RECONSTRUCTING PARTON DISTRIBUTION FUNCTIONS FROM THEIR COORDINATE SPACE BEHAVIOR

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Will try to provide an answer to Emanuele's questions:

i) What information from PDF fits is relevant to constrain/test/validate lattice calculations?

ii) What PDF-related quantities we would like lattice QCD to compute?

…

3) Can we quantify the accuracy that lattice calculations should have in order to have a direct impact on the PDF fit?

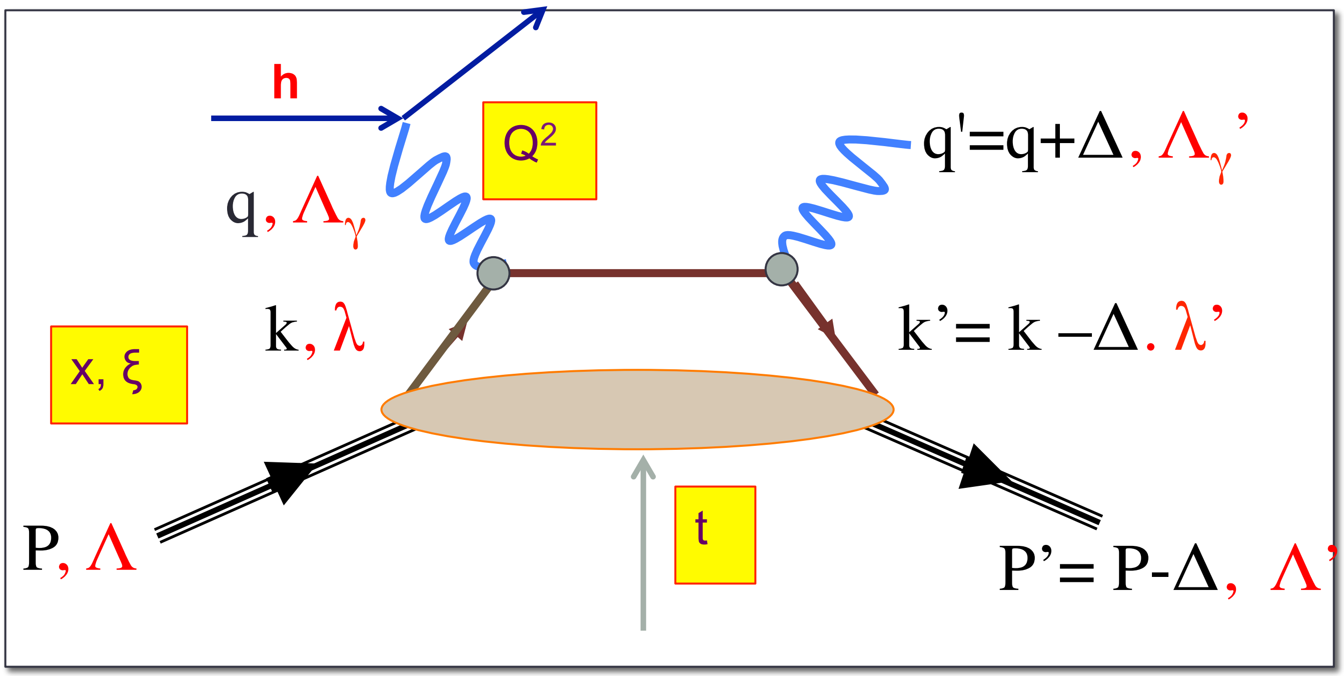
4) To which extent available lattice results agree with the results of global PDF fits?

Extend to GPDs!

1. DEFINITIONS

GPD/PDF CORRELATION FUNCTION

$$W^{\Gamma}_{\Lambda'\Lambda}(x,\xi,\Delta) = \frac{1}{2}\int \frac{dz^-}{(2\pi)} e^{ixP^+z^-} \langle P-\Delta, \Lambda' \mid \bar{q}\,(0)\,\Gamma\mathcal{W}\left(0, z^-\right) q\left(z^-\right) \mid P, \Lambda\rangle \Big|_{z_T=0, z^+=0}$$



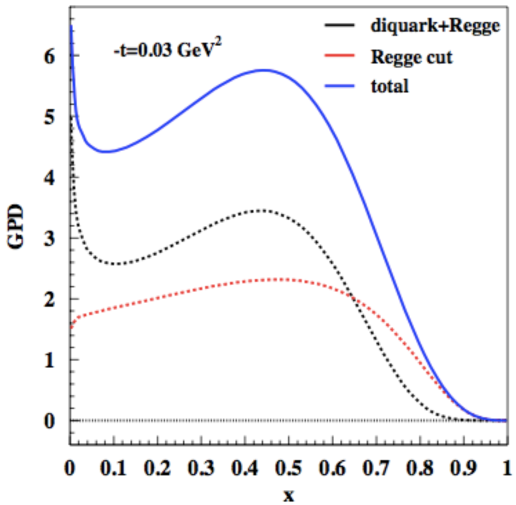
GPDs involve two types of distance

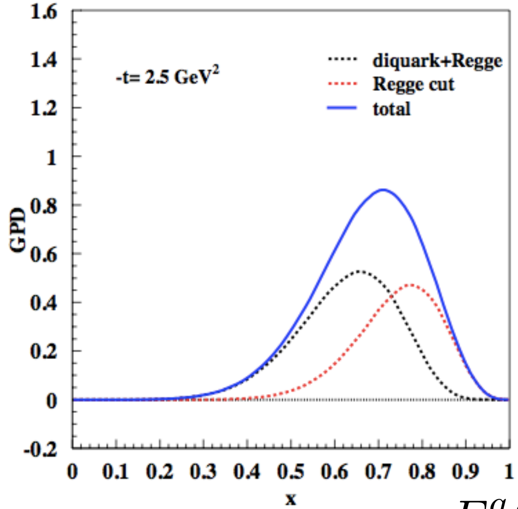
$$H^q(x,0,\Delta) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P-\Delta,\Lambda' \mid \bar{q}(0)\gamma^+ q(z^-) \mid P,\Lambda\rangle_{\mathbf{z}_T=0}$$

"Fourier transform of non-diagonal density distribution in **z^-** and diagonal density distribution in conjugate of **Δ➔ b"**

$$\bar{q}^\dagger_+(0,b)q_+(z^-,b) \rightarrow \rho(0,b;z^-,b)$$

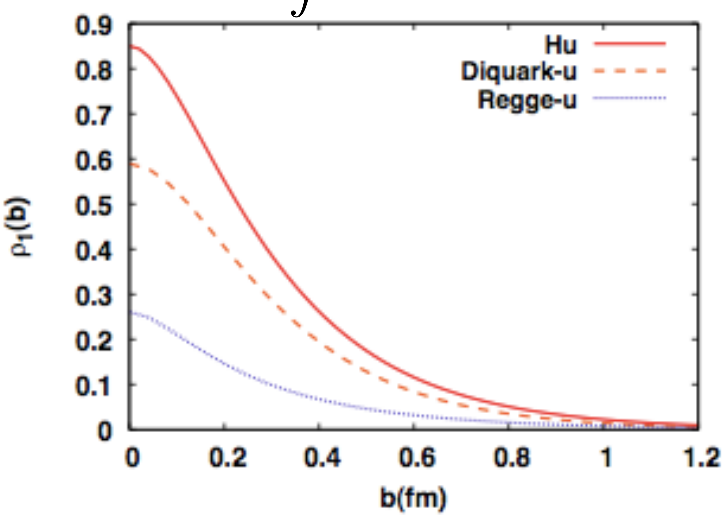
$H^u(x,0,\Delta)$

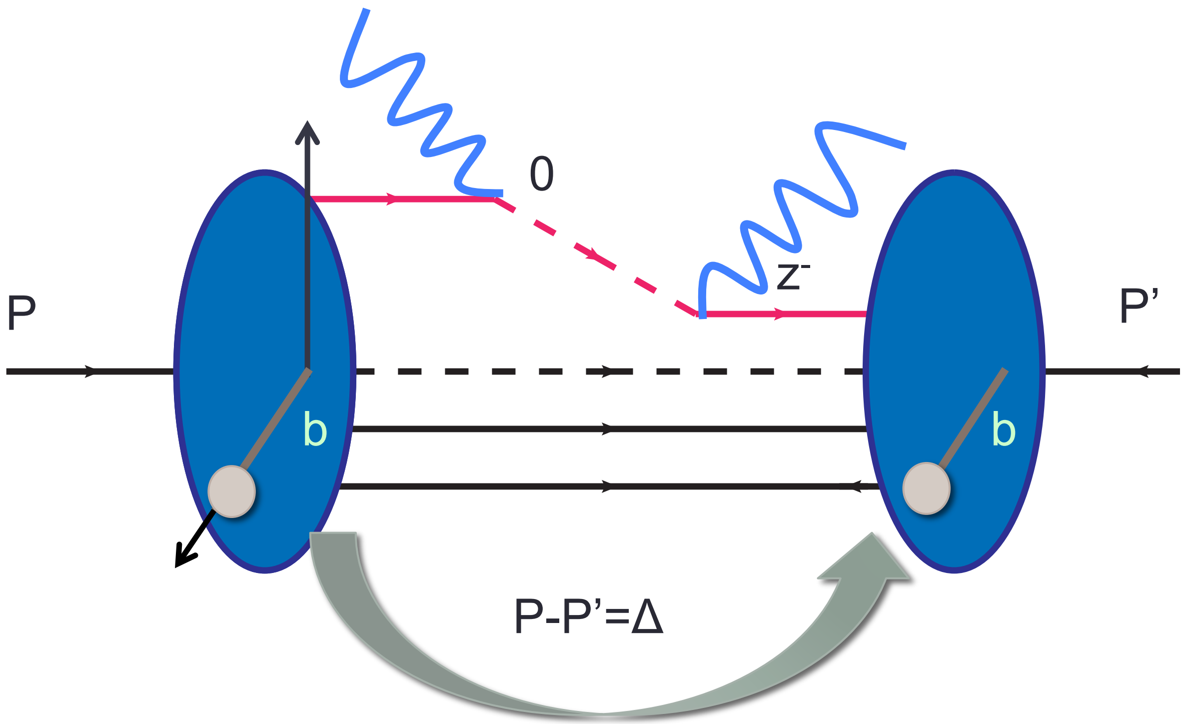




$$F^q(t \equiv \Delta^2) = \int dx H^q(x, 0, \Delta) \rightarrow \rho^q(b)$$

O. Gonzalez-Hernandez et al., PRC88 (2013)





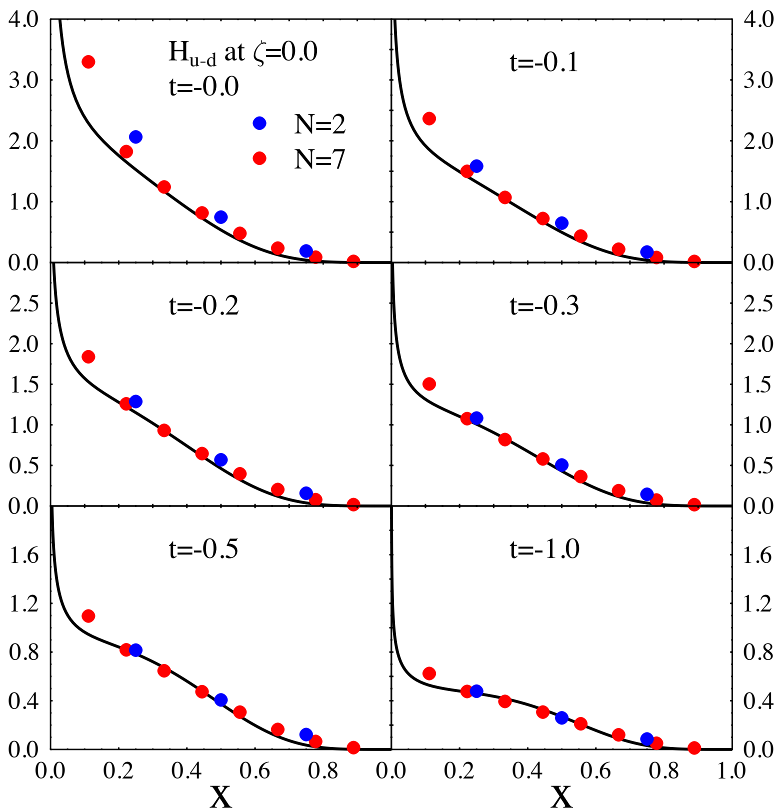
$$\langle P-\Delta, \Lambda' \mid \bar{q}(0)\gamma^{+}\mathcal{W}(0,z)q(z^{-}) \mid P, \Lambda\rangle_{\mathbf{z}_T=0}$$

Reconstructing GPDs from a finite number of moments

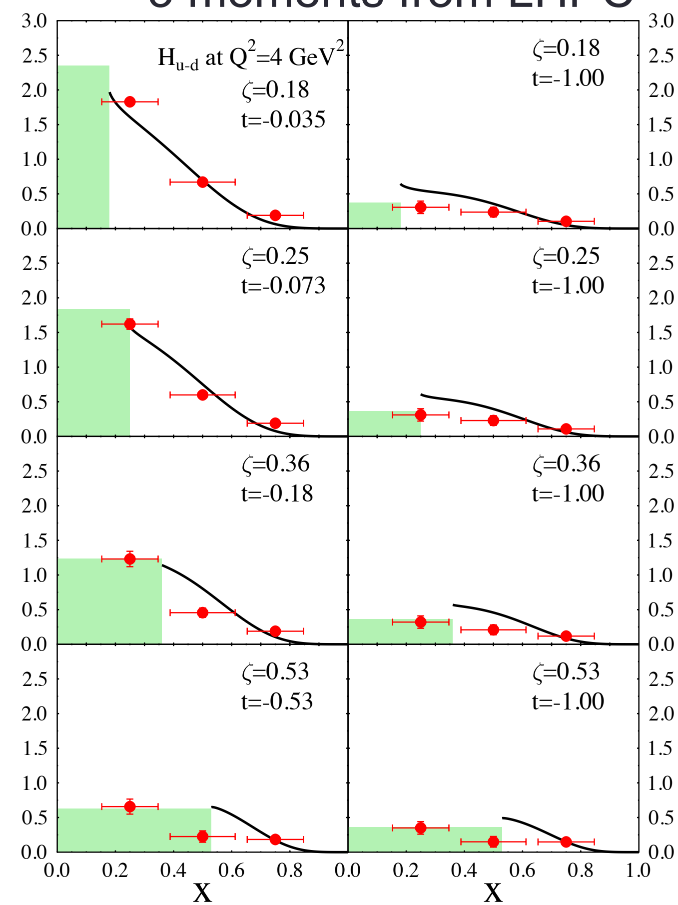
(Using lattice moments from LHPC, Ph. Hägler et al., 2005, 2006, 2008)

Some result from Bernstein polynomials based analysis H. Honkanen et al., EPJC(2007)

8 moments



3 moments from LHPC



2. DETERMINATION OF PDF FROM COORDINATE SPACE BEHAVIOR

Space-time picture of the correlation function

- At small distances - $z^- < R$ - the virtual photon scatters incoherently from the individual partons. This region is dominated by bulk properties such as the average momentum.

- At larger distances - $z^- >> R$ - the interaction is given by coherent scattering over several partons. The virtual photon converts into a q-qbar pair covering a distance z^-

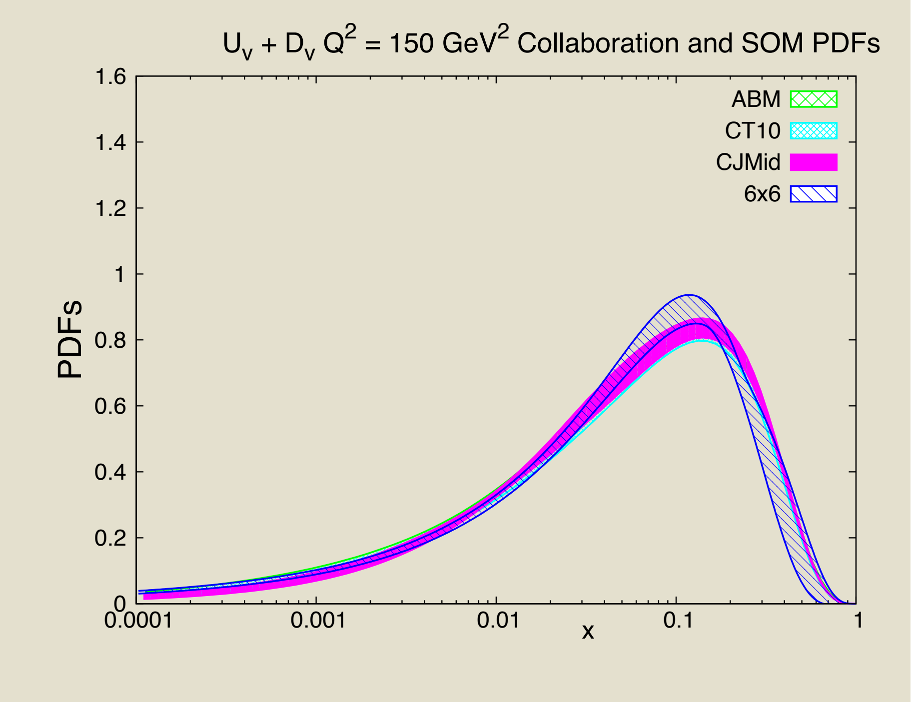
Example: Unpolarized PDFs

$$f_1^q(x) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P, \Lambda \mid \bar{q}(0)\gamma^+ q(z^-) \mid P, \Lambda \rangle_{\mathbf{z}_T = 0}$$

Parton x-momentum distribution is the Fourier transform of a non-diagonal one body density distribution in coordinate z-space

$$\bar{q}_+^\dagger(0) q_+(z^-) \to \rho(0, z^-)$$

$$F_2(x) = \sum_q x f_1^q(x)$$



E. Askanazi, K. Holcomb, S. L., J. Phys. G 42, no. 3, 034030 (2015) [arXiv:1411.2487 [hep-ph]].

$$\langle P,\Lambda' \mid \bar{q}(0)\,\gamma^{+}\overbrace{q\left(z^{-}\right) \mid P,\Lambda\rangle + \langle P,\Lambda' \mid \bar{q}(0)}^{\mathrm{T_s}}\,\gamma^{+} q\left(-z^{-}\right) \mid P,\Lambda\rangle = \frac{i}{2P^{+}} \int_0^1 dx\, q_s(x)\sin(xz)$$

$$\langle P,\Lambda' \mid \bar{q}(0)\,\gamma^{+}\overbrace{q\left(z^{-}\right) \mid P,\Lambda\rangle - \langle P,\Lambda' \mid \bar{q}(0)}^{\mathrm{T_c}}\,\gamma^{+} q\left(-z^{-}\right) \mid P,\Lambda\rangle = \frac{1}{2P^{+}} \int_0^1 dx\, q_v(x)\cos(xz)$$

Symmetry: C-parity

Collins and Soper, ‘80s

$$\bar{q}(x) = -q(-x)$$

C-even $$q(x) + \bar{q}(x)$$ Anti-symmetric with respect to x=0

C-odd $$q(x) - \bar{q}(x)$$ Symmetric with respect to x=0

$$\boxed{T_s(z) \quad = \frac{i}{2M}\int_0^1 dx q(x)\sin(xz) = \frac{i}{2M}\left[M_2 z + \frac{1}{3!}M_4 z^3 + ...\right]}$$

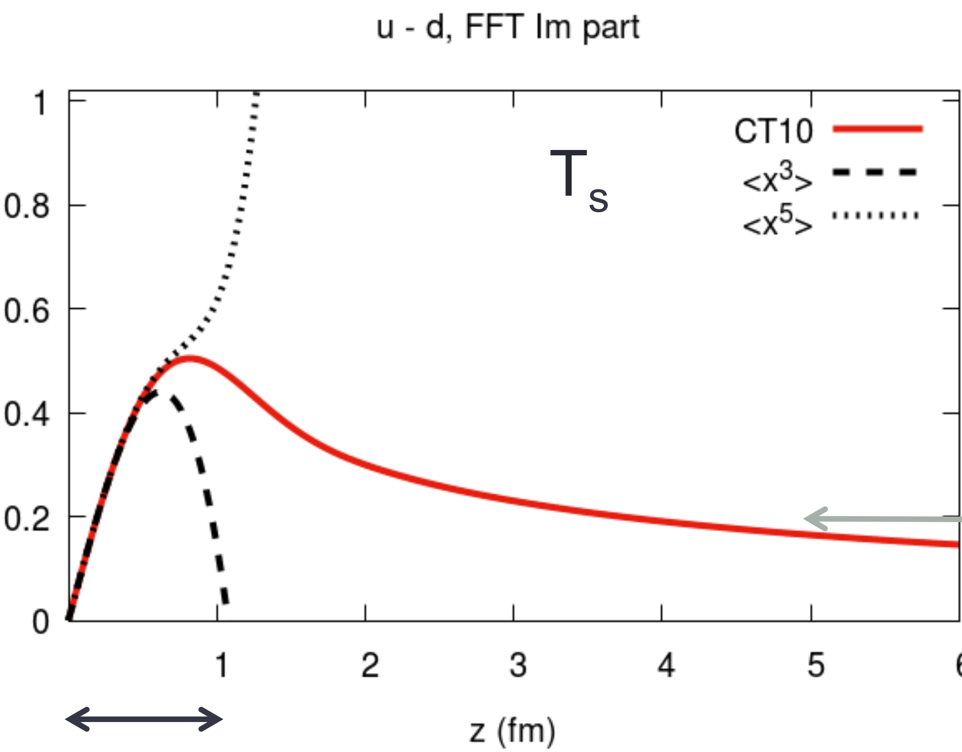
$$M_2 = \int_0^1 dx x\, q(x)$$

$$M_4 = \int_0^1 dx x^3 q(x)$$

$$\boxed{T_c(z) \quad = \frac{1}{2M}\int_0^1 dx q_s(x)\cos(xz) = \frac{i}{2M}\left[M_1 + \frac{1}{2}M_3 z^2 + ...\right]}$$

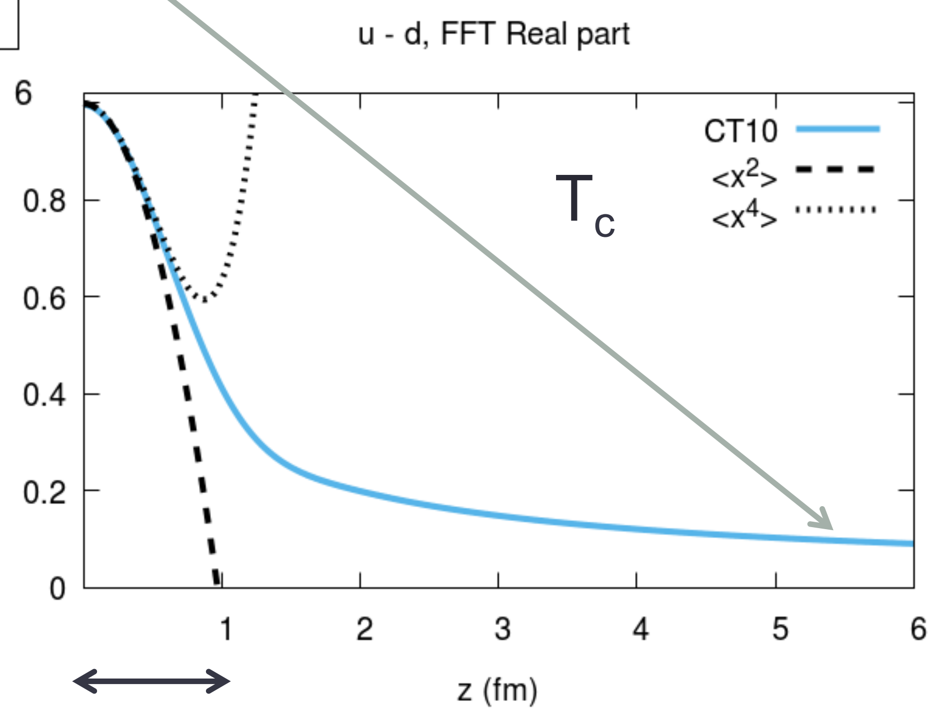
$$M_1 = \int_0^1 dx \;\, q(x)$$

$$M_3 = \int_0^1 dx x^2 q(x)$$



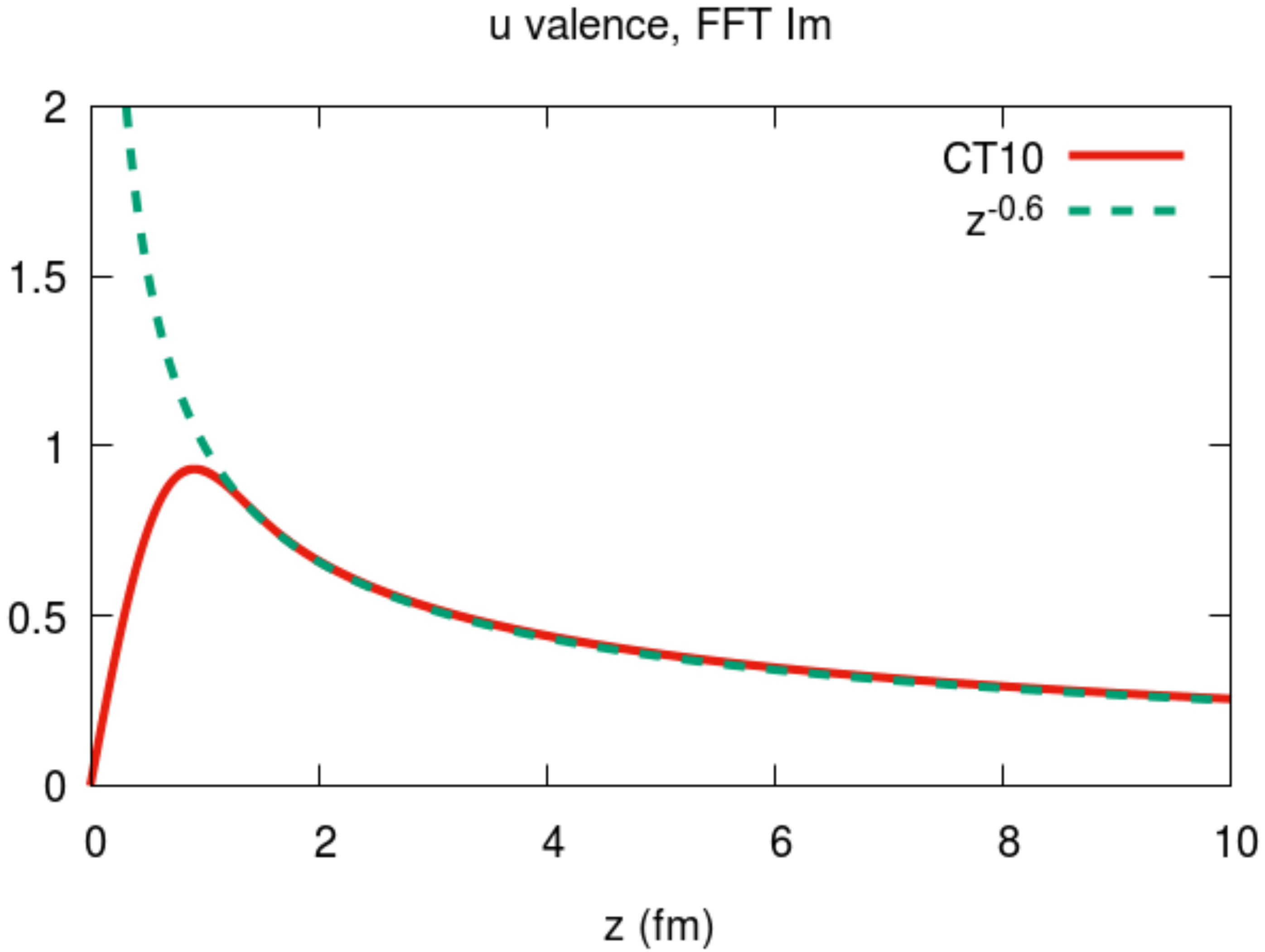
proton size

Regge tail ➡ $z^{-\alpha+1}$

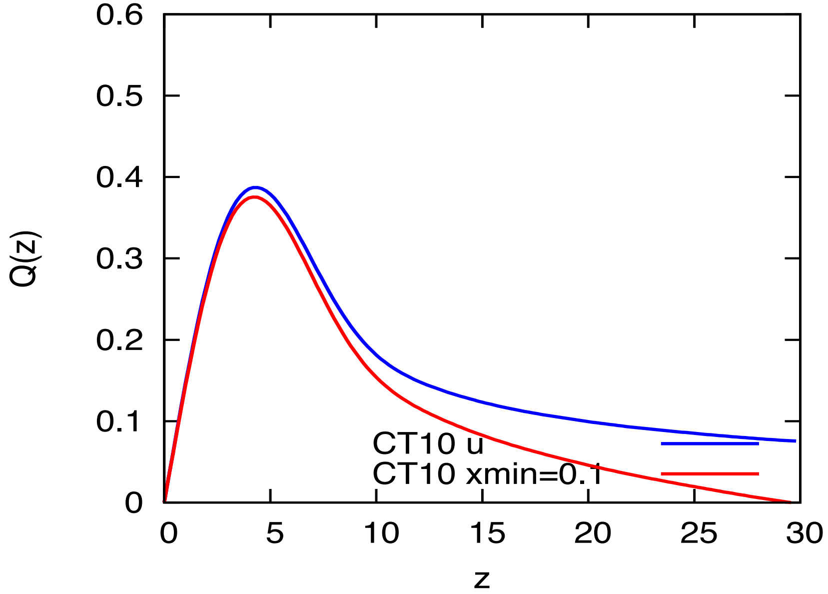


proton size

Regge term



Effect of Regge term



2. HOW WELL CAN WE RECONSTRUCT THE X DEPENDENT FUNCTION (PDF/ GPD)?

ii), 3) 4)

Information from lattice

u-d

W. Detmold et al., Eur.Phys.J.3 (2001), Mod.Phys.Lett. A18 (2003)

Moment u-d μ^2= 4 GeV^2	**Linear extrapolation**	**Chiral extrapolation**	**Phenomenology CT10**
M_1	1	1	1
M_2	0.262	0.18(3)	0.169
M_3	0.0843	0.05(2)	0.0536
M_4	0.0340	0.02(1)	0.0221

LHPC (Ph. Hägler et al.) Phys.Rev. D77 (2008)

u-d

Moment u-d μ^2= 4 GeV^2	m_π=352 MeV	Chiral extrapolation	Phenomenology CT10
M_1	1	1	1
M_2	0.206(14)	0.157(10)	0.169
M_3	0.078(16)	/	0.0536
M_4	/	/	0.0221

G. Bali et al. PoS LATTICE2015 (2016)

Moment u-d μ^2= 4 GeV^2	Linear extrapolation	Chiral extrapolation	Phenomenology CT10
M_1	1	1	1
M_2	/	0.200(-7/+9)	0.169
M_3	/	/	0.0536
M_4	/	/	/

ETMC (C. Alexandrou et al., Phys Rev D93, 2016)

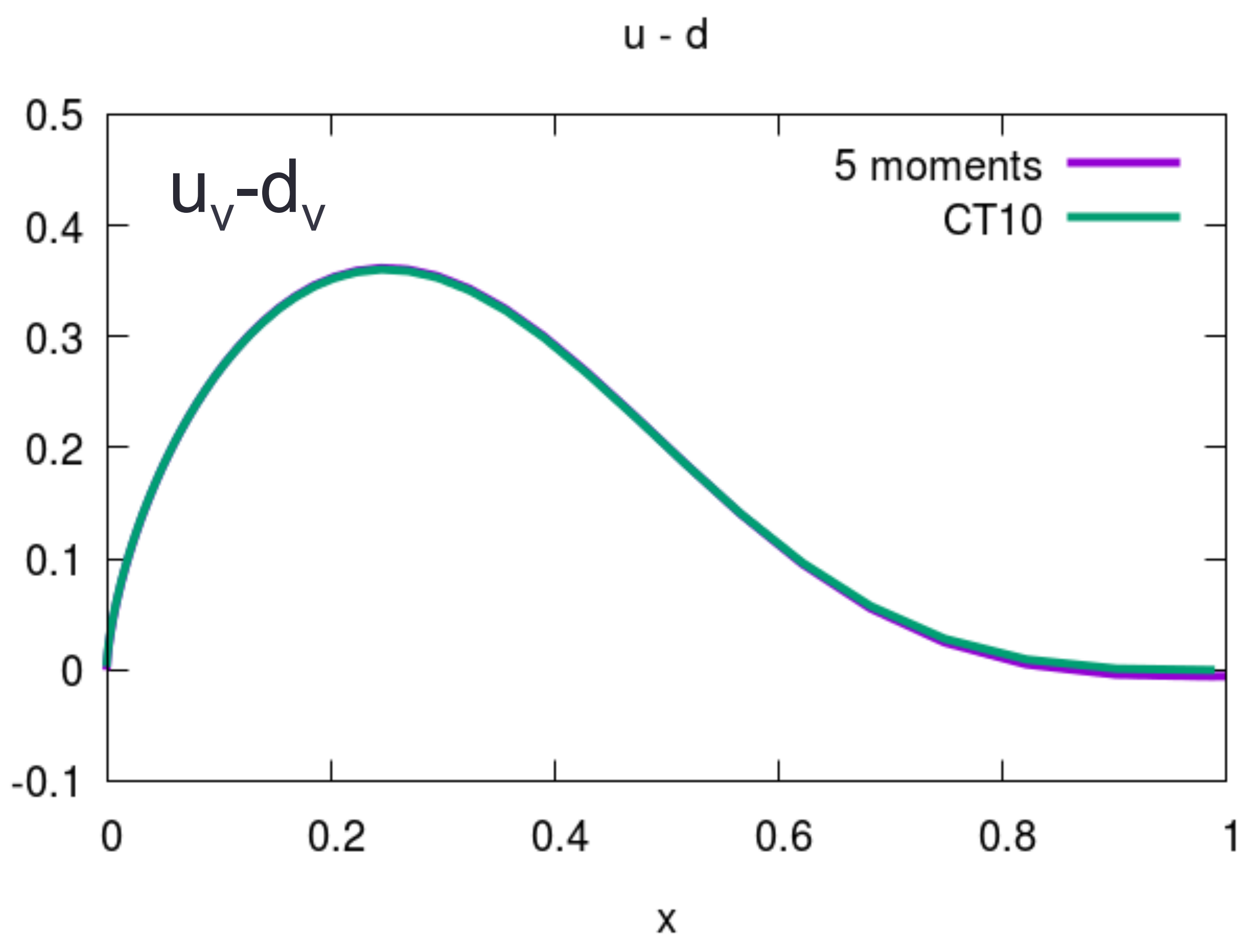
u-d

Moment u-d μ^2= 4 GeV^2	m_π=physical (largest source sink)	Hägler: Chiral extrapolation	Phenomenology CT10
M_1	1	1	1
M_2	0.208(24)	0.157(10)	0.169
M_3	/	/	0.0536
M_4	/	/	0.0221

LHPC (J. Green et al., Phys. Lett. B, 2014)

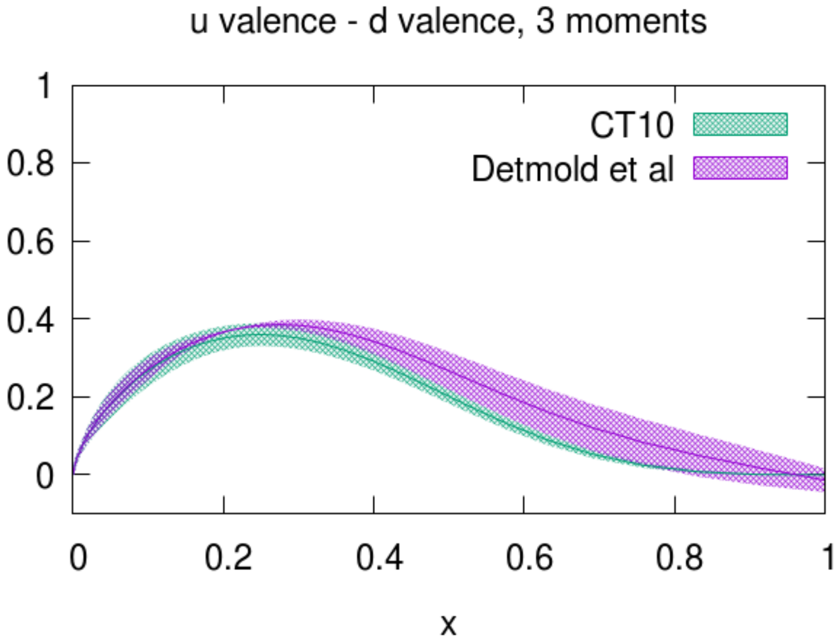
Moment u-d μ^2= 4 GeV^2	m_π=physical	Hägler: Chiral extrapolation	Phenomenology CT10
M_1	1	1	1
M_2	0.140(21)	0.157(10)	0.169
M_3	/	/	0.0536
M_4	/	/	0.0221

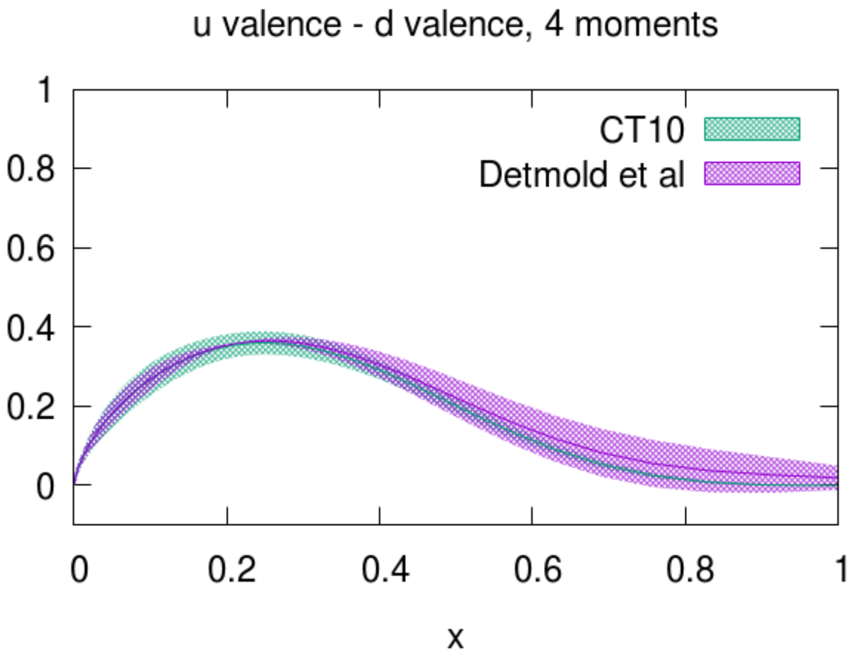
Reconstructed pdf: using its own moments up to x^5



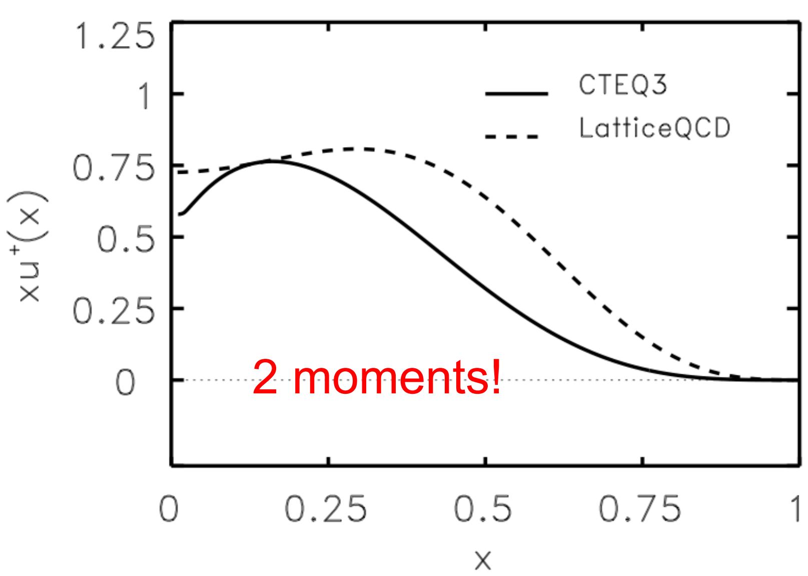
no error shown

Using Detmold et al.

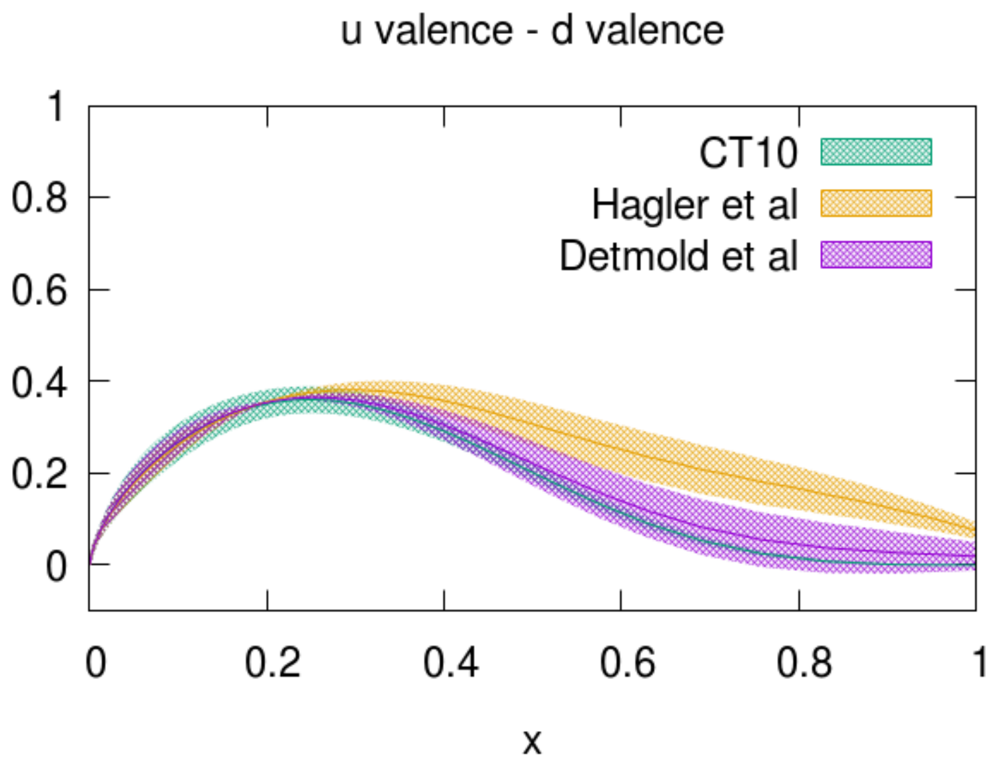




Weigl and Mankiewicz, PLB 1996



Comparison Detmold and Hägler

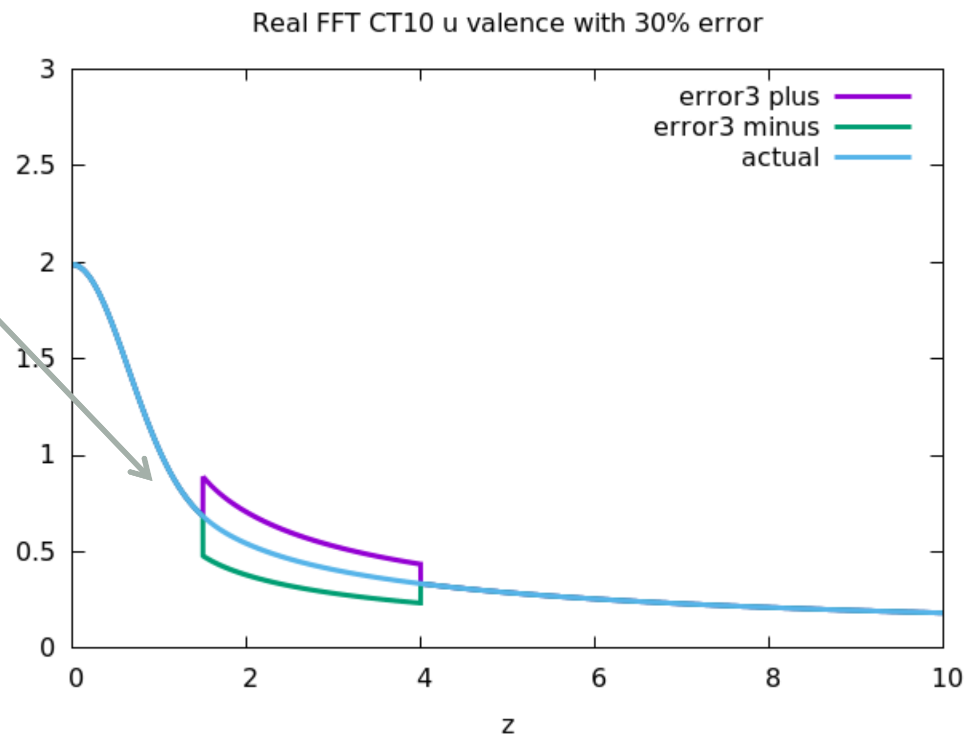


Error analysis in progress

ii) What PDF-related quantities would we like lattice QCD to compute?

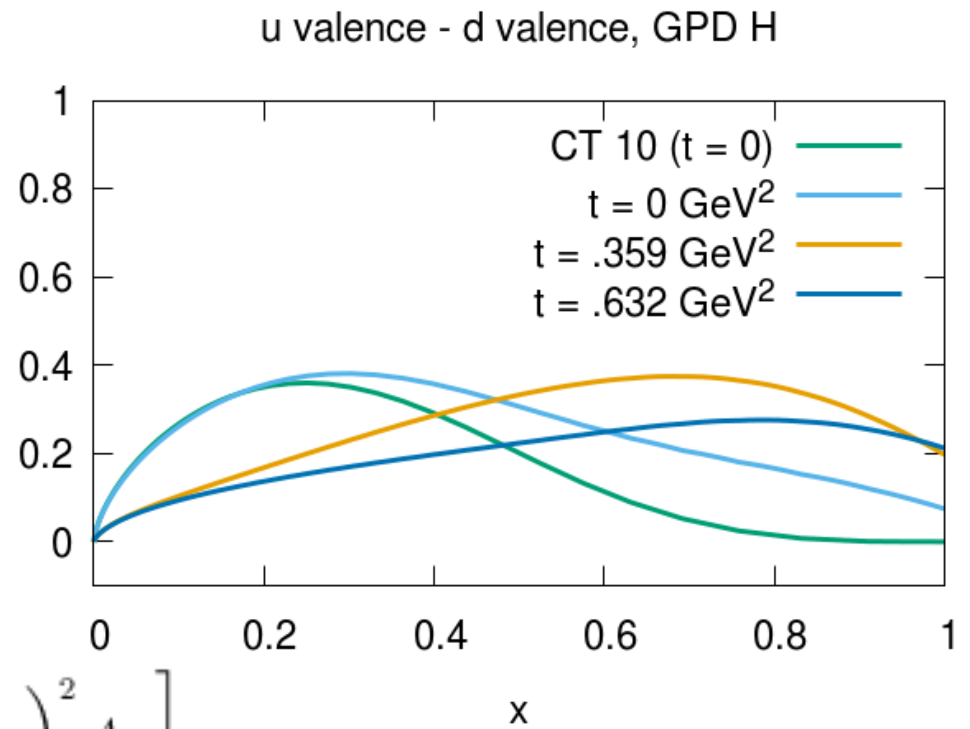
Sources of error:

- lattice moments
- Regge tail (CT10)
- unknown region



A careful analysis needs to be done and it's on its way!

First stab at GPDs…

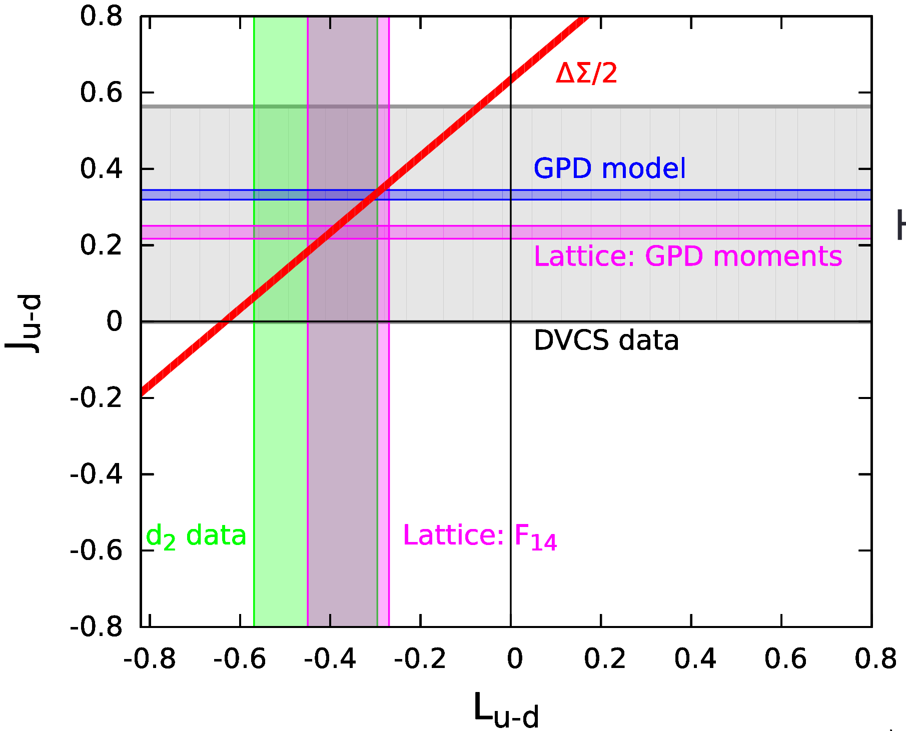


$$\overline{H}_{02}(X_{02}) = 3A_{10} - 6A_{20} + 3\left[A_{30} + \left(\frac{2\zeta}{2-\zeta}\right)^2 A_{32}\right],$$

$$\overline{H}_{12}(X_{12}) = 6A_{20} - 6\left[A_{30} + \left(\frac{2\zeta}{2-\zeta}\right)^2 A_{32}\right],$$

$$\overline{H}_{22}(X_{22}) = 3A_{30} + \left[\left(\frac{2\zeta}{2-\zeta}\right)^2 A_{32}\right].$$

Extra slides from the discussion….



Hägler et al.

Abha Rajan et al., PRD2016

Aurore Courtoy et al., PRL2015