Recent results for the 2nd moments of distribution amplitudes from RQCD (difficulties and prospects)

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RQCD (G. Bali, V. Braun, P. Bruns, B. Gläßle, M. Göckeler, M. Gruber, F. Hutzler, B. Lang, B. Musch, P. Perez-Rubio, R. Rödl, A. Schäfer, R. Schiel, J. Simeth, W. Söldner, A. Sternbeck, P. Wein, J. Zhang) **+ J. Gracey**.

PDFLattice 2017, 22-24 March, Oxford, UK.

Outline

★ Introduction and Motivation.

★ Moments of meson DAs: Pion and rho.

★ Moments of baryon DAs: Nucleon and octet.

★ Status and difficulties.

★ Perspectives: Direct determination of pion DA.

Meson and baryon distribution amplitudes

Intuitive picture of a hadron in the infinite momentum frame: superposition of Fock states with different numbers of quarks and gluons

$|B\rangle = |qqq\rangle + |qqqg\rangle + |qqq\bar{q}q\rangle + \ldots, \; |M\rangle = |\bar{q}q\rangle + |\bar{q}qg\rangle + |\bar{q}q\bar{q}q\rangle + \ldots.$

Within **Fock state decomposition**: **(light cone) distribution amplitudes (DAs)**
$\rightarrow$ **Hadron wave functions at small transverse distances of the constituents**
$\rightarrow$ describe the distribution of longitudinal momentum.

In hard exclusive processes higher Fock states are power-suppressed $\rightarrow$ **at high momentum transfer the valence contribution plays the most important role.**

DAs complementary to parton distribution functions (PDFs)

PDFs: single-particle probabilities (or densities).

"Directly" extracted from fits to DIS and SIDIS data.

DAs: amplitudes (or wavefunctions).

Appear always in convolutions in expressions for hard exclusive processes. **Difficult to extract from experiment** without contamination from other hadronic uncertainties.

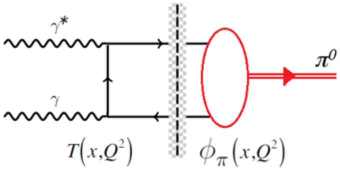
Meson and baryon distribution amplitudes

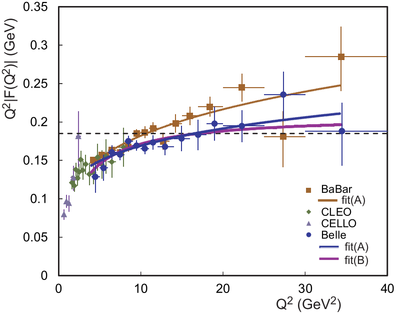
DAs: nonperturbative input for the theoretical description of hard exclusive processes.

Collinear factorisation at large Q^2.

Pion: $\gamma\gamma^* \to \pi$ form factor

[Belle,1205.3249]





$$\mathbf{F_{\pi\gamma}(Q^2)} = \frac{2f_\pi}{3}\int_0^1 dx\, \underbrace{T^H_{\gamma\pi}(x,\mu,Q^2)}_{\text{hard}}\,\underbrace{\phi_\pi(x,\mu)}_{\text{soft}}$$

Pion distribution amplitudes

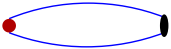
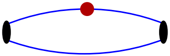
DA's: non-local matrix elements involving light-like quark separation,

$$\langle 0|\bar{\boldsymbol{d}}\boldsymbol{(-z)}\gamma_\mu\gamma_5[-z,z]\boldsymbol{u(z)}|\pi^+\rangle = if_\pi p_\mu \int_0^1 d\xi\, e^{-i\xi p\cdot z}\phi_\pi(\xi,\mu),$$

f_π is the pion decay constant, z is a light-like vector, $\xi = x-(1-x) = 2x-1$ and $[-z,z]$ is a Wilson line connecting the u and $\bar{d}$ fields.

Moments of DA's: are related to matrix elements of leading twist local operators.

$$\langle\xi^n\rangle = \int d\xi\,\xi^n\phi(\xi,\mu), \qquad \langle\xi^0\rangle = 1,\ \ \langle\xi^2\rangle \to \langle 0|\bar{d}\,\overleftrightarrow{D}_{(\mu}\overleftrightarrow{D}_{\nu}\,\gamma_{\rho)}\gamma_5 u|\pi(p)\rangle\ldots$$

Extracted from:  c.f. for moments of pdf's 

Gegenbauer expansion:

$$\phi(\xi,\mu) = \phi(2x-1,\mu) = 6x(1-x)\left(1+\sum_{n=2,4,\ldots} a_n^\pi(\mu)C_n^{3/2}(2x-1)\right)$$

[Collinear conformal symmetry, $SL(2,\mathbb{R})$, $C_n^{3/2}(\xi)$ analogous to $Y^{lm}(\theta,\phi)$ in $O(3)$.]

$\langle\xi^n\rangle$ and a_n^π related by simple algebraic expressions.

Meson and baryon distribution amplitudes

DAs are universal: involved in many processes

Pseudoscalar mesons:

Pion form factor to 6 GeV^2 (**JLab Hall C**)

Weak exclusive B and Λ_b decays (**LHCb**).

Vector mesons:

Deeply-virtual exclusive ρ-meson production ($eN \to eN\rho$) (**JLab, EIC**).

Weak exclusive $B/B_s \to V\mu^+\mu^-, V\ell\nu_\ell, V\pi$ decays (**LHCb**),
e.g. $B \to K^*\mu^+\mu^-$, $B_s \to \phi\mu^+\mu^-$.

Baryons:

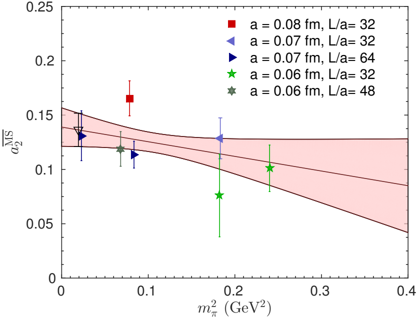
Electric and magnetic nucleon form factor $Q^2 \sim 14$ GeV ^{2} (**JLab, FAIR**)

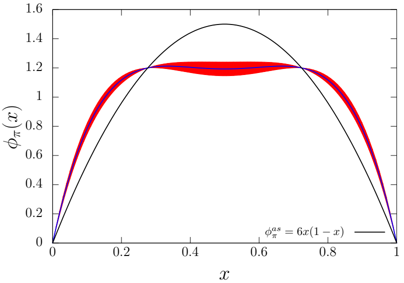
Electric neutron form factor $Q^2 \sim 8$ GeV ^{2} (**JLab, FAIR**)

Electroproduction of nucleon resonances at large $Q^2 \sim 14$ GeV^2 (**JLab**)

Pion DAs: a_2^π, $\langle \xi^2 \rangle$ from $\langle 0 | \bar{d} \overleftrightarrow{D}_{(\mu} \overleftrightarrow{D}_\nu \gamma_{\rho)} \gamma_5 u | \pi(p) \rangle$

RQCD: [Braun,1503.03656], $N_f = 2$, $m_\pi = 150 - 490$ MeV, $Lm_\pi = 3.4 - 6.7$.





$$\phi_\pi(\mathbf{x}) = 6x(1-x)\left[1 + \mathbf{a_2^\pi}(\mu)\, C_2^{3/2}(2x-1)\right], \qquad \xi = 2x - 1$$

RQCD $a_2^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.136(15)(15)$,

RBC/UKQCD [Arthur,1011.5906] $a_2^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.233(30)(60)$

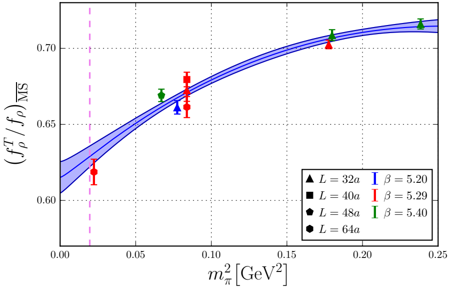
QCDSF/UKQCD [Braun,hep-lat/0606012] $a_2^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.211(114)$

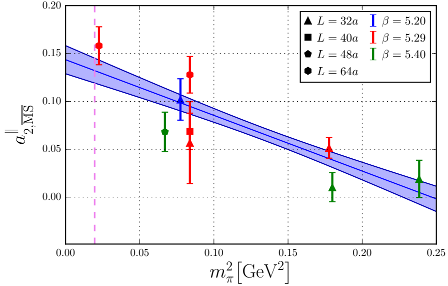
Rho DAs: a_2, $\langle \xi^2 \rangle$ from $\langle 0|\bar{d}\, \overleftrightarrow{D}_{(\mu}\overleftrightarrow{D}_{\nu}\, \gamma_{\rho)} u|\rho(p)\rangle$ etc

$$\langle 0|\bar{d}(z) n\cdot\gamma[-z,z] u(z)|\rho^+(p,\lambda)\rangle = m_\rho f_\rho (e^{(\lambda)}\cdot n)\int_0^1 d\xi\, e^{-i\xi p\cdot z}\phi_\rho^{\parallel}(\xi,\mu)$$

$$e_{\perp,\mu}^{(\lambda')} n_\nu \langle 0|\bar{d}(-z)\sigma^{\mu\nu}[-z,z] u(z)|\rho^+(p,\lambda)\rangle = i f_\rho^T \left(e_\perp^{(\lambda')}\cdot e_\perp^{(\lambda)}\right)(p\cdot n)\int_0^1 d\xi\, e^{-i\xi p\cdot z}\phi_\rho^{\perp}(\xi,\mu),$$

RQCD: [Braun,1612.02955], $N_f = 2$, $m_\pi = 150 - 490$ MeV, $Lm_\pi = 3.4 - 6.7$.





RQCD $f_\rho^T/f_\rho = 0.629(8)$, $a_2^{\parallel,\overline{\rm MS}}(2\,\mathrm{GeV}) = 0.132(27)$, $a_2^{\perp,\overline{\rm MS}}(2\,\mathrm{GeV}) = 0.101(22)$

RBC/UKQCD [Arthur,1011.5906] $a_2^{\parallel,\overline{\rm MS}}(2\,\mathrm{GeV}) = 0.20(6)$

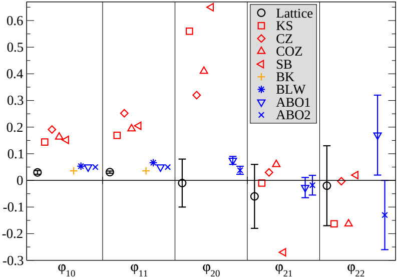
Also $f_\rho^T/f_\rho = 0.72(3)$ [Becirevic,hep-lat/0301020], 0.742(14) [Braun,hep-lat/0306006], 0.687(27) [Allton,hep-lat/0509196],

Nucleon

RQCD: [Braun,1403.4189], $N_f = 2$, $m_\pi = 150 - 490$ MeV, $Lm_\pi = 3.4 - 6.7$.

Leading twist distribution amplitude:

$$\phi_N(x_i,\mu) = 120 x_1 x_2 x_3 \Big\{ 1 + \phi_{11}(\mu) 7(x_1 - 2x_2 + x_3) + \phi_{10}(\mu) 21(x_1 - x_3)$$
$$+ \phi_{20}(\mu) \frac{63}{10}[3(x_1 - x_3)^2 - 3x_2(x_1 + x_3) + 2x_2^2] + \phi_{21}(\mu) \frac{63}{2}(x_1 - 3x_2 + x_3)(x_1 - x_3)$$
$$+ \phi_{22}(\mu) \frac{9}{5}[x_1^2 + 9x_2(x_1 + x_3) - 12 x_1 x_3 - 6x_2^2 + x_3^2] + \dots \Big\}$$



"Momentum fractions" in the proton:

$\langle x_1 \rangle = 0.372(7)$,
$\langle x_2 \rangle = 0.314(3)$,
$\langle x_3 \rangle = 0.314(7)$.

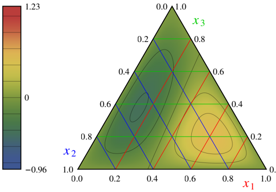
$q_1^\uparrow (q_2^\downarrow q_3^\uparrow)$.

Earlier work: [QCDSF,0804.1877], [QCDSF,0811.2712].

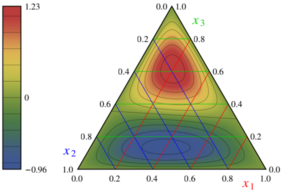
Baryon octet

RQCD: [Bali,1512.02050], $N_f = 2+1$, $a = 0.086$ **fm**, $m_\pi = 220 - 420$ MeV.

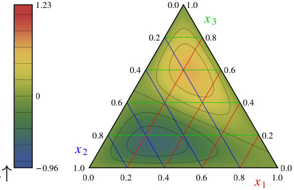
Barycentric plots ($x_1 + x_2 + x_3 = 1$) showing deviation from asymptotic shape $\phi^{as} \equiv 120 x_1 x_2 x_3$.



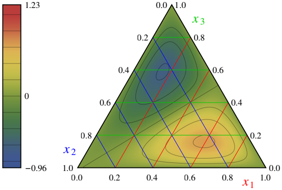
N, $u^\uparrow u^\downarrow d^\uparrow$



Λ, $u^\uparrow d^\downarrow s^\uparrow$



Σ, $d^\uparrow d^\downarrow s^\uparrow$



Ξ, $s^\uparrow s^\downarrow u^\uparrow$

N, Σ, Ξ: q_1 favoured over q_2, strange quarks carry more momentum.

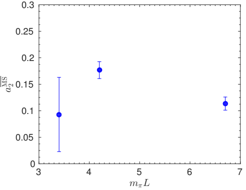
Λ: maximum of distribution shifted to strange quark.

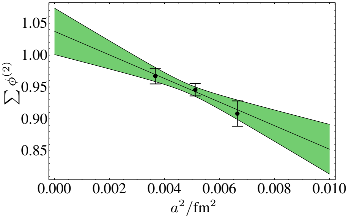
Status and difficulties

Steady progress in computing the 1st and 2nd moments of hadron DAs.

Statistics vs systematics: finite volume ($Lm_\pi \gtrsim 4$), unphysical quark mass ($m_\pi \sim 135 - 150$ **MeV**), discretisation effects ($4a$**'s**), (non-perturbative) renormalisation ...

(Left) pion, $a = 0.07$ fm, $m_\pi = 290$ MeV. [Braun,1503.03656]
(Right) nucleon, constraint: $x_1 + x_2 + x_3 = 1$, $m_\pi = 280 - 290$ MeV. [Braun,1403.4189]





Future work includes: K, K^*, η, η', ...

Resonances need to be treated with a finite volume analysis.

Difficulty in calculating higher moments

- Statistics: higher n require matrix elements of operators with more derivatives. Signal/noise is low, at present $n = 1, 2$ calculated.
- Renormalisation: mixing of operators under renormalisation due to reduced symmetry on the lattice (rotational sym.$\to$ hypercubic). Worse for higher n.
  $O_i(\mu) = Z_{ij}(\mu, a) O_j(a)$, $\quad Z_{\overline{MS}} = Z^{P}_{\overline{MS},MOM} Z^{NP}_{MOM}$

Direct determination of distribution amplitudes

Large-momentum effective field theory (LatMet)[Ji,1305.1539]: compute quasi correlation[Zhang,1702.00008],

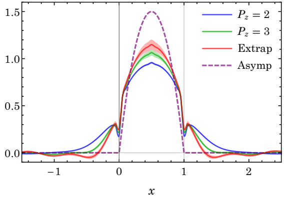
$$\tilde{\phi}(x,P_z) = \frac{i}{f_\pi}\int \frac{dz}{2\pi} e^{-i(x-1)P_z z}\langle \pi(P)|\bar{\psi}(0)\gamma^z\gamma_5\Gamma(0,z)\psi(z)|0\rangle,$$

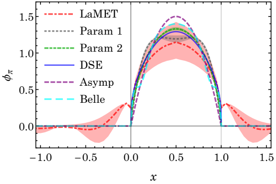
with quark fields separated along the spatial z direction.

Can be matched to pion DA[Ji,1506.00248]:

$$\tilde{\phi}(x,\Lambda,P_z) = \int_0^1 dy\, Z_\phi(x,y,\Lambda,\mu,P_z)\phi(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{P_z^2},\frac{m_\pi^2}{P_z^2}\right),$$

with $\Lambda = \pi/a$. $m_\pi = 310$ MeV, $a = 0.12$ fm, $Lm_\pi = 4.5$.





High momenta required: signal/noise deteriorates, discretisation effects ($\mathcal{O}(ap)$ and $\mathcal{O}(a^2p^2)$).

Distribution amplitudes from a position space method

[Braun and Mueller,0709.1348]+[Aglietti,hep-ph/9806277]:

Example: pion, lattice "data" $M(\vec{y}\cdot\vec{p}, |\vec{y}|^2, \mu)$,

$$M^{\overline{MS}}(\vec{y}\cdot\vec{p}, |\vec{y}|^2, \mu) = \langle \pi^0(p)|\bar{\boldsymbol{q}}(\boldsymbol{y}/2)\boldsymbol{\Gamma_1 q}(\boldsymbol{y}/2)\bar{\boldsymbol{q}}(-\boldsymbol{y}/2)\boldsymbol{\Gamma_2 q}(-\boldsymbol{y}/2)|0\rangle$$

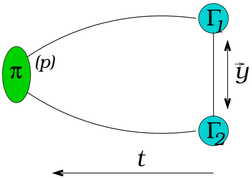
For $2/|\vec{y}|$ perturbative and $\gg m_\pi$,

$$M^{\overline{MS}}(\vec{y}\cdot\vec{p}, \vec{y}^2, \mu) = Z(\frac{2}{|\vec{y}|}, \mu)\int_0^1 dx\, e^{i(2x-1)p\cdot y/2}\, T_H(x, \mu_F, \frac{2}{|\vec{y}|})\phi_\pi(x, \mu_F)$$

To leading order and leading twist:

$$M = \frac{y\cdot p}{\pi^2 y^4} f_\pi \int_0^1 dx\, e^{i(2x-1)p\cdot y/2}\phi_\pi(\mathbf{x}, \mu) = \frac{y\cdot p}{\pi^2 y^4}\mathcal{F}(\mathbf{p}\cdot\mathbf{y})$$

With $\Gamma_1 = S = \mathbb{1}$, $\Gamma_2 = P = \gamma_5$, $\mu_F = \mu =$ 2 GeV.

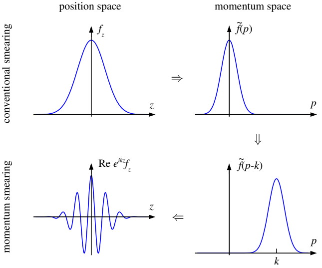


$$\mathcal{F}(\mathbf{p}\cdot\mathbf{y}, \mathbf{t}) \propto Z_S Z_P \frac{\pi^2 y^4}{y\cdot p}\frac{\mathbf{C}^{(\mathbf{p})}_{\pi\mathbf{SP}}(\mathbf{t})\, e^{E_\pi t/2}}{\sqrt{2E_\pi C^{(p)}_\pi(t)}}$$

Pion two-point function: $C^{(p)}_\pi(t)$

Local operator renorm. factors

Achieving a good signal at high momenta

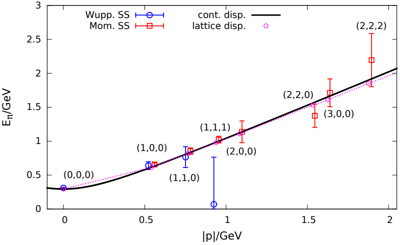


RQCD [Bali,1602.05525]:

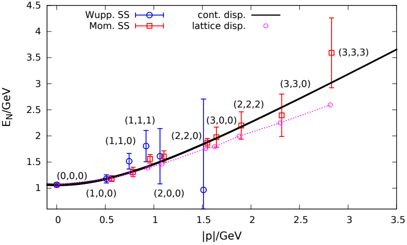
Quark smearing that maintains small statistical errors and good overlaps with ground state wavefunctions.

Test case: $N_f = 2$, $m_\pi = 295$ MeV, $a = 0.07$ fm. Momenta up to 2.8 GeV achieved.

Pion



Nucleon



Pion distribution amplitude from a position space method

RQCD: G. Bali, V. Braun, M. Göckeler, B. Lang, B. Musch, A. Schäfer, P. Wein, J. Zhang.

Test case: $N_f = 2$, $m_\pi = 295$ MeV, $a = 0.07$ fm.

$\vec{p} = (2, 0, 0)2\pi/L$, different $\vec{x}$ (fixed $|\vec{x}|^2$) give range $\vec{p} \cdot \vec{x}$

Keep $2/|\vec{x}|$ large, need large $\vec{p}$ for sensitivity to parameterisation of ϕ_π.