# Black Hole Accretion and Feedback

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#### black holes

escape velocity from surface of a star is  $v = (2GM/R)^{1/2}$ 

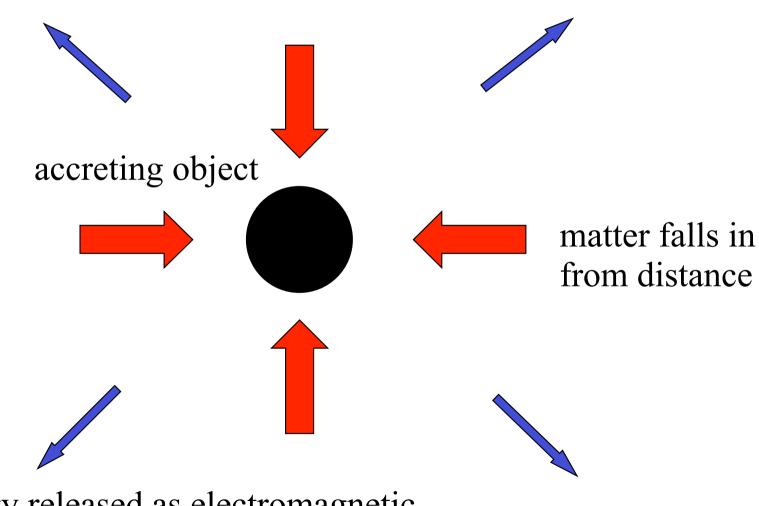
this reaches c for

$$R = \frac{2GM}{c^2} = 3\frac{M}{M_{\odot}} \text{ km}$$

Schwarzschild radius

#### black holes

accretion = release of gravitational energy from infalling matter



energy released as electromagnetic (or other) radiation

#### accretion energy release

for accretor of mass M, radius R, gravitational energy release per unit mass is

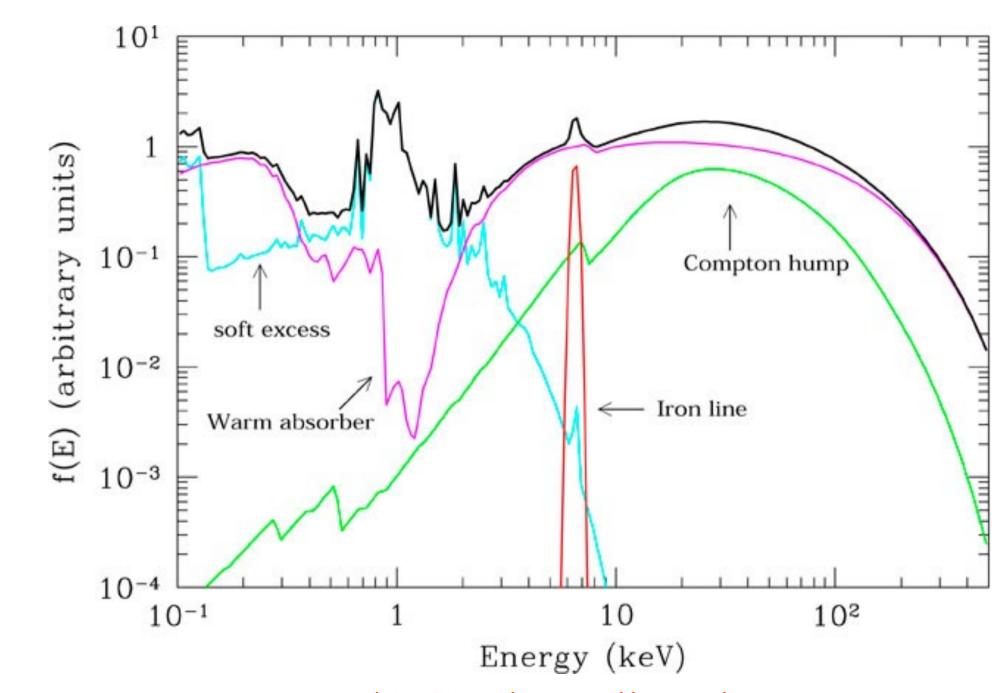
$$\Delta E_{\rm acc} = \frac{GM}{R}$$

black hole:  $R = 2GM/c^2$ , so  $\Delta E_{\rm acc} = c^2/2$ 

compare with nuclear yield (hydrogen burning):  $\Delta E_{\rm nuc} = 0.007c^2$ 

accretion on to a black hole is the most efficient way of getting energy from normal matter: GR => accretion efficiency  $\eta$  is  $\sim 0.1$ 

it must power the most luminous objects in the Universe e.g. quasars, active galactic nuclei (AGN)



quasars/AGN broadband spectrum

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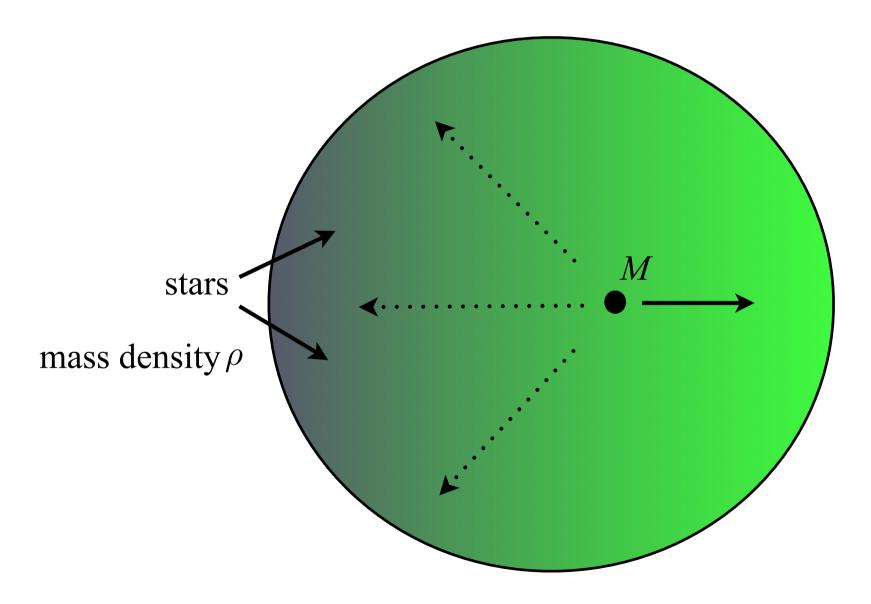
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#### almost every galaxy hosts an SMBH

#### where are the holes? dynamical friction



moving mass M slowed by raising gravitational 'wake' in star motions

#### dynamical friction

drag force gives equation of motion of moving mass M as

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{4\pi CG^2 M\rho}{v^2}$$
 (e.g. Sparke & Gallagher, pp 224 - 5)

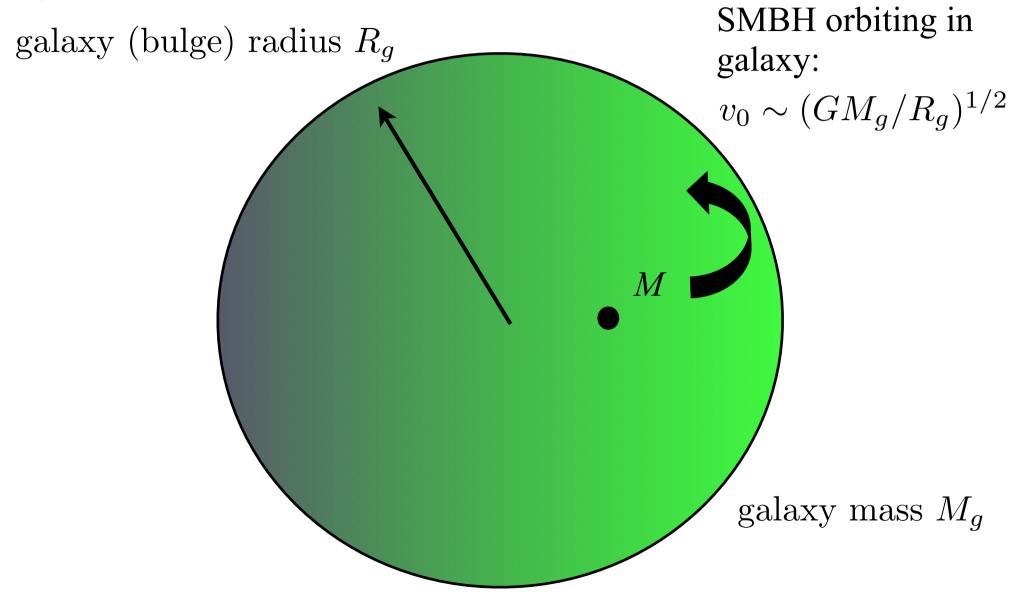
with  $C \simeq 10$ , giving

$$v^3 = v_0^3 \left( 1 - \frac{t}{t_{\text{fric}}} \right)$$

mass halts (spirals in to centre of mass of stellar distribution) after a time

$$t_{\rm fric} = \frac{v_0^3}{12\pi CG^2 M\rho}$$

# dynamical friction



# dynamical friction

with  $\rho = 3M_g/4\pi R_g^3$  we find

$$t_{\rm fric} = \frac{1}{9C} \frac{M_g}{M} \left(\frac{R_g^3}{GM_g}\right)^{1/2}$$

and with  $M = 10^8 M_{\odot}$ ,  $M_g = 10^{11} M_{\odot}$ ,  $R_g = 10$  kpc we get

$$t_{\rm fric} \sim 10^8 \ {\rm yr}$$

short compared with age of galaxy - SMBH at centre of host

#### Books

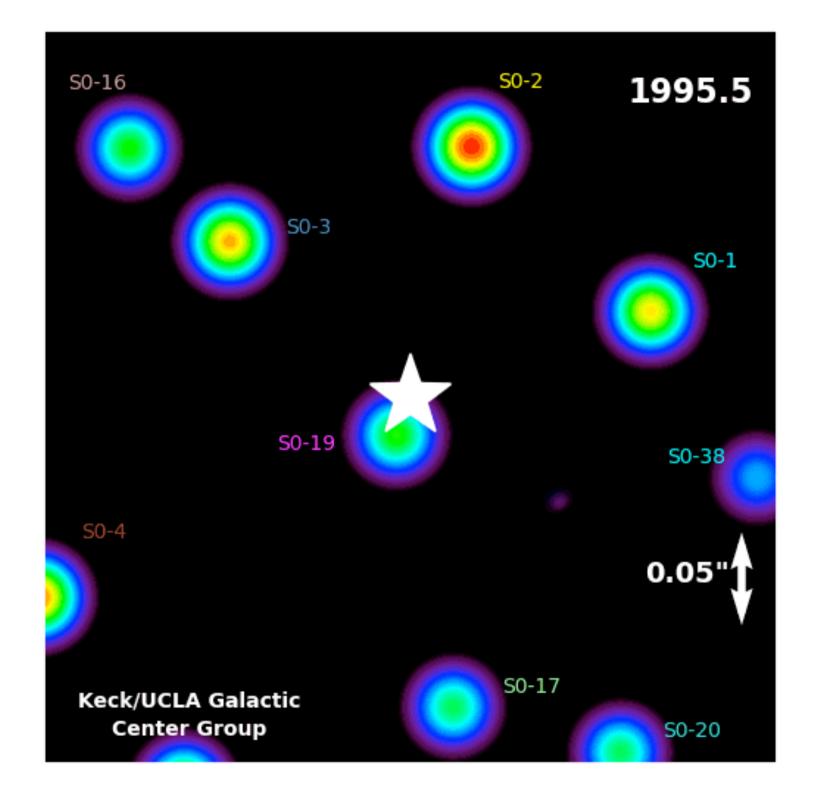
- Introduction to Active Galaxies, Peterson
  - good introduction to observed AGN properties, measurements etc
- Galaxies in the Universe, Sparke and Gallagher
  - introduction to galaxy properties and evolution
- Accretion Power in Astrophysics, 3rd Ed., Frank, King & Raine (APIA)
  - standard reference for basic accretion theory (SMBH part now rather dated)

#### centre of Milky Way

infrared source Sgr A\* in Galactic Centre shows clear dynamical evidence of SMBH

$$M \simeq 4 \times 10^6 M_{\odot}$$

from motions of surrounding stars



#### motion near a point mass

(e.g. APIA pp 234 - 236)

energy equation for Newtonian point mass is

$$\frac{v^2}{2} - \frac{GM}{r} = E$$

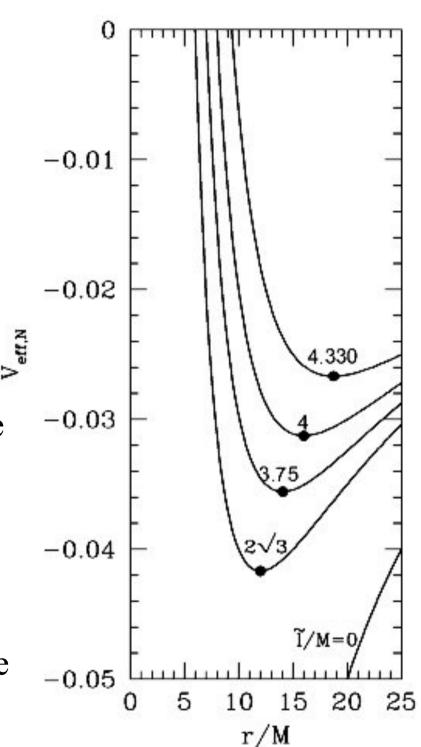
now with  $v^2 = \dot{r}^2 + r^2 \dot{\theta}^2$ and  $r^2 \dot{\theta} = J$  (specific a.m.) we have

$$\frac{1}{2}\dot{r}^2 + V(r) = E$$

where

$$V(r) = \frac{J^2}{2r^2} - \frac{GM}{r}$$

is the effective potential for a particle of fixed angular momentum J



# motion near a point mass

- J = 0: particle falls in to origin
- nonzero J: particle escapes i.e.  $r \to \infty$ , if E > 0, but is bound (r stays finite) if E < 0
- circular orbit requires  $\dot{r} = 0$ , so

$$V(r) = E = \text{constant}$$

and

$$\frac{\mathrm{d}V}{\mathrm{d}r} = 0$$

circular orbits are possible at minima of effective potential

-0.01-0.02-0.03-0.04-0.0520 r/M

circular speeds grow as radius drops

#### motion near a black hole

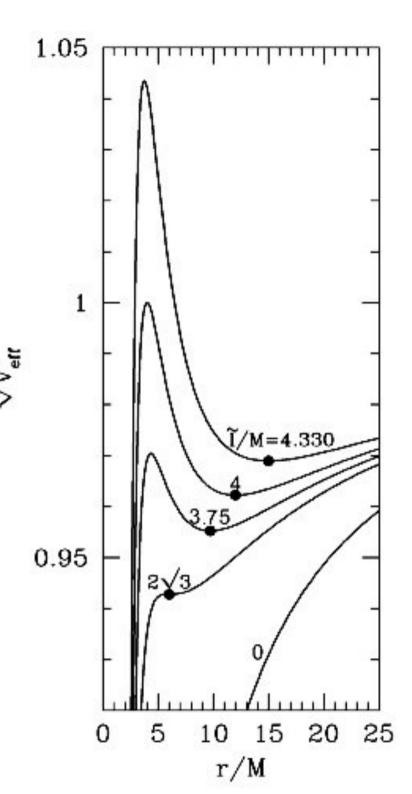
circular speed cannot exceed *c*, so effective potential *does not have minima* inside a certain radius:

innermost stable circular orbit: 'ISCO'

ISCO radius depends on spin angular momentum  $J_h$  of hole

$$J_h = \frac{GM^2}{c}a$$

a is (dimensionless) Kerr parameter



# spinning (Kerr) black hole

- characterized *completely* by mass M and spin parameter a
- black holes with same M, a are identical 'a black hole has no hair'
- spin a.m. is limited ('breakup') by -1 < a < 1
- negative *a* implies spin in opposite sense to a.m. of test particle orbit, i.e. orbit is retrograde
- circular orbits have unique importance because matter has angular momentum and accretes on to black hole through a disc
- Kerr a specifies ISCO radius: specific binding energy of this specifies accretion energy yield per unit mass  $\eta$

# ISCOs and accretion yields

$$R_{\rm ISCO} = z \frac{GM}{c^2}$$

$$\eta = 1 - \left[1 - \frac{2}{3z}\right]^{1/2}$$

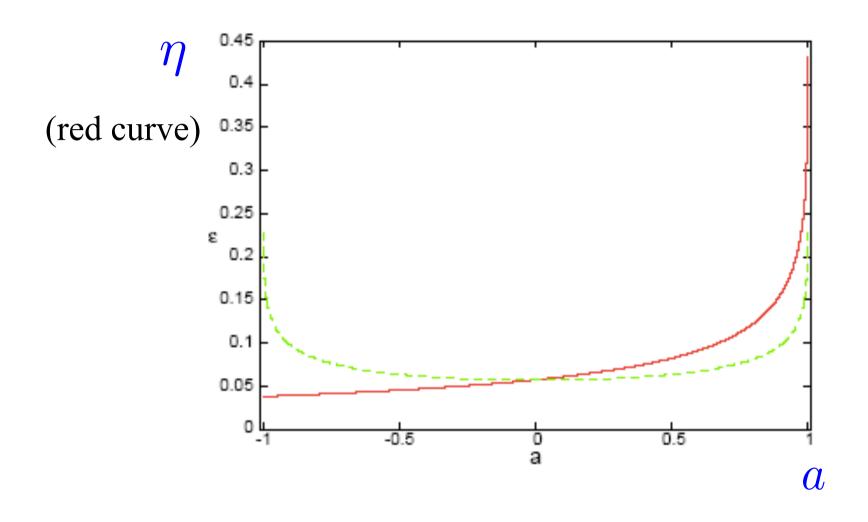
maximal retrograde

Schwarzschild

maximal prograde

a	z	$\eta$
-1	9	0.038
0	6	0.057
1	1	0.422

# BH spin parameter a determines efficiency $\eta$



#### accretion yields

- efficiency  $\eta = 0.1$  is reasonable
- => accretion luminosity  $L=\eta c^2\dot{M}=10^8L_\odot$  for accretion rate  $\dot{M}=1M_\odot\,{\rm yr}^{-1}$
- accretion on to SMBH explains quasar luminosities

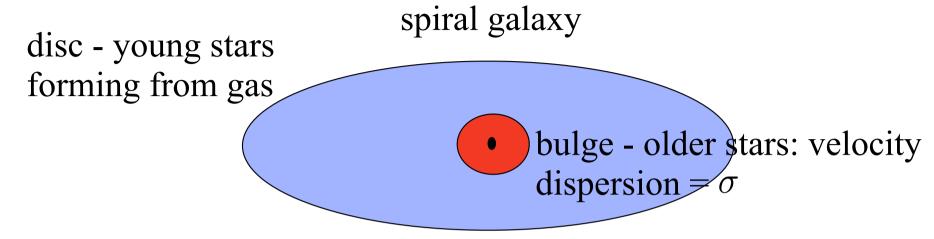
#### accretion on to SMBH power AGN and quasars

what is their significance for the Universe – where do they fit?

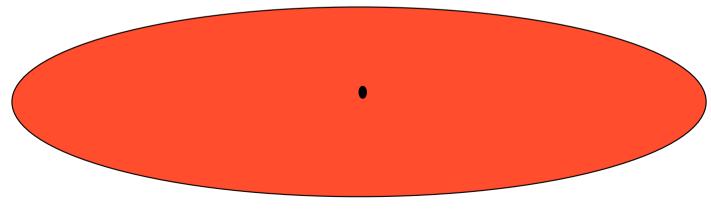
AGN are the growth phases of the SMBH in ALL galaxies

peak of quasar activity = peak of SMBH growth at  $z \sim 2$ 

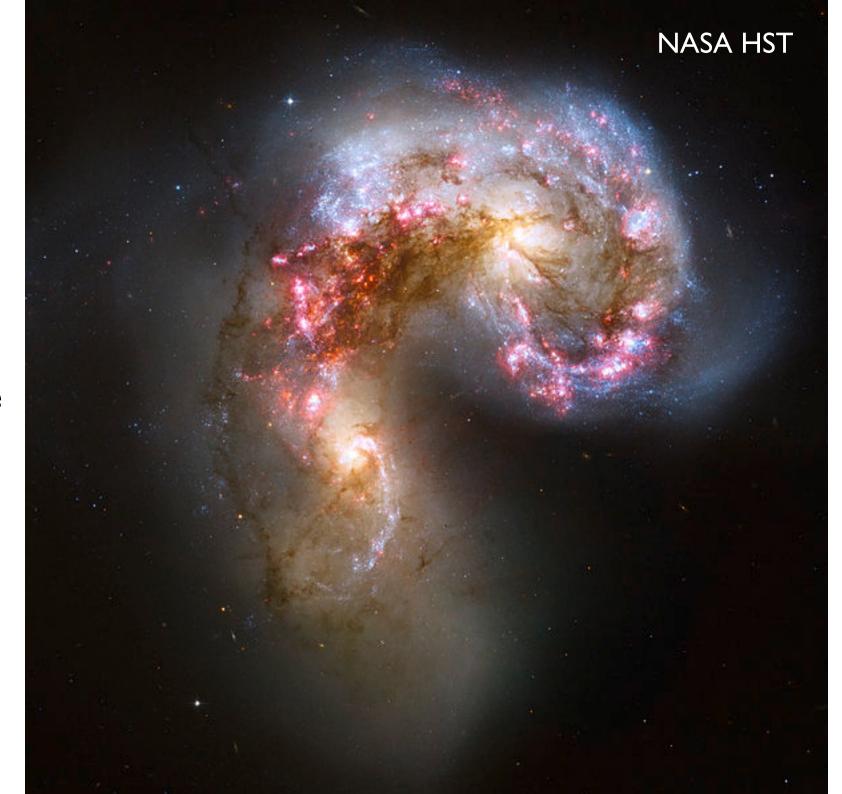
# galaxies



elliptical galaxy - merger of spirals?

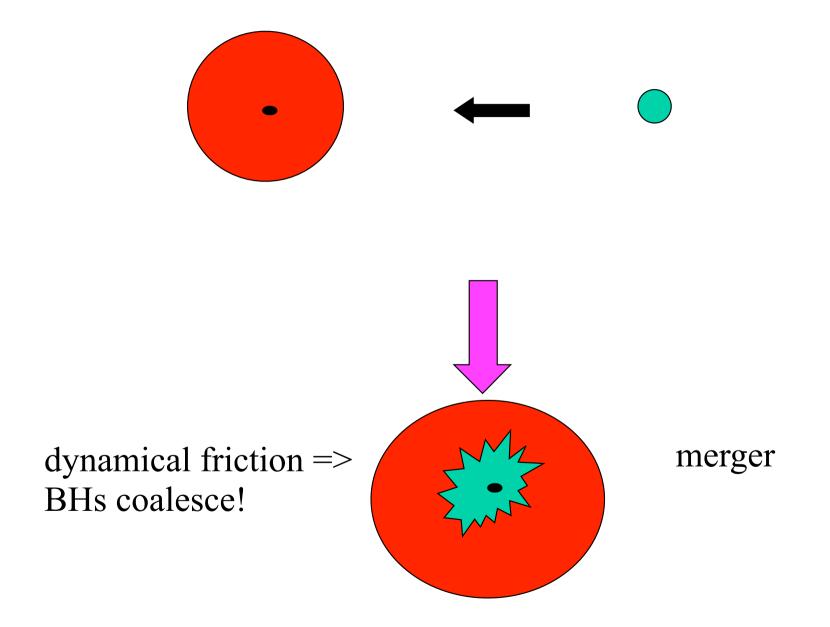


little gas left - old stars only 'red and dead'

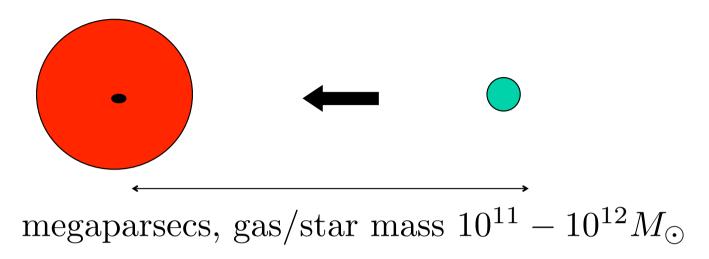


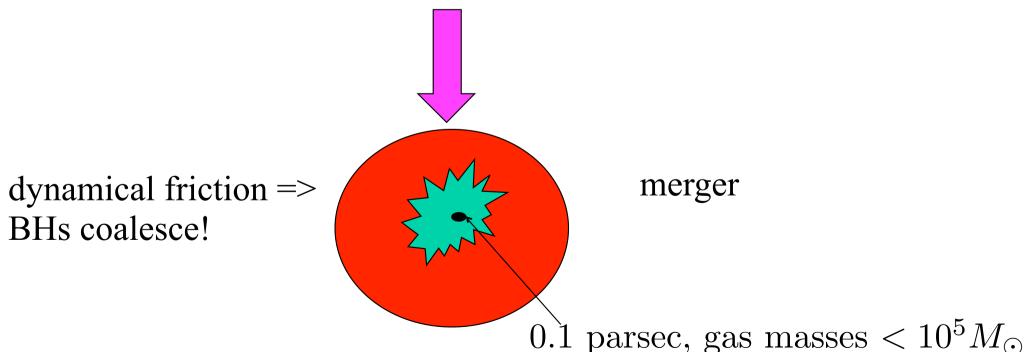
galaxies merge

# cosmological picture of growth: big galaxy swallows small



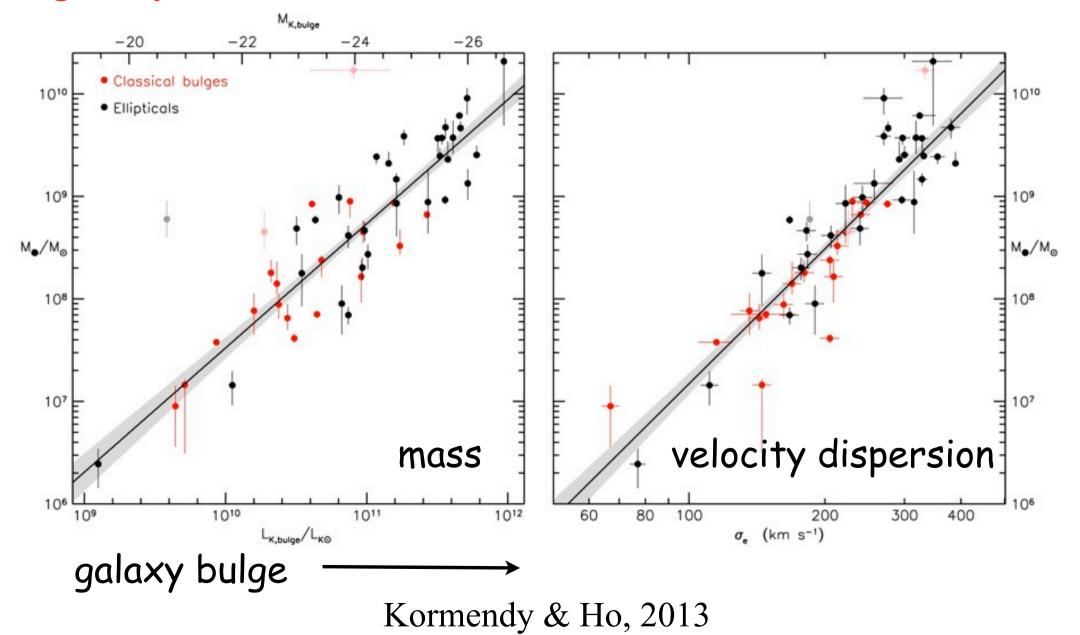
#### cosmological picture of growth: big galaxy swallows small





huge range of mass and length scales: numerical treatment impossible

#### galaxy knows about central SMBH



#### how?

SMBH mass is completely insignificant:  $M \sim 10^{-3} M_{\rm bulge}$ ,

so its gravity affects only a region

$$R_{\rm inf} = \frac{GM}{\sigma^2} \sim 10 \frac{M_8}{\sigma_{200}^2} \text{ parsec}$$

$$(M_8 = M/10^8 M_{\odot}, \sigma_{200} = \sigma/200 \text{ km s}^{-1})$$

- far smaller than bulge

why does the galaxy notice the hole?

well....

SMBH releases accretion energy  $\sim 0.1 M_{BH} c^2 \sim 10^{61}$  erg galaxy bulge binding energy  $M_b \sigma^2 \sim 10^{58}$  erg

galaxy notices hole through energy release:

'feedback'

well....

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galaxy notices hole through energy release:

`feedback'

black hole is dangerous for galaxy!

#### feeding the hole

transferred mass does not hit black hole in general, but must orbit it



— initial orbit is a rosette, but self—intersections → dissipation → energy loss, but no angular momentum loss

Kepler orbit with lowest energy for fixed a.m. is a circle

thus orbit circularizes, with radius such that it retains its orginal specific angular momentum

further energy loss only possible if angular momentum can be removed

#### accretion disc

#### disc formation is unavoidable

all accreting gas has enough angular momentum to orbit the hole, so a disc always forms

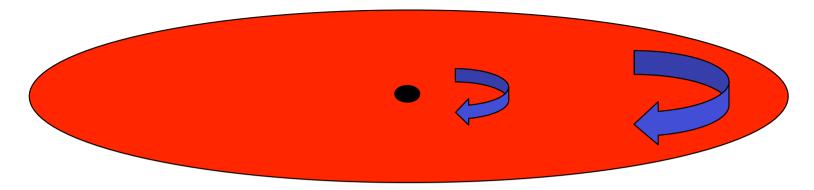
disc must be small enough for matter to accrete on reasonable timescales, i.e.  $\sim 0.1$  pc

this requires any feeding mechanism to produce an accurate 'shot' towards the black hole

feeding SMBH is difficult

this may be why  $M_{\rm BH} \simeq 10^{-3} M_{\rm bulge}$ 

## accretion disc structure (see APIA Ch 5)



flat, differentially rotating gas disc, thickness H(R)

surface density (mass/area)  $\Sigma(R) = \rho H$ 

rotational angular velocity  $\Omega(R)$  increases towards centre

angular momentum  $R^2\Omega(R)$  decreases towards centre

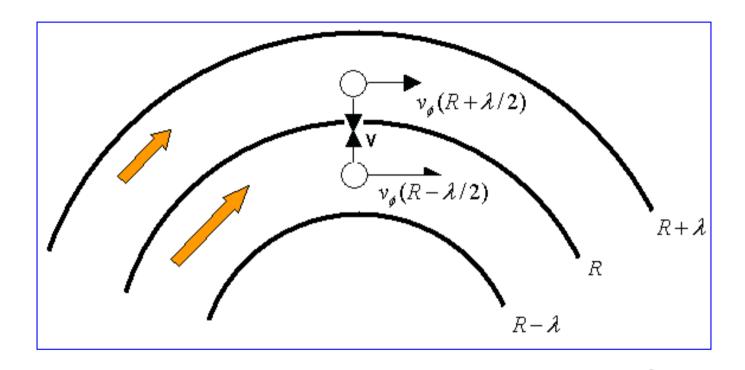
disc is thin, 
$$\frac{H}{R} \sim \frac{c_s}{v_{\rm K}} << 1$$
, Keplerian  $R\Omega(R) = v_K = \left(\frac{GM}{R}\right)^{1/2}$ 

(pressure forces small) if and only if it can cool

#### accretion disc structure

- driver of accretion is 'viscosity' some dissipative process which transports angular momentum outwards, against a.m. gradient
- currently unknown but may be magnetic
- characterized by a lengthscale  $\lambda$  and a speed v describing random motions around mean streaming (fluid) motion
- e.g. molecular viscosity has  $\lambda =$  mean free path, v = thermal speed of molecules (sound): other processes have larger  $\lambda$ , e.g. turbulence
- a viscosity transports fluid momentum and angular momentum within it
- gas spirals in, losing angular momentum and energy

#### accretion disc structure



torque of inner ring on outer one is  $G(R)=2\pi\nu\Sigma R^3\frac{\mathrm{d}\Omega}{\mathrm{d}R}$ , with  $\nu\sim\lambda\nu$  dissipation per unit disc face area of a steady thin disc is

$$D(R) = \frac{3GM\dot{M}}{8\pi R^3} \left[ 1 - \left(\frac{R_{\rm in}}{R}\right)^{1/2} \right]$$

viscous timescale to lose angular momentum and spiral in is long: disc surface density  $\Sigma(R,t)$  obeys a diffusion equation

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left( R^{1/2} \frac{\partial}{\partial R} \left[ \nu \Sigma R^{1/2} \right] \right)$$

where  $\nu$  is 'kinematic viscosity': parametrize as  $\nu = \alpha c_s H$ , with  $\alpha < 1$ ,  $\dot{M}(R,t) = 3\pi\nu\Sigma$ 

 $\Sigma$  spreads on viscous timescale

$$t_{\rm visc} = \frac{R^2}{\nu} = \frac{1}{\alpha} \left(\frac{R}{H}\right)^2 t_{\rm dyn}$$

where  $t_{\rm dyn}$  is the dynamical timescale  $R/v_K = (R^3/GM)^{1/2}$ 

this is long:  $t_{\rm visc} \simeq 10^{10} \ {\rm yr} \ {\rm for} \ R \sim 1 \ {\rm pc} \ (H/R \lesssim 10^{-2})$ 

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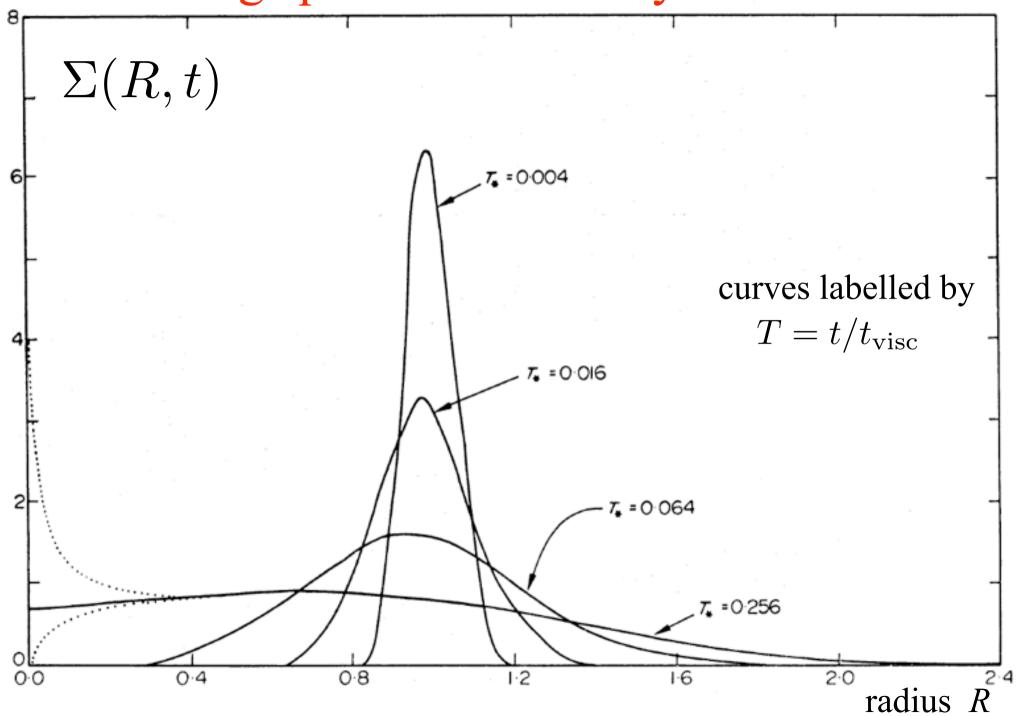
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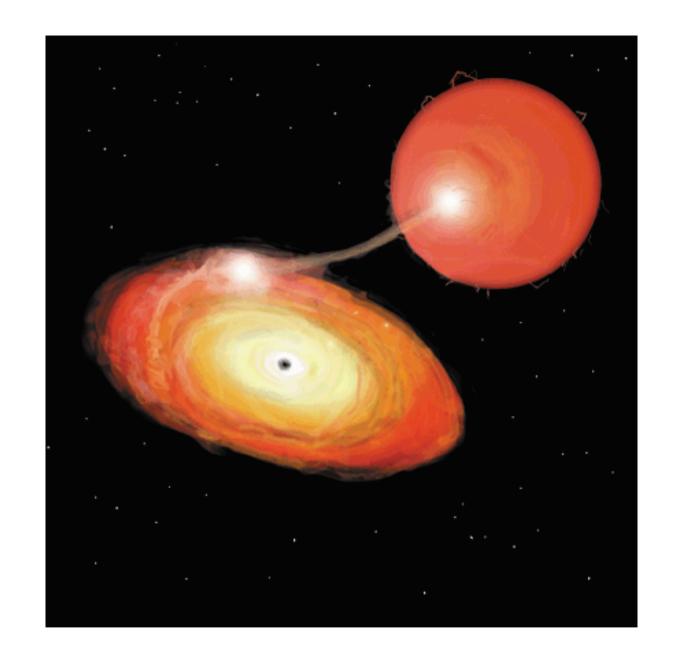
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# initial ring spreads diffusively to make a disc





close binary system with an accretion disc

some phenomena qualitatively independent of viscosity: only specifies *overall timescale* as

$$t_{\rm visc} \sim \frac{R^2}{\nu}$$

e.g. superhumps: requires orbital resonances within disc (Whitehurst & King, 1991; Lubow, 1991, 1992)

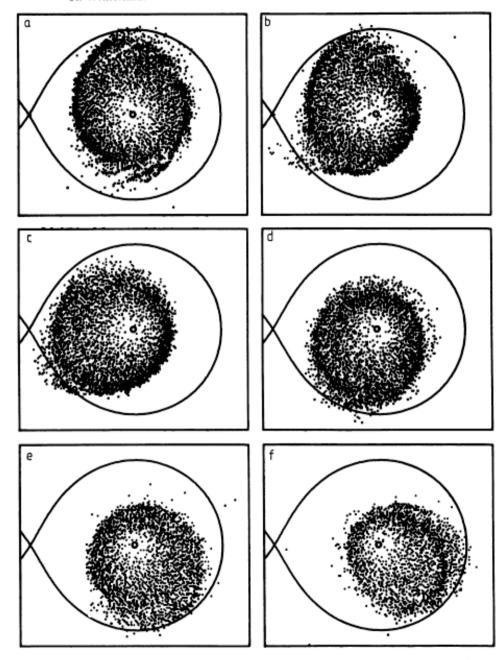
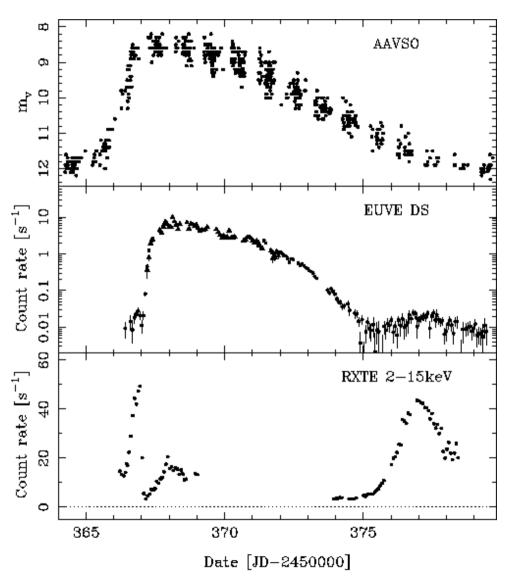


Figure 2. Transformation of the disc from the initial corotating mode to the eccentric 1::1 resonant mode. Each snapshot is precisely three orbital periods apart. Note that the rotation of the disc is clear in snapshots (d), (e) and (f).

#### or disc instabilities



X-ray Nova
GS 2000 + 25

May 1 June 1 July 1 1988

soft X-ray transient (irradiated disc)

dwarf nova (unirradiated disc)

unirradiated (dwarf nova) case: disc evolves viscously for short time, themal evolution (cooling wave) quickly cuts outburst off (Meyer & Meyer-Hofmeister, 1982)

irradiated (SXT) case: central X-ray irradiation prevents cooling wave, and traps disc in hot state (King & Ritter, 1998) until much of central disc mass depleted – much longer outbursts in SXTs than in dwarf novae, despite similar size discs

exponential outburst if disc fully irradiated (short orbital periods (K & R 98)

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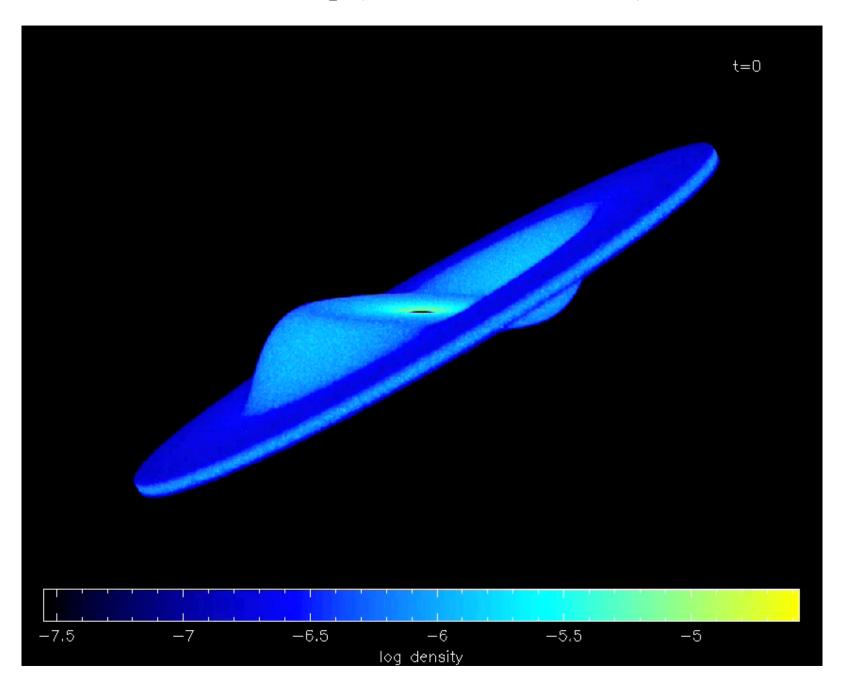
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#### warps

but disc is often *warped*: plane changes with radius, often because accretor is not purely spherical, e.g. accretor is a spinning black hole (Lense-Thirring effect) or binary black hole (quadrupole)

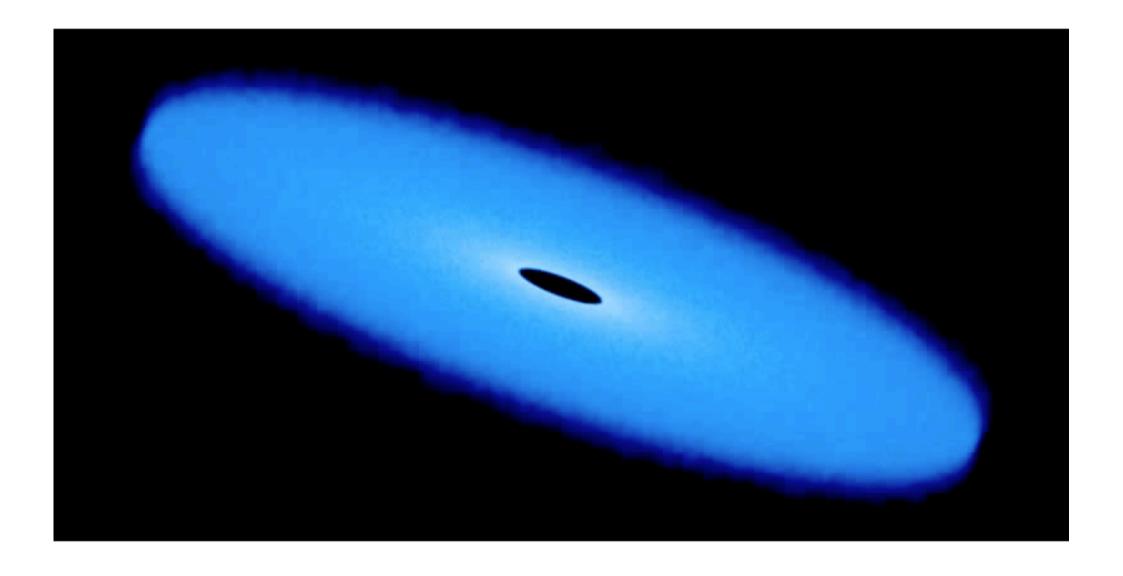
often the disc can accomodate this in a steady warp ('Bardeen - Petterson effect')

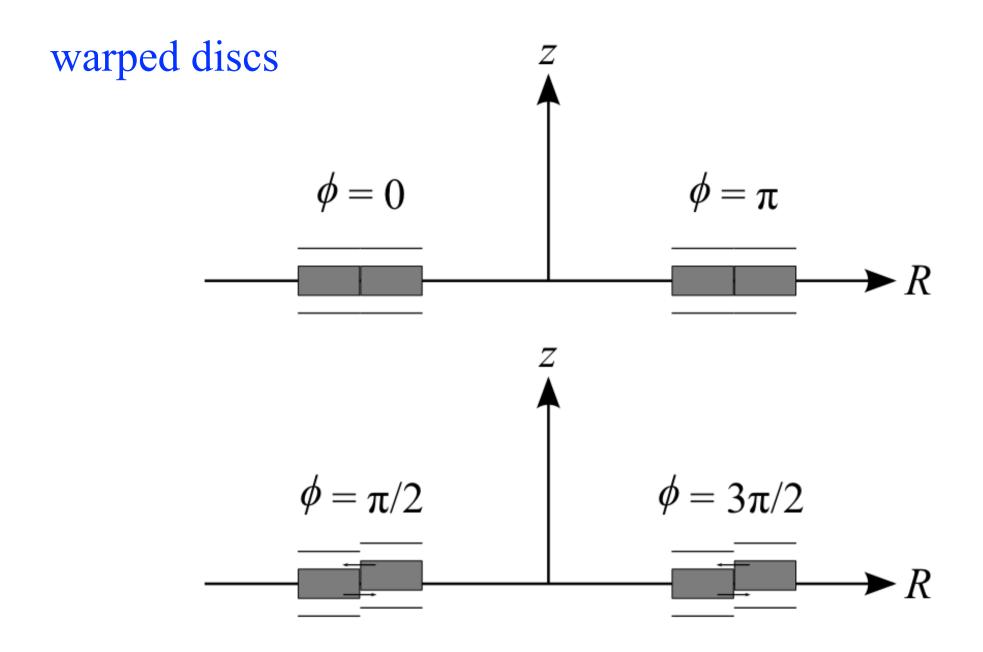
### assumed warp (Lodato & Price 2010)



strong warp, significant viscosity

induced warp: Lense-Thirring with small tilt (Nixon & King, 2011)





a warped disc: the shaded areas have higher pressure: arrows show pressure gradients induced by the warp: an orbiting fluid element feels a phase-dependent pressure gradient whose amplitude is a function of height

## warped discs

but pressure oscillations are *resonant* (epicyclic freq) => large effect

result: if viscosity is locally isotropic, forces trying to hold the disc together actually *weaken* for larger warps

so for a sufficiently large amplitude warp the disc *breaks* 

# general nonlinear theory

Equation (38) at  $O(\epsilon^{s+3})$ :

$$\left(\Omega \partial_{\phi} - \frac{v_{\theta 1}}{r} \partial_{\zeta}\right) p_{1} + \left(v_{r 1} \partial_{r} - \frac{v_{\theta 2}}{r} \partial_{\zeta} + \frac{v_{\phi 1}}{r} \partial_{\phi}\right) p_{0} = \frac{\Gamma p_{1}}{r} \partial_{\zeta} v_{\theta 1} - \Gamma p_{0} \left[\frac{1}{r^{2}} \partial_{r} (r^{2} v_{r 1}) - \frac{1}{r} \partial_{\zeta} v_{\theta 2} + \frac{1}{r} \partial_{\phi} v_{\phi 1}\right]. \tag{74}$$

Equation (41) at  $O(\epsilon^{s+2})$ :

$$\begin{split} &\rho_0 \left( \Omega \partial_{\phi} - \frac{v_{\theta 1}}{r} \partial_{\varsigma} \right) v_{r2} + \rho_0 \left( v_{r1} \partial_r - \frac{v_{\theta 2}}{r} \partial_{\varsigma} + \frac{v_{\phi 1}}{r} \partial_{\phi} \right) v_{r1} + \rho_1 \left( \Omega \partial_{\phi} - \frac{v_{\theta 1}}{r} \partial_{\varsigma} \right) v_{r1} - \frac{\rho_0}{r} \left[ v_{\theta 1} + r v_{r1} (\beta' \cos \phi + \gamma' \sin \beta \sin \phi) \right]^2 \\ &- 2\rho_0 \Omega \left[ -\frac{1}{2} r \Omega_{\varsigma}^{\varsigma 2} + v_{\phi 2} + r (\dot{\gamma} + v_{r2} \gamma') \cos \beta - r v_{r1} (\beta' \sin \phi - \gamma' \sin \beta \cos \phi) \dot{\varsigma} \right] - \frac{\rho_0}{r} (v_{\phi 1} + r v_{r1} \gamma' \cos \beta)^2 \\ &- 2\rho_1 \Omega (v_{\phi 1} + r v_{r1} \gamma' \cos \beta) = - (\beta' \cos \phi + \gamma' \sin \beta \sin \phi) \partial_{\varsigma} \left\{ \rho_1 - \left( \mu_{b0} + \frac{1}{3} \mu_0 \right) \left[ \frac{1}{r^2} \partial_r (r^2 v_{r1}) - \frac{1}{r} \partial_{\varsigma} v_{\theta 2} + \frac{1}{r} \partial_{\phi} v_{\phi 1} \right] \right. \\ &+ \left. \left( \mu_{b1} + \frac{1}{3} \mu_1 \right) \frac{1}{r} \partial_{\varsigma} v_{\theta 1} \right\} - (\partial_r - \gamma' \cos \beta \partial_{\phi}) \left[ \rho_0 + (\mu_{b0} + \frac{1}{3} \mu_0) \frac{1}{r} \partial_{\varsigma} v_{\theta 1} \right] \\ &+ \left. \left[ \frac{1}{r^2} + (\beta' \cos \phi + \gamma' \sin \beta \sin \phi)^2 \right] \partial_{\varsigma} (\mu_0 \partial_{\varsigma} v_{r2} + \mu_1 \partial_{\varsigma} v_{r1}) + (\beta' \cos \phi + \gamma' \sin \beta \sin \phi) \partial_{\varsigma} \left[ \mu_0 (\partial_r - \gamma' \cos \beta \partial_{\phi}) v_{r1} \right] \right. \\ &+ \frac{1}{r^2} (\partial_r - \gamma' \cos \beta \partial_{\phi}) \left[ \mu_0 r^2 (\beta' \cos \phi + \gamma' \sin \beta \sin \phi) \partial_{\varsigma} v_{r1} \right] - \frac{2v_{r1}}{r} (\beta' \cos \phi + \gamma' \sin \beta \sin \phi) \partial_{\varsigma} \mu_0 \\ &+ \frac{1}{r^3} \left[ (\partial_r - \gamma' \cos \beta \partial_{\phi}) (\mu_0 r^2) \right] \partial_{\varsigma} \left[ v_{\theta 1} + r v_{r1} (\beta' \cos \phi + \gamma' \sin \beta \sin \phi) \right] \\ &- \left. \left( \partial_{\varsigma} \mu_0 (\partial_r - \gamma' \cos \beta \partial_{\phi}) \left[ \frac{v_{\theta 1}}{r} + v_{r1} (\beta' \cos \phi + \gamma' \sin \beta \sin \phi) \right] \right. \\ &+ \frac{1}{r} (\partial_{\varsigma} \mu_0) \partial_{\phi} \left[ (\beta' \cos \phi + \gamma' \sin \beta \sin \phi) (v_{\phi 1} + r v_{r1} \gamma' \cos \beta) \right] + \Omega' \partial_{\phi} \mu_0 \\ &+ \frac{1}{r} (\beta' \cos \phi + \gamma' \sin \beta \sin \phi) \partial_{\phi} v_{\theta 1} + r v_{r1} (\beta' \cos \phi + \gamma' \sin \beta \cos \phi) \partial_{\varsigma} v_{\theta 1}. \end{split}$$

Equation (42) at  $O(\epsilon^{s+2})$ :

$$\begin{split} &\rho_0 \Big( \Omega \partial_\phi - \frac{v_{\theta 1}}{r} \partial_\zeta \Big) \Big[ v_{\theta 2} + r (\dot{\beta} + v_{r2} \beta') \cos \phi + r (\dot{\gamma} + v_{r2} \gamma') \sin \beta \sin \phi \Big] \\ &+ \rho_0 \Big( v_{r1} \partial_r - \frac{v_{\theta 2}}{r} \partial_\zeta + \frac{v_{\phi 1}}{r} \partial_\phi \Big) \Big[ v_{\theta 1} + r v_{r1} (\beta' \cos \phi + \gamma' \sin \beta \sin \phi) \Big] \end{split}$$

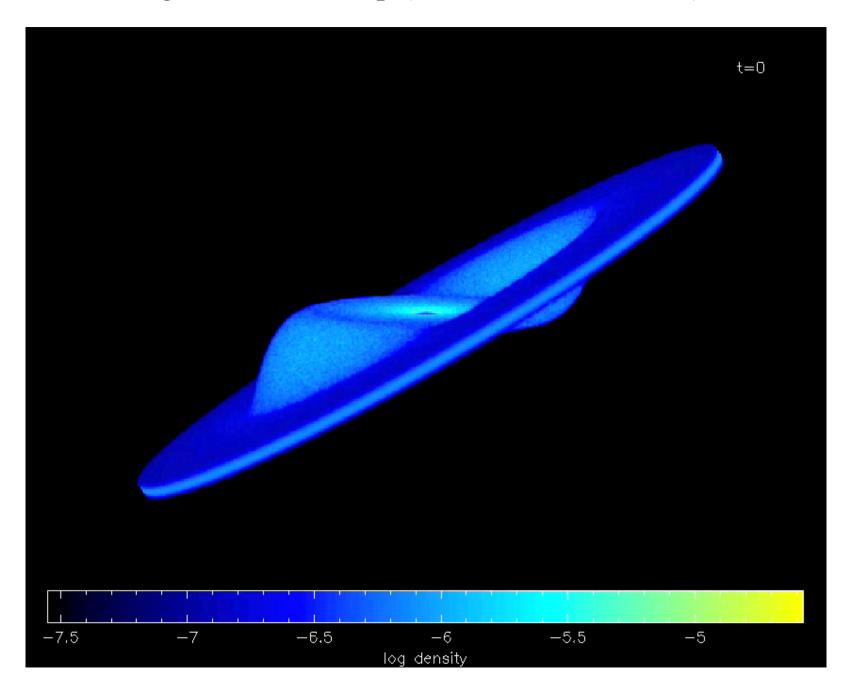
$$-\frac{1}{r}(\partial_{\zeta}\mu_{0})\partial_{\phi}\left[(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)(v_{\phi 1} + rv_{r1}\gamma'\cos\beta)\right] + \Omega'\partial_{\phi}\mu_{0}$$

$$+\frac{1}{r}(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)(\partial_{\phi}\mu_{0})\partial_{\zeta}(v_{\phi 1} + rv_{r1}\gamma'\cos\beta) + \Omega(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)\partial_{\zeta}\mu_{1}.$$
(75)

Equation (42) at  $O(\epsilon^{s+2})$ :

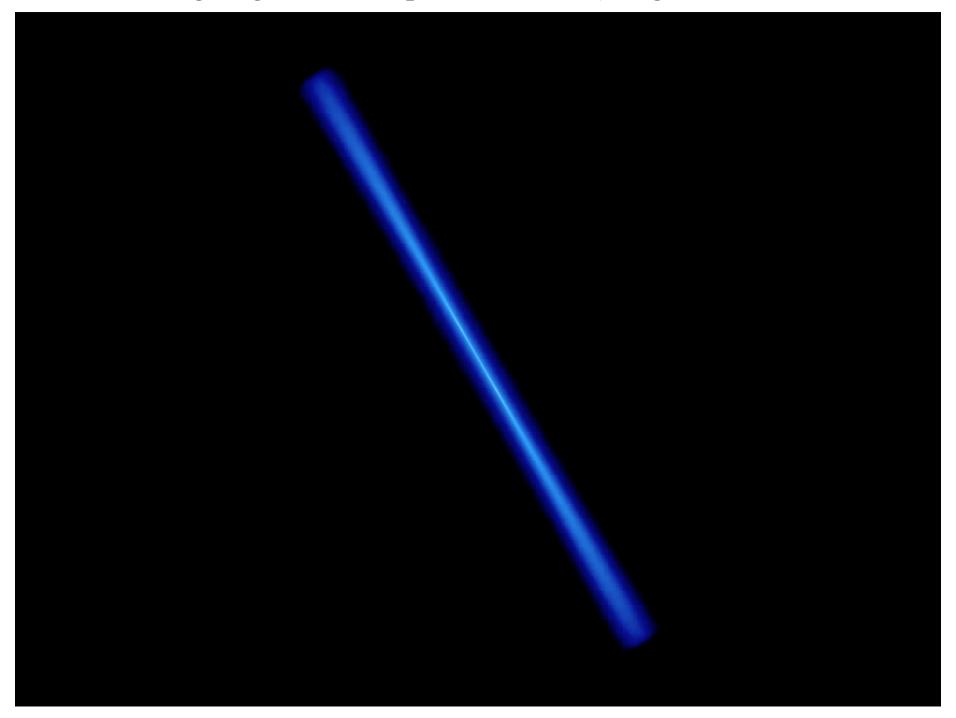
$$\begin{split} &\rho_0\left(\Omega\partial_\phi - \frac{v_{\theta_1}}{r}\partial_\xi\right)\left[v_{\theta_2} + r(\beta + v_{r2}\beta')\cos\phi + r(\gamma + v_{r2}\gamma')\sin\beta\sin\phi\right] \\ &+ \rho_0\left(v_{r1}\partial_r - \frac{v_{\theta_2}}{r}\partial_\xi + \frac{v_{\theta_1}}{r}\partial_\phi\right)\left[v_{\theta_1} + rv_{r1}(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\right] \\ &+ \rho_1\left(\Omega\partial_\phi - \frac{v_{\theta_1}}{r}\partial_\xi\right)\left[v_{\theta_1} + rv_{r1}(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\right] + \frac{\rho_0v_{r1}}{r}\left[v_{\theta_1} + rv_{r1}(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\right] \\ &- \rho_0\Omega\left[\left(v_{\phi_1} + rv_{r1}\gamma'\cos\beta\right)\xi + r(\beta + v_{r2}\beta')\sin\phi - r(\gamma + v_{r2}\gamma')\sin\beta\cos\phi\right] \\ &- \frac{\rho_0}{r}\left(v_{\phi_1} + rv_{r1}\gamma'\cos\beta\right)\left[r\Omega_\xi' + rv_{r1}(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)\right] - \rho_1\Omega\left[r\Omega_\xi' + rv_{r1}(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)\right] \\ &= \frac{1}{r}\partial_\xi\left\{p_1 - \left(\mu_{b0} + \frac{1}{3}\mu_0\right)\left[\frac{1}{r^2}\partial_r(r^2v_{r1}) - \frac{1}{r}\partial_\xi v_{\theta_2} + \frac{1}{r}\partial_\phi v_{\phi_1}\right] + \left(\mu_{b1} + \frac{1}{3}\mu_1\right)\frac{1}{r}\partial_\xi v_{\theta_1}\right\} \\ &+ \left[\frac{1}{r^2} + (\beta'\cos\phi + \gamma'\sin\beta\sin\phi)^2\right]\partial_\xi\left\{\mu_0\partial_\xi\left[v_{\theta_2} + r(\beta + v_{r2}\beta')\cos\phi + r(\gamma + v_{r2}\gamma')\sin\beta\sin\phi\right] \\ &+ \mu_1\partial_\xi\left[v_{\theta_1} + rv_{r1}(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\right]\right\} \\ &+ \left[\frac{1}{r^2}(\partial_r - \gamma'\cos\beta\partial_\phi)\left\{\mu_0r^2(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\partial_\xi\left[v_{\theta_1} + rv_{r1}(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\right]\right\} \\ &+ \frac{1}{r^2}(\partial_r - \gamma'\cos\beta\partial_\phi)\left\{\mu_0r^2(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\partial_\xi\left[v_{\theta_1} + rv_{r1}(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\right]\right\} \\ &- \frac{1}{r}(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\partial_\xi\mu_0\left[v_{\theta_1} + rv_{r1}(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\right] - \frac{1}{r^3}(\partial_\xi v_{r1})(\partial_r - \gamma'\cos\beta\partial_\phi)(\mu_0r^2) \\ &+ \frac{1}{r}(\partial_\xi\mu_0)(\partial_r - \gamma'\cos\beta\partial_\phi)v_{r1} + \frac{1}{r^2}(\partial_\xi\mu_0)\partial_\phi(v_{\phi_1} + rv_{r1}\gamma'\cos\beta) \\ &- \frac{1}{r^2}(\partial_\phi\mu_0)\partial_\xi(v_{\phi_1} + rv_{r1}\gamma'\cos\beta) - (\beta'\sin\phi-\gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\partial_\xi\left[\mu_0(v_{\phi_1} + rv_{r1}\gamma'\cos\beta)\right] \\ &- r\Omega(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi)\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 \\ &- r\Omega(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 \\ &- \rho_0(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 \\ &- \rho_0(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 \\ &- \rho_0(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\cos\phi) \\ &- \rho_0(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi)\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 \\ &- \rho_0(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\cos\phi) \\ &- \rho_0(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\cos\phi) \\ &- \rho_0(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\cos\phi) \\ &- \rho_0(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\sin\phi\partial_\xi\mu_0 + \gamma'\sin\beta\cos\phi) \\ &- \rho_0(\beta'\sin\phi - \gamma'\sin\beta\cos\phi)(\beta'\cos\phi + \gamma'\sin\beta\sin\phi\partial_\xi\mu$$

larger assumed warp (Lodato & Price, 2010)



strong warp, viscosity relatively weaker: disc breaks!

Lense-Thirring: big tilt ==> rapid accretion (King & Nixon 2012



#### broken discs tear

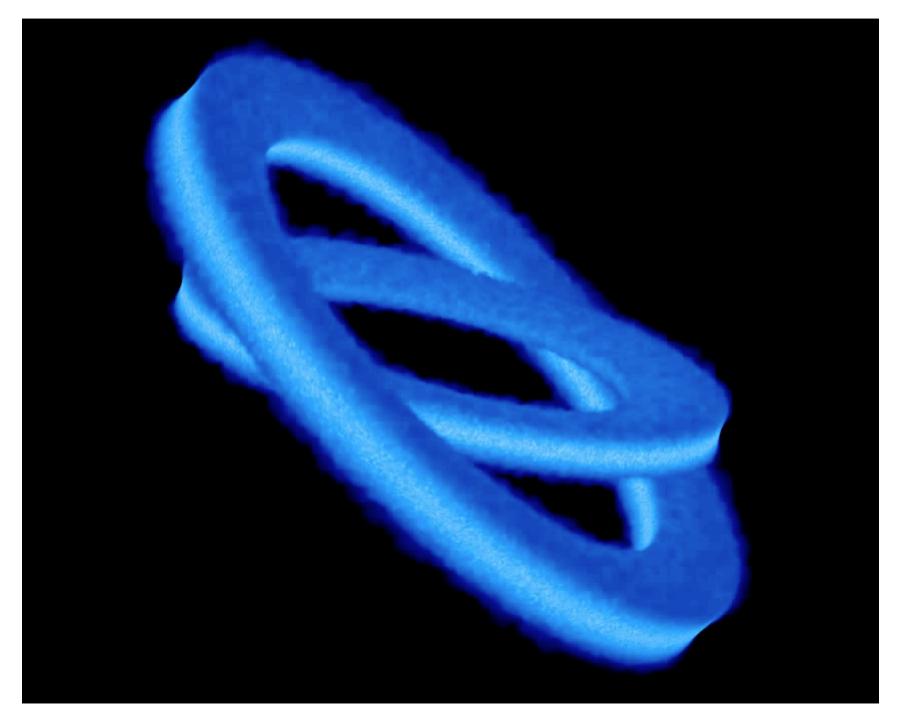
why does disc behave in this complex way? why is accretion so rapid?

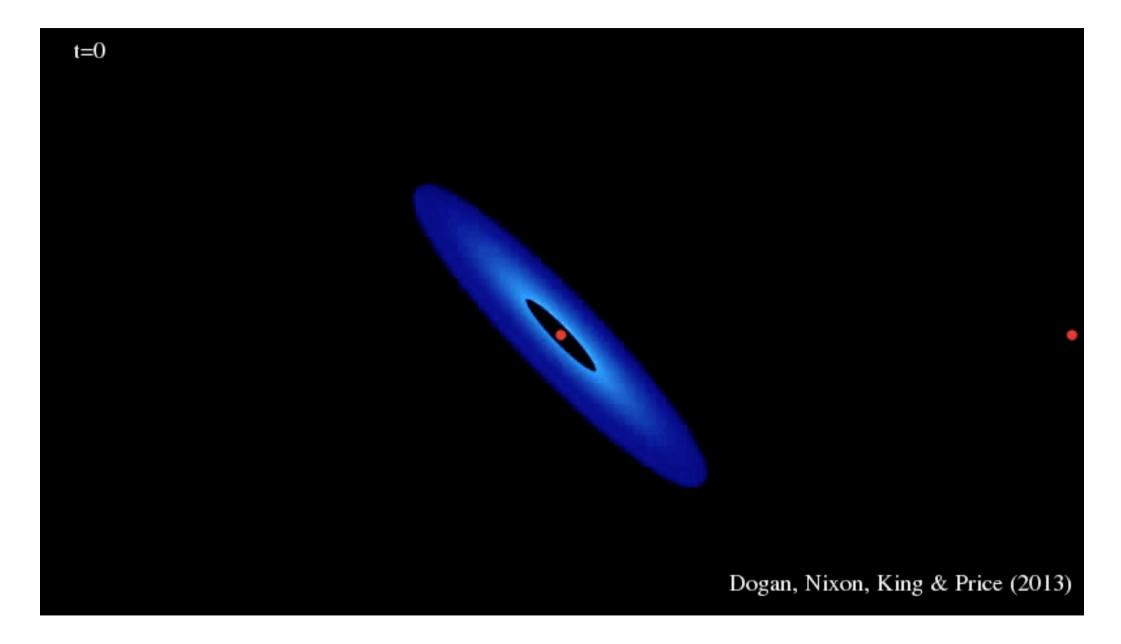
inclined discs => disc breaking

now broken disc components precess independently

=> opposed accretion => rapid accretion: 'tearing'

rapid accretion: counterrotating discs (Nixon & King, 2012)





# warped, broken and tearing discs

mild warps => smooth bending of disc plane (Bardeen-Petterson effect)

but for a sufficiently large (few degrees) warp the disc breaks

broken disc components precess into opposition

rapid (dynamical) accretion

disc gas has borrowed angular momentum from the accretor in order to cancel its own, and so fall in dynamically

can overcome angular momentum barrier to black hole growth

many more consequences, e.g. state changes in X-ray binaries, merging of supermassive black hole binaries......

accretion produces radiation: radiation makes pressure – can this inhibit further accretion?

radiation pressure acts on electrons; but electrons and ions (protons) cannot separate because of Coulomb force: radiation pressure force on an electron is

$$F_{\rm rad} = \frac{L\sigma_T}{4\pi c r^2}$$

(in spherical symmetry).
gravitational force on electron—proton pair is

$$F_{\text{grav}} = \frac{GM(m_p + m_e)}{r^2} \simeq \frac{GMm_p}{r^2}, \ (m_p >> m_e)$$

# Eddington limit

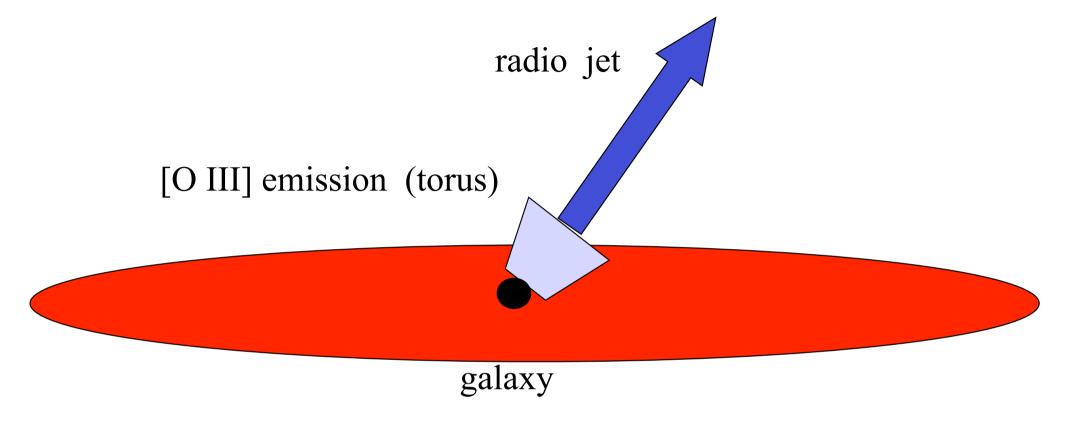
accretion is inhibited once  $F_{\rm rad} \geq F_{\rm grav}$ , i.e. once

$$L \ge L_{\rm Edd} = \frac{4\pi GMc}{\kappa} = 1.3 \times 10^{46} M_8 \text{ erg s}^{-1}$$

Eddington limit: luminosity requires minimum mass

$$(\kappa = \sigma_T/m_p = electron\ scattering\ opacity \simeq 0.34\ cm^2\ g^{-1})$$

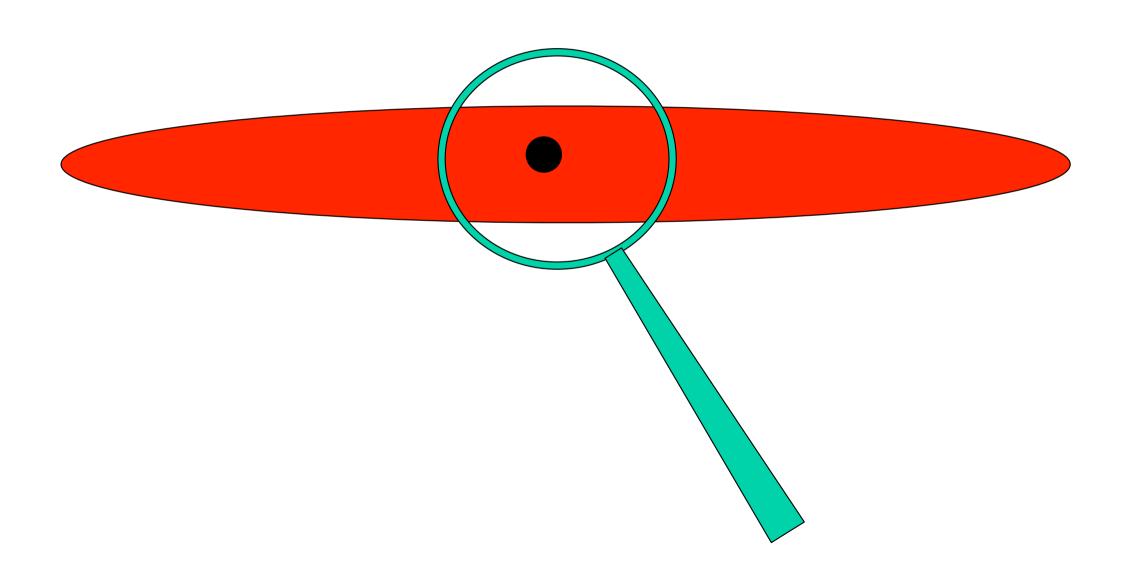
#### orientations



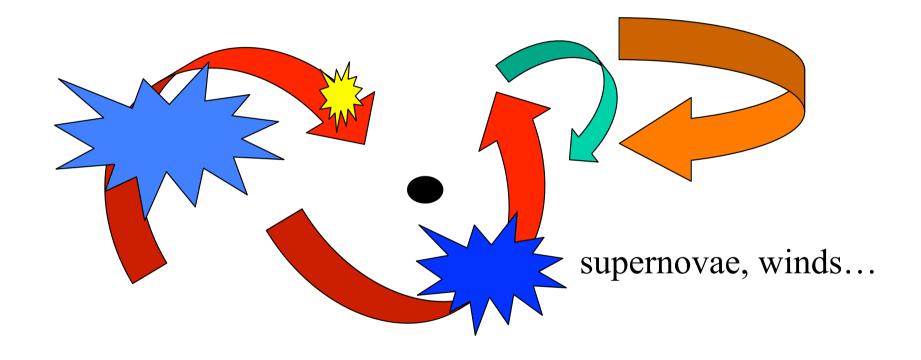
jet and torus directions correlate with each other, but are uncorrelated with galaxy major axis
(Kinney et al., 2000; Nagar & Wilson, 1999; Schmitt et al, 2003)

central disc flow has angular momentum unrelated to host accretion disc is 'warped' - centre and edge have different planes

# zoom in to nucleus



#### central accretion flow



chaotic – no relation to large—scale structure of host accretion is via a sequence of randomly oriented discs

huge range of length and mass scales: numerical treatment impossible

# black hole growth

can BH grow in line with galaxy?

can we grow masses  $M>5 imes10^9~M_{\odot}$ 

at redshifts z = 6 (Barth et al., 2003; Willott et al., 2003),

only 10<sup>9</sup> years after the Big Bang?

accretion rate limited by Eddington (radiation pressure) limit:

$$\eta c^2 \dot{M}_{\rm acc} \le L_{\rm Edd} = \frac{4\pi GMc}{\kappa}$$

and some of rest-mass energy goes into radiation, i.e.

$$\dot{M} = (1 - \eta)\dot{M}_{\rm acc}$$

$$=>\dot{M}\leq \frac{1-\eta}{\eta}\frac{M}{t_{\rm Edd}}$$

where

$$t_{\rm Edd} = \frac{\kappa c}{4\pi G} = 4.5 \times 10^8 \text{ yr}$$

SO

$$\frac{M}{M_0} \le \exp\left[-\left(\frac{1}{\eta} - 1\right) \frac{t}{t_{\rm Edd}}\right]$$

$$\frac{M}{M_0} \le \exp\left[\left(\frac{1}{\eta} - 1\right) \frac{t}{t_{\rm Edd}}\right]$$

SO

$$a = 1 => \eta = 0.42 => \frac{M}{M_0} \le 21$$

whereas

$$a = 0 => \eta = 0.057 => \frac{M}{M_0} \le 8 \times 10^{15} !$$

$$\frac{M}{M_0} \le \exp\left[\left(\frac{1}{\eta} - 1\right) \frac{t}{t_{\rm Edd}}\right]$$

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holes with high spin require `seeds' with masses > 0.05 x current mass — greater than some current SMBH!

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# holes with low spin can grow *much* faster — (and are easier to retain if they coalesce)

$$\frac{M}{M_0} \le \exp\left[\left(\frac{1}{\eta} - 1\right) \frac{t}{t_{\rm Edd}}\right]$$

SO

$$a = 1 => \eta = 0.42 => \frac{M}{M_0} \le 21$$
  $(e^3)$ 

whereas

$$a = 0 => \eta = 0.057 => \frac{M}{M_0} \le 8 \times 10^{15} !$$
 (e<sup>36</sup>)

holes with high spin require `seeds' with masses > 0.05 x current mass — greater than some current SMBH!

# holes with low spin can grow much faster –

(and are easier to retain if they coalesce)

so need to understand spin evolution of SMBH

$$\eta = 0.1$$
:

$$\frac{M}{M_0} \le \exp\left[2.2\left(\frac{1}{\eta} - 1\right)\right]$$

$$=e^{19.8}$$

$$= 4 \times 10^8$$

so the standard 'ballpark'  $\eta=0.1$  is borderline for growing large QSO masses at z=6 from stellar seeds, with maximal feeding

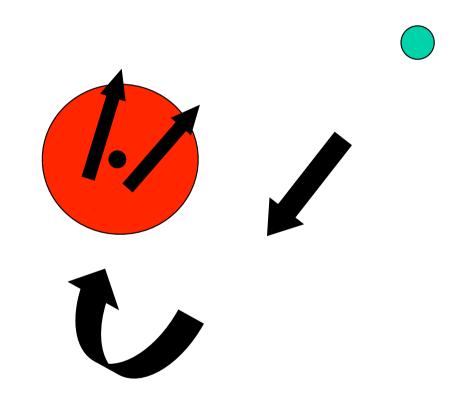
#### accretion to central black hole

central object gains a.m. and spins up

 $\rightarrow$  reaches maximum spin rate  $a \sim 1$  after accreting  $\sim M$ , if accretion always has same sense

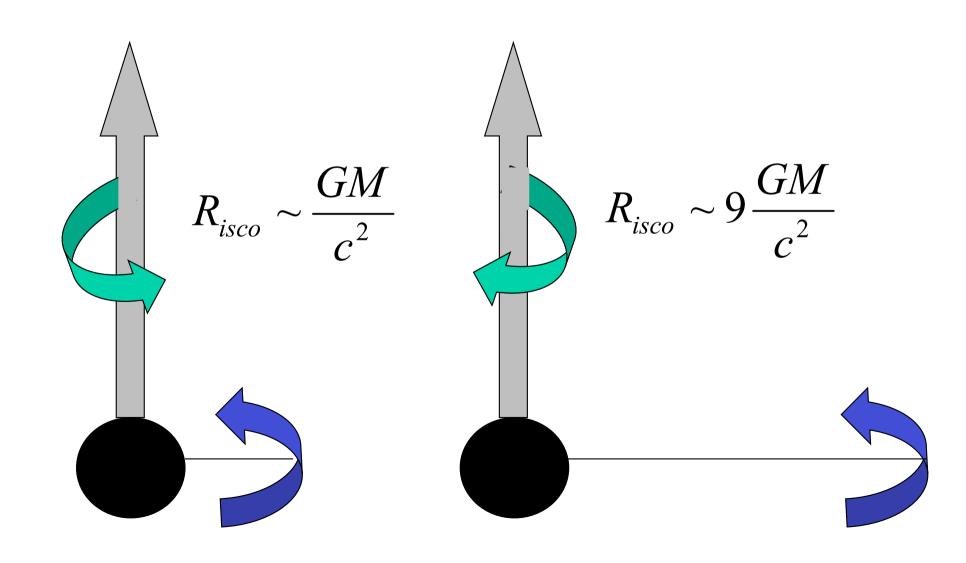
hole gains mass significantly – does it spin up?

# gas infall can be both prograde and retrograde



with equal probability

accreting (or coalescing) from a retrograde orbit has a bigger effect since last stable orbit has larger lever arm than prograde one



#### BH area theorem

BH event horizon area is

$$A = 8\pi \left(\frac{GM}{c^2}\right)^2 \left[1 + (1 - a^2)^{1/2}\right]$$

this behaves like (is!) the *entropy* of the hole - so A cannot decrease

e.g. spin up from a=0 to a=1:M must increase to prevent decrease of A - rotational energy adds to mass!

spindown - extraction of rotational energy, so M decreases: but cannot decrease so much that area A drops, hence maximum mass decrease is from M to  $M/\sqrt{2}$  (as a goes from 1 to 0)

maximum rotational energy extraction is  $(1 - 1/\sqrt{2})Mc^2 \simeq 0.29Mc^2$ 

thus can give up angular momentum and still increase area, i.e. release rotational energy – e.g. as gravitational radiation

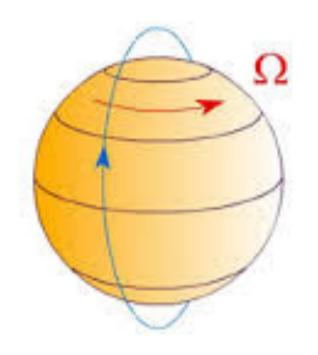
then mass M decreases!

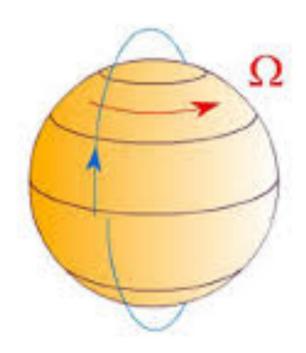
BH coalescence can be both prograde and retrograde wrt spin of merged hole, i.e. orbital opposite to spin a.m.

long—term effect of black-hole coalescences is spindown, since last stable circular orbit has larger a.m. in retrograde case.

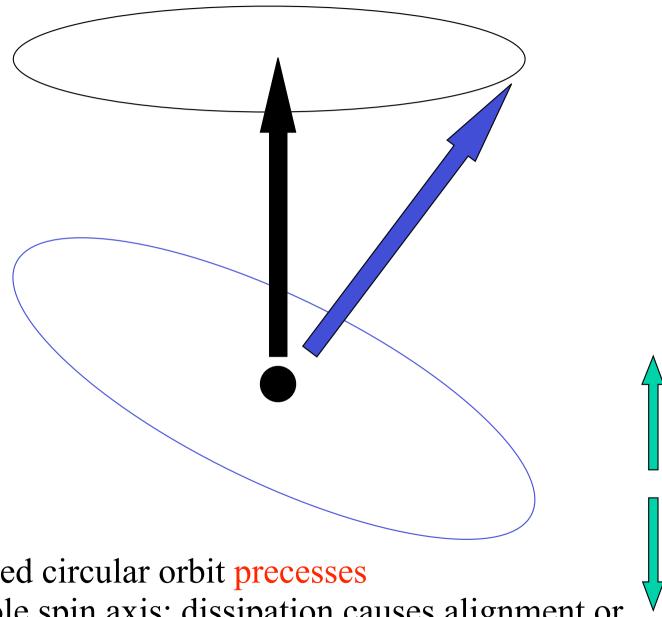
- black hole coalescences cause net spindown because of this
- is this true of accretion?
- actually *yes*, but disc warping effect complicates things

# Lense-Thirring effect





# Lense-Thirring effect



plane of inclined circular orbit precesses about black hole spin axis: dissipation causes alignment or counteralignment

### gas torques on hole:

- two main types: 1. accretion spinup or spindown hole mass has to double to change a significantly — slow
  - 2. Lense—Thirring from misaligned disc

viscous timescale — fast in inner disc

- old argument: alignment via Lense—Thirring occurs rapidly, hole spins up to keep a  $\sim 1$ , accretion efficiency is high
- but L—T also vanishes for counteralignment
- alignment or not? (King, Lubow, Ogilvie & Pringle 05)

torque on hole is pure precession, so orthogonal to spin.

thus general equation for spin evolution is

$$\frac{d\mathbf{J}_h}{dt} = -K_1[\mathbf{J_h} \wedge \mathbf{J_d}] - K_2[\mathbf{J}_h \wedge (\mathbf{J}_h \wedge \mathbf{J}_d)]$$

here  $K_1, K_2 > 0$  depend on disc properties: first term specifies precession, second alignment

clearly magnitude  $J_h$  is constant, and vector sum  $J_t$  of  $J_h$ ,  $J_d$  is constant. Thus  $J_t$  stays fixed, while tip of  $J_h$  moves on a sphere during alignment

using these, we have

$$\frac{d}{dt}(\mathbf{J}_h.\mathbf{J}_t) = \mathbf{J}_t.\frac{d\mathbf{J}_h}{dt} = \mathbf{J}_d.\frac{d\mathbf{J}_h}{dt}$$

SO

$$\frac{d}{dt}(\mathbf{J}_h.\mathbf{J}_t) = K_2[J_d^2J_h^2 - (\mathbf{J}_d.\mathbf{J}_h)^2] > 0 \quad \text{Exercise!}$$

but  $J_h, J_t$  are constant, so angle  $\theta_h$  between them obeys

$$\frac{d}{dt}(\cos\theta_h) > 0$$

— hole spin always aligns with total angular momentum

can further show that  $J_d^2$  always decreases during this process – dissipation

thus viewed in frame precessing with  $J_h$ ,  $J_d$ ,

 $J_t$  stays fixed:  $J_h$  aligns with it while keeping its length constant

 $J_d^2$  decreases monotonically because of dissipation

since if  $\theta$  is the angle between  $\mathbf{J}_h$  and  $\mathbf{J}_d$  the cosine theorem gives

$$J_t^2 = J_h^2 + J_d^2 + 2J_h J_d \cos(\theta)$$

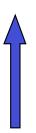
disc and hole a.m. counteralign  $(J_t^2 < J_h^2)$  if and only if

$$\cos\theta < -\frac{J_d}{2J_h}$$

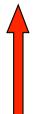
counteralignment occurs often if the disc's angular momentum is always comparable to or smaller than the hole's, i.e. in most cases where  $\mathbf{J}_d$  and  $\mathbf{J}_h$  are initially opposed (i.e.  $\theta > \pi/2$ )

older treatments assumed disc fixed, i.e.  $J_d \to \infty$  so  $J_t^2 > J_h^2$  always: so always found alignment!





$$J_d =$$



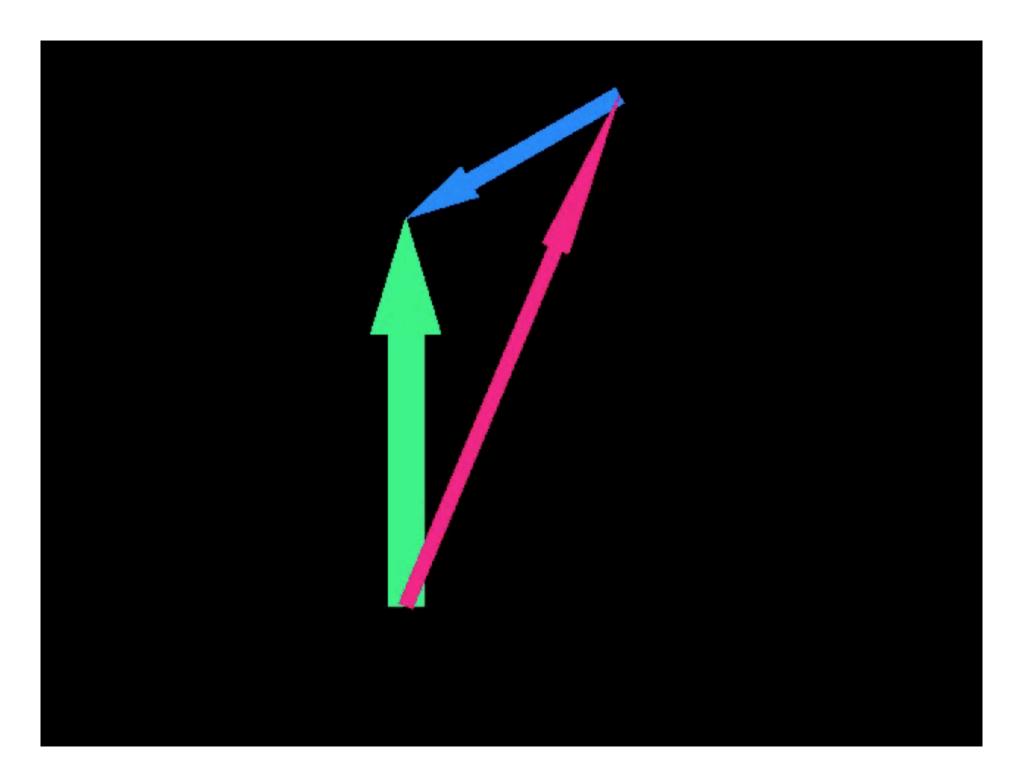
$$oldsymbol{J}_t = oldsymbol{J}_h + oldsymbol{J}_d =$$



so if

$$\cos \theta > -\frac{J_d}{2J_h}$$

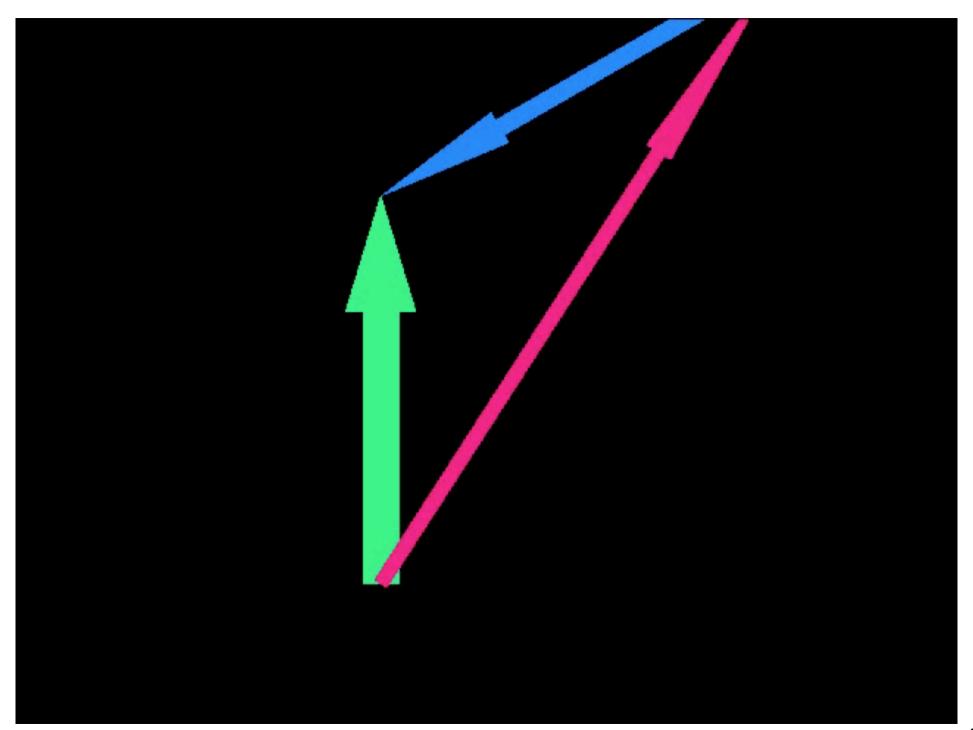
alignment follows

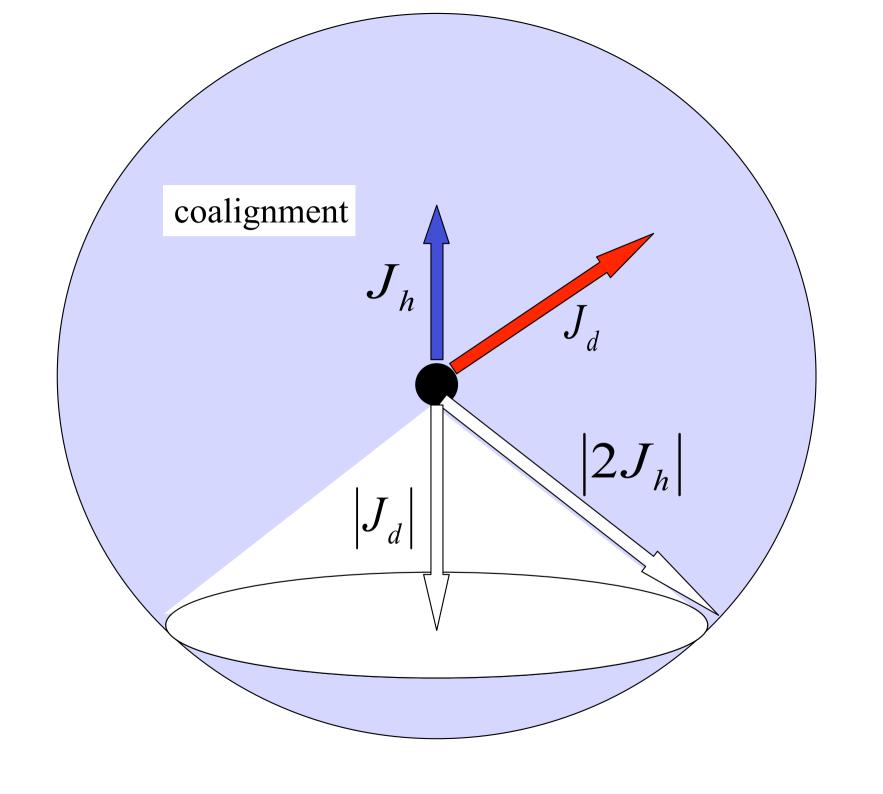


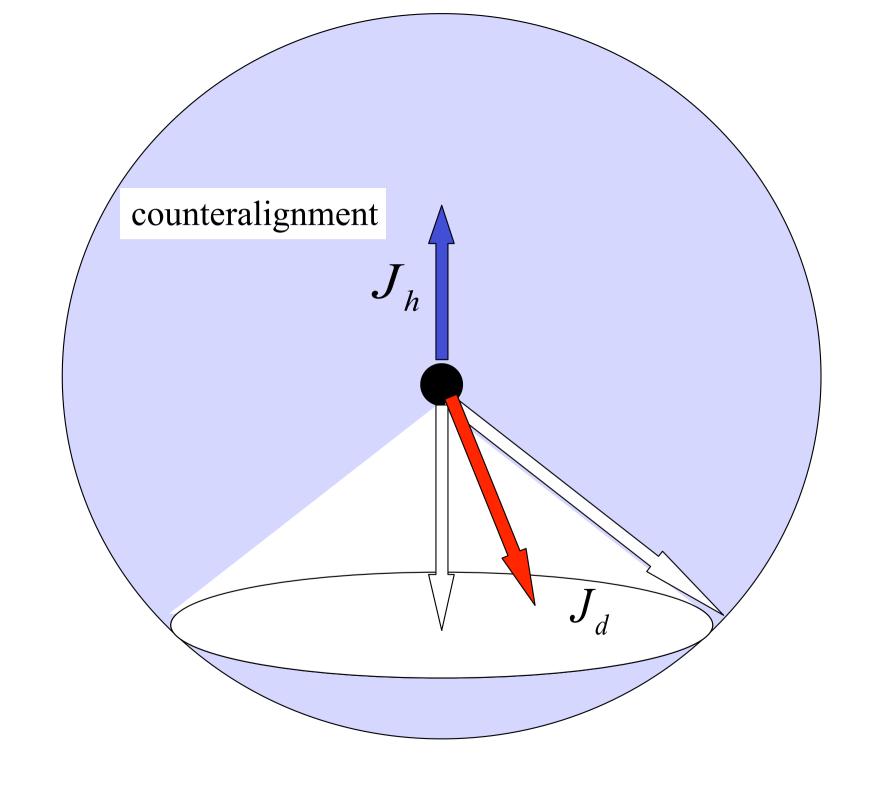
#### but if instead

$$\cos\theta < -\frac{J_d}{2J_h}$$

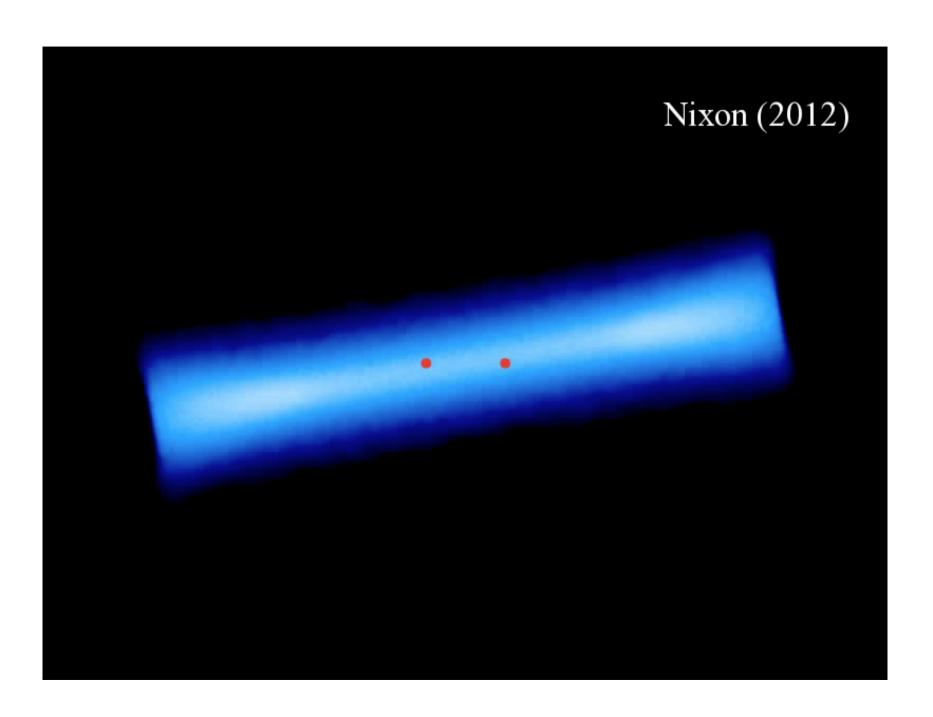
counteralignment follows







alignment/counteralignment with a binary: Nixon (2012)



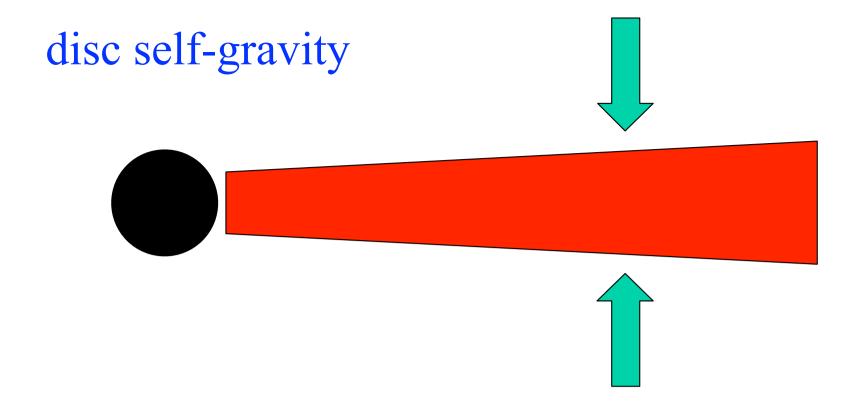
#### accretion to central black hole

central object gains a.m. and spins up

 $\rightarrow$  reaches maximum spin rate  $a \sim 1$  after accreting  $\sim M$ , if accretion always has same sense

hole gains mass significantly – does it spin up?

alignment/counteralignment depends on  $J_d/J_h$  so how large is this quantity?

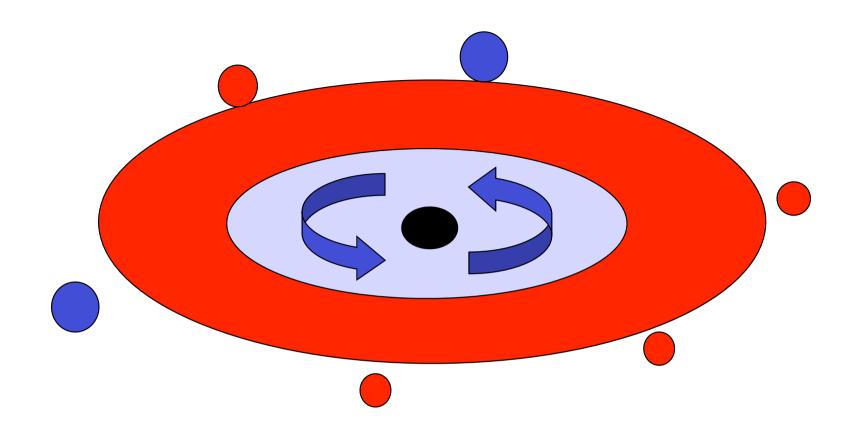


important if gravity force from local disc matter > BH tidal field, i.e.

$$\frac{G\rho H^3}{H^2} > \frac{GMH}{R^3} => \rho > \frac{M}{R^3}$$

so self gravity is important (disc may fragment) if

$$M_{\rm disc} > R^2 H \rho \gtrsim \frac{H}{R} M$$

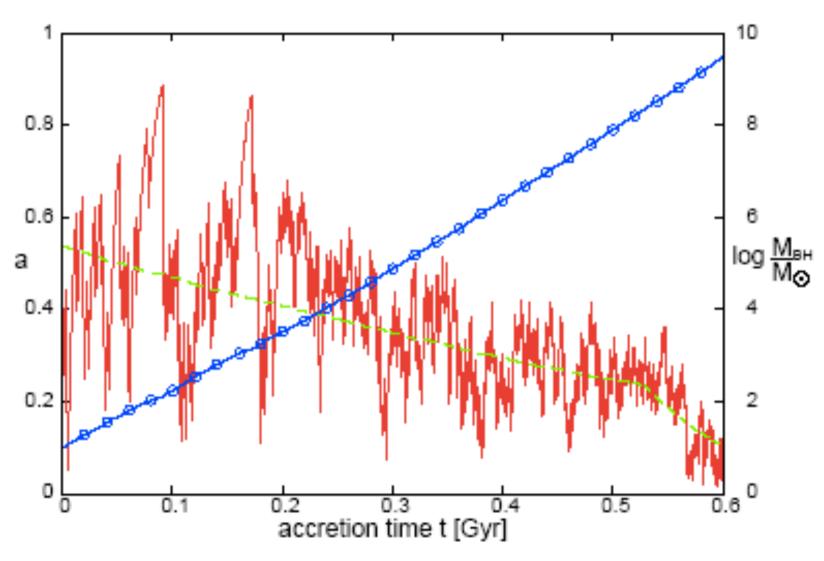


self—gravity limit on disc mass and size implies  $J_d/J_h < 1$  gas outside  $R_{\rm sg}$  forms stars and disconnects from the disc

condition 
$$\cos \theta < -\frac{J_d}{2J_h}$$
 is easy to satisfy

spinup and spindown alternate – spin becomes small

so given a sufficient mass supply, black holes can grow to observed high—redshift masses from small beginnings



King, Pringle & Hofmann, MNRAS 2008

- coalescences lift SMBH discontinuously above curve
- curve is an attractor, so SMBH return to it once they have doubled their masses by accretion
- doubling is unlikely for largest SMBH giant ellipticals
- so *some* of these galaxies *can* have SMBH with *high spin*

# how big can a black hole grow?

self-gravity radius is almost independent of parameters:

$$R_{\rm sg} = 3 \times 10^{14} \alpha_{0.1}^{14/27} \eta_{0.1}^{8/27} (L/L_{\rm Edd})^{-8/27} M_8^{1/27} \,\mathrm{m}$$

or

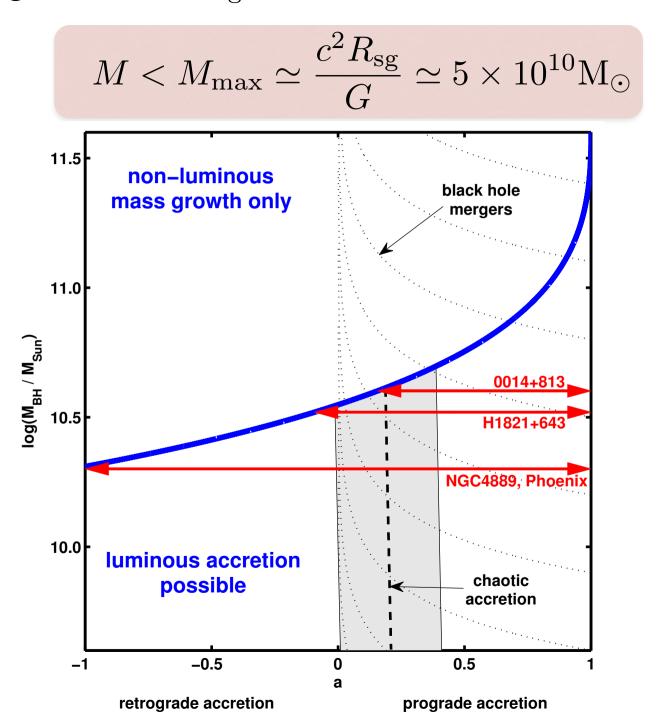
$$R_{\rm sg} \simeq C \simeq 3 \times 10^{14} \, \mathrm{m}$$

so disc radius must be smaller than this

but disc must be bigger than ISCO:

$$R_{\rm ISCO} = f(a) \frac{GM}{c^2} = 7.7 \times 10^{11} M_8 f_5 \,\mathrm{m}$$

requiring  $R_{\rm ISCO} < R_{\rm sg}$  shows that to have a luminous disc



#### SMBH – host connection?

SMBH in every large galaxy (Soltan)

but only a small fraction of galaxies are AGN

→ SMBH grow at Eddington rate in AGN

→ AGN should show outflows

# Eddington limit

accretion is inhibited once  $F_{\rm rad} \geq F_{\rm grav}$ , i.e. once

$$L \ge L_{\rm Edd} = \frac{4\pi GMc}{\kappa} = 1.3 \times 10^{46} M_8 \text{ erg s}^{-1}$$

Eddington limit: luminosity requires minimum mass

$$(\kappa = \sigma_T/m_p = electron\ scattering\ opacity \simeq 0.34\ cm^2\ g^{-1})$$

### Can a black hole *ignore* the Eddington limit?

accretion would *not* be limited by the Eddington rate

$$\dot{M}_{\mathrm{Edd}} = rac{L_{\mathrm{Edd}}}{\eta c^2}$$

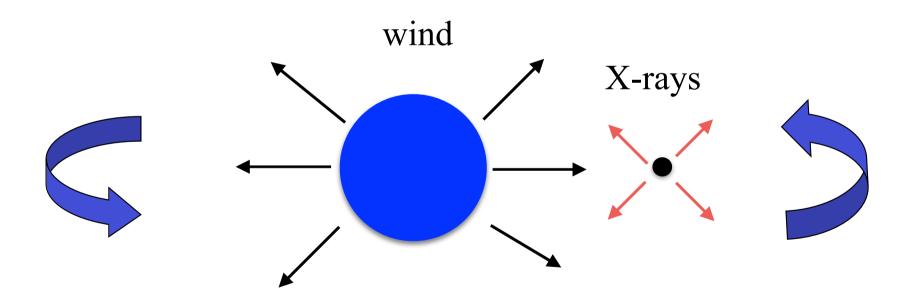
if radiation could somehow escape without pushing matter away.

radiative transfer calculations sometimes suggest this

### Can a black hole *ignore* the Eddington limit?

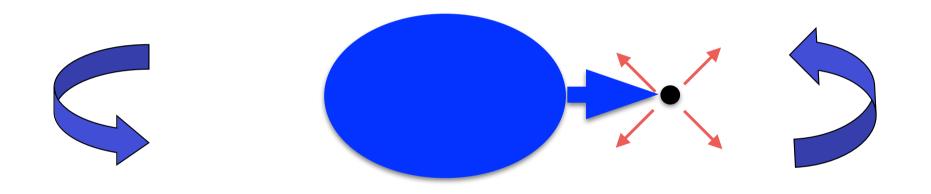
observational constraint: compact binary systems in our Galaxy do not do this

high-mass X-ray binary (HMXB): BH or NS accretes from stellar wind of blue supergiant



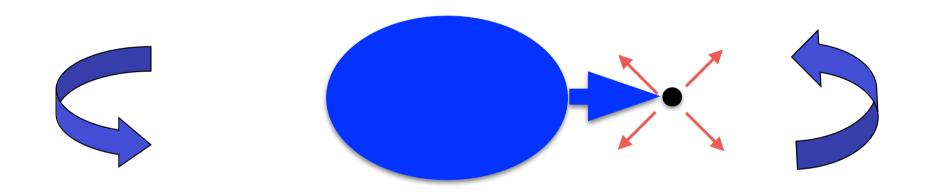
 $\sim 30$  such systems in the Galaxy: they live  $\sim 10^5\,\rm yr$  , with luminosities  $\sim 10^{37}\,\rm erg\,s^{-1}$ 

eventually supergiant fills its Roche lobe and transfers mass at a rate  $\sim 10^5\,{\rm M_\odot\,yr^{-1}}$  to the BH/NS, i.e  $\sim 10^2-10^3\dot{M}_{\rm Edd}$ : lifetime in this state is  $\sim 10^6\,{\rm yr}$ 



so if BH can ignore the Eddington limit, there should be  $\sim$  300 binaries in the Galaxy with  $L>>L_{\rm Edd}$ 

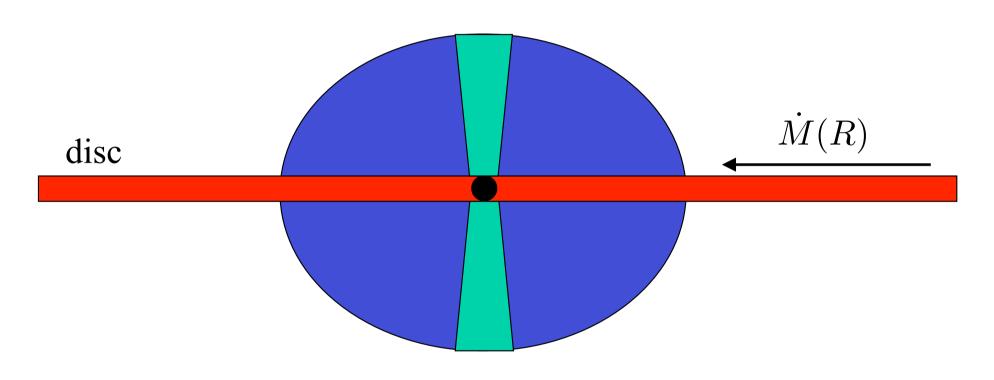
eventually supergiant fills its Roche lobe and transfers mass at a rate  $\sim 10^5 \, \rm M_{\odot} \, yr^{-1}$  to the BH/NS, i.e  $\sim 10^2 - 10^3 \dot{M}_{\rm Edd}$ : lifetime in this state is  $\sim 10^6 \, \rm yr$ 



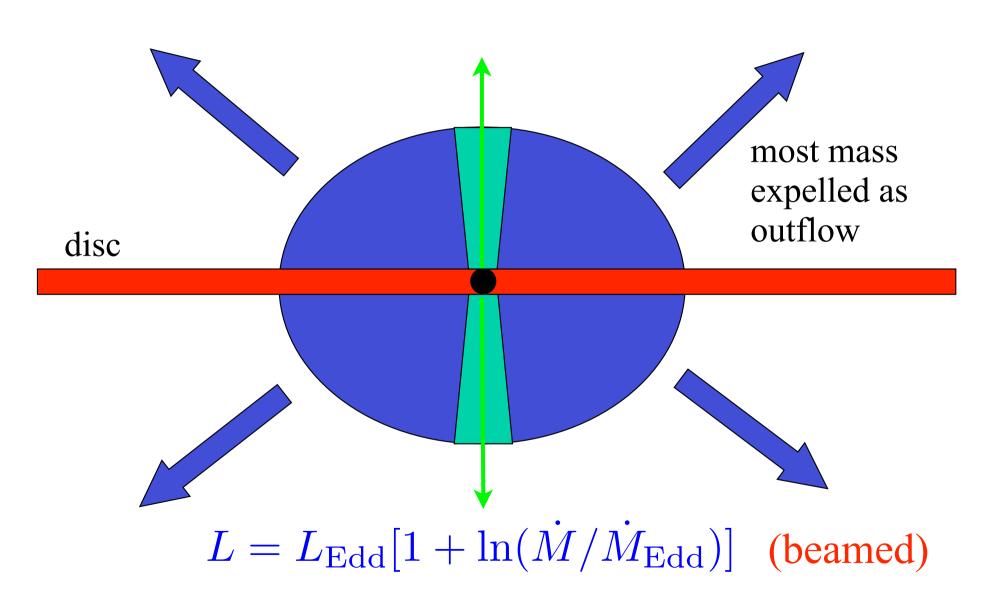
so if BH can ignore the Eddington limit, there should be  $\sim$  300 binaries in the Galaxy with  $L>>L_{\rm Edd}$  there are none:

# luminosities do not dramatically exceed $L_{\rm Edd}$ [ULXs are beamed, so *intrinsic* luminosities are $\lesssim L_{\rm Edd}$ ]

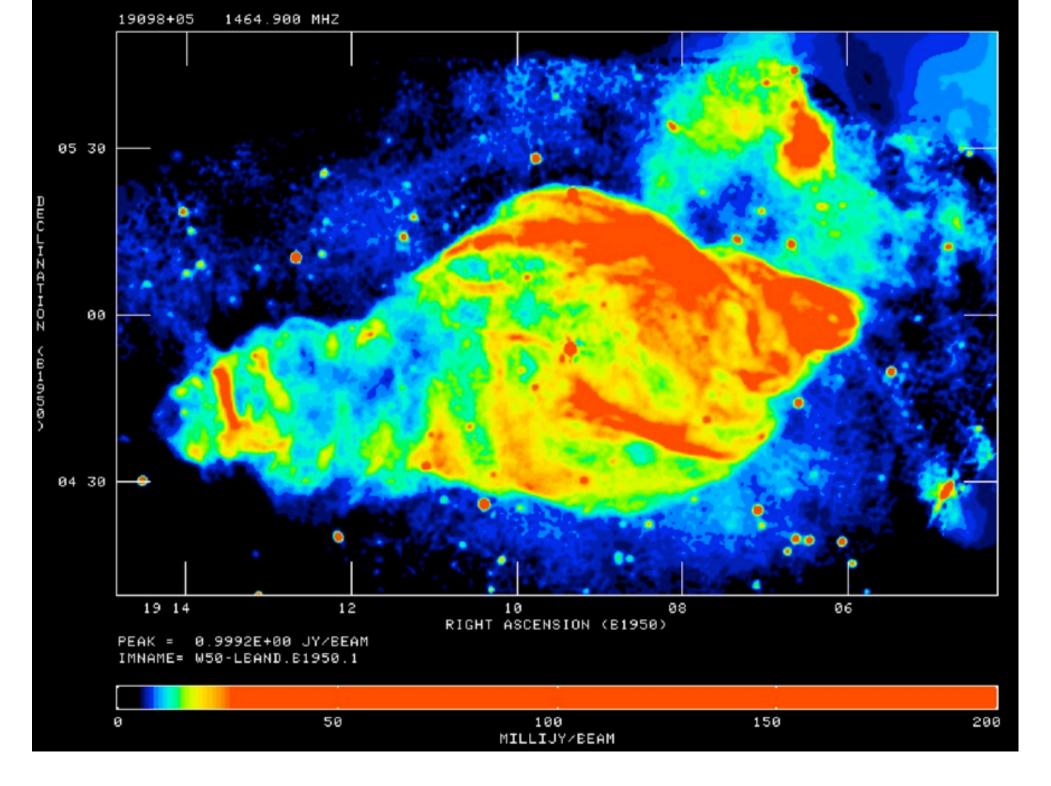
 $\dot{M}(R)$  adjusts to keep  $GM\dot{M}(R)/R = L_{\rm Edd}$ 

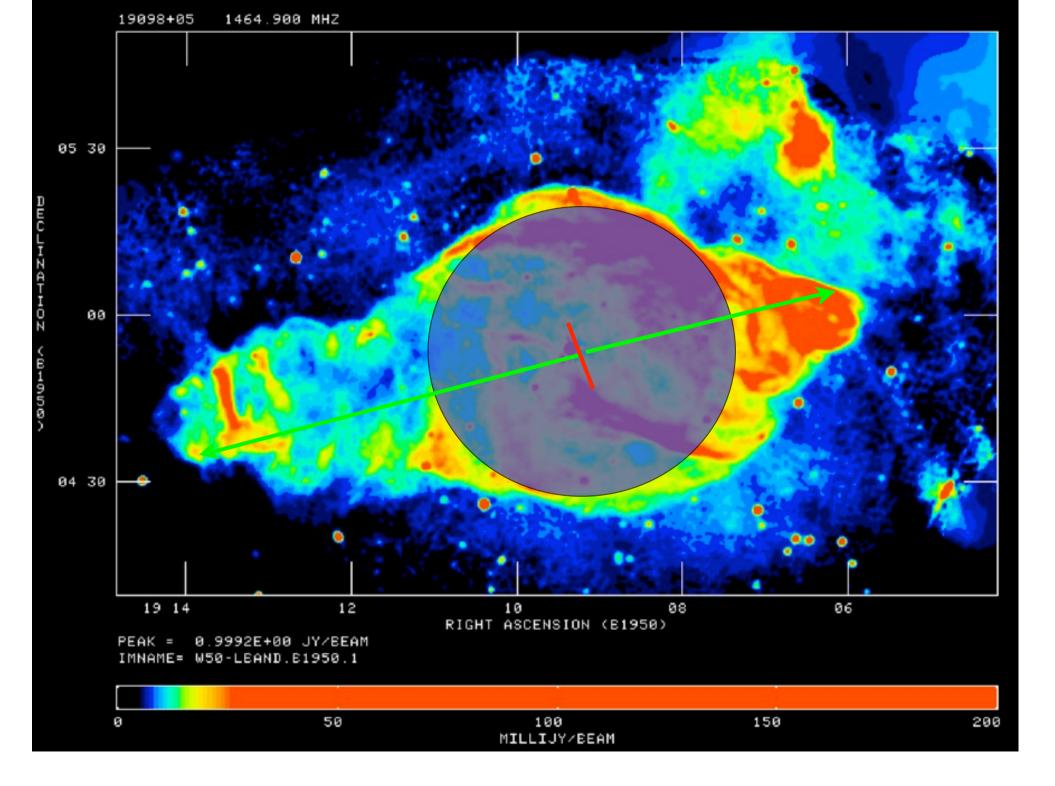


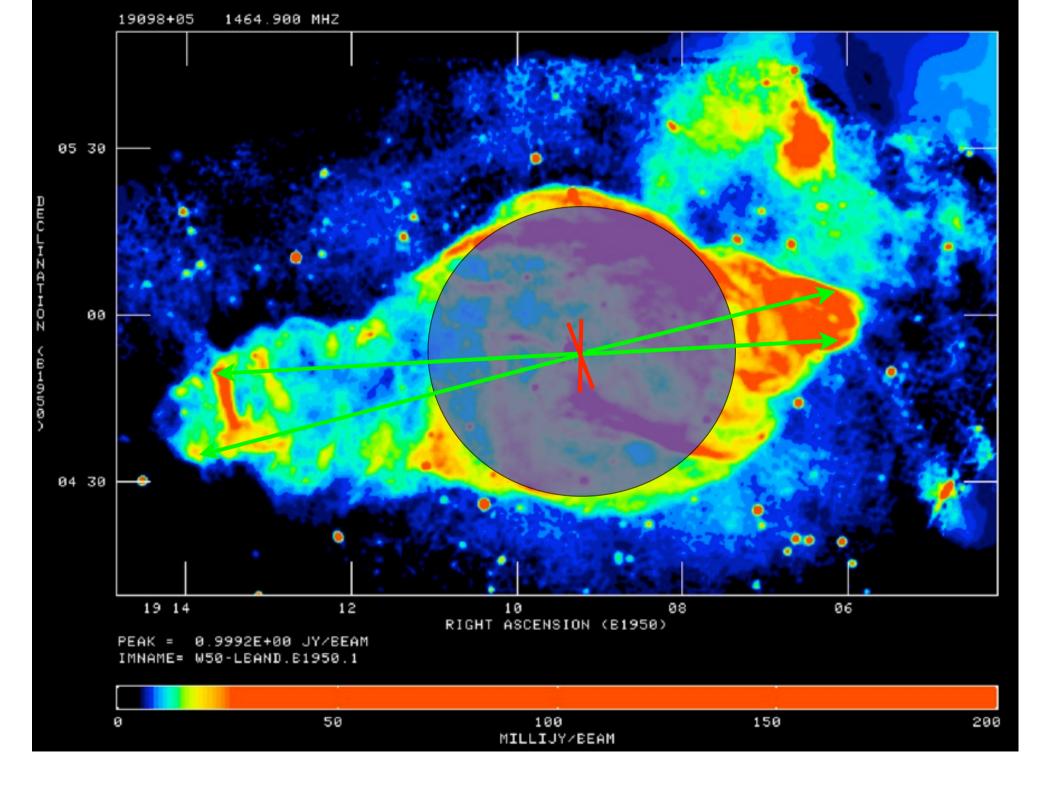
so 
$$\dot{M}(R) \propto R$$

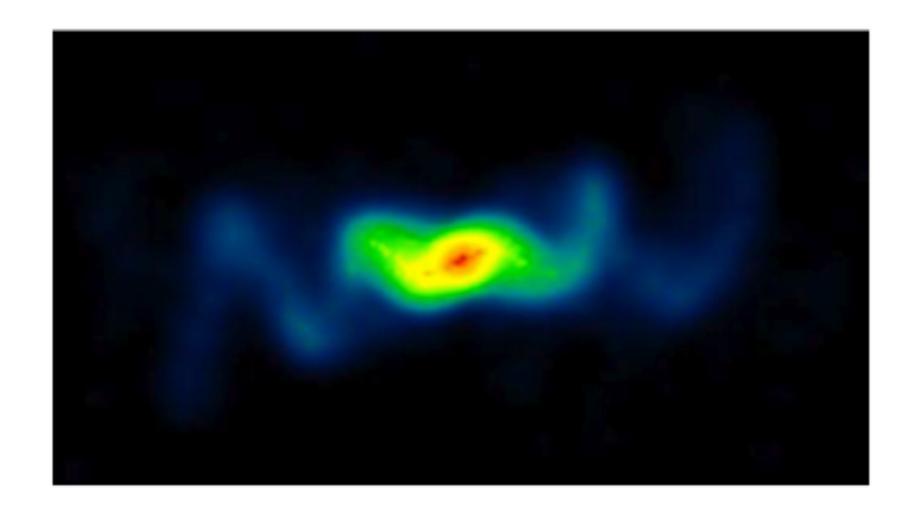


compare with SS433

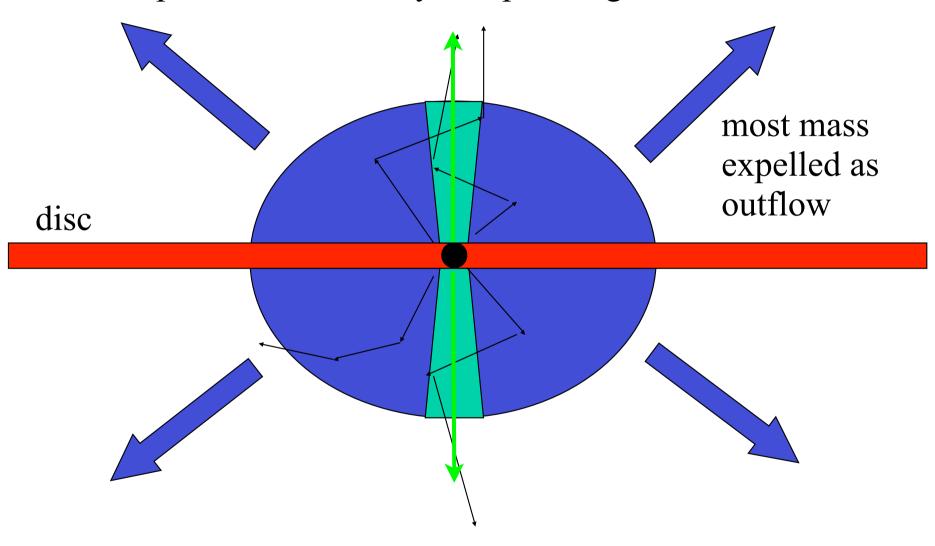




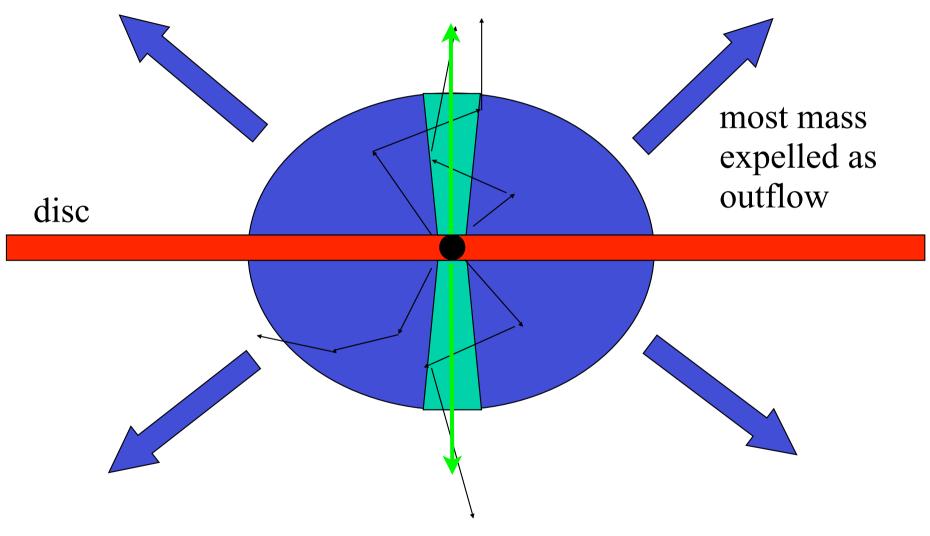




most photons eventually escape along cones near axis

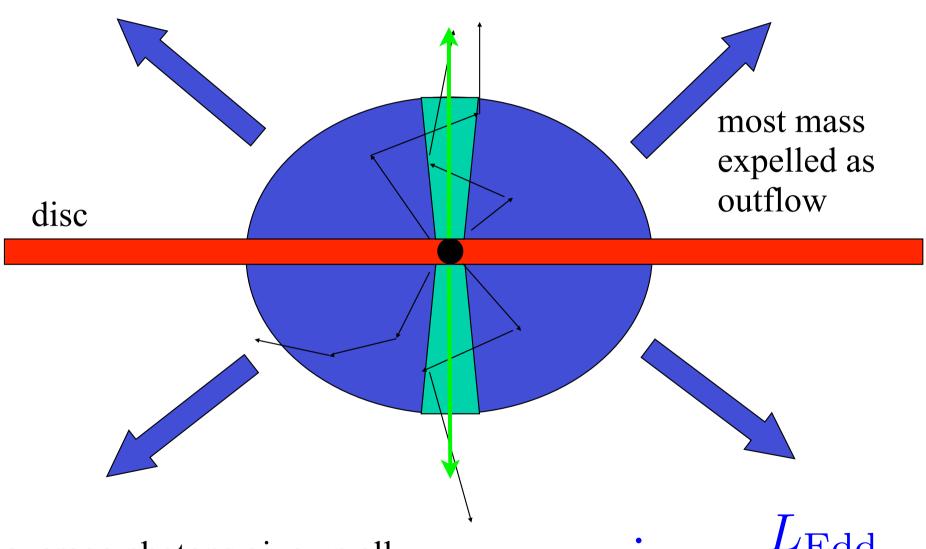


most photons eventually escape along cones near axis



on average photons give up all momentum to outflow after ~ 1 scattering

most photons eventually escape along cones near axis

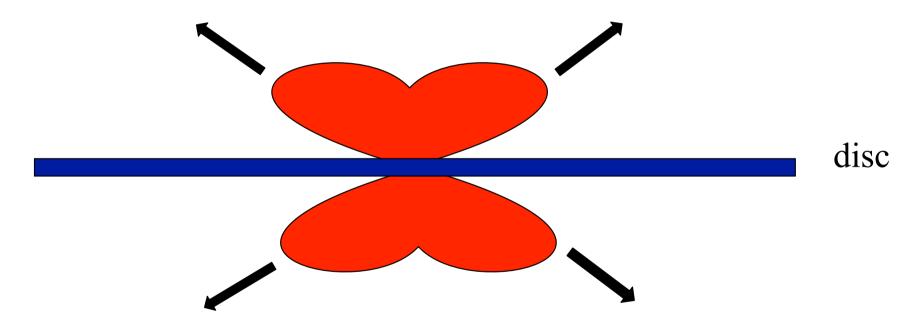


on average photons give up all momentum to outflow after ~ 1 scattering

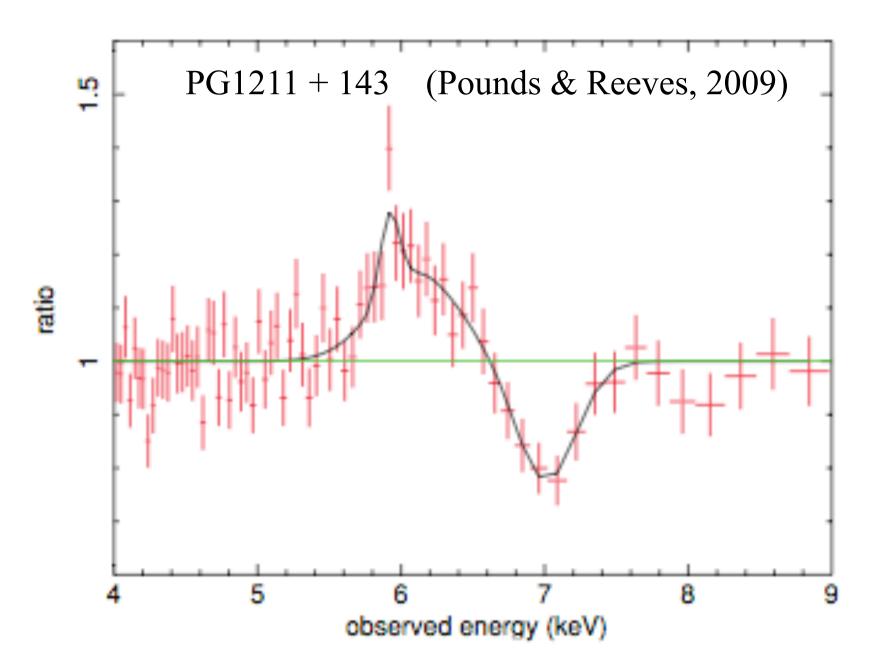
$$\dot{M}v = \frac{L_{\rm Edd}}{c}$$

### outflows have effectively spherical geometry since

(a) basic outflow pattern is roughly spherical



(b) disc axis moves randomly as accretion orientation changes



P Cygni profile of iron K- alpha: outflow with  $v \simeq 0.1c$  `ultrafast outflow' -- `UFO'

## mass outflow rate

 $\bullet$  measure velocity v directly from blueshift of absorption line

- ionization state of wind gas determined by the quantity  $\xi = L_i/NR^2$ , where  $L_i$  is the luminosity able to produce a given ion, N is the number density of the gas, and R the distance from the ionizing source (i.e. the quasar)
- measure  $L_i$  directly from quasar spectrum
- combining these gives mass outflow rate

$$\dot{M}_{\rm out} = 4\pi b m_p N R^2 v \sim 1 \rm M_{\odot} \, yr^{-1} \sim \dot{M}_{\rm Edd}$$

where the wind has solid angle  $4\pi b$ :  $b \sim 1$  since most local AGN show UFO--type outflows

## outflow affects galaxy bulge

outflow energy  $\sim 0.1 M_{BH} c^2$  is  $\sim 10^{61}$  erg for  $10^8 M_{\odot}$  black hole binding energy of bulge of mass  $10^{11} M_{\odot}$  and  $\sigma = 200$  km s<sup>-1</sup> is  $10^{58}$  erg

more than enough energy to unbind bulge – only a fraction used

galaxy must notice presence of hole

# Eddington outflows: summary

#### momentum outflow rate

$$\dot{M}_{
m out}v = rac{L_{
m Edd}}{c}$$

speed

$$v = \frac{L_{\rm Edd}}{\dot{M}_{\rm out}c} = \frac{\eta c}{\dot{m}} \sim 0.1c$$

where 
$$\dot{m} = \dot{M}_{\rm out} / \dot{M}_{\rm Edd} \sim 1$$

#### energy outflow rate

$$\frac{1}{2}\dot{M}_{\rm out}v^2 = \frac{\eta}{2}.\eta c^2\dot{M}_{\rm out} = \frac{\eta}{2}L_{\rm Edd} \simeq 0.05L_{\rm Edd}$$

where 
$$\dot{m} = \dot{M}_{\rm out} / \dot{M}_{\rm Edd} \sim 1$$

### outflow shock

outflow must collide with bulge gas, and shock – what happens?

either

(a) shocked gas cools:

'momentum—driven flow' negligible thermal pressure

or

(b) shocked gas does not cool:

'energy—driven flow' thermal pressure > ram pressure

Compton cooling by quasar radiation field very effective out to large bulge radii (cf Ciotti & Ostriker, 1997, 2001)

expansion into bulge gas is driven by momentum  $\frac{L}{2}$ 

 $\frac{L_{\mathrm{Edd}}}{}$