Event-by-event picture for medium-induced jet evolution

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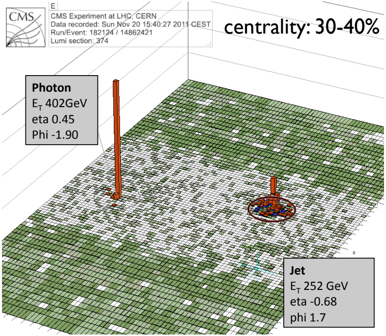

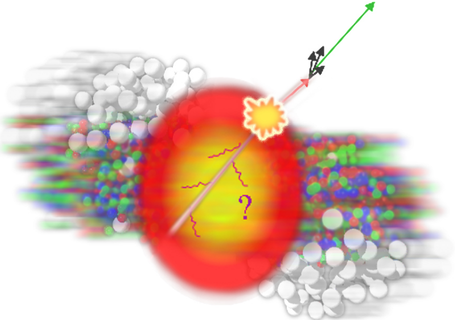

Outline

- Motivation: di-jet asymmetry at the LHC
- Medium-induced radiation: qualitative discussion (pQCD)

  characteristic scales, multiple branching, physical picture
- Quantitative discussion: a Markovian branching process

  gluon distribution, energy loss, multiplicities, & fluctuations

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  characteristic scales, multiple branching, physical picture
- Quantitative discussion: a Markovian branching process

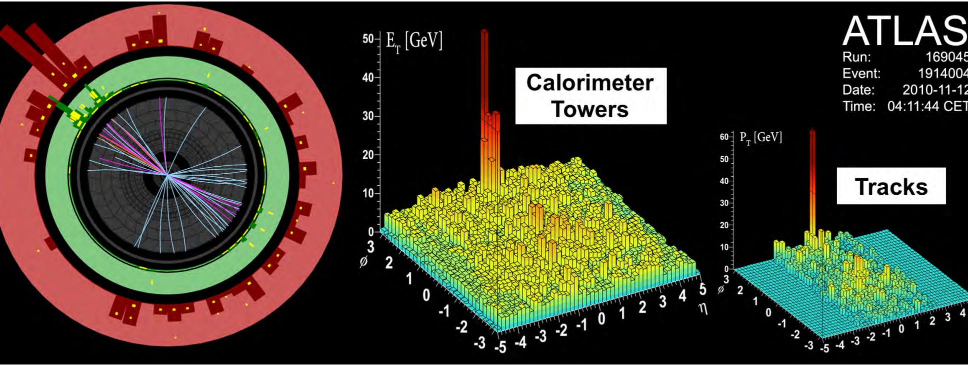
  gluon distribution, energy loss, multiplicities, & fluctuations
- The average picture is by now rather well understood

  Blaizot, Dominguez, E.I., Mehtar-Tani, B. Wu (2012-15)

  Apolinário, Armesto, Milhano, Salgado (2014); Kurkela, Wiedemann (2014)
- The importance of fluctuations started being understood only recently

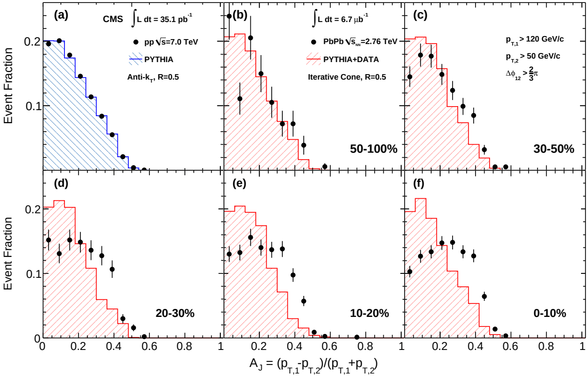
  Milhano and Zapp (2015); Escobedo and E.I. (2016)

“Mono-jets” in Pb+Pb collisions



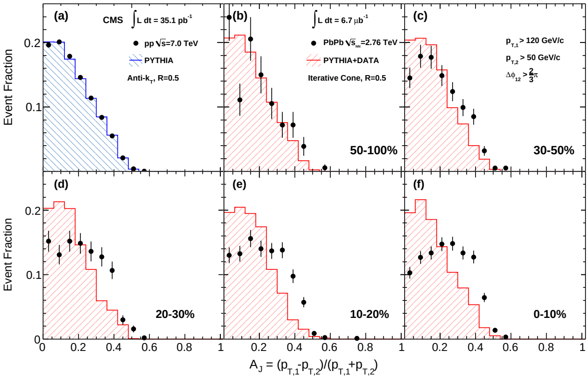
- Central Pb+Pb: ‘mono–jet’ events
- The secondary jet can barely be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

Di–jet asymmetry : A_J



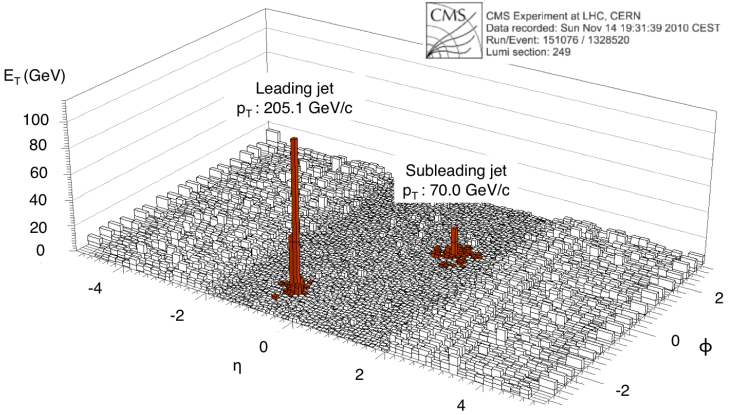
- Event fraction as a function of the di–jet energy imbalance in p+p (a) and Pb+Pb (b–f) collisions for different bins of centrality

$$A_J = \frac{E_1 - E_2}{E_1 + E_2} \qquad (E_i \equiv p_{T,i} = \text{ jet energies})$$



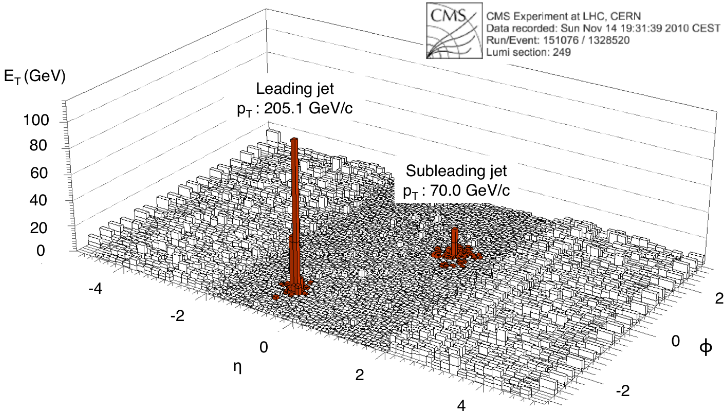
- N.B. A pronounced asymmetry already in p+p collisions !
    - 3-jets events, fluctuations in the branching process
- Central Pb+Pb : the asymmetric events occur more often

Di–jet asymmetry at the LHC



- The 'missing energy' is found in the underlying event:
  - many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles
- Soft hadrons can be easily deviated towards large angles
  - elastic scatterings with the medium constituents

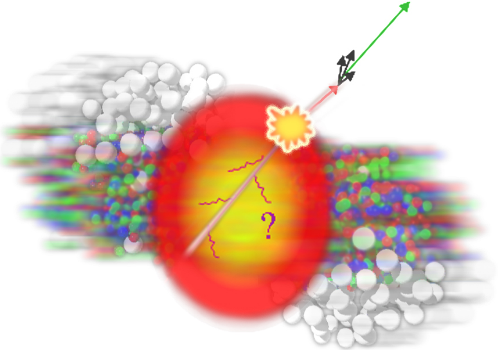
Di–jet asymmetry at the LHC



- The main question: how is that possible that a significant fraction of the jet energy be carried by its soft constituents ?
- Very different from the usual jet fragmentation pattern in the vacuum
  - bremsstrahlung favors collinear splittings ⇒ jets are collimated
  - many soft gluons ... but energy remains in the few partons at large x

The generally expected picture

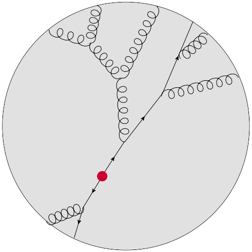
- “One jet crosses the medium along a distance longer than the other”



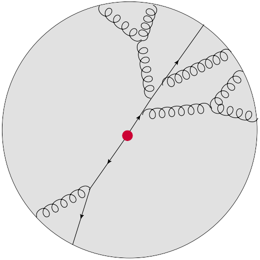
- Implicit assumption: fluctuations in energy loss are small
  - “the energy loss is always the same for a fixed medium size”
- Fluctuations are known to be important for a branching process

The role of fluctuations

- Different path lengths



- Fluctuations in the branching pattern



- Fluctuations in the energy loss are as large as the average value

  (M. Escobedo and E.I., arXiv:1601.03629 & 1609.06104)

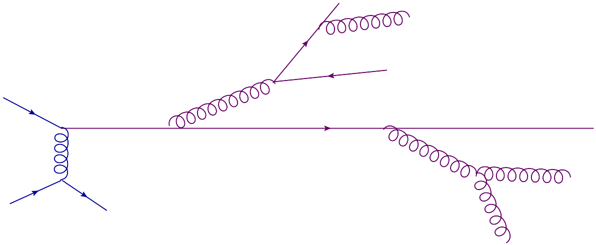
- Similar conclusion independently reached by a Monte-Carlo study

  (Milhano and Zapp, arXiv:1512.08107, "JEWEL")

- What is the event-by-event picture of the in-medium jet evolution ?

Medium-induced jet evolution

- The leading particle (LP) is produced by a hard scattering
- It subsequently evolves via radiation (branchings) ...



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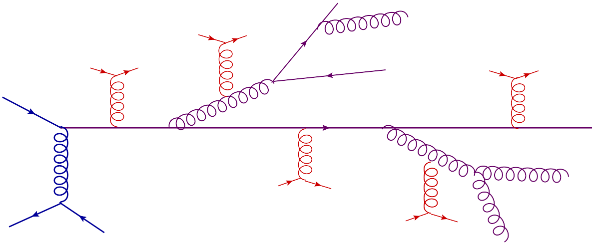

- ... and via collisions off the medium constituents

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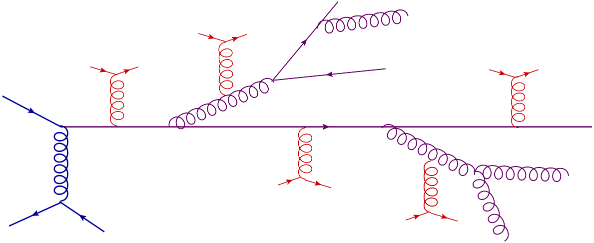

- ... and via collisions off the medium constituents
- Collisions can have several effects
  - broaden the p_T–distribution of the jet constituents
  - trigger additional radiation ('medium-induced branching')
  - thermalize the (soft) products of this radiation

Medium-induced jet evolution

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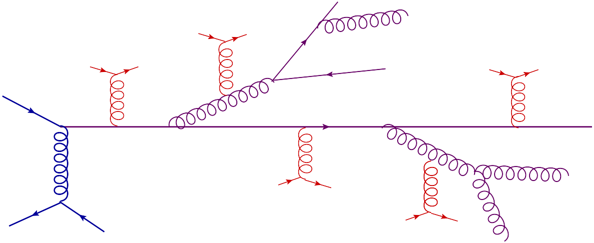


- ... and via collisions off the medium constituents
- BDMPS–Z mechanism for medium-induced radiation in pQCD

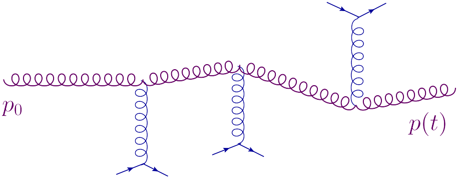
  Baier, Dokshitzer, Mueller, Peigné, Schiff; Zakharov (1996-97)

  Wiedemann (2000), "Bottom-up" (2001), Arnold, Moore, Yaffe (2002-03) ...

  - gluon emission is linked to transverse momentum broadening

Formation time

- Independent multiple scattering $\Longrightarrow$ a random walk in $p_\perp$

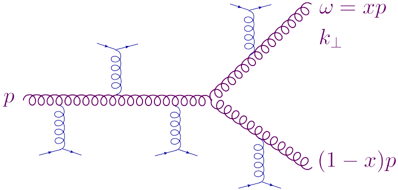


$$\langle p_\perp^2 \rangle \simeq \hat{q}\,\Delta t$$

$$\hat{q} \simeq \frac{m_D^2}{\lambda} \sim \alpha_s^2 T^3 \ln \frac{1}{\alpha_s}$$

$$\hat{q} \simeq 1 \div 2\,\mathrm{GeV}^2/\mathrm{fm}$$

- Collisions destroy quantum coherence and thus trigger emissions

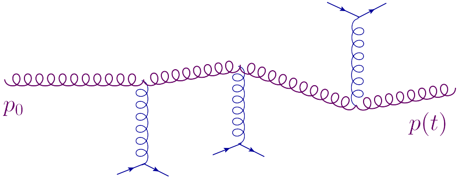


formation time

$$t_\mathrm{f} \simeq \frac{1}{\Delta E} \simeq \frac{\omega}{k_\perp^2}$$

- During formation, the gluon acquires a momentum $k_\perp^2 \sim \hat{q} t_\mathrm{f}$

- Independent multiple scattering $\Longrightarrow$ a random walk in $p_\perp$



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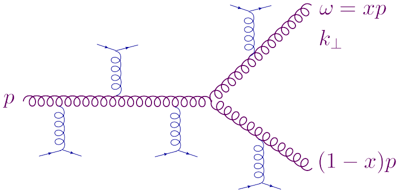


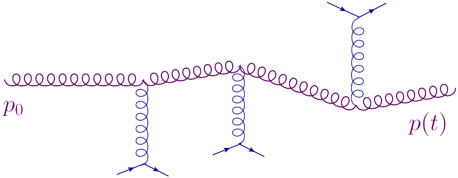
formation time

$$t_\mathrm{f} \simeq \frac{\omega}{\hat{q} t_\mathrm{f}} \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

- Assumptions: $\lambda < t_\mathrm{f}(\omega) < L \Longrightarrow T \lesssim \omega \leq \omega_c \equiv \hat{q}L^2$

Formation time

- Independent multiple scattering $\Longrightarrow$ a random walk in $p_\perp$



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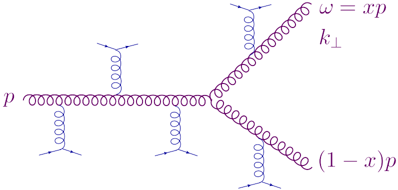


formation time

$$t_{\mathrm{f}} \simeq \frac{\omega}{\hat{q} t_{\mathrm{f}}} \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

- Soft gluons ($\omega \ll \omega_c$) have short formation times: $t_{\mathrm{f}}(\omega) \ll L$

Multiple branchings

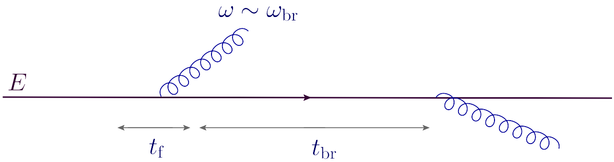
- Probability for emitting a gluon with energy $\geq \omega$ during a time L

$$\mathcal{P}(\omega, L) \simeq \alpha_s \frac{L}{t_{\rm f}(\omega)} \simeq \alpha_s L \sqrt{\frac{\hat{q}}{\omega}}$$

- When $\mathcal{P}(\omega, L) \sim 1$, multiple branching becomes important

$$\omega \lesssim \omega_{\rm br}(L) \equiv \alpha_s^2 \hat{q} L^2 \quad \Longleftrightarrow \quad L \gtrsim t_{\rm br}(\omega) \equiv \frac{1}{\alpha_s}\sqrt{\frac{\omega}{\hat{q}}}$$

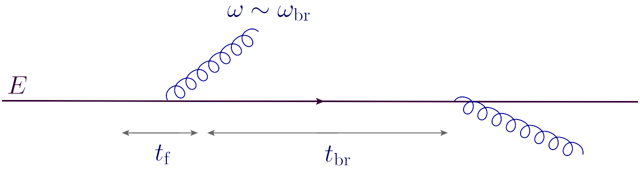
  - $\omega_{\rm br} = \alpha_s^2 \omega_c$: characteristic energy for the onset of multiple branching



  - $t_{\rm br} = \frac{1}{\alpha_s} t_{\rm f}$: typical distance between 2 successive branchings

Energy loss by the leading particle

- LHC: the leading particle has $E \sim 100\,\mathrm{GeV} \gg \omega_{\mathrm{br}} \sim 5\,\mathrm{GeV}$



- In a typical event, the LP emits ...
  - a number of $\mathcal{O}(1)$ of gluons with $\omega \sim \omega_{\mathrm{br}}$

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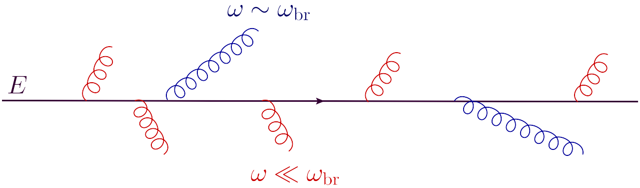

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- The energy loss is controlled by the hardest primary emissions

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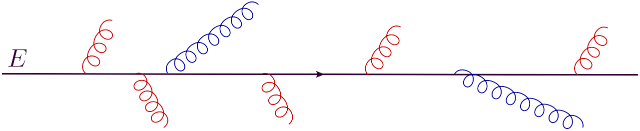

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- Not exactly the experimental picture for di-jet asymmetry ....
  - a small number ($\mathcal{O}(1)$) of relatively hard ($\omega \sim \omega_{\rm br}$) gluons which propagate at rather small angles

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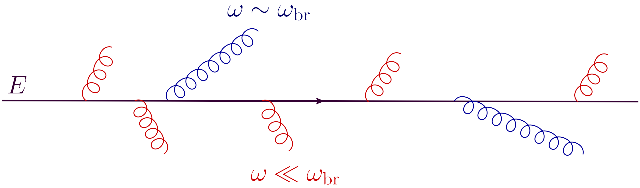

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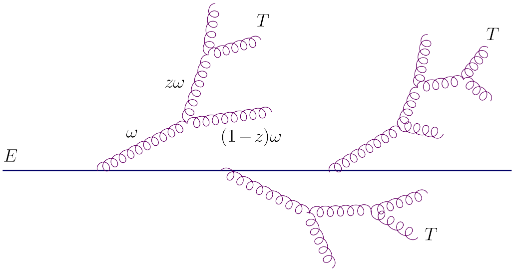
- Not exactly the experimental picture for di-jet asymmetry ....
  - a small number $(\mathcal{O}(1))$ of relatively hard $(\omega \sim \omega_{\rm br})$ gluons which propagate at rather small angles
- ... but this is not our final picture !

Democratic branchings

Blaizot, E. I., Mehtar-Tani, 2013; Kurkela, Wiedemann, 2014

- The primary gluons generate 'mini-jets' via democratic branchings
  - daughter gluons carry comparable energy fractions: $z \sim 1 - z \sim 1/2$

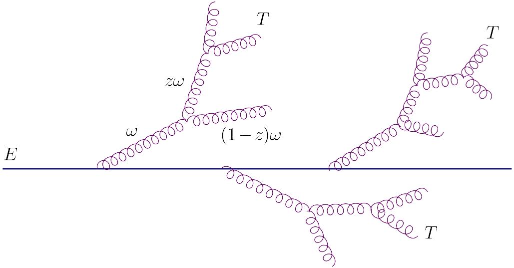


$$\mathcal{P}(z\omega, L) \simeq \frac{L}{t_{\rm br}(z\omega)} \simeq \alpha_s\, L\sqrt{\frac{\hat{q}}{z\omega}}$$

  - when $\omega \sim \omega_{\rm br}$, $\mathcal{P}(z\omega, L) \sim 1$ independently of the value of z
- Very different from usual bremsstrahlung which "likes" $z \ll 1$

Energy loss by the jet

- A mini-jet with $\omega \lesssim \omega_{\rm br}$ decays over a time $t_{\rm br}(\omega) \lesssim L$
- Via democratic branchings, the energy is successively transmitted to softer and softer gluons, down to $\omega \sim T$
- The soft gluons thermalize via elastic collisions *(E.I. and Bin Wu, 2015)*
- The energy appears in many soft quanta propagating at large angles



- What is the average energy loss and its fluctuations ?

Probabilistic picture

- Medium-induced jet evolution $\approx$ a Markovien stochastic process
  - successive branchings are non-overlapping: $t_{\rm br} \sim \frac{1}{\alpha_s}\, t_{\rm f}$
  - interference phenomena could complicate the picture ...
    (in the vacuum, they lead to angular ordering)
  - ... but they are suppressed by rescattering in the medium
    Casalderrey-Solana, E.I. (2011);
    Blaizot, Dominguez, E.I., Mehtar-Tani (2012)
    Apolinário, Armesto, Milhano, Salgado (2014)
- Hierarchy of equations for n-point correlation functions ($x \equiv \omega/E$)

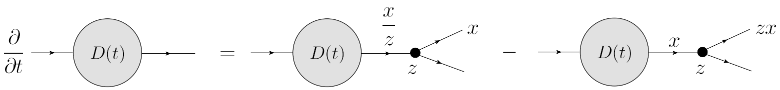
$$D(x,t) \equiv x \left\langle \frac{{\rm d}N}{{\rm d}x}(t) \right\rangle , \qquad D^{(2)}(x,x',t) \equiv xx' \left\langle \frac{{\rm d}N_{\rm pair}}{{\rm d}x\,{\rm d}x'}(t) \right\rangle$$

- Analytic solutions *(Blaizot, E. I., Mehtar-Tani, '13; Escobedo, E.I., '16)*
- New phenomena: wave turbulence, KNO scaling, large fluctuations

Gluon spectrum: the average energy loss

J.-P. Blaizot, E. I., Y. Mehtar-Tani, 2013

- Kinetic equation for $D(x,t) = x(\mathrm{d}N/\mathrm{d}x)$: 'gain' - 'loss'



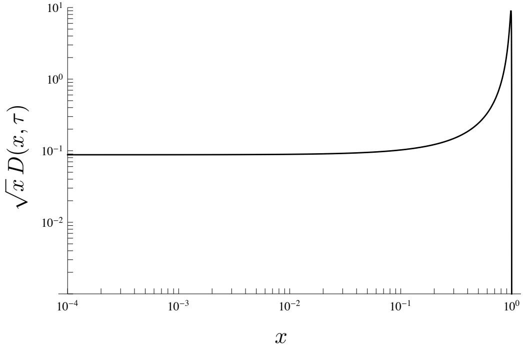
- Exact solution with initial condition $D(x, t = 0) = \delta(x - 1)$

$$D(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\, \mathrm{e}^{-\pi \frac{\tau^2}{1-x}}\,, \qquad x \equiv \frac{\omega}{E}\,, \quad \tau \equiv \frac{t}{t_{\rm br}(E)}$$

- $t_{\rm br}(E)$: the lifetime of the LP until its first democratic branching
  - power-law spectrum $D \propto \frac{1}{\sqrt{x}}$ at $x \ll 1$ for any τ
  - Kolmogorov fixed point: wave turbulence

Gluon spectrum

$$D(x,\tau) \;=\; \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\;\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}\,, \qquad x\equiv\frac{\omega}{E}\,,\quad \tau\equiv\frac{t}{t_{\rm br}(E)}$$



$\tau = 0.1$

- Pronounced LP peak at small times $t \ll t_{\rm br}(E)$

Gluon spectrum

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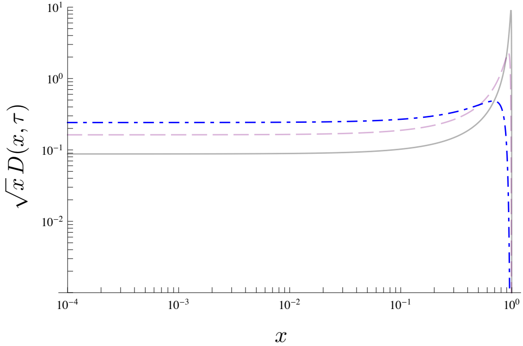
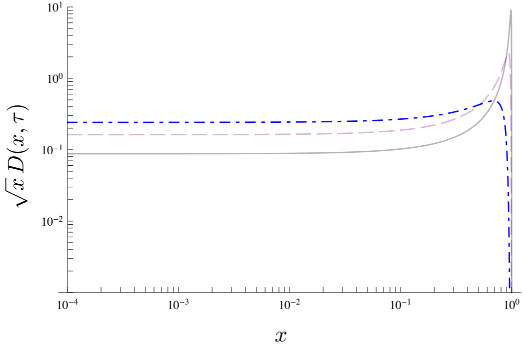


$\tau = 0.1,$ 0.2, 0.4

- Increasing t: the LP peaks decreases, broadens, and moves to the left

Gluon spectrum

$$\frac{\tau^2}{1-x} = \frac{\omega_{\rm br}(t)}{E(1-x)} = \frac{\omega_{\rm br}(t)}{\Delta E}, \qquad \omega_{\rm br}(t) = \alpha_s^2 \hat{q} t^2$$

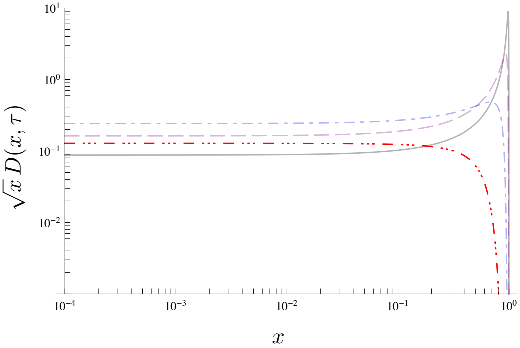


$\tau = 0.1,\ 0.2,\ 0.4,\ 0.8$

- After a time t, the LP loses an energy $\Delta E \sim \omega_{\rm br}(t)$
  - energy loss is controlled by the primary emissions with $\omega \sim \omega_{\rm br}$

Gluon spectrum

$$\frac{\tau^2}{1-x} = \frac{\omega_{\rm br}(t)}{E(1-x)} = \frac{\omega_{\rm br}(t)}{\Delta E}\,, \qquad \omega_{\rm br}(t) = \alpha_s^2 \hat{q} t^2$$

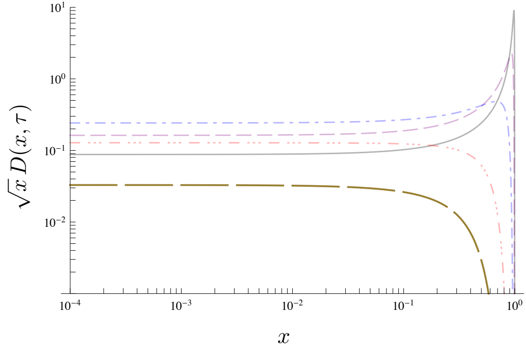


$\tau = 0.1,\ 0.2,\ 0.4,\ 0.8$

- $\tau \sim 1$, i.e. $t \sim t_{\rm br}(E)$: the LP disappears via a democratic branching

Turbulent energy flow

- The energy contained in the spectrum: $\int_0^1 \mathrm{d}x\, D(x,\tau) = \mathrm{e}^{-\pi\tau^2}$



$\tau = 0.1$, 0.2, 0.4, 0.8, 1

- Energy flux is independent of x : wave turbulence
- Where does the energy go ?

Turbulent energy flow

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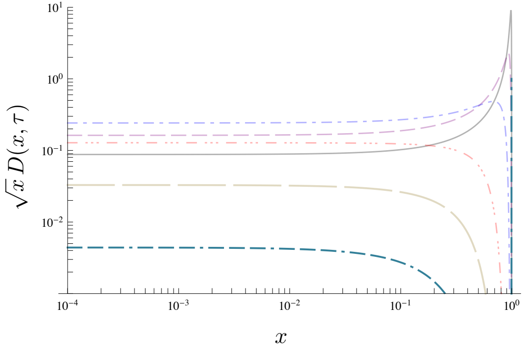


$\tau = 0.1,\ 0.2,\ 0.4,\ 0.8,\ 1,\ 1.2$

- Formally, it accumulates into a condensate at $x = 0$
- Physically, it is transmitted to the medium, via thermalization

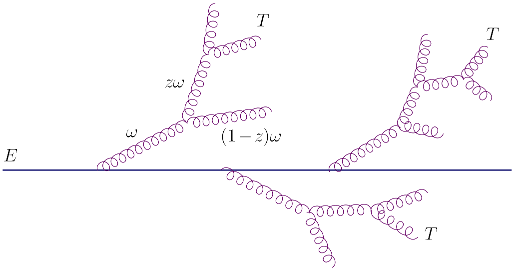
The average energy loss

$$\langle \Delta E \rangle = E\left(1 - \mathrm{e}^{-\pi\tau^2}\right) = E\left[1 - \mathrm{e}^{-\pi\frac{\omega_{\rm br}}{E}}\right]$$

- LHC: $E \sim 100\,\mathrm{GeV} \gg \omega_{\rm br} \sim 5 \div 10\,\mathrm{GeV}$

$$\langle \Delta E \rangle \simeq \pi\omega_{\rm br} = \pi\alpha_s^2\hat{q}L^2$$

- The energy lost by the LP is also lost by the jet as a whole

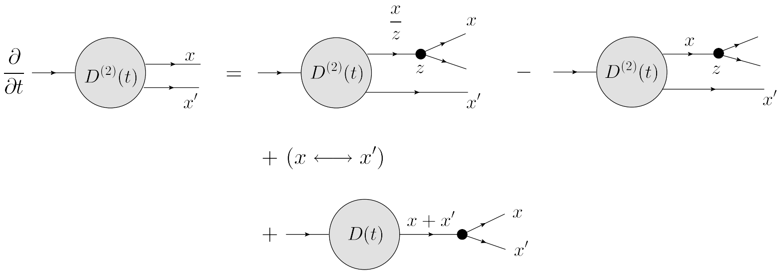


- The primary gluons with $\omega \sim \omega_{\rm br}$ disappear via democratic cascades

Fluctuations in the energy loss

M.A. Escobedo and E. I., arXiv:1601.03629, arXiv:1609.06104

- $\sigma^2 \equiv \langle \Delta E^2 \rangle - \langle \Delta E \rangle^2$ is related to the gluon pair density $D^{(2)}(x, x', t)$
- Kinetic equation for $D^{(2)}(x, x', t)$:



- The 1-body density $D(x + x', t)$ acts as a source for the 2-body density

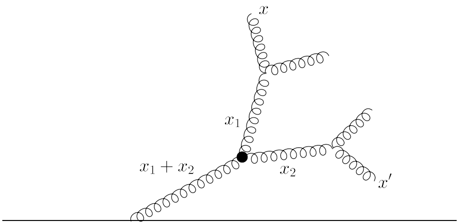
Fluctuations in the energy loss

M.A. Escobedo and E. I., arXiv:1601.03629, arXiv:1609.06104

- $\sigma^2 \equiv \langle \Delta E^2 \rangle - \langle \Delta E \rangle^2$ is related to the gluon pair density $D^{(2)}(x, x', t)$
- Exact solution: correlations due to common ancestors

$$D^{(2)}(x, x', \tau) = \frac{1}{2\pi} \frac{1}{\sqrt{x x' (1 - x - x')}} \left[\mathrm{e}^{-\frac{\pi \tau^2}{1-x-x'}} - \mathrm{e}^{-\frac{4\pi \tau^2}{1-x-x'}} \right]$$

- The 2 measured gluons x and x' have a last common ancestor $x_1 + x_2$

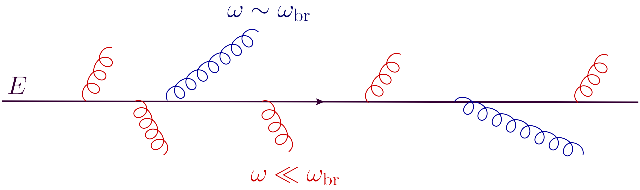


Dispersion in the energy loss

- Small time/high energy $E \gg \omega_{\rm br}$ (LHC) :

$$\sigma^2 \equiv \langle \Delta E^2 \rangle - \langle \Delta E \rangle^2 \simeq \frac{\pi^2}{3}\,\omega_{\rm br}^2 = \frac{1}{3}\langle \Delta E \rangle^2$$

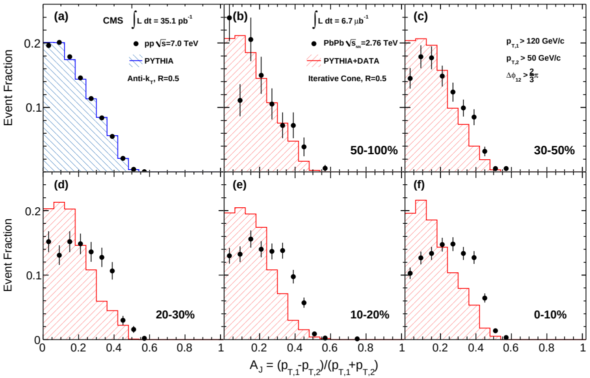
- Fluctuations in the energy loss are comparable with the average value
- Recall: the probability for a primary emission with $\omega \sim \omega_{\rm br}$ is of $\mathcal{O}(1)$



  - the average number of such emissions is of $\mathcal{O}(1)$ (indeed, it is π)
  - successive such emissions are quasi-independent $(E \gg \omega_{\rm br})$

- Fluctuations in the number of such emissions must be of $\mathcal{O}(1)$ as well

Di–jet asymmetry from fluctuations

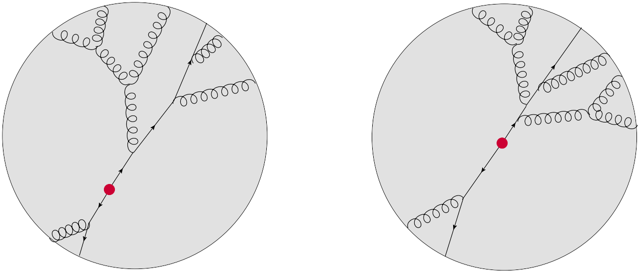


- Event fraction as a function of the di–jet energy imbalance

$$A_{\rm J}=\frac{|E_1-E_2|}{E_1+E_2}$$

- Fluctuations cannot cancel since $A_{\rm J}$ is positive-definite, by construction

Di-jet asymmetry from fluctuations



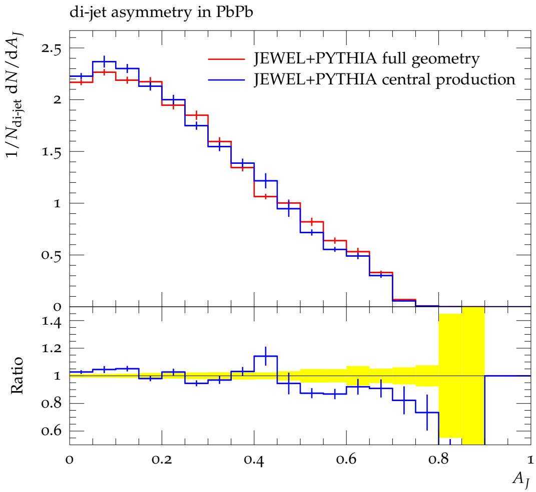
- A relatively large value A_{J} can either correspond to a peripheral di-jet, or (more often) to a large fluctuation in the branching pattern

$$\langle (E_1 - E_2)^2 \rangle - \langle E_1 - E_2 \rangle^2 = \sigma_1^2 + \sigma_2^2 \propto \langle L_1^4 + L_2^4 \rangle$$

- Fluctuations dominate whenever $L_1 \sim L_2$ (the typical situation)
- Difficult to check: no direct experimental control of L_1 and L_2

Monte-Carlo studies (JEWEL)

(Milhano and Zapp, arXiv:1512.08107)



- Central production $(L_1 = L_2)$ vs. randomly distributed production points ("full geometry") : no significant difference !

Particle multiplicities

- The average multiplicities and their fluctuations are dominated by very soft gluons :

$$\frac{\mathrm{d}N}{\mathrm{d}\omega} = \frac{1}{\omega} D(\omega) \propto \frac{1}{\omega^{3/2}}$$

- Number of gluons with $\omega \geq \omega_0$, where $\omega_0 \ll E$:

$$\langle N(\omega_0)\rangle = \int_{\omega_0}^{E} \mathrm{d}\omega \, \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq 1 + 2\left[\frac{\omega_{\rm br}}{\omega_0}\right]^{1/2} \quad \text{(LP + radiation)}$$

  - $\langle N(\omega_0)\rangle \simeq 1$ when $\omega_0 \gg \omega_{\rm br}$: just the LP
  - $\langle N(\omega_0)\rangle \gg 1$ when $\omega_0 \ll \omega_{\rm br}$: multiple branching

- All the higher moments $\langle N^p\rangle$ have been similarly computed

  M.A. Escobedo and E. I., arXiv:1609.06104

Koba-Nielsen-Olesen scaling

- All the higher moments $\langle N^p \rangle$ have been similarly computed

$$\frac{\langle N^2 \rangle}{\langle N \rangle^2} \simeq \frac{3}{2}, \qquad \frac{\langle N^p \rangle}{\langle N \rangle^p} \simeq \frac{(p+1)!}{2^p}$$

- KNO scaling : the reduced moments are pure numbers
- A special negative binomial distribution (parameter $r = 2$)
  - huge fluctuations (say, as compared to a Poissonian distribution)

$$\frac{\sigma_N}{\langle N \rangle} = \frac{1}{\sqrt{2}} \qquad \text{vs.} \qquad \frac{\sigma_N}{\langle N \rangle} = \frac{1}{\sqrt{\langle N \rangle}}$$

  - fluctuations are stronger than for jets in the vacuum $(r = 3)$

$$\frac{\sigma_N}{\langle N \rangle} = \frac{1}{\sqrt{2}} \qquad \text{vs.} \qquad \frac{\sigma_N}{\langle N \rangle} = \frac{1}{\sqrt{3}}$$

- Difficult to check against the data: huge backgrounds at soft energies

Conclusions

- Effective theory and physical picture for jet quenching from pQCD
  - democratic branchings leading to wave turbulence
  - thermalization of the soft branching products with $p \sim T$
  - efficient transmission of energy to large angles
  - wide probability distribution, strong fluctuations, KNO scaling
- Di-jet asymmetry : geometry (path length difference) competes with fluctuations
- Qualitative and semi-quantitative agreement with the phenomenology of di-jet asymmetry at the LHC
- Important dynamical information still missing: vacuum-like radiation (parton virtualities), medium expansion ...