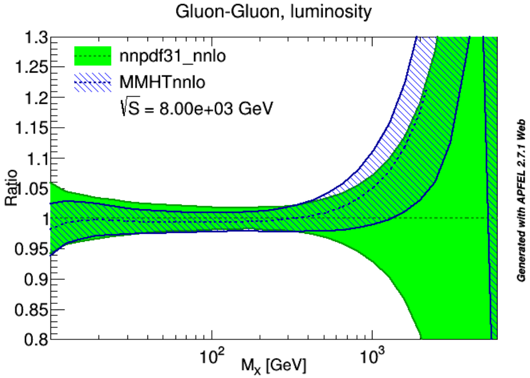
$$\frac{dL_{ij}}{d\hat{s}dy} = \frac{1}{s}\kappa_{ij}\left[f_i(x_1,\mu_F^2)f_j(x_2,\mu_F^2) + (1 \rightleftharpoons 2)\right]$$

$$\boxed{\sigma(H_1 H_2 \to X) = \sum_{i,j} \int \left(\frac{d\hat{s}}{\hat{s}} dy\right) \left(\frac{dL_{ij}}{d\hat{s}dy}\right) \left(\hat{s}\hat{\sigma}_{ij\to X}\right)}$$

Integrated luminosity

$$\frac{dL_{ij}}{d\hat{s}} = \int dy\, \frac{dL_{ij}}{d\hat{s}dy}$$

One example



Gluon-Gluon, luminosity
nnpdf31_nnlo
MMHTnnlo
√S = 8.00e+03 GeV
Ratio
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
10^2
10^3
M_X [GeV]
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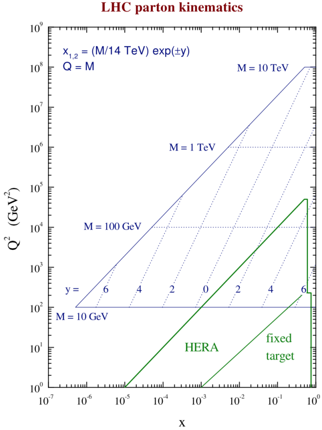
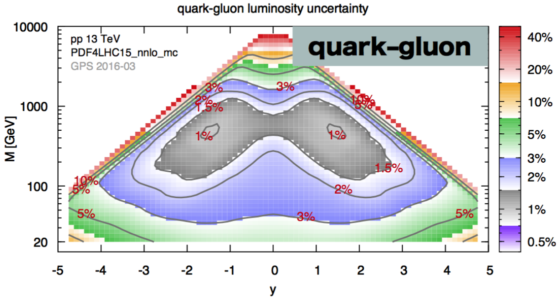
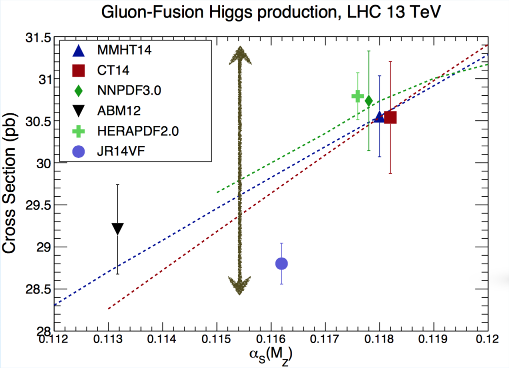
LHC kinematics



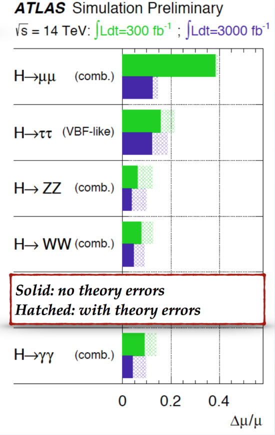


PDF uncertainties for Higgs physics

Uncertainties from Parton Distributions** are one of the limiting factors of theory predictions of Higgs production, **degrading the exploration of the Higgs sector

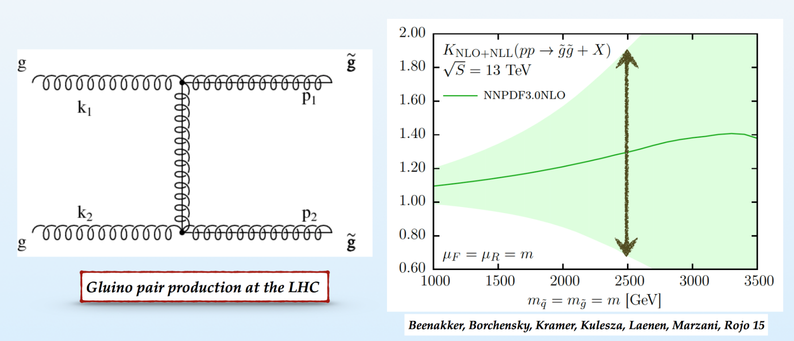


Higgs Cross-Section Working Group YR4 16



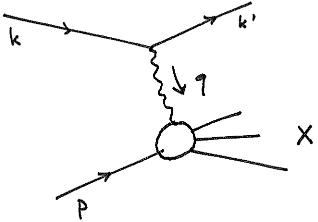
Solid: no theory errors
Hatched: with theory errors

PDF uncertainties for new physics



systematic/quantitative assessment?

PDF from DIS



kinematics:

$$k^\mu = k'^\mu + q^\mu$$
$$Q^2 = -q^2$$
$$\nu = p \cdot q$$

in the lab frame: $p = (m_N, 0)$

$$Q^2 = 2EE'(1 - \cos\theta)\,, \quad \nu = m_N \left(E - E'\right)$$
$$x = Q^2/(2\nu)\,, \quad y = \frac{q \cdot p}{k \cdot p} = 1 - E'/E$$

$$\boxed{d\sigma = \frac{d^3k'}{|\mathbf{k}'|} \frac{1}{2s(Q^2 - m_N^2)} L^{\mu\nu}(k, k') W_{\mu\nu}(p, q)}$$

Hadronic tensor

$$W_{\mu\nu}(p,q) = \frac{1}{4\pi}\sum_X \langle p|j_\mu(0)^\dagger|X\rangle\langle X|j_\nu(0)|p\rangle(2\pi)^4\delta\left(p_X - p - q\right)$$
$$= \frac{1}{4\pi}\int d^4y\, e^{iq\cdot y}\, \langle p|j_\mu(x)^\dagger j_\nu(0)|p\rangle$$

$W_{\mu\nu}$ parametrized in terms of **structure functions**

$$W_{\mu\nu}(p,q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(Q^2,\nu) +$$
$$+ \left(p_\mu - q_\mu\frac{\nu}{q^2}\right)\left(p_\nu - q_\nu\frac{\nu}{q^2}\right) W_2(Q^2,\nu)$$

$$F_1(x,Q^2) = W_1(Q^2,\nu)$$
$$F_2(x,Q^2) = \nu W_2(Q^2,\nu)$$

Factorization theorem

$$\boxed{\begin{aligned} F(x,Q^2) &= \sum_i \int \frac{dz}{z}\, C_i\left(z, Q^2/\mu_F^2, \alpha_S(\mu_F^2)\right)\, f_i(\frac{x}{z}, \mu_F^2) \\ &= \sum_i C_i\left(x, Q^2/\mu_F^2, \alpha_S(\mu_F^2)\right) \otimes f_i(x, \mu_F^2) \end{aligned}}$$

- C_i coefficient function, process specific - computed in perturbation theory in a given **scheme**
- $f_i(x, \mu_F^2)$ PDF, **universal**, nonperturbative dynamics
- using the equation above, f_i can be fitted from experimental data

DGLAP evolution

• Independence of F of the factorization scale:

$$\mu_F^2 \frac{d}{d\mu_F^2} F(x, Q^2) = 0$$

hence:

$$\mu_F^2 \frac{d}{d\mu_F^2} f_i(x, \mu_F^2) = \sum_j P_{ij}\left(x, \alpha_s(\mu_F^2)\right) \otimes f_j(x, \mu_F^2)$$

•$P_{ij}\left(x, \alpha_s(\mu_F^2)\right)$: Altarelli-Parisi splitting functions - computed in perturbation theory -

• Solution of the evolution equation:

$$\boxed{f_i(x, \mu_F^2) = \sum_j \Gamma_{ij}(x, \alpha_s, \alpha_s^0) \otimes f_j(x, \mu_{F,0}^2)}$$

Non-singlet PDF

Simple example:

$$\begin{aligned} F_2^{\mathrm{NS}}(x,Q^2) &= F_2^{\mathrm{p}}(x,Q^2) - F_2^{\mathrm{d}}(x,Q^2) \\ &= \int_x^1 \frac{dy}{y}\, C_{\mathrm{NS}}(y,Q^2)\, q_{\mathrm{NS}}\left(\frac{x}{y},Q^2\right) \end{aligned}$$

where

$$q_{\mathrm{NS}}(x,Q^2) = \left[\left(u(x,Q^2)+\bar{u}(x,Q^2)\right) - \left(d(x,Q^2)+\bar{d}(x,Q^2)\right)\right]$$

combining the evolution and the coefficient function

$$F_2^{\mathrm{NS}}(x,Q^2) = \int_x^1 \frac{dy}{y}\, K_{\mathrm{NS}}\left(y,\alpha_s\left(Q^2\right),\alpha_s\left(Q_0^2\right)\right)\, q_{\mathrm{NS}}\left(\frac{x}{y},Q_0^2\right)$$

Fitting the data

- parametrize the PDF at some initial scale by some set of coefficients ω:

$$q_{\mathrm{NS}}\left(x, Q_0^2\right) = \phi\left(x; \omega\right)$$

ϕ denotes the parametrization (NNPDF, fixed functional forms)

- minimize the χ^2 function:

$$E[\omega] = \sum_{I,J=1}^{N_{\mathrm{dat}}} \left(F_I^{(\mathrm{dat})} - F_I^{\mathrm{NS}}[\omega]\right) \left(\mathrm{cov}^{-1}\right)_{IJ} \left(F_J^{(\mathrm{dat})} - F_J^{\mathrm{NS}}[\omega]\right)$$

where:

$$F_I^{\mathrm{NS}}[\omega] = K_{\mathrm{NS}}(x_I, \alpha_{s,I}, \alpha_s^0) \otimes \phi\left(x_I; \omega\right)$$

Error propagation

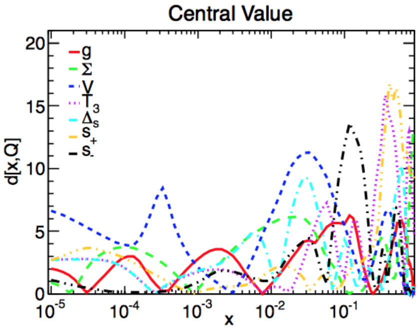
- PDF fitted from data $\Longrightarrow$ data error propagates
- MC method (NNPDF): create an ensemble of replicas of data distriburted according to the correlated errors of the real data. Perform a fit for each replica, and study the distribution of the replicas.
- Hessian method: quadratic expansion of the χ^2 around its minimum
- systematic errors/bias
- lattice determination independent of the data

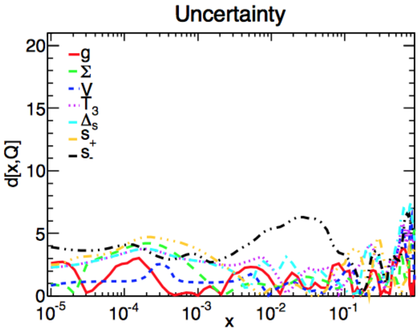
Current dataset

	LO			NLO			NNLO		
	$N_{\rm dat}$	$\chi^2_{\rm exp}$	$\chi^2_{t_0}$	$N_{\rm dat}$	$\chi^2_{\rm exp}$	$\chi^2_{t_0}$	$N_{\rm dat}$	$\chi^2_{\rm exp}$	$\chi^2_{t_0}$
Total	4258	2.42	2.17	4276	1.23	1.25	4078	1.29	1.27
NMC d/p	132	1.41	1.09	132	0.92	0.92	132	0.93	0.93
NMC	224	2.83	3.3	224	1.63	1.66	224	1.52	1.55
SLAC	74	3.29	2.96	74	1.59	1.62	74	1.13	1.17
BCDMS	581	1.78	1.78	581	1.22	1.27	581	1.29	1.35
CHORUS	862	1.55	1.16	862	1.11	1.15	862	1.09	1.13
NuTeV	79	0.97	1.03	79	0.70	0.66	79	0.86	0.81
HERA-I	592	1.75	1.51	592	1.05	1.16	592	1.04	1.12
ZEUS HERA-II	252	1.94	1.44	252	1.40	1.49	252	1.48	1.52
H1 HERA-II	511	3.28	2.09	511	1.65	1.65	511	1.79	1.76
HERA $\sigma^c_{\rm NC}$	38	1.80	2.69	47	1.27	1.12	47	1.28	1.20
E886 d/p	15	2.04	1.10	15	0.53	0.54	15	0.48	0.48
E886 p	184	0.98	1.64	184	1.19	1.11	184	1.55	1.17
E605	119	0.67	1.07	119	0.78	0.79	119	0.90	0.72
CDF Z rapidity	29	2.02	3.88	29	1.33	1.55	29	1.53	1.62
CDF Run-II k_t jets	76	1.51	2.12	76	0.96	1.05	52	1.80	1.20
D0 Z rapidity	28	1.35	2.48	28	0.57	0.68	28	0.61	0.65
ATLAS W, Z 2010	30	5.94	3.20	30	1.19	1.25	30	1.23	1.18
ATLAS 7 TeV jets 2010	90	2.31	0.62	90	1.07	0.52	9	1.36	0.85
ATLAS 2.76 TeV jets	59	3.88	0.61	59	1.29	0.65	3	0.33	0.33
ATLAS high-mass DY	5	13.0	15.6	5	2.06	2.84	5	1.45	1.81
ATLAS W p_T	-	-	-	9	1.13	1.28	-	-	-
CMS W electron asy	11	10.9	0.95	11	0.87	0.79	11	0.73	0.70
CMS W muon asy	11	76.8	2.25	11	1.81	1.80	11	1.72	1.72
CMS jets 2011	133	1.83	1.74	133	0.96	0.91	83	1.9	1.07
CMS $W+c$ total	5	11.2	25.8	5	0.96	1.30	5	0.84	1.11
CMS $W+c$ ratio	5	2.04	2.17	5	2.02	2.02	5	1.77	1.77
CMS 2D DY 2011	88	4.11	12.8	88	1.23	1.56	110	1.36	1.59
LHCb W rapidity	10	3.17	4.01	10	0.71	0.69	10	0.72	0.63
LHCb Z rapidity	9	5.14	6.17	9	1.10	1.34	9	1.59	1.80
$\sigma(t\bar{t})$	6	42.1	115	6	1.43	1.68	6	0.66	0.61

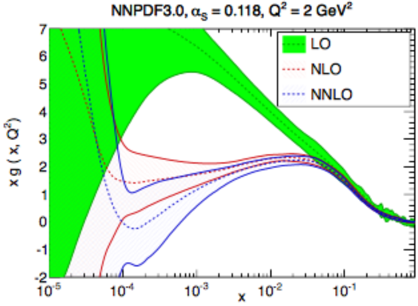
Change from new data/methodology

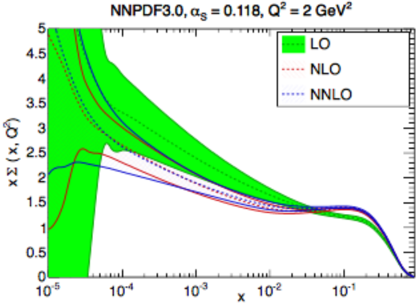
comparing NNPDF2.3 and NNPDF3.0

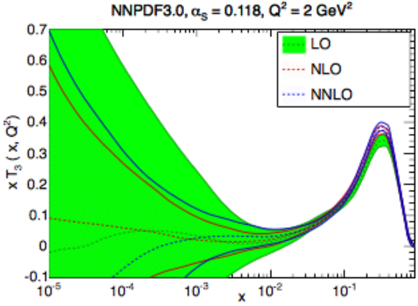


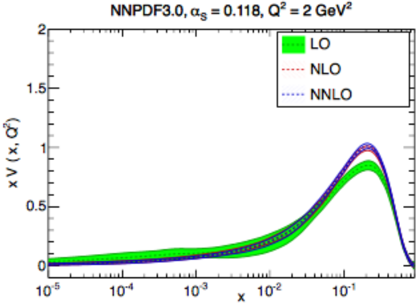


Different perturbative orders









DIS and OPE: analyticity

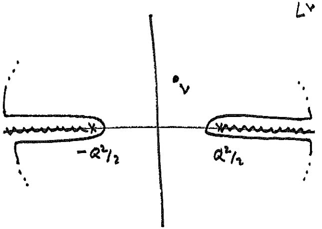
- deep-inelastic limit of $W_{\mu\nu}$ related to the short-distance limit of $T_{\mu\nu}$:

$$T_{\mu\nu} = \int d^4y \; e^{iq\cdot y} \, \langle p | T j_\mu(y) j_\nu(0) | p \rangle$$

$$W_{\mu\nu} = 2 \, \mathrm{Im} \, T_{\mu\nu}$$

- $T_{\mu\nu}$ can be expressed in terms of form factors T_i

- For fixed Q^2, T_i are analytic in the complex ν plane,
  cuts from the threshold $\nu = \pm Q^2/2$



DIS and OPE: dispersion relations

- Cauchy's theorem:

$$T_i(Q^2,\nu) = \frac{1}{2\pi i}\int \frac{d\nu'}{\nu'-\nu}\, T_i(Q^2,\nu')$$

- Expanding T_i in $1/x = 2\nu/Q^2$

$$T_i = \sum_n T_{i,n}(Q^2) x^{-n}$$

yields

$$T_{i,n}(Q^2) = \frac{1}{2\pi i}\int \frac{d\nu'}{\nu'}\, \left(\frac{Q^2}{2\nu'}\right)^n\, T_i(Q^2,\nu')$$

and finally

$$T_{i,n}(Q^2) = -2i\int dx'\, x'^{n-1}\left[F_i(x',Q^2) \pm \bar{F}_i(x',Q^2)\right] = F_{i,n}(Q^2) \pm \bar{F}_{i,n}(Q^2)$$

DIS and OPE: small-distance expansion

$$T_{\mu\nu} \sim -2i \int d^4y\, e^{iq\cdot y} \sum_{Ja} \langle p|[\mathcal{O}^{Ja}_{\mu_1\ldots\mu_J}]|p\rangle\, (-i)^J \times$$
$$\times \Bigg[y^{\mu_1} \ldots y^{\mu_J} \left(-g_{\mu\nu} \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial y^\mu \partial y^\nu} \right) C_1^{Ja}(y^2, \alpha_s, \mu) +$$
$$+ y^{\mu_3} \ldots y^{\mu_J} \left(g_\mu^{\mu_1} \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial y^\mu \partial y_{\mu_1}} \right) \ldots C_2^{Ja}(y^2, \alpha_s, \mu) + \ldots \Bigg]$$

where

$$\mathcal{O}^{Ja}_{\mu_1\ldots\mu_J} = \bar{\psi} \gamma_{\mu_1} i \overleftrightarrow{D}_{\mu_2} \ldots i \overleftrightarrow{D}_{\mu_J} \lambda^a \psi$$
$$\mathcal{O}^{Jg}_{\mu_1\ldots\mu_J} = G_{\mu_1\nu} i D_{\mu_2} \ldots i D_{\mu_{J-1}} G^\nu_{\mu_J}$$
$$\langle p| \left[\mathcal{O}^{Ja}_{\mu_1\ldots\mu_J} \right] |p\rangle = \langle p| \left[\mathcal{O}^{Ja} \right] |p\rangle\, p_{\mu_1} \ldots p_{\mu_J}$$

moments of structure functions

transforming to momentum space

$$T_{\mu\nu} \sim \left(-g_{\mu\nu} + q_\mu q_\nu/q^2\right) \left[-2i \sum_{Ja} x^{-J} \langle p| \left[\mathcal{O}^{Ja}\right] |p\rangle C_1^{Ja}(Q^2/\mu^2, \alpha_s)\right] +$$
$$+ \left(p_\mu - q_\mu \nu/q^2\right) \left(p_\nu - q_\nu \nu/q^2\right) \times$$
$$\times \left[-2i \sum_{Ja} x^{1-J} \langle p| \left[\mathcal{O}^{Ja}\right] |p\rangle C_2^{Ja}(Q^2/\mu^2, \alpha_s)\right]$$

hence

$$F_{2,J-1}(Q^2) \pm \bar{F}_{2,J-1}(Q^2) = \sum_a C_2^{Ja}(Q^2) \langle p|[\mathcal{O}^{Ja}]|p\rangle$$

PDF moments

introducing Mellin moments

$$f^{Ja}(\mu_F^2) = \int dz\, z^{J-1} f_a(z, \mu_F^2)$$

$$C^{Ja}(Q^2/\mu_F^2, \alpha_s) = \int dz\, z^{J-1} C_i(z, Q^2/\mu_F^2, \alpha_s)$$

$$F_J(Q^2) = \sum_a f^{Ja}(\mu_F^2)\, C^{Ja}(\alpha_s, Q^2/\mu_F^2)$$

$$\langle p| \left[\mathcal{O}^{Ja}\right] |p\rangle = \int dx\, x^{J-1} \left[f_a(x,\mu^2) + f_{\bar{a}}(x,\mu^2)\right]$$

renormalized operators

$$[\mathcal{O}^{J,a}] = Z_{ab}\mathcal{O}^{Jb}\,, \quad \beta(g)\frac{\partial}{\partial g} Z_{ab} = \gamma_{ac} Z_{cb}$$

renormalization of the lattice operators

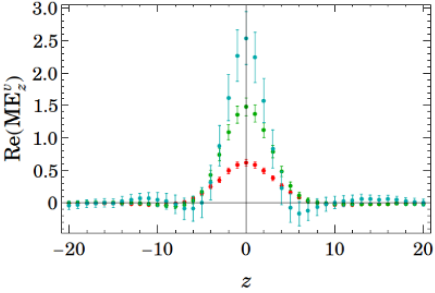
PDFs from non-local operators

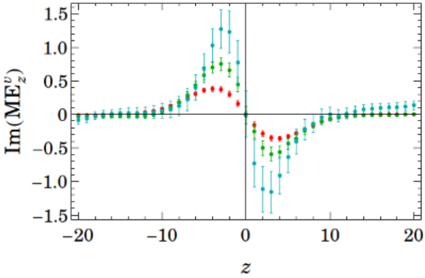
$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi}\, e^{-ix\xi^- P^+}\, \langle P|\bar{\psi}(\xi)\gamma^+ \times$$
$$\times\, \mathrm{P}\exp\left(-ig\int d\eta^-\, A^+(\eta^-)\right)\psi(0)|P\rangle$$

time-independent non-local expression:

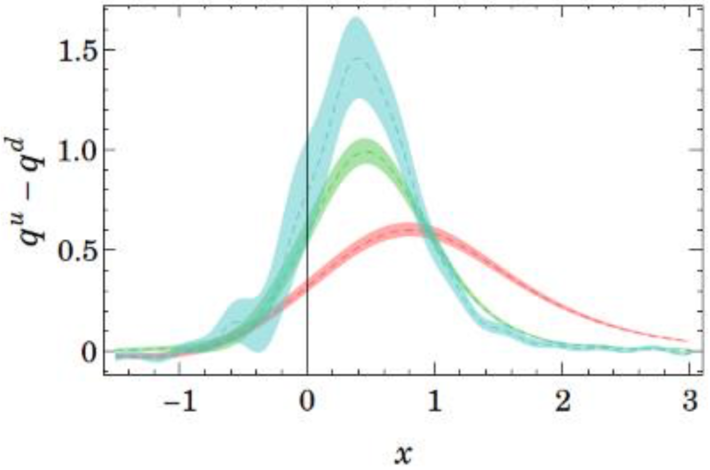
$$q(x,\mu_F,P_z) = \int \frac{dz}{4\pi}\, e^{-ixzP_z}\, \langle P|\bar{\psi}(z)\gamma^z \times$$
$$\times\, \mathrm{P}\exp\left(-ig\int dz'\, A^z(z')\right)\psi(0)|P\rangle$$

Lattice correlators





Bare quasi-distribution



Connection with PDFs

One loop calculation at the partonic level

$$q(x,\mu^2,P_z) = \frac{\alpha_s}{2\pi}\left[T(x)\frac{\Lambda}{P_z} + P(x)\log\frac{\Lambda P_z}{\lambda^2}\right]$$

where

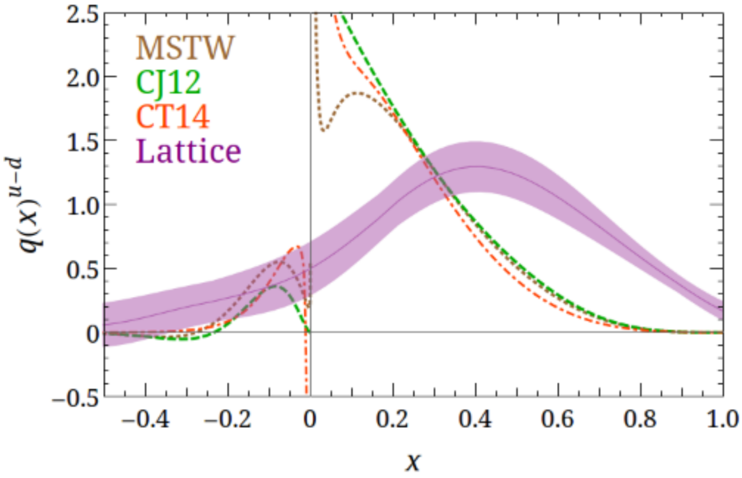
$$\Lambda = \sqrt{\mu^2 + (1-x)^2 P_z^2} - (1-x)P_z$$

limit $P_z \to \infty$: recover the standard Altarelli-Parisi kernel
limit $\mu^2 \to \infty$: linear divergence

“factorization theorem”:

$$q(x,\mu^2,P_z) = \int \frac{dy}{y}\, Z\left(\frac{x}{y}, \frac{\mu^2}{P_z^2}\right) q(y,\mu^2) + O(m_N^2/P_z^2) + O(\Lambda_{\text{QCD}}^2/P_z^2)$$

Renormalized quasi-distribution



interesting direction...

Systematic errors

- $Z\left(\frac{x}{y}, \frac{\mu^2}{P_z^2}\right)$ computed in perturbation theory at one-loop
  subtraction of power divergences? matching to which scheme?
- extrapolation in m_N^2/P_z^2, $\Lambda_{\mathrm{QCD}}^2/P_z^2$
  large physical P_z are needed, fine lattices? noisy observables?
- quark mass? continuum limit, (aP) artefacts?
- Minkowski to Euclidean rotation - recent paper by Briceno et al

Outlook

- precision determinations of α_s and PDFs needed for LHC
- kinematic regions not constrained by data - quantify this statement?
- renormalization and matching to MS scheme
- what is actually being computed?
- lattice input incorporated in global fits?
- limitations/opportunities in the computation of moments
- limitations/opportunities in the computation of quasi-distributions