Observations on renormalization patterns of quark bilocal operators

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Lattice TMD Collaboration

Renormalization patterns – Expectations

Quark bilocal operator: $\overline{\psi}(-b/2)\ \Gamma\ \mathcal{U}[-b/2, b/2]\ \psi(b/2)$

Expectation:

- Renormalization is multiplicative – renormalization factors can be canceled in matrix element ratios
- Renormalization factors become independent of Γ at large separations b; $Z = Z_\psi S_\mathcal{U} Z_\psi$

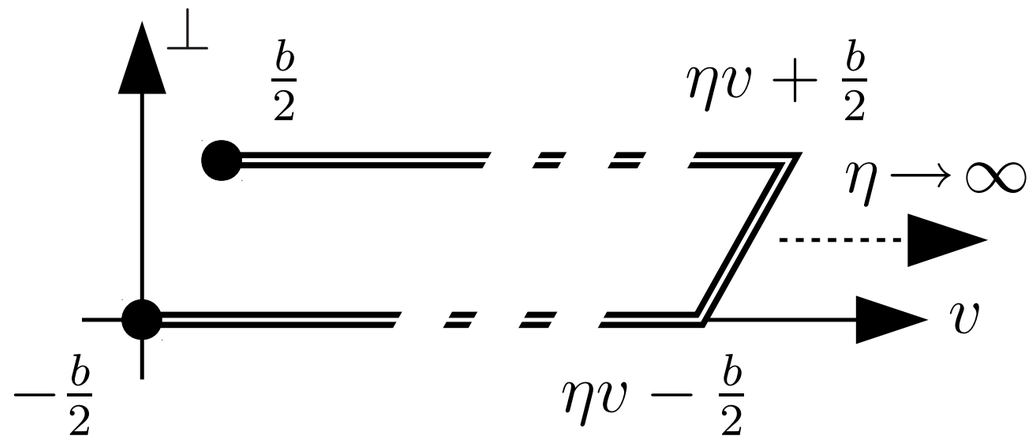
Compare two lattice ensembles:

- DWF, $a = 0.084$ fm, $m_\pi = 297$ MeV
- Clover, $a = 0.114$ fm, $m_\pi = 317$ MeV

Do ratios of matrix elements coincide?

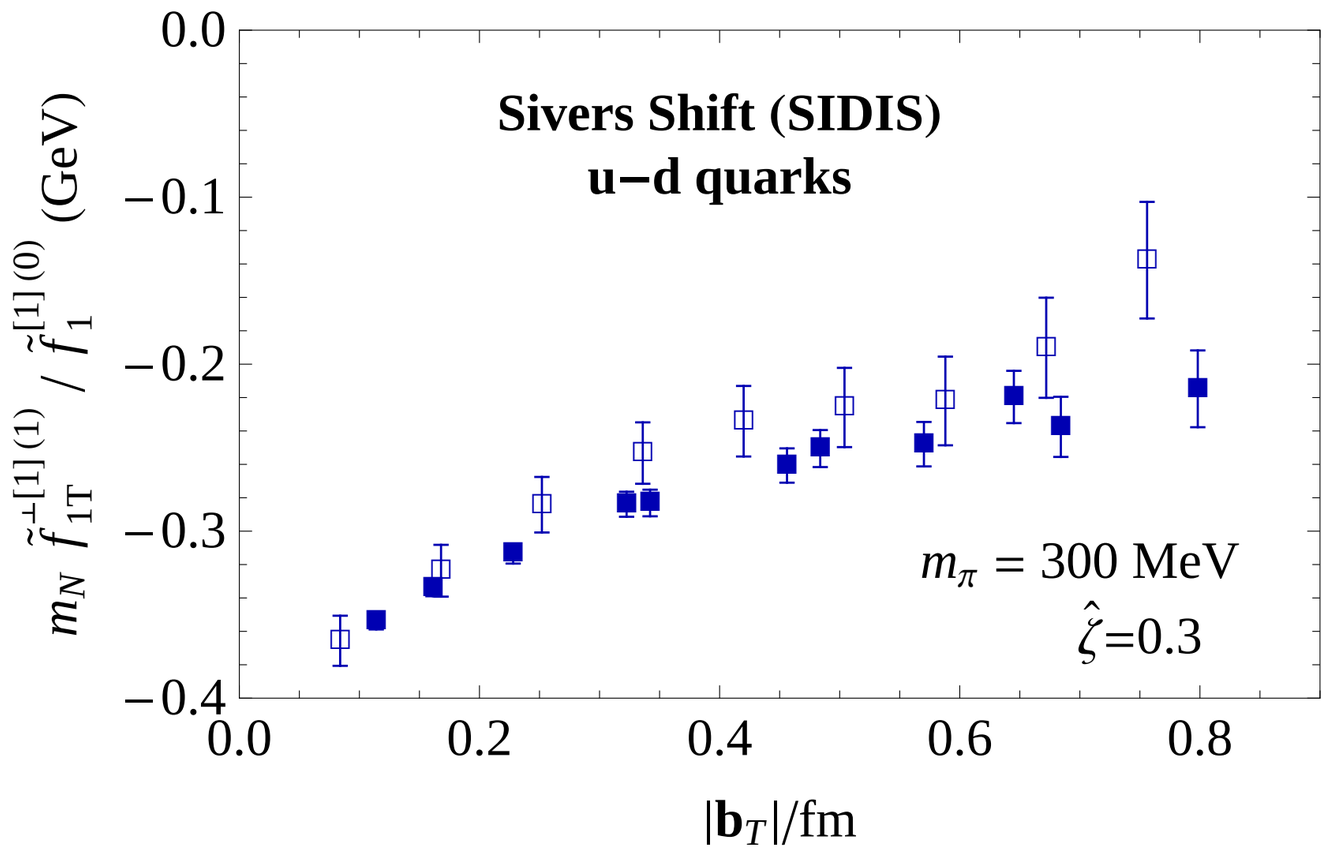
Geometry

Path of gauge link $\mathcal{U}$:



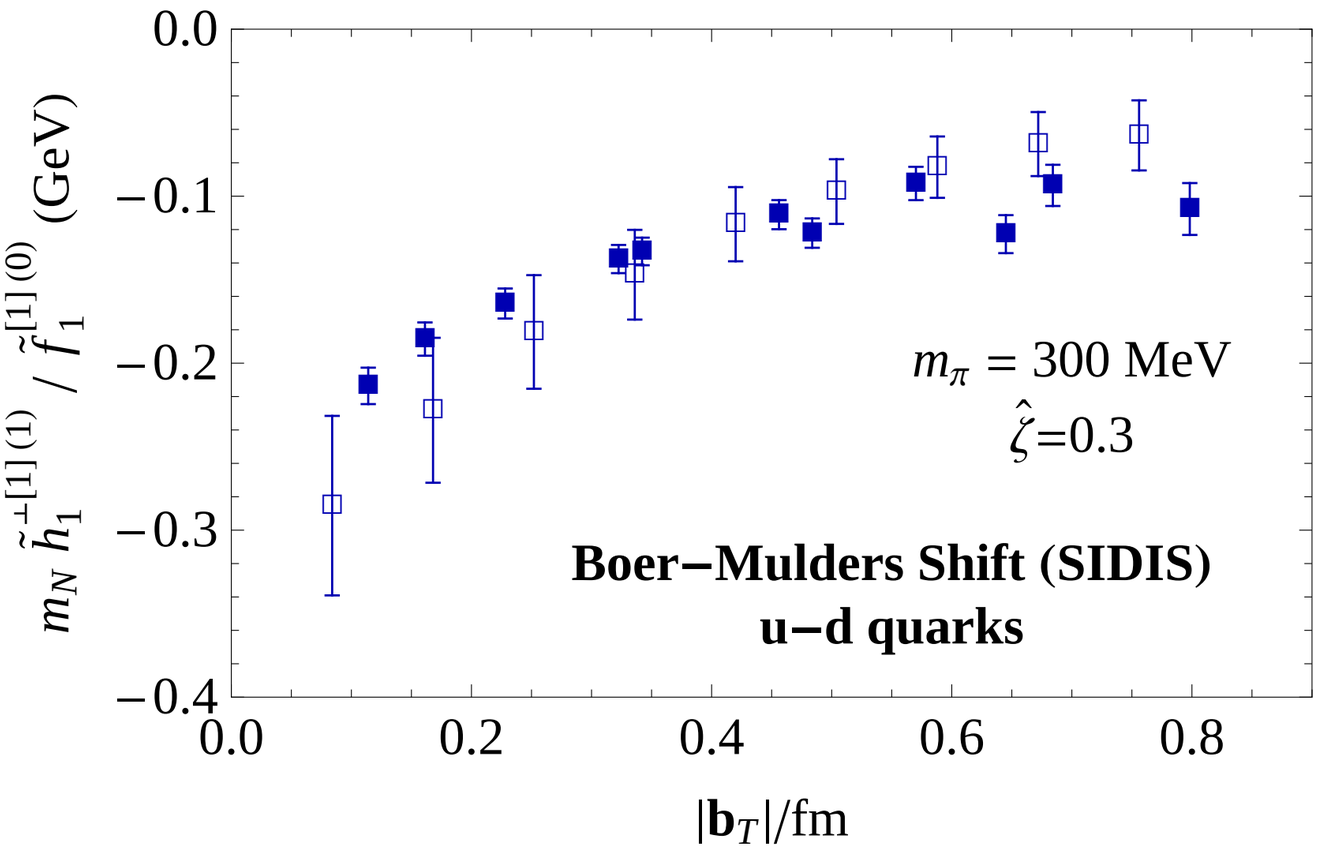
- Have generated data for variable η, including $\eta = 0$ (straight link)
- Separation b is transverse to proton momentum P; also $P = 0$ data available

Sivers Ratio



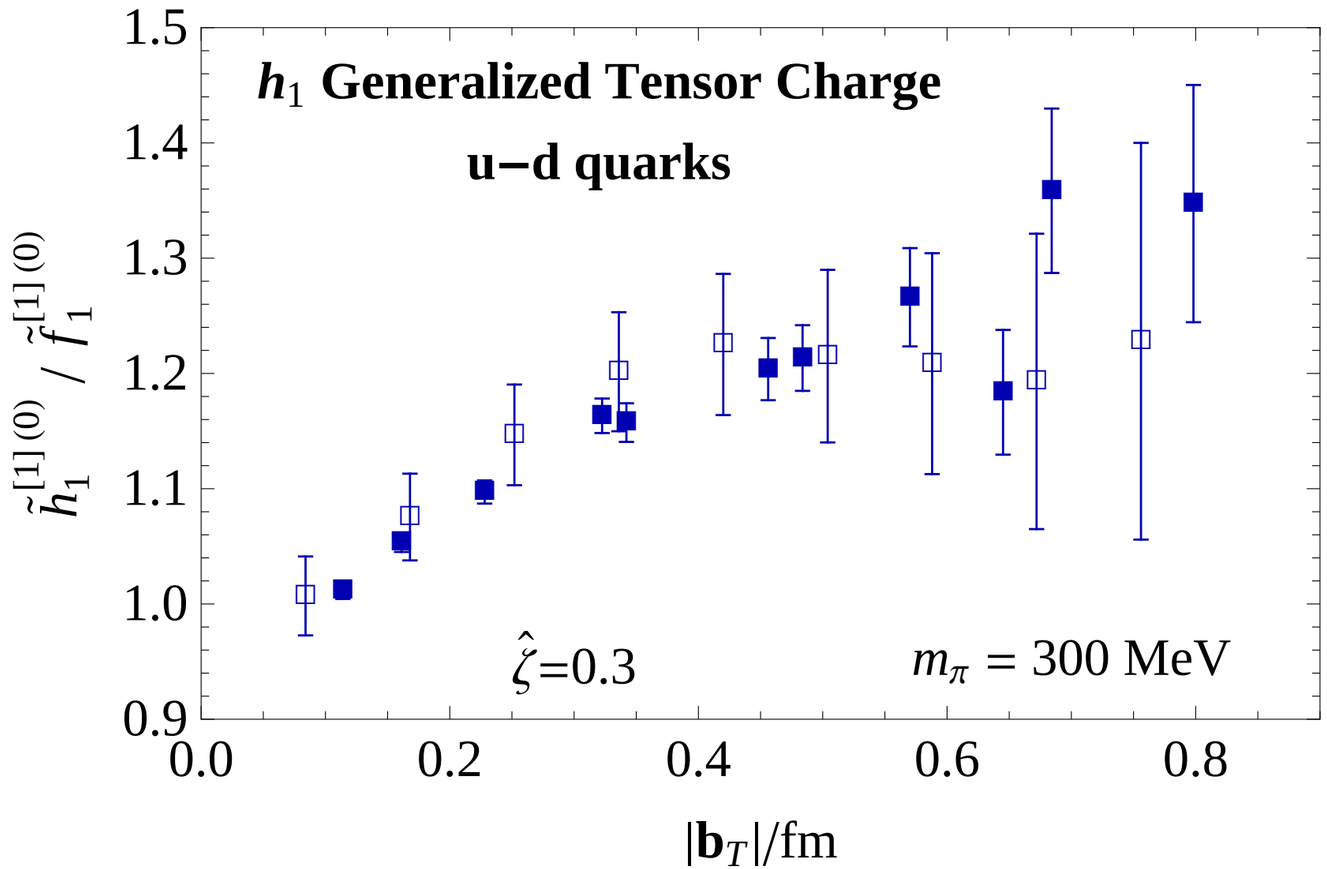
Im $\langle\gamma^+\rangle$/Re$\langle\gamma^+\rangle$

Boer-Mulders Ratio



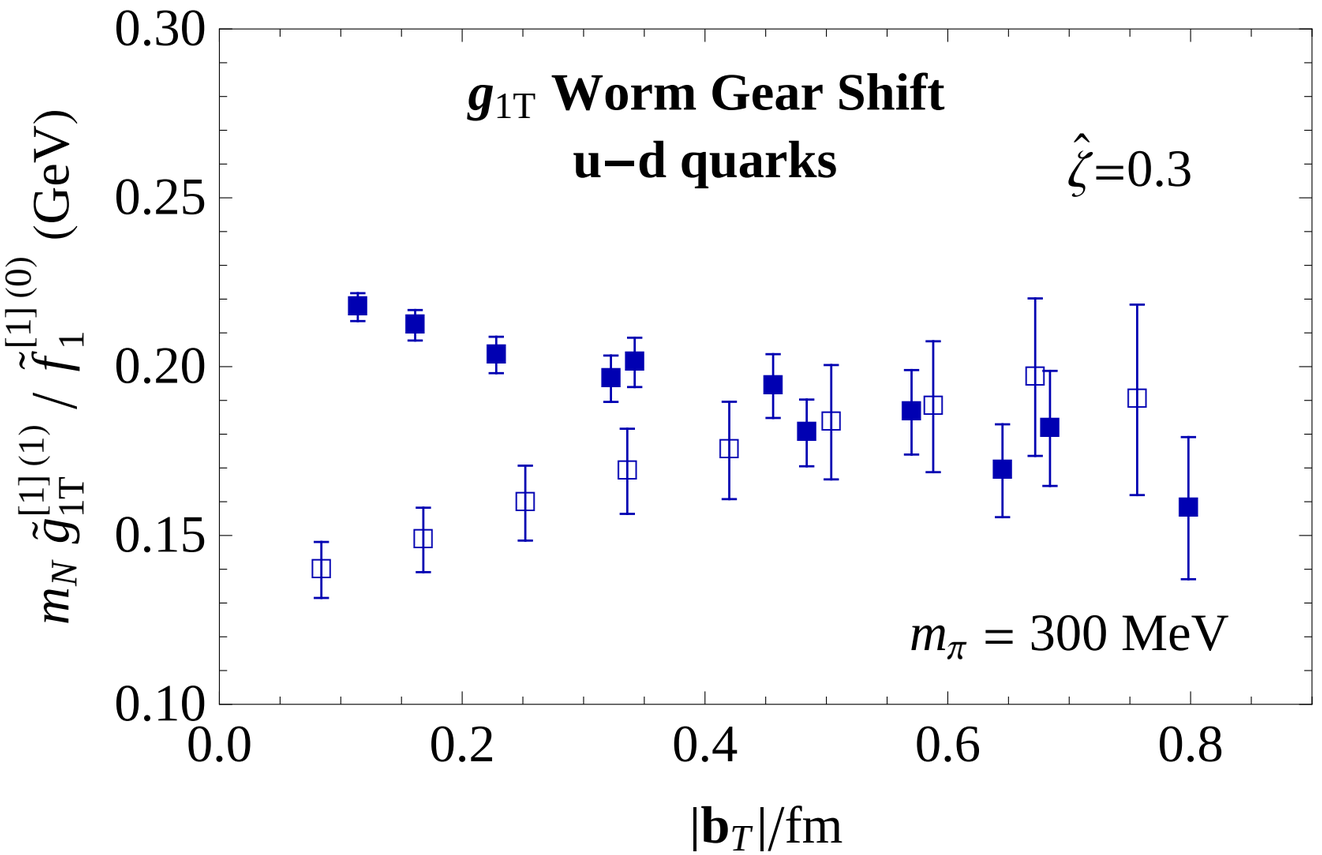
$\langle i\sigma^{i+}\gamma^5\rangle / \mathrm{Re}\langle\gamma^+\rangle$

Generalized Tensor Charge



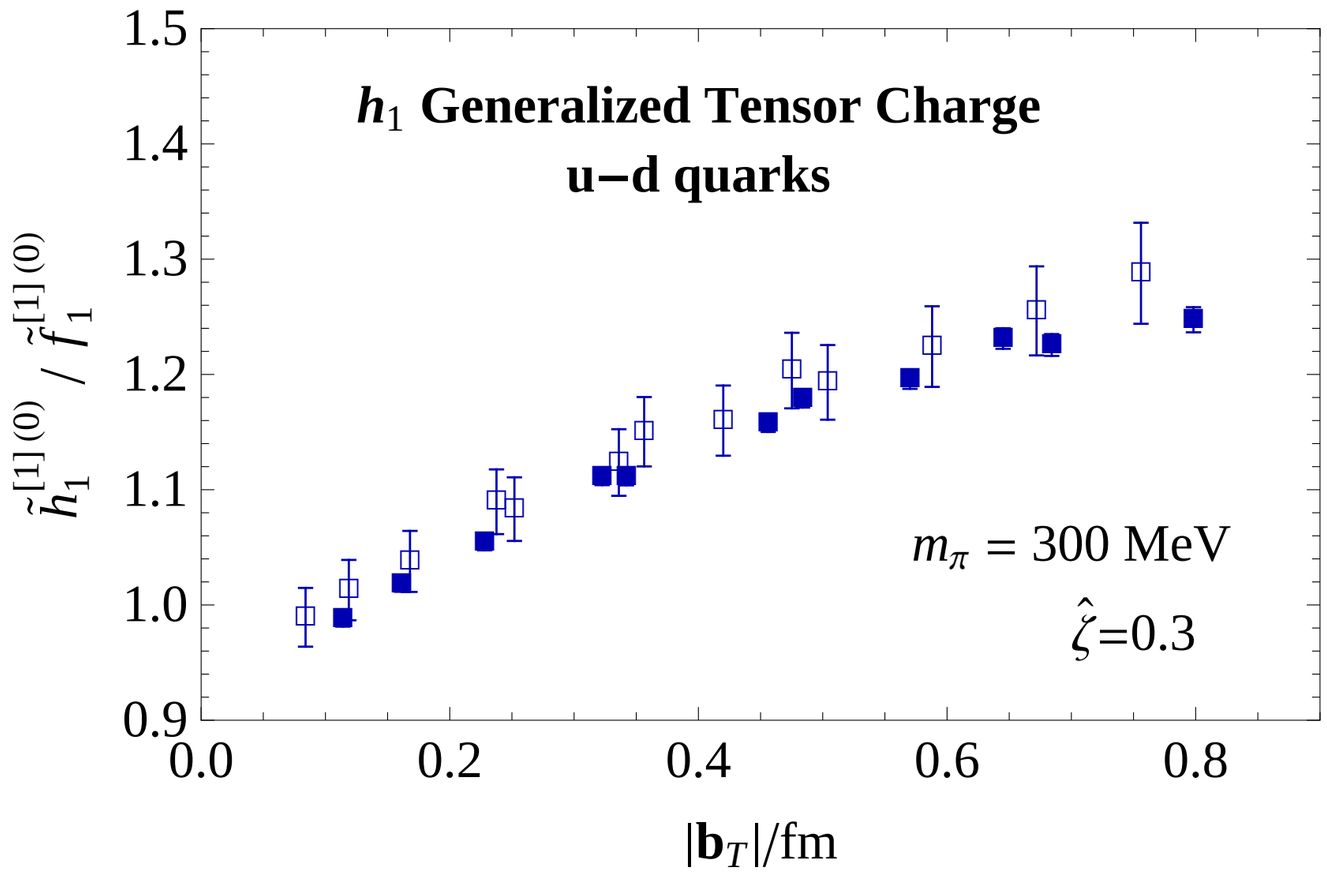
$\langle i\sigma^{i+}\gamma^5\rangle / \mathrm{Re}\langle\gamma^+\rangle$

g_{1T} Worm Gear Ratio



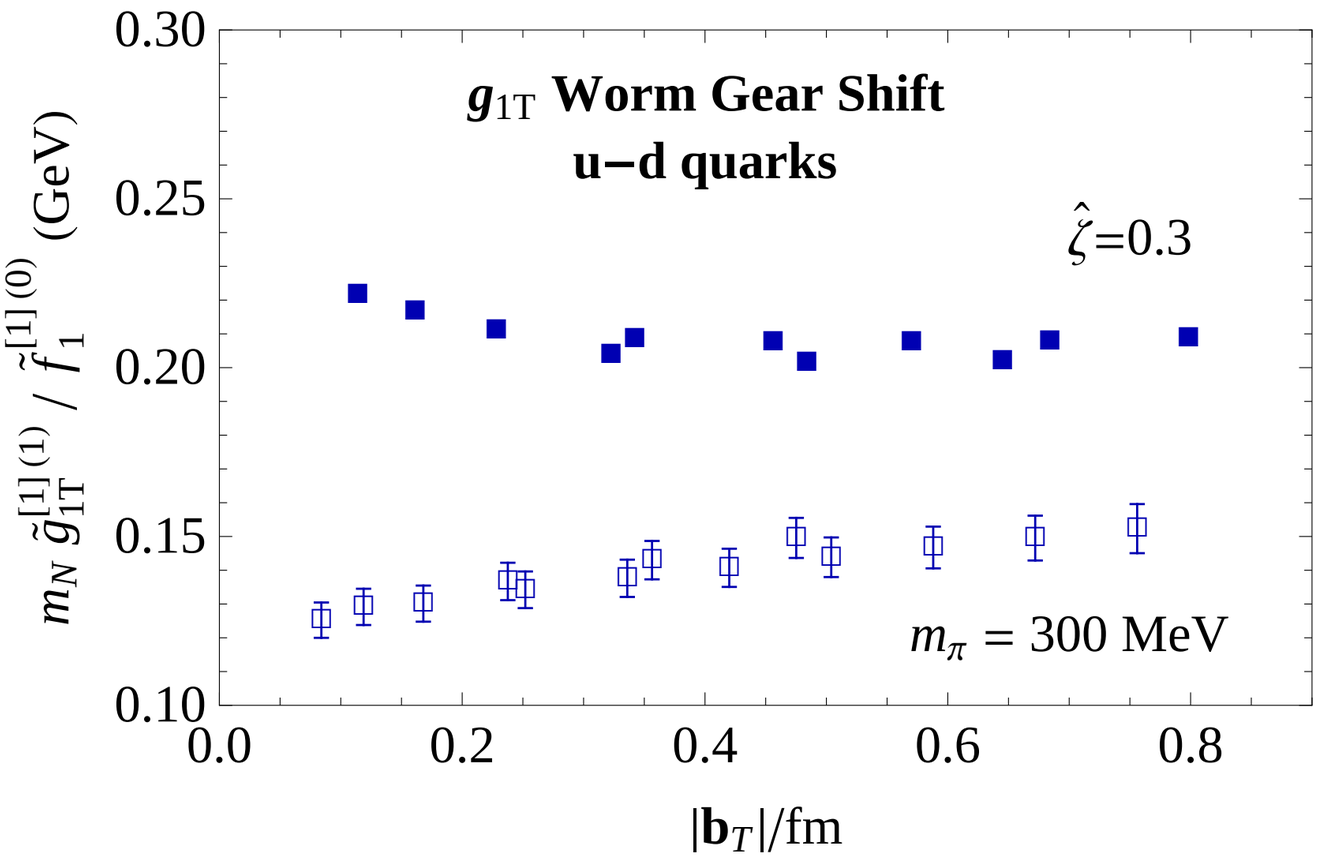
$\langle\gamma^+\gamma^5\rangle/\mathrm{Re}\langle\gamma^+\rangle$

Generalized Tensor Charge: $\eta = 0$



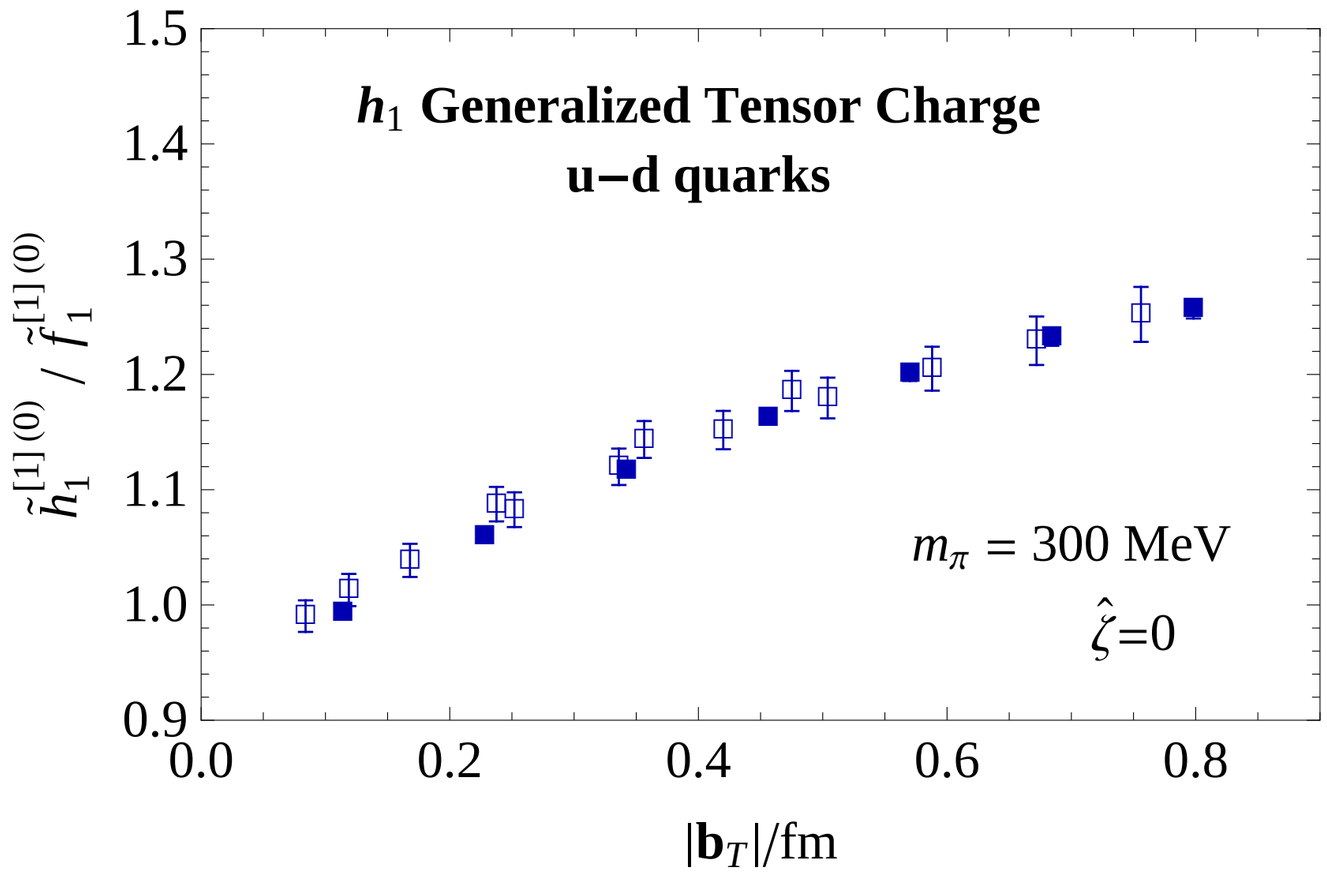
$\langle i\sigma^{i+}\gamma^5\rangle / \mathrm{Re}\langle\gamma^+\rangle$

g_{1T} Worm Gear Ratio: $\eta = 0$



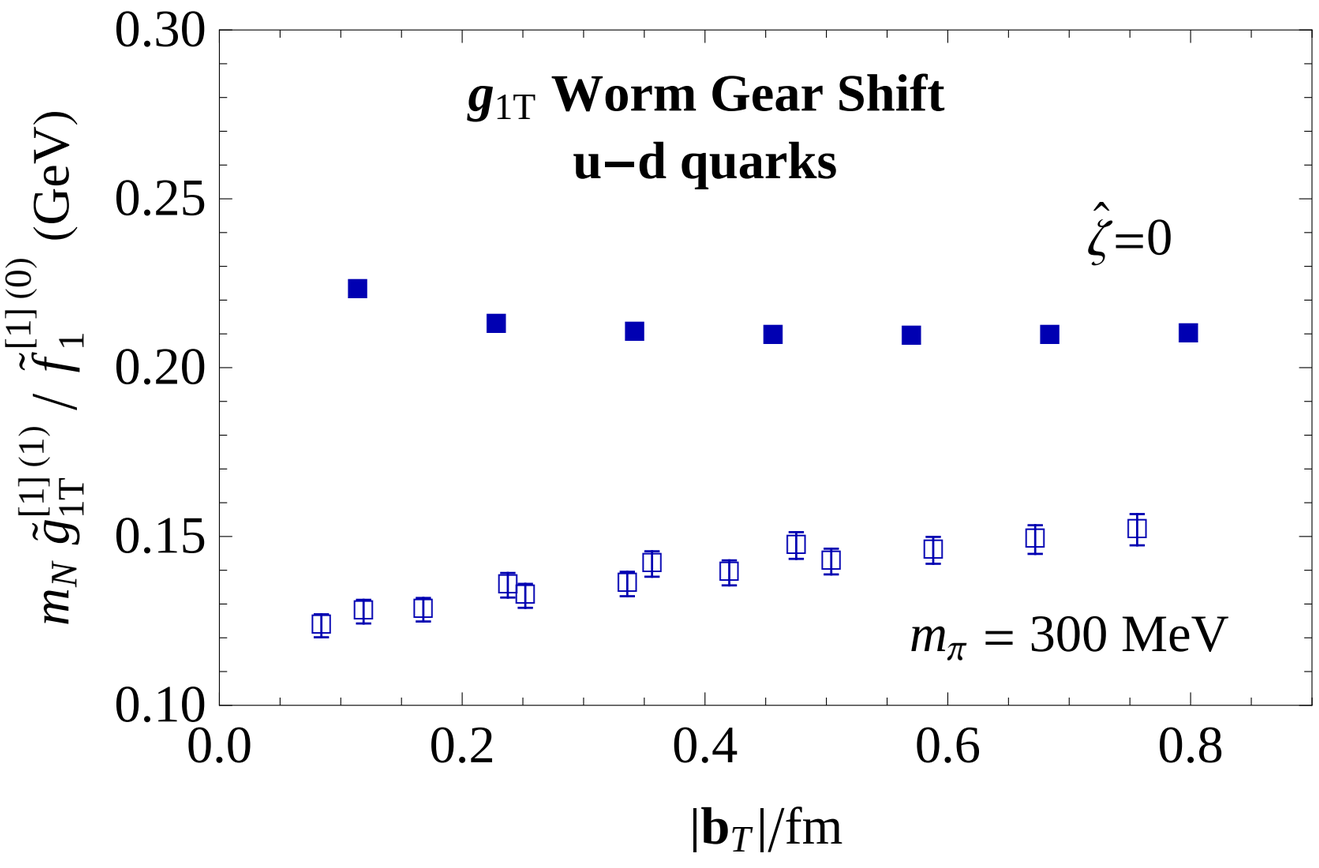
$\langle \gamma^+ \gamma^5 \rangle / \mathrm{Re} \langle \gamma^+ \rangle$

Generalized Tensor Charge: $\eta = 0$



$\langle i\sigma^{i+}\gamma^5\rangle / \mathrm{Re}\langle\gamma^+\rangle$

g_{1T} Worm Gear Ratio: $\eta = 0$



$\langle \gamma^+ \gamma^5 \rangle / \mathrm{Re} \langle \gamma^+ \rangle$

Conclusions

- There is something still to be understood here.
- Is the behavior of the straight-link worm gear ratio at large distances an effect of lattice renormalization?
- How is it caused in detail?
- What other quantities are affected?