Astrophysical Rate and Population Constraints

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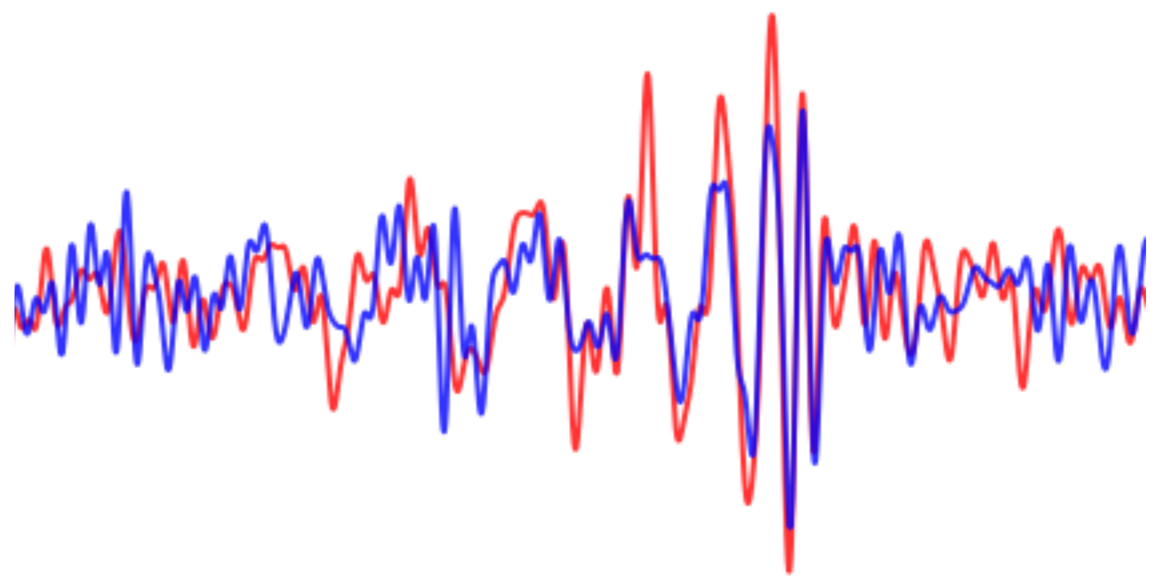
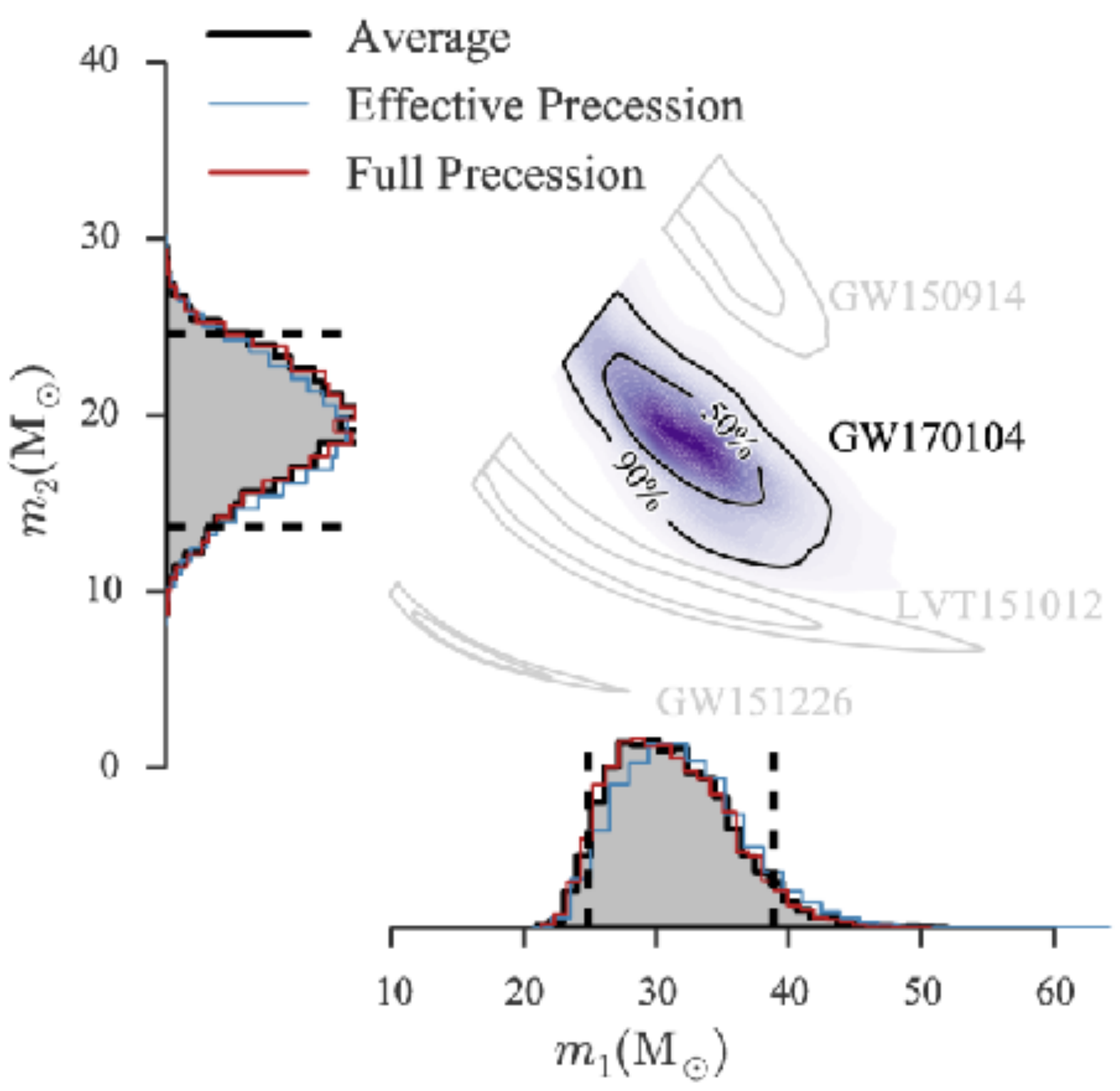
Astrophysical inference

Gravitational wave data



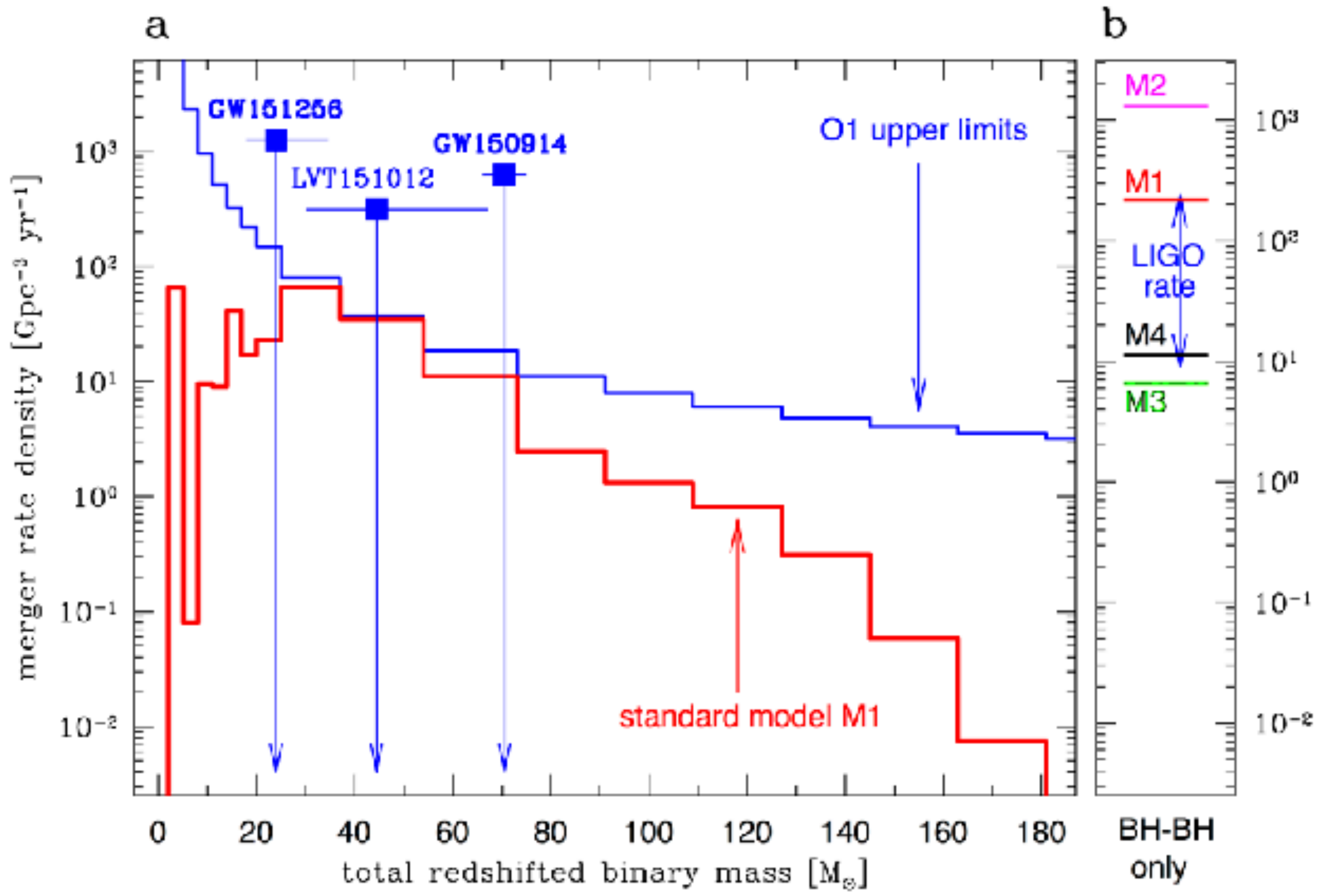
Detector operation

Detection confidence,
Parameter Inference



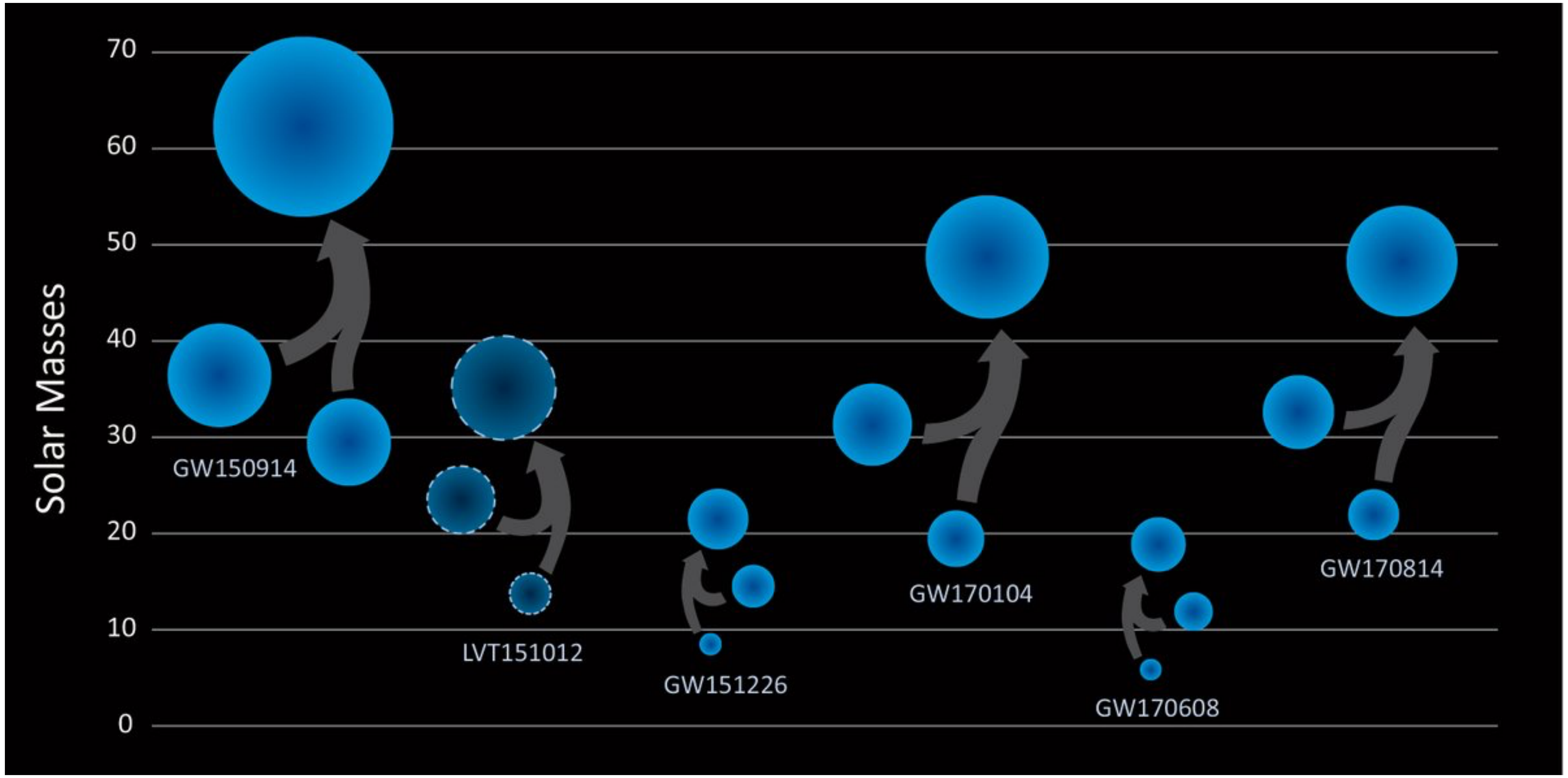
Waveform models,
GW data analysis

Astrophysical rate and
population inference



Astrophysical models,
population synthesis

Astrophysical Inference



Would like to know merger rate to constrain population synthesis models. Even better, would like to know merger rate as a function of mass, spin, redshift etc

Have to account for uncertainties in detections, both probability an event is real, and uncertainties in the inferred parameters. Also have to account for selection bias

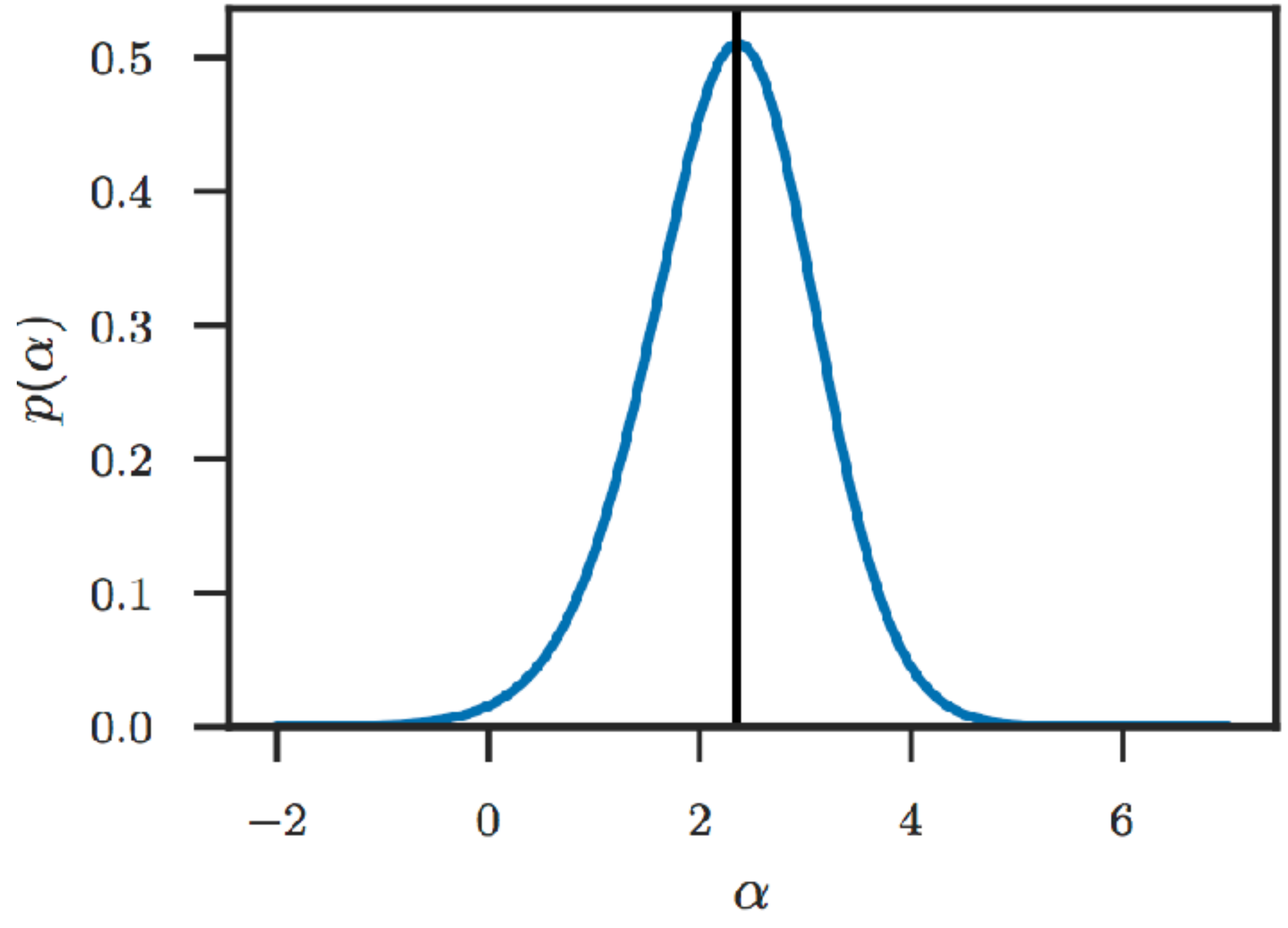
Many cool techniques being developed to do this using methods such as mixture models, Gaussian processes etc

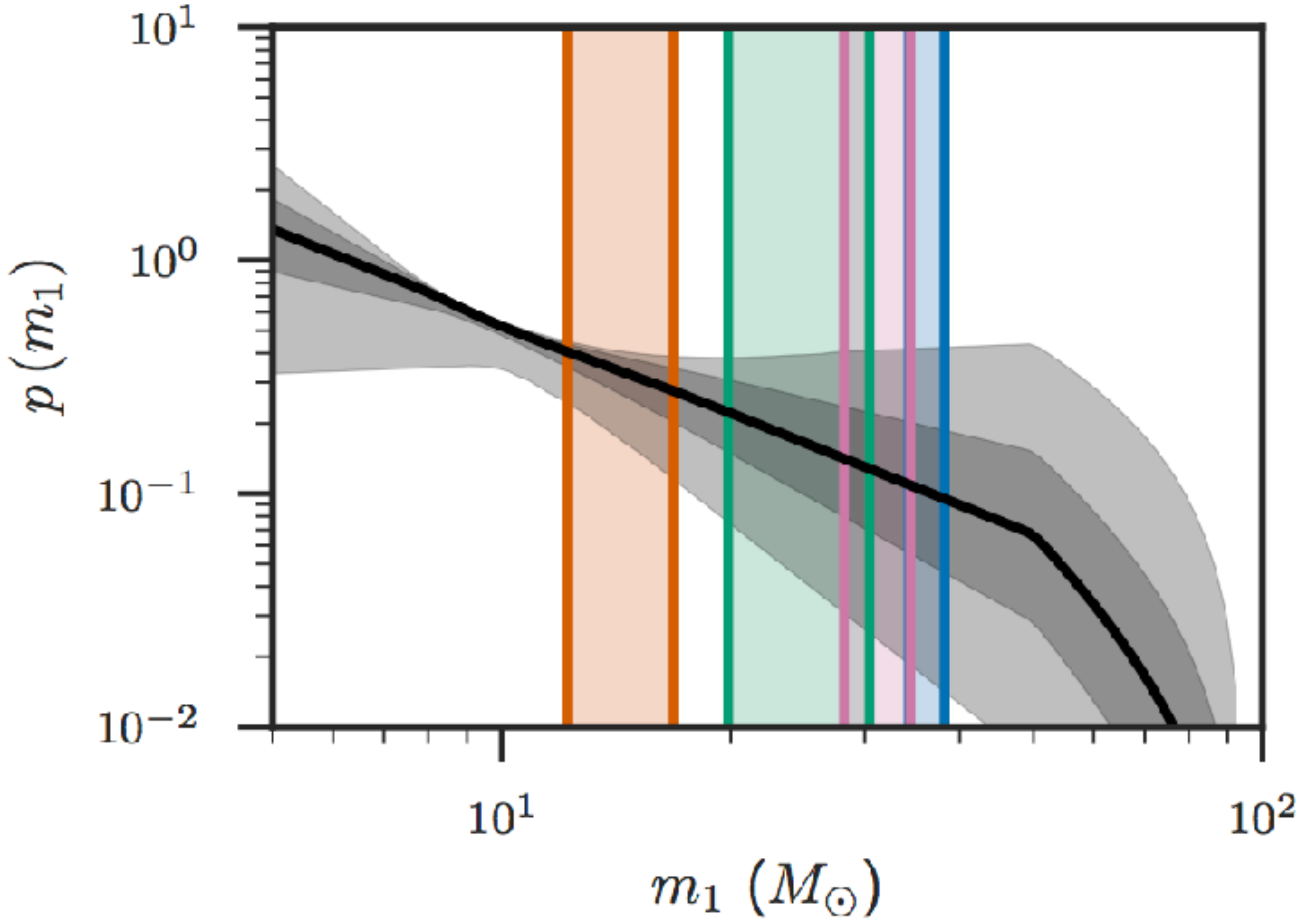
Population inference

Can only constrain very simple models with just the first 4 detections

e.g. power law in the mass of the heaviest component, flat in mass ratio $\quad p(m_1, m_2) \propto \frac{m_1^{-\alpha}}{m_1 - m_{\rm min}}$

$$m_{\rm min} = 5 M_\odot \qquad\qquad m_1 + m_2 < 100 M_\odot$$





Astrophysical Merger Rate: Simple Version

Even without a detection we can produce interesting astrophysical results such as bounds on the binary merger rate for NS-NS

Expect binary mergers to be a Poisson process. If the expected number of events is λ, then the probability of detecting k events is

$$p(k|\lambda) = \lambda^k e^{-\lambda}/k!$$

If the event rate is $R \;\; [\mathrm{Mpc}^{-3} \; \mathrm{year}^{-1}]$, and the observable 4-volume is $VT \;\; [\mathrm{Mpc}^3 \; \mathrm{year}]$

$$\lambda = RVT$$

The probability of observing zero events (k=0) is then

$$p(R) = VT \, e^{-RVT}$$

Follows from $p(0|\lambda) = p(R)\, dR = e^{-RVT}$

Astrophysical Rate Limits

$$p(R) = VT\, e^{-RVT}$$

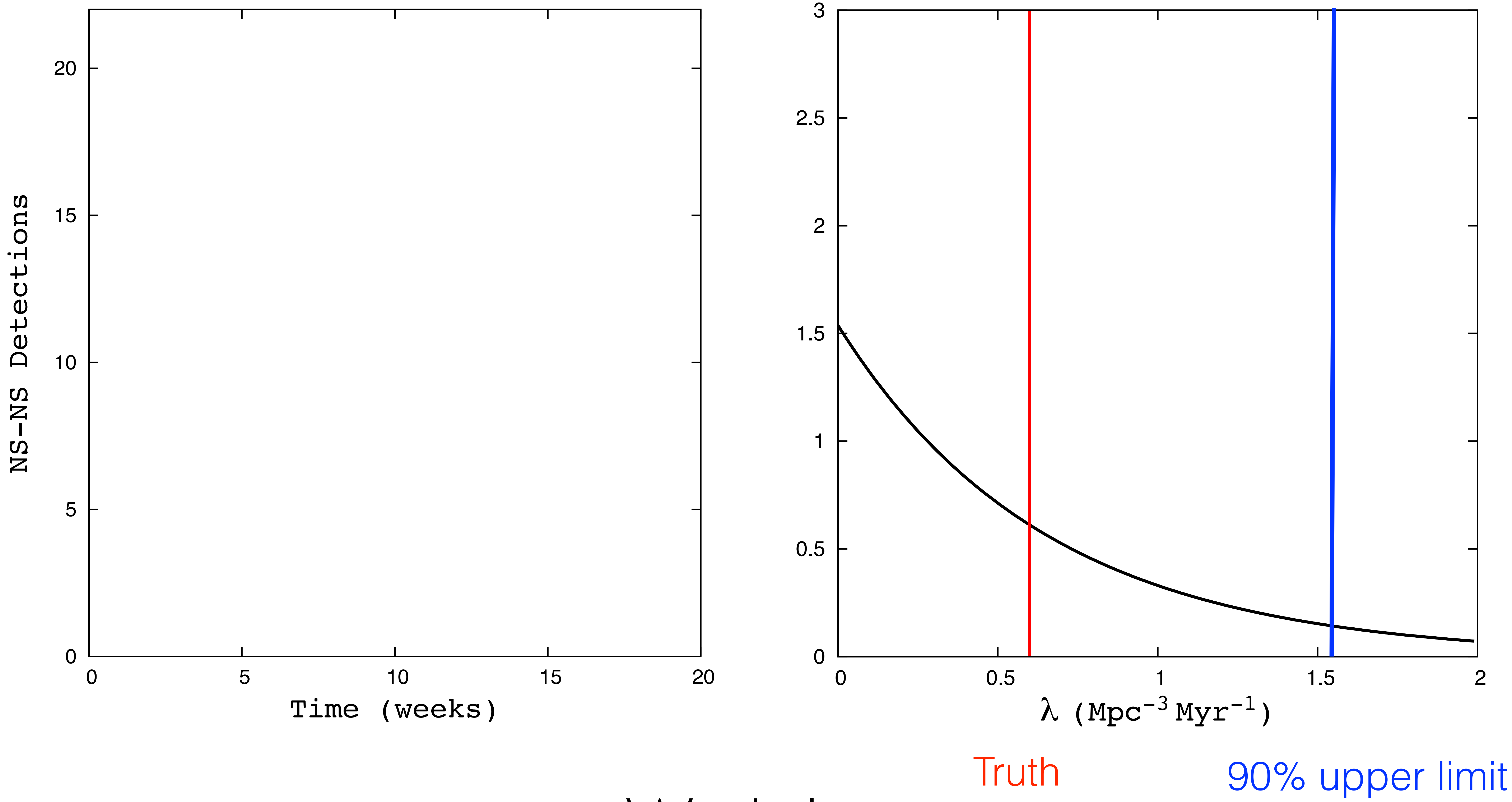
The probability distribution is peaked at a rate of zero. A 90% rate upper limit can be computed:

$$\int_0^{R_*} p(R) dR = 1 - e^{-R_* VT} = 0.9$$

$$\Rightarrow \quad R_* VT = \ln(0.1)$$

$$\Rightarrow \quad R_* = \frac{2.3}{VT}$$

e.g. NS-NS Merger Rate, aLIGO at design sensitivity $V = \frac{4\pi}{3}(200\,\mathrm{Mpc})^3$



Week 1

Astrophysical Rate Limits

When a signal is detected the probability distribution for the rate is no longer peaked at zero. For example, with a single detection (k=1) we have

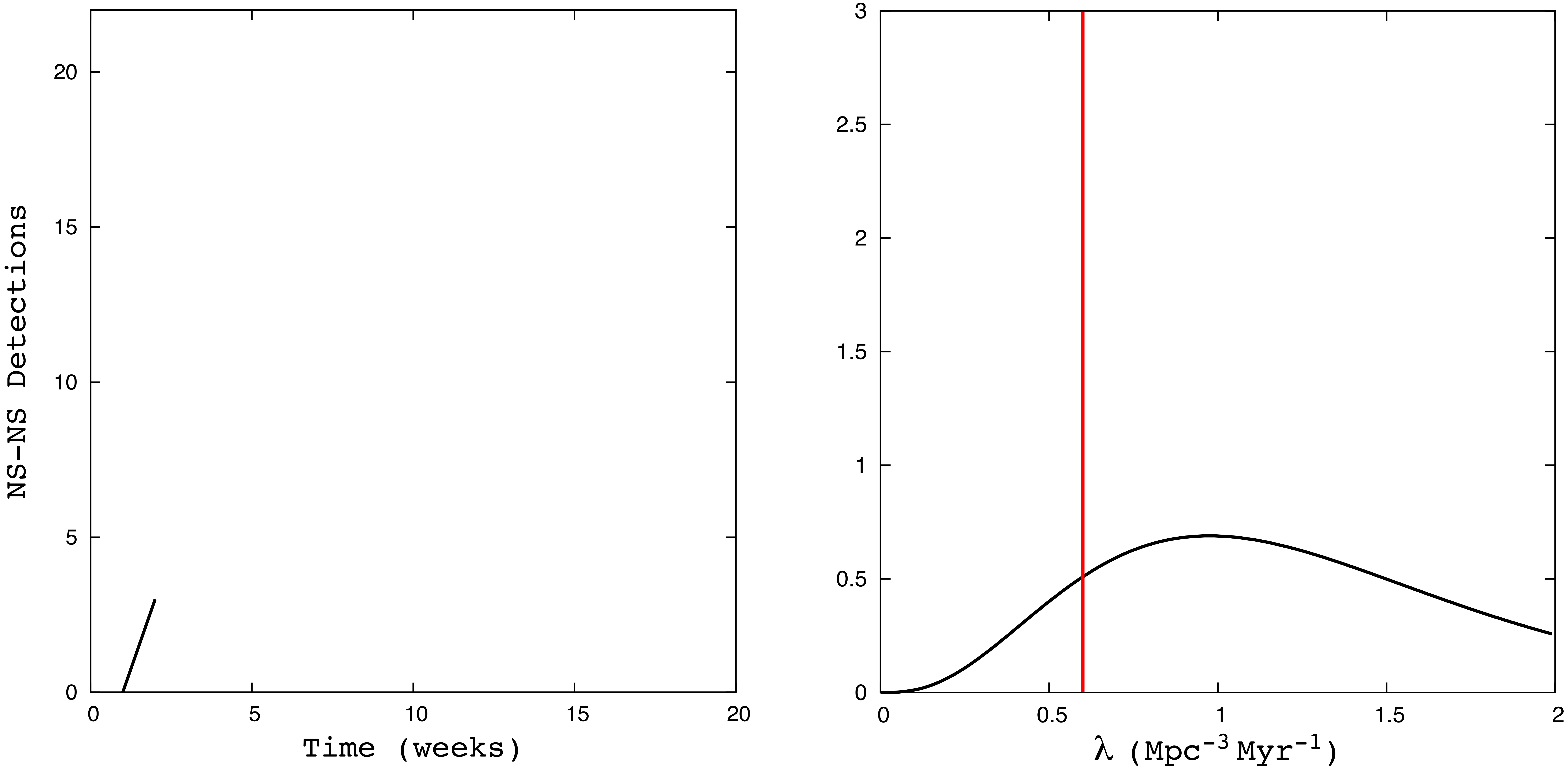
$$p(R) = R(VT)^2\, e^{-RVT}$$

This distribution is peaked at

$$R = \frac{1}{VT}$$

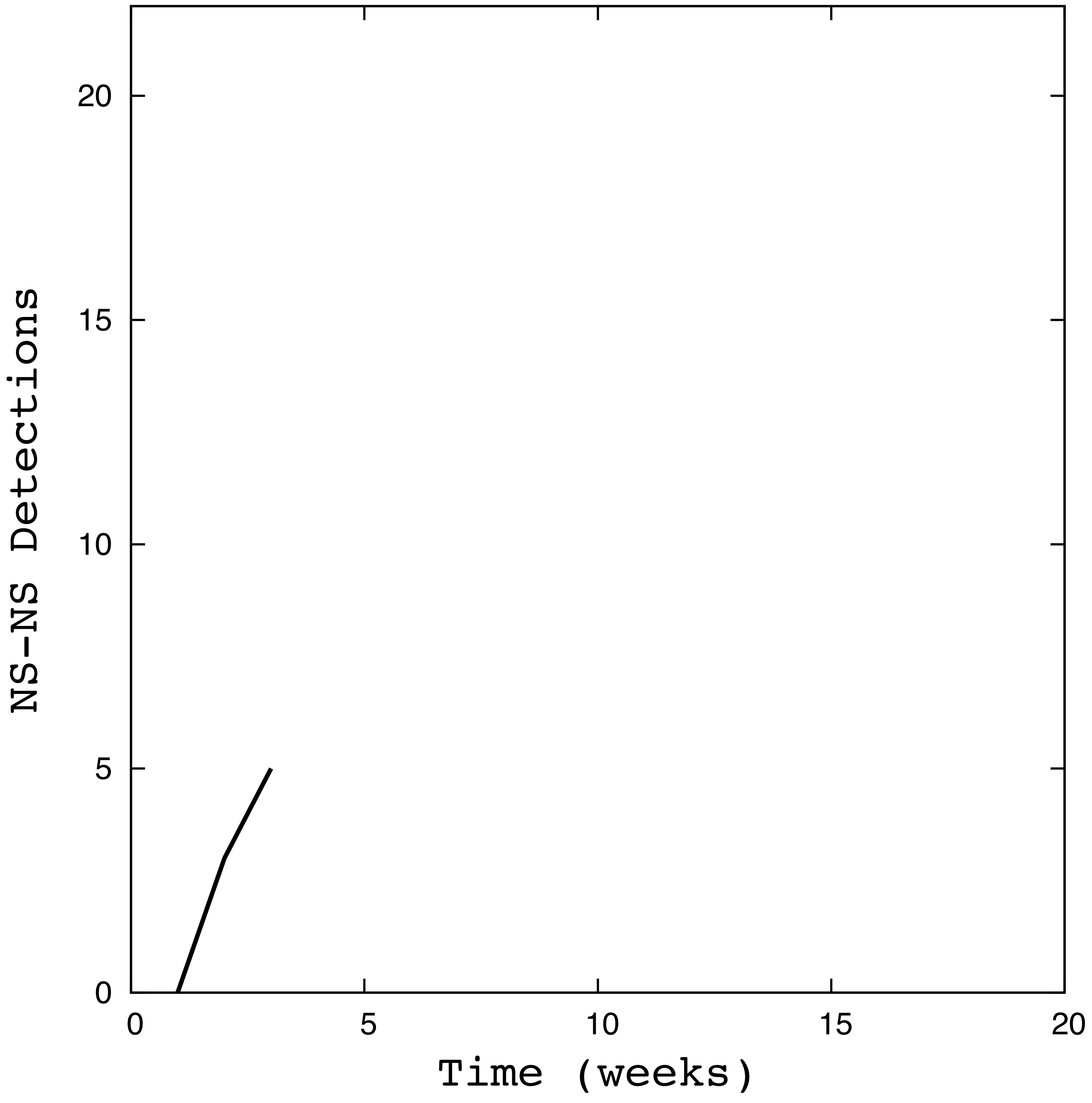
The 90% confidence interval now sets upper ***and*** lower limits on the merger rate.

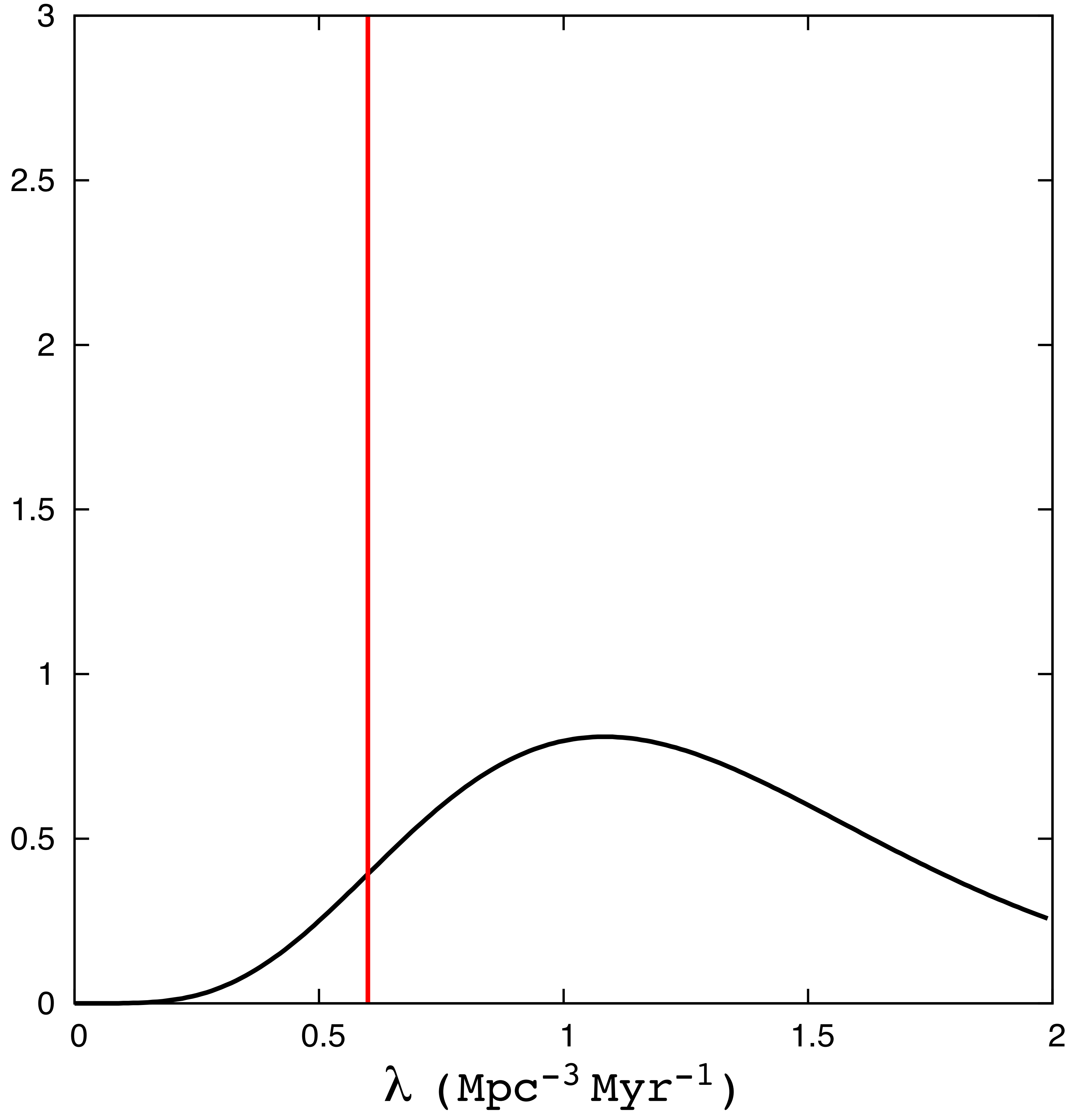
Simulated NS-NS Merger Rate Constraints



Week 2

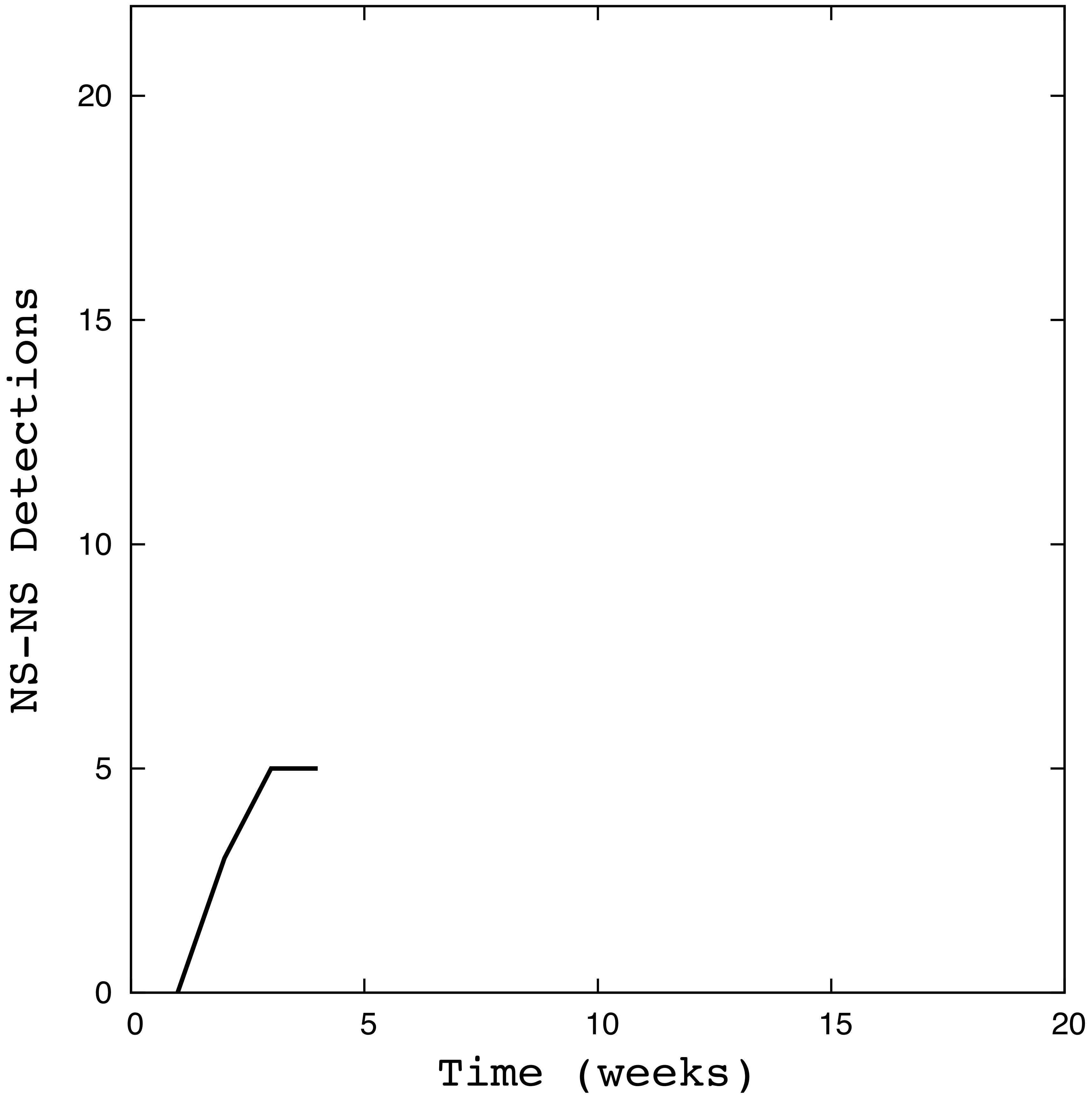
Simulated NS-NS Merger Rate Constraints

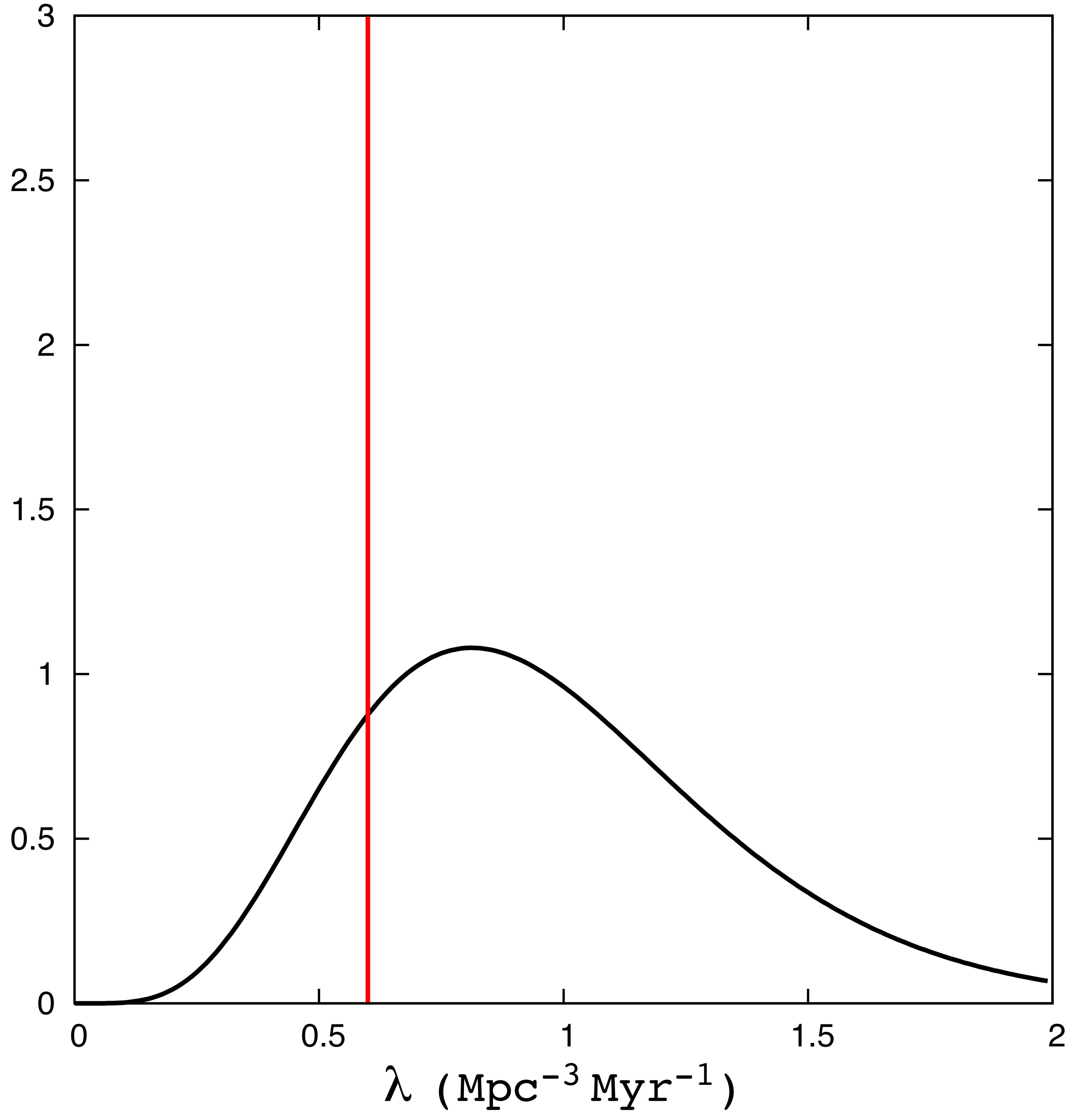




Week 3

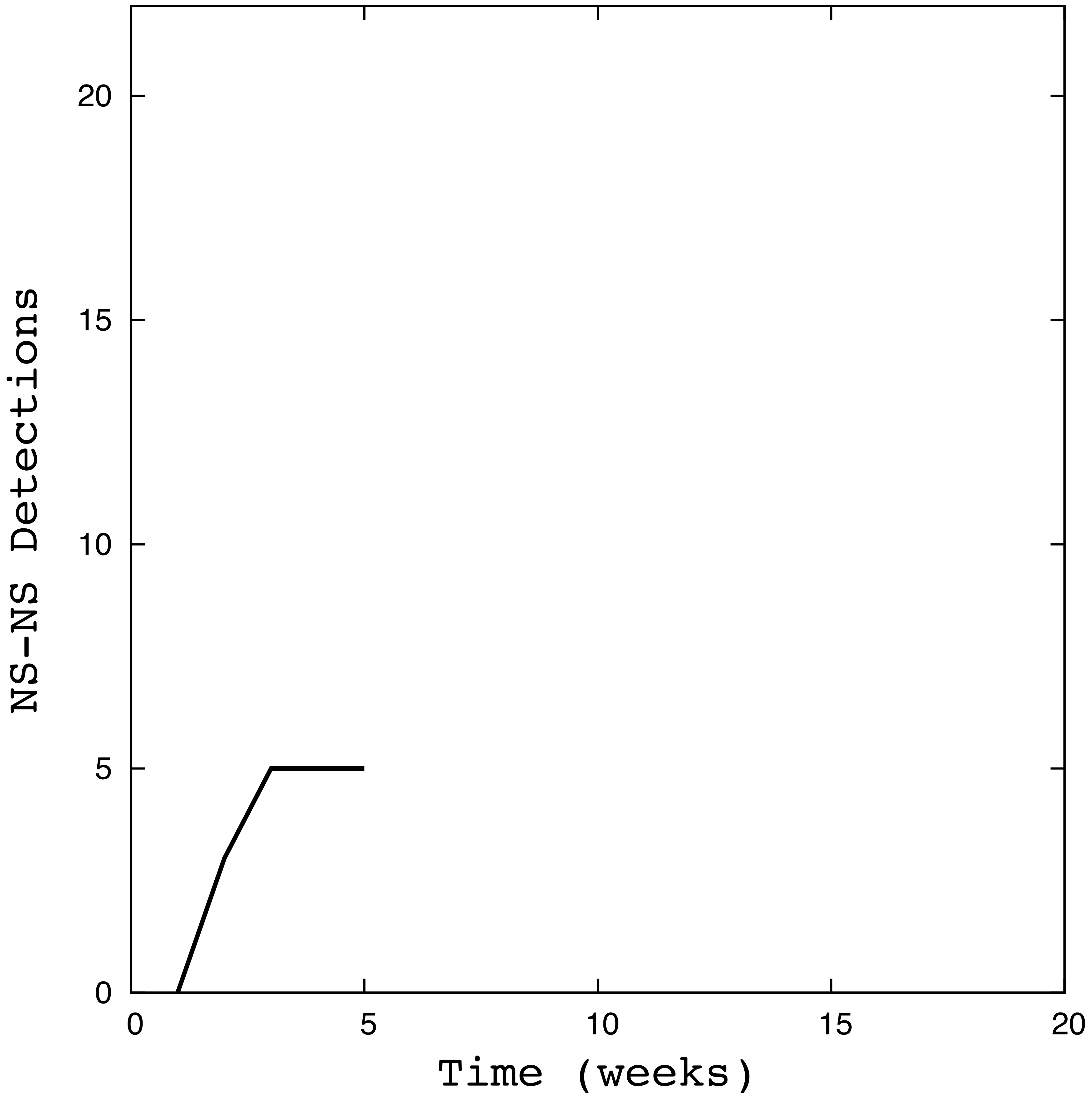
Simulated NS-NS Merger Rate Constraints

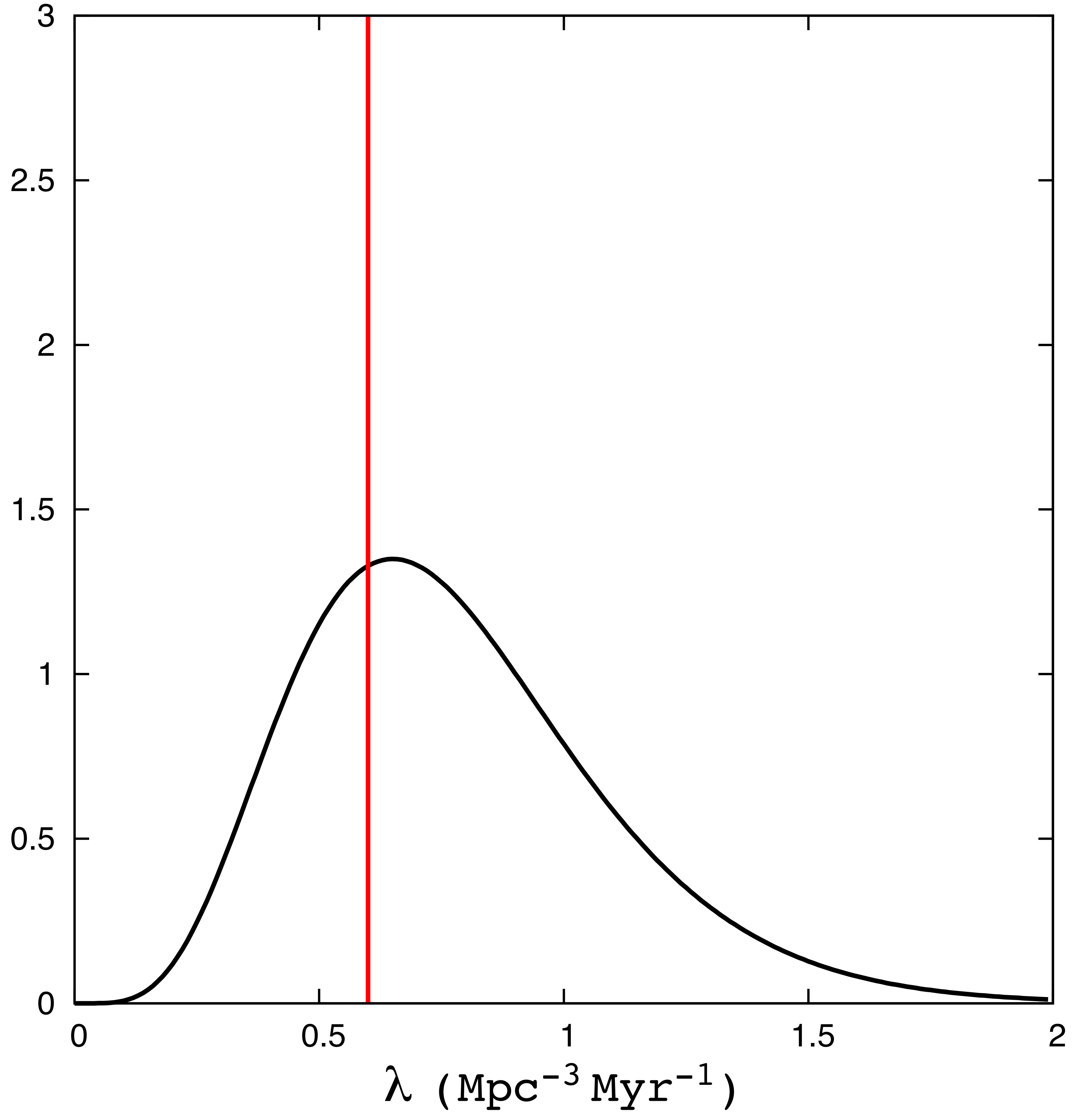




Week 4

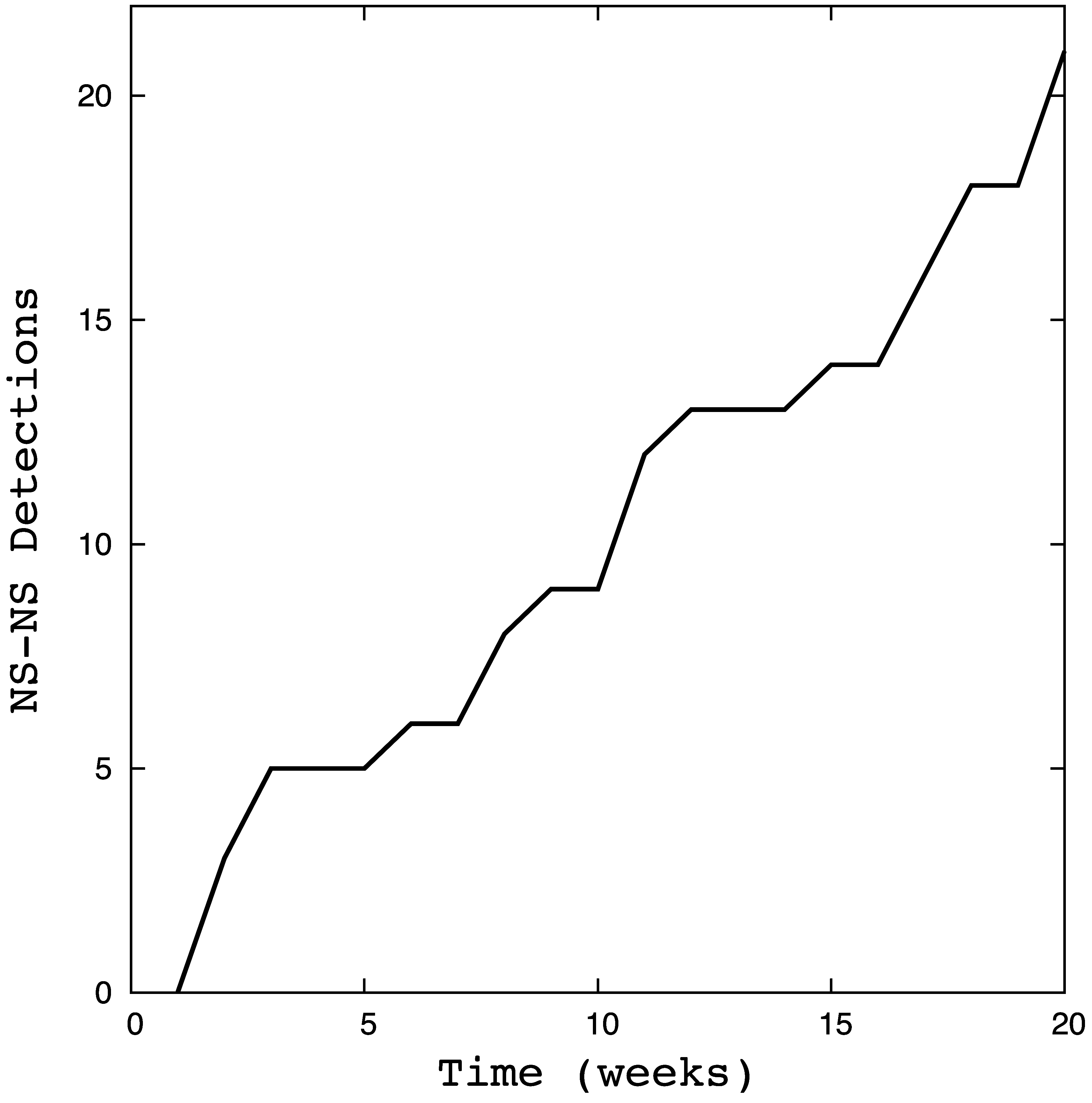
Simulated NS-NS Merger Rate Constraints

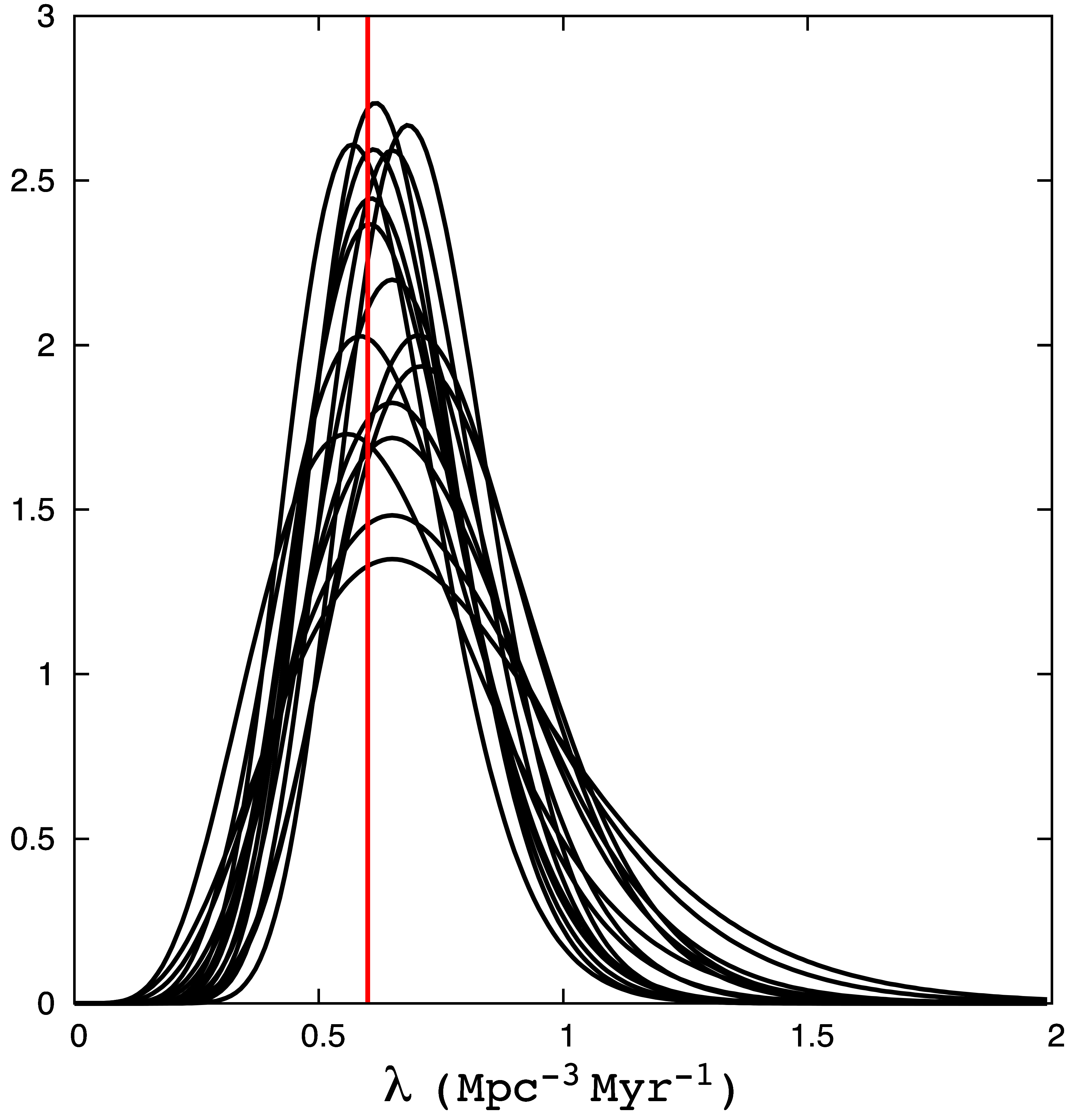




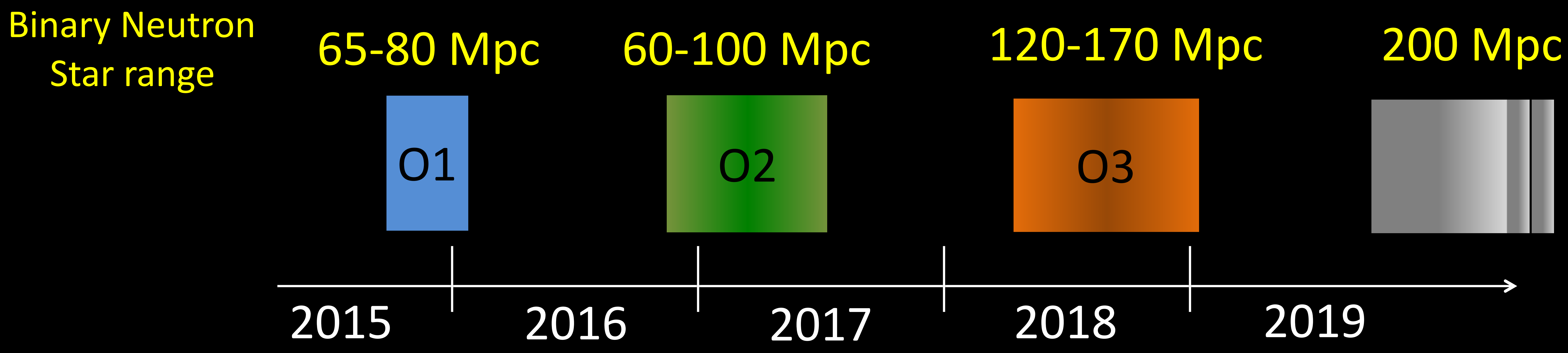
Week 5

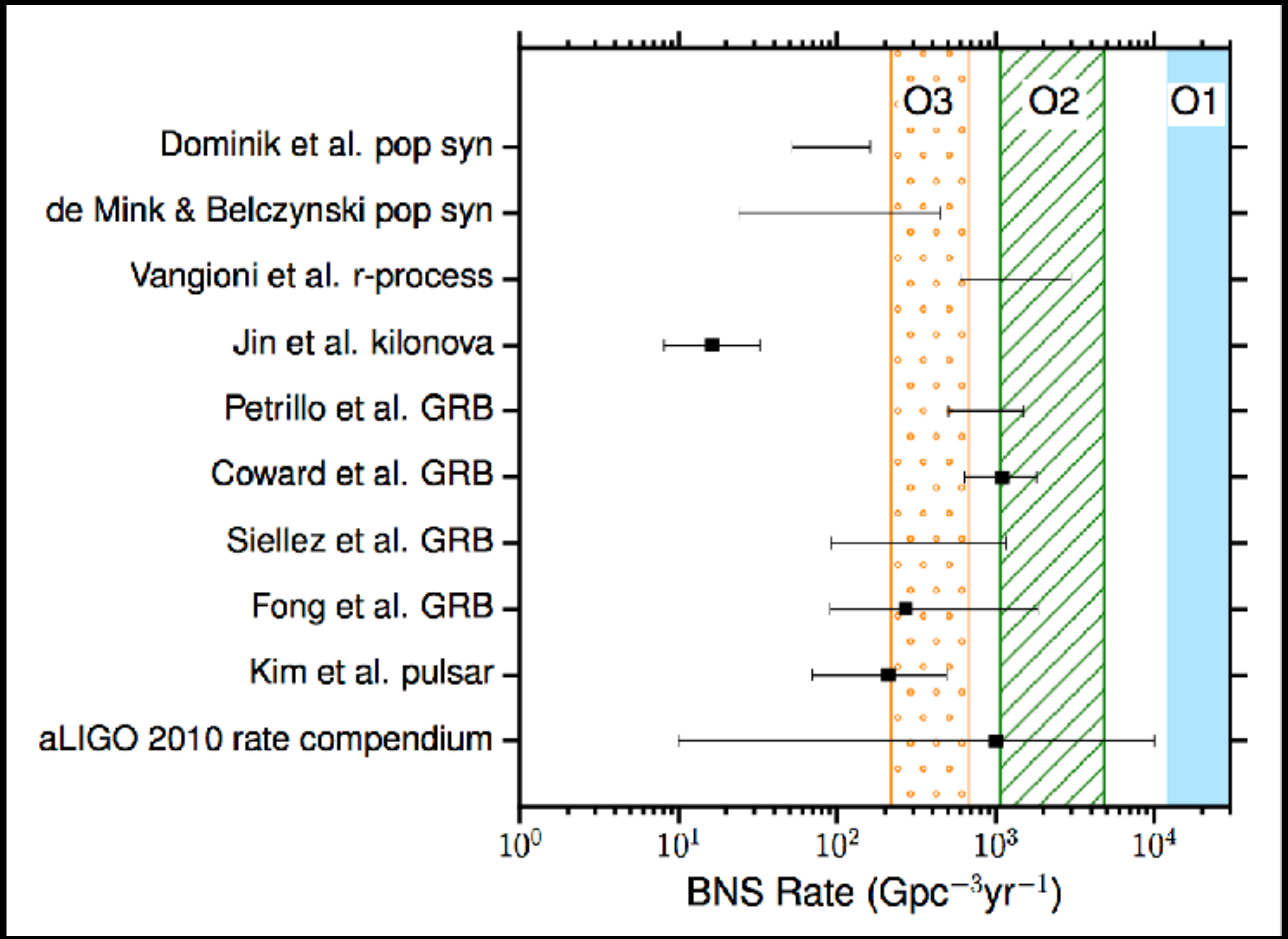
Simulated NS-NS Merger Rate Constraints

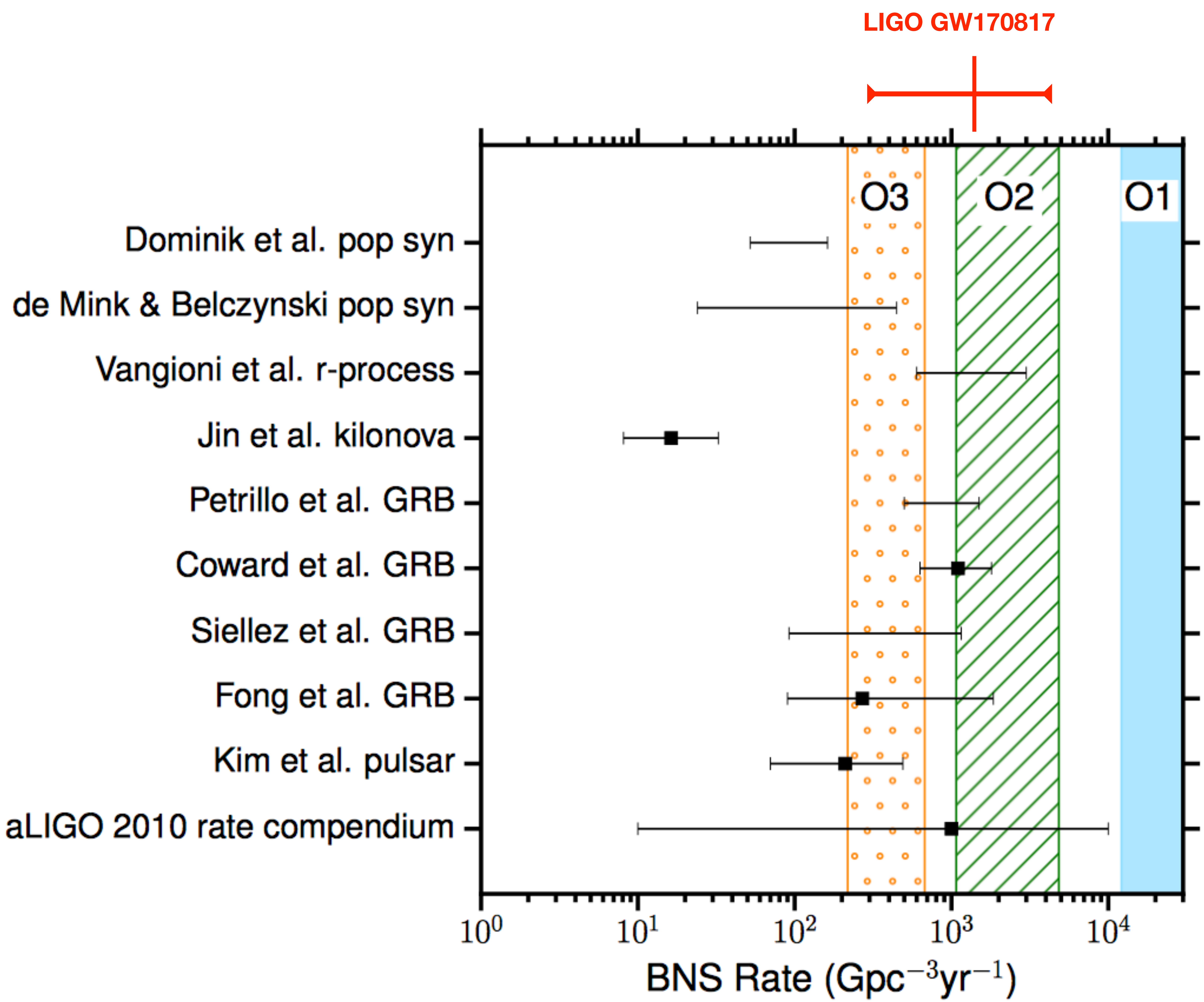




Plausible Observing Run Timeline

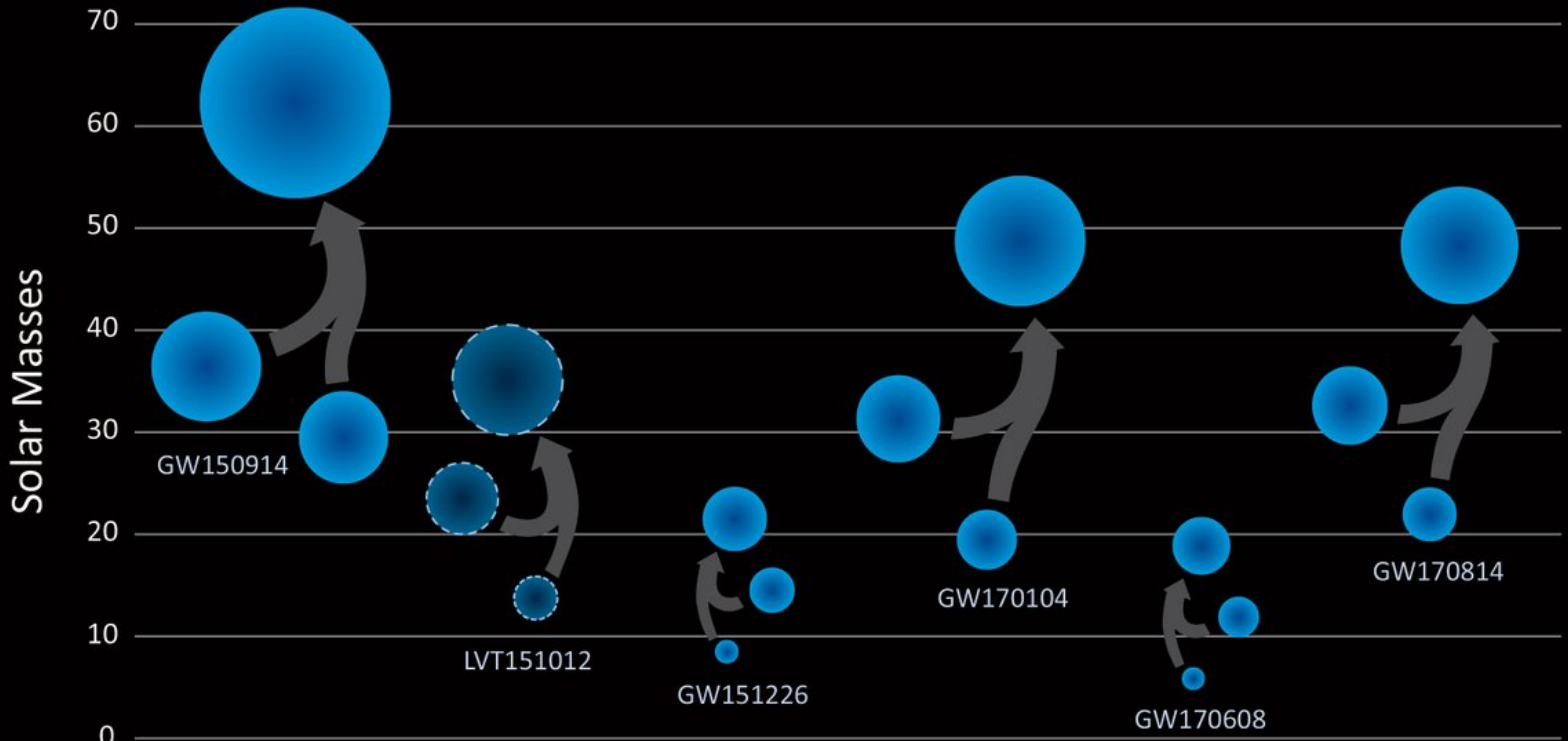






LIGO GW170817
O3
O2
O1
Dominik et al. pop syn
de Mink & Belczynski pop syn
Vangioni et al. r-process
Jin et al. kilonova
Petrillo et al. GRB
Coward et al. GRB
Siellez et al. GRB
Fong et al. GRB
Kim et al. pulsar
aLIGO 2010 rate compendium
10^0
10^1
10^2
10^3
10^4
BNS Rate ($Gpc^{-3}yr^{-1}$)

How to account for uncertainty and selection effects?



Selection Effects: Binary Systems

$$\rho^2 \sim \frac{\mathcal{M}^{5/3}}{D_L^2}\left(1 + 6\cos^2\iota + \cos^4\iota\right)$$

(assuming the mass is not so large that the signal gets cut-off in the detection band)

- More sensitive to nearby sources

- More sensitive to high mass systems

- More sensitive to face on/off systems

Can avoid any biases in population inference by including foreground and background events and their parameter dependencies

Incorporating Detection Uncertainty and Parameter Dependence in Merger Rate Inference

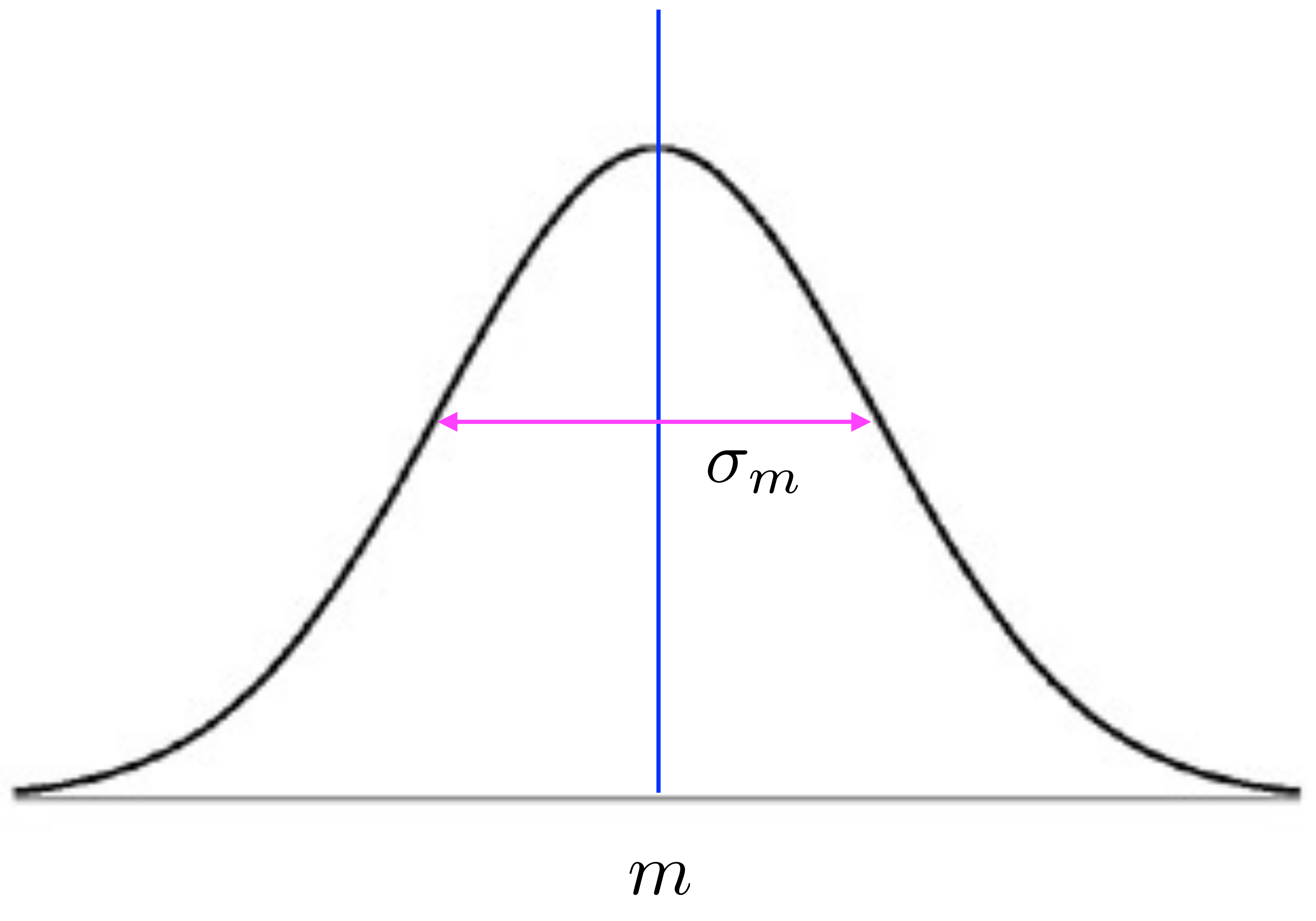
[Messenger & Veitch, New Journal of Physics, 15, 053027 (2013)]
[Farr, Gair, Mandel & Cutler, Phys. Rev. D 91, 023005, (2015)]

Any analysis which attempts to draw inferences from a population of signals or events which come from a search with a detection threshold are vulnerable to selection bias if the population of detected signals does not match that of the underlying sources

Bias can be avoided by accounting for events that are thrown away (which will include signals and noise) and the uncertain nature of events that are kept (which will include signals and noise)

Toy Example: Neutron Star Mergers

[Messenger & Veitch, New Journal of Physics, 15, 053027 (2013)]



Neutron star population with mean mass $m = 1.2\,M_\odot$ and variance $\sigma_m = 0.1\,M_\odot$

Merger rate $R = 10^{-7}\,\mathrm{Mpc}^{-3}\,\mathrm{yr}^{-1}$

Assume zero uncertainty in measured masses, but include uncertainty in detection (false positives and false dismissals)

Assume SNR based detection threshold
$$\rho^2_{\mathrm{opt}} = \frac{5\,\mathcal{M}^{5/3}}{96\,\pi^{4/3}\,D_L^2}\int_{f_{\min}}^{f_{\max}} \frac{f^{-7/3}}{S_n(f)}\,df$$

Toy Example: Neutron Star Mergers

Most events will have low SNR, and will be thrown away if we set a high detection threshold - this wastes information

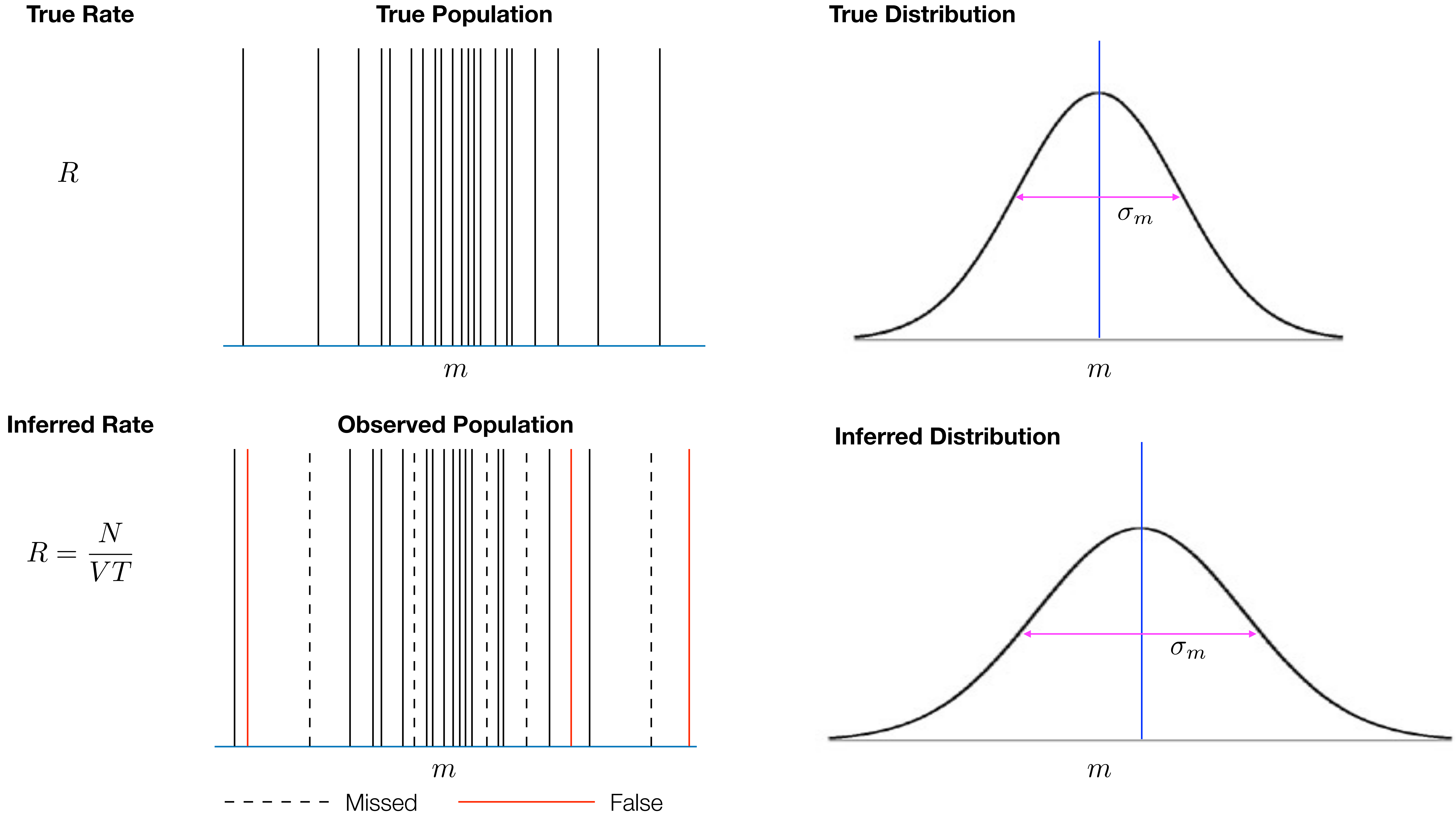
$$\rho_{\mathrm{opt}} \sim \frac{1}{D_L} \qquad p(D_L) \sim D_L^2 \qquad \Rightarrow \quad p(\rho_{\mathrm{opt}}) \sim \frac{1}{\rho_{\mathrm{opt}}^4}$$

Break the analysis up into K small time intervals so the probability of detection in each interval Δt is small:

$$p(\mathcal{H}^+|\vec{\Lambda}) = RV\Delta t$$ Prior probability of a detection

$$p(\mathcal{H}^-|\vec{\Lambda}) = 1 - RV\Delta t$$ Prior probability of no detection

$$\vec{\Lambda} \to (R, m, \sigma_m)$$ Model parameters (merger rate, average mass, spread in masses)



True Rate
True Population
True Distribution
R
m
σ_m
m
Inferred Rate
Observed Population
Inferred Distribution
$R = \frac{N}{VT}$
σ_m
m
m
Missed
False

Toy Example: Neutron Star Mergers

True detection **False detection**

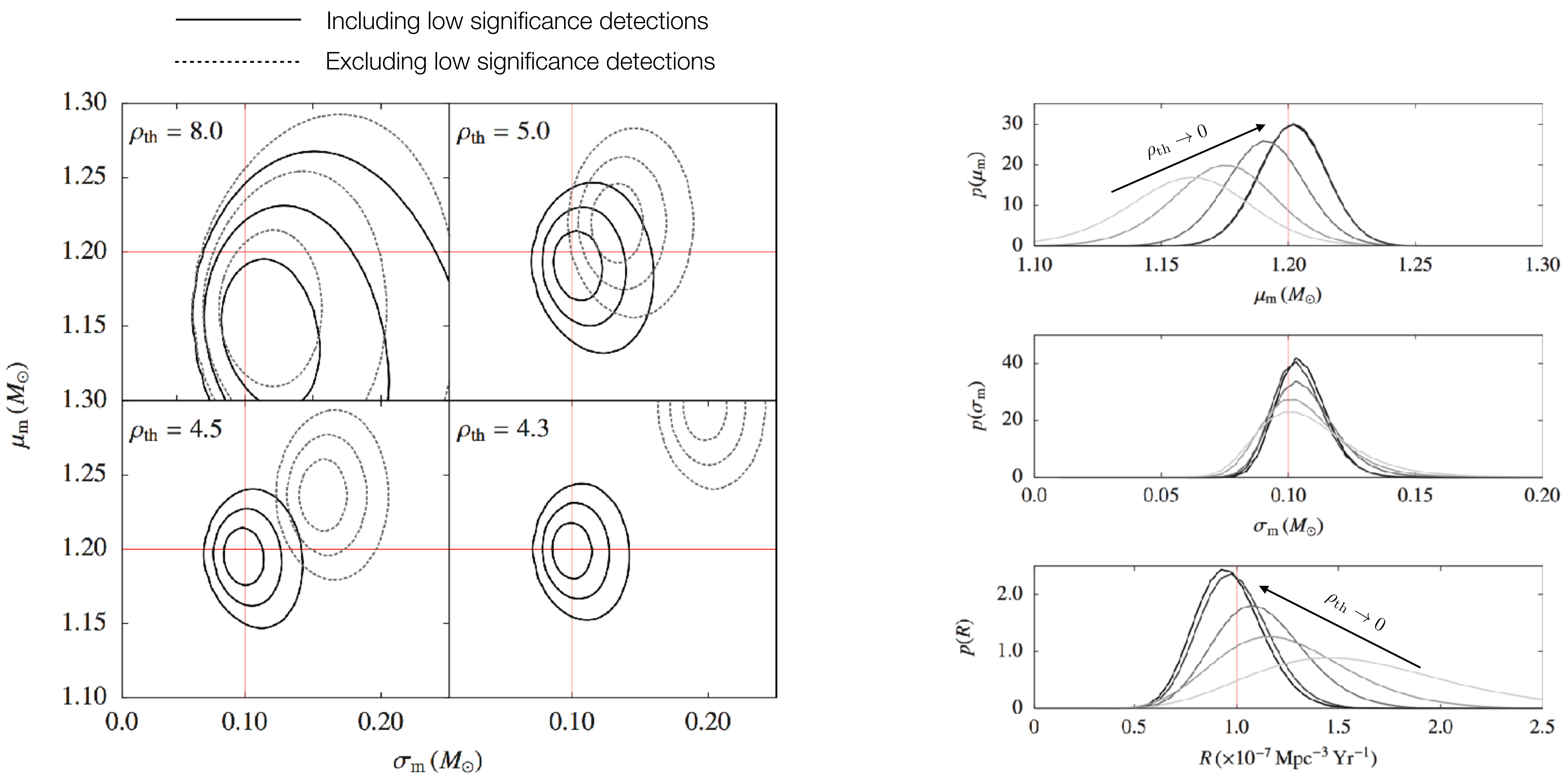
$$p(\vec{\Lambda}|D) = \frac{p(\vec{\Lambda})}{P(D)} \prod_{j=1}^{n} \Big[p(D^+, \rho_j|\vec{\Lambda}, \mathcal{H}^+) p(\mathcal{H}^+|\vec{\Lambda}) + p(D^+, \rho_j|\vec{\Lambda}, \mathcal{H}^-) p(\mathcal{H}^-|\vec{\Lambda}) \Big]$$

$$\times \prod_{k=1}^{K-n} \Big[p(D^-|\vec{\Lambda}, \mathcal{H}^+) p(\mathcal{H}^+|\vec{\Lambda}) + p(D^-|\vec{\Lambda}, \mathcal{H}^-) p(\mathcal{H}^-|\vec{\Lambda}) \Big]$$

False dismissal **True dismissal**

Need to include all the information, detections and non-detections, whether true or not

Toy Example: Neutron Star Mergers



Bottom line: Much better to use all the detections, confident to not

What next for gravitational wave astronomy?

What are the big unsolved problems?