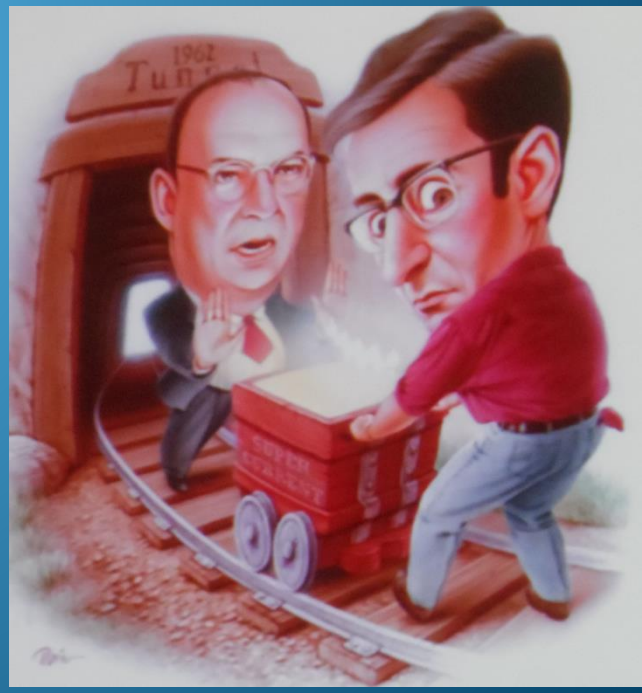
Superconducting electronics

-from Josephson effects to quantum computing

-by Pascal Febvre and Paul Seidel

Superconducting electronics – Part 1 Josephson effects



Superconductors

Cooper pairs (CP)

CP density n_s

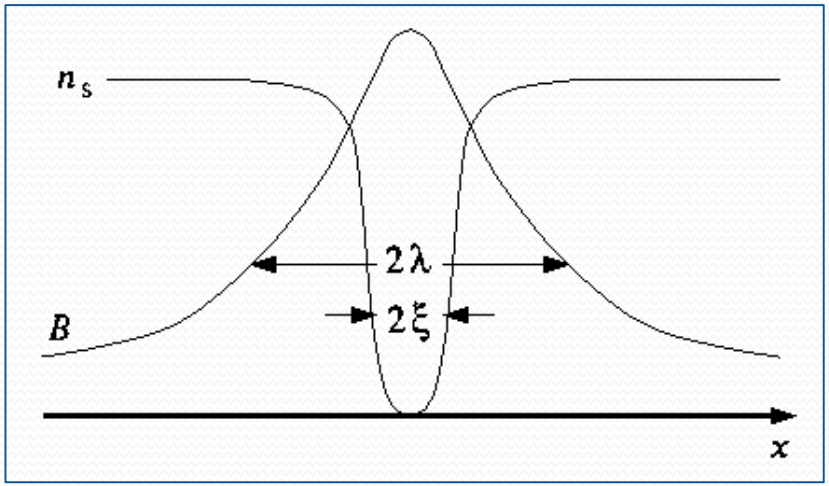
Common wave function ψ

Quantum mechanical phase

London penetration depth λ

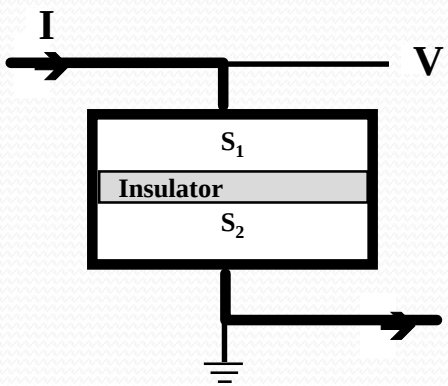
Coherence length ξ

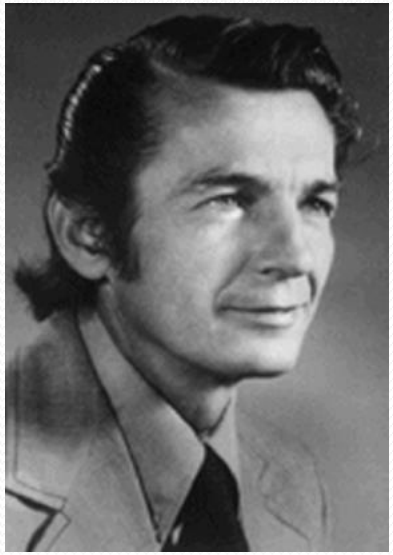
Abrikosov vortex



Tunneling effects

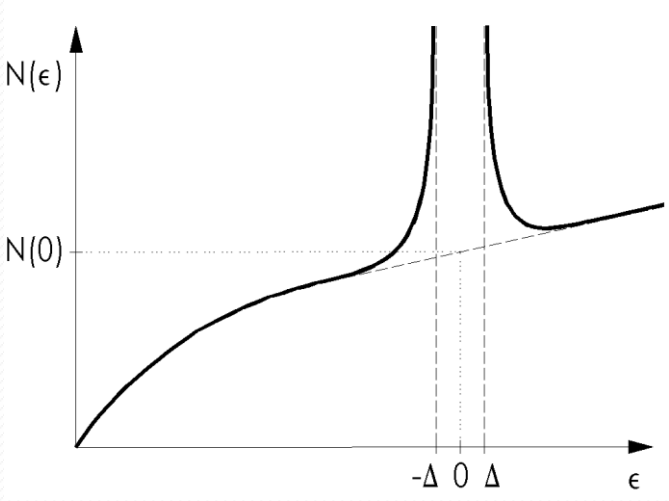
- 1961 Ivar Giaever
- Quasiparticle tunneling and energy gap



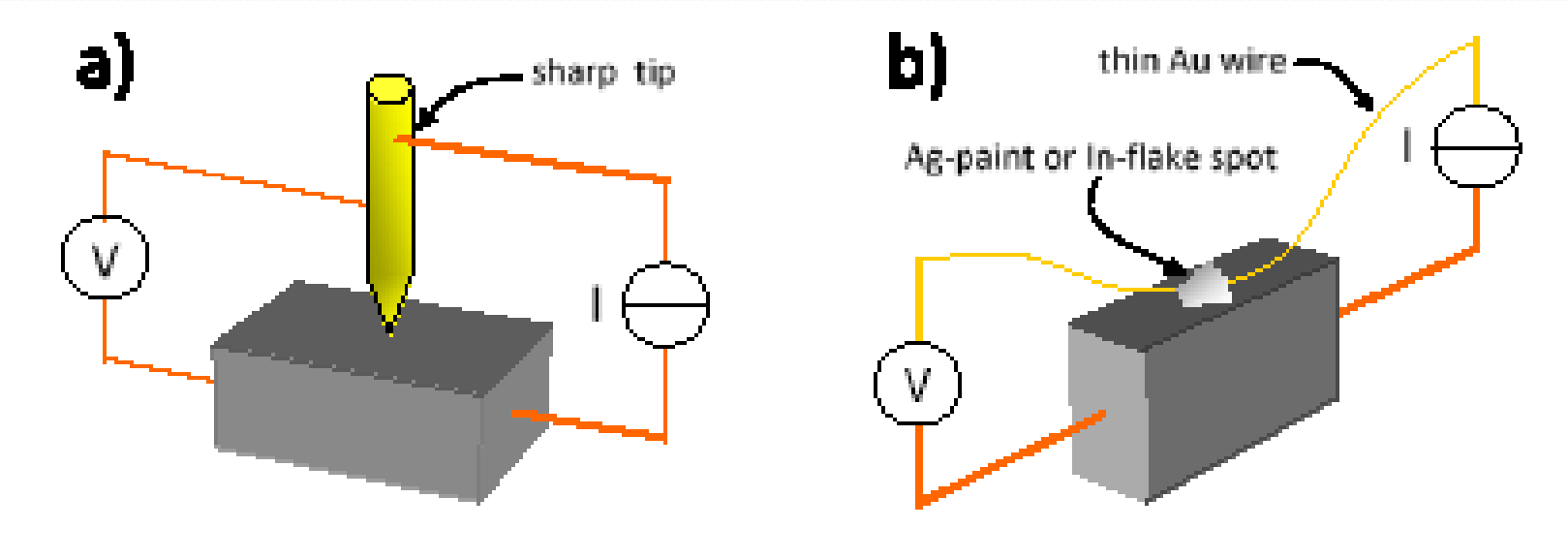


$$N_N(\varepsilon) \approx N_N(0)$$

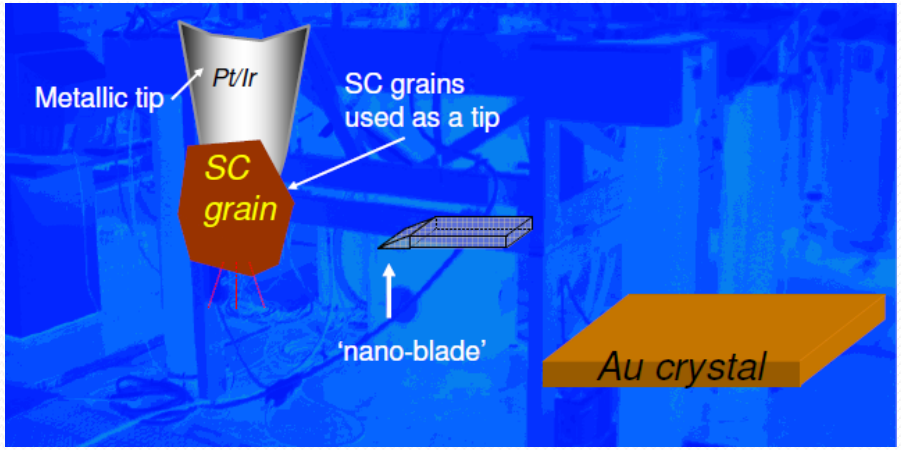
$$N_S(\varepsilon) = N_N(0)\,\mathrm{Re}\left[\frac{|\varepsilon|}{\left[\varepsilon^2 - \Delta^2\right]^{\frac{1}{2}}}\right]$$



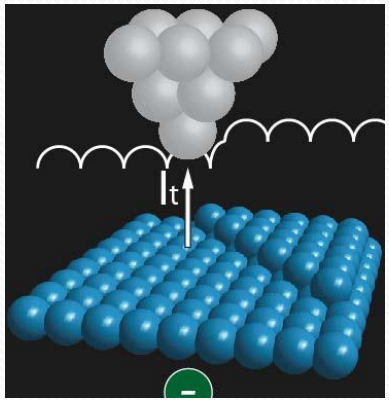
Tunneling experiments



[Daghero, Gonelli]



[Y. Noat]



[F. Massee]

Tunneling spectroscopy

$$I_T \sim \int_{-\infty}^{\infty} dE\, N_1(E)\, N_2(E+eU)\, [f_1(E)-f_2(E+eU)]$$

für T = o, S-I-N, $N_N(E) \equiv N(o)$ = const.

$$I_T \sim \int_0^{eU} dE\, N_S(E+eU)$$

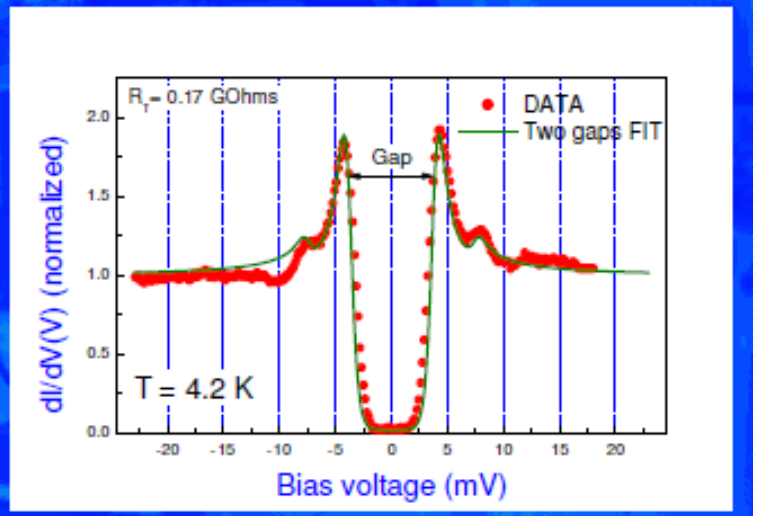
$$\rightarrow \quad \frac{dI_T}{dU} \sim N_S(E+eU)$$ "Tunneling spectrum"

MgB_2: Simplest two-gap model

Partial DOS in the two bands

$$N_S^{\,L}(E) = \Re e\left[\frac{E-i\Gamma}{\sqrt{(E-i\Gamma)^2-\Delta_L^{\,2}}}\right]$$

$$N_S^{\,S}(E) = \Re e\left[\frac{E-i\Gamma}{\sqrt{(E-i\Gamma)^2-\Delta_S^{\,2}}}\right]$$



Measured tunneling DOS

$$N(E) = \alpha N_S^S + (1-\alpha) N_S^L$$

Tunneling selectivity

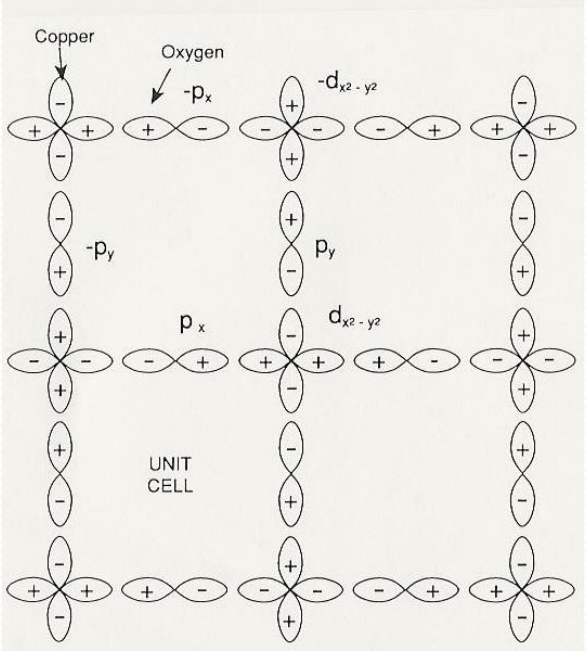
$\alpha = 0.13 \Rightarrow \Delta_L = 7.5$ mV

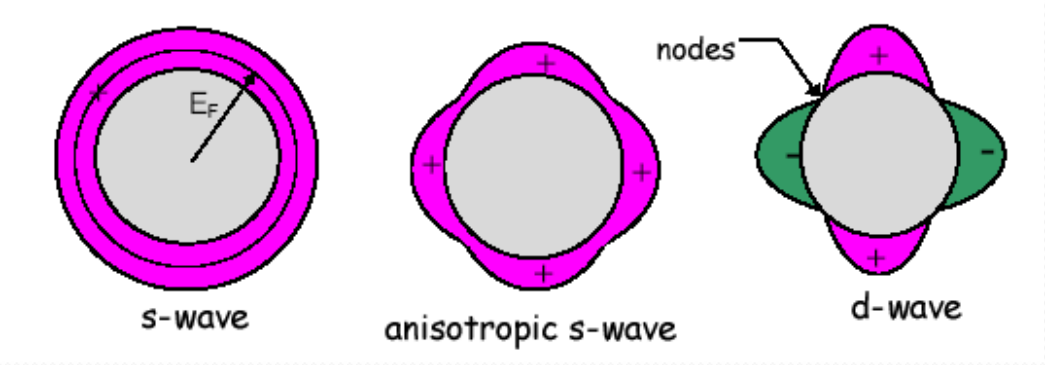
$1-\alpha = 0.87 \Rightarrow \Delta_S = 3.8$ mV

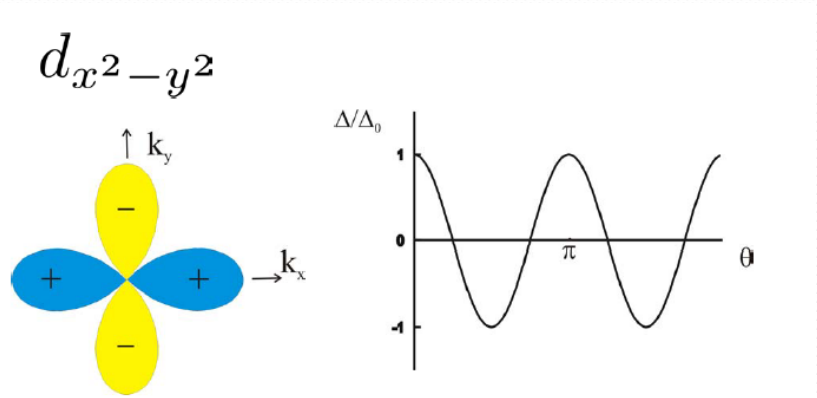
$\Gamma = 0.15$ mV $\ll \Delta_S, \Delta_L$

[Y. Noat]

YBCO as d-wave superconductor

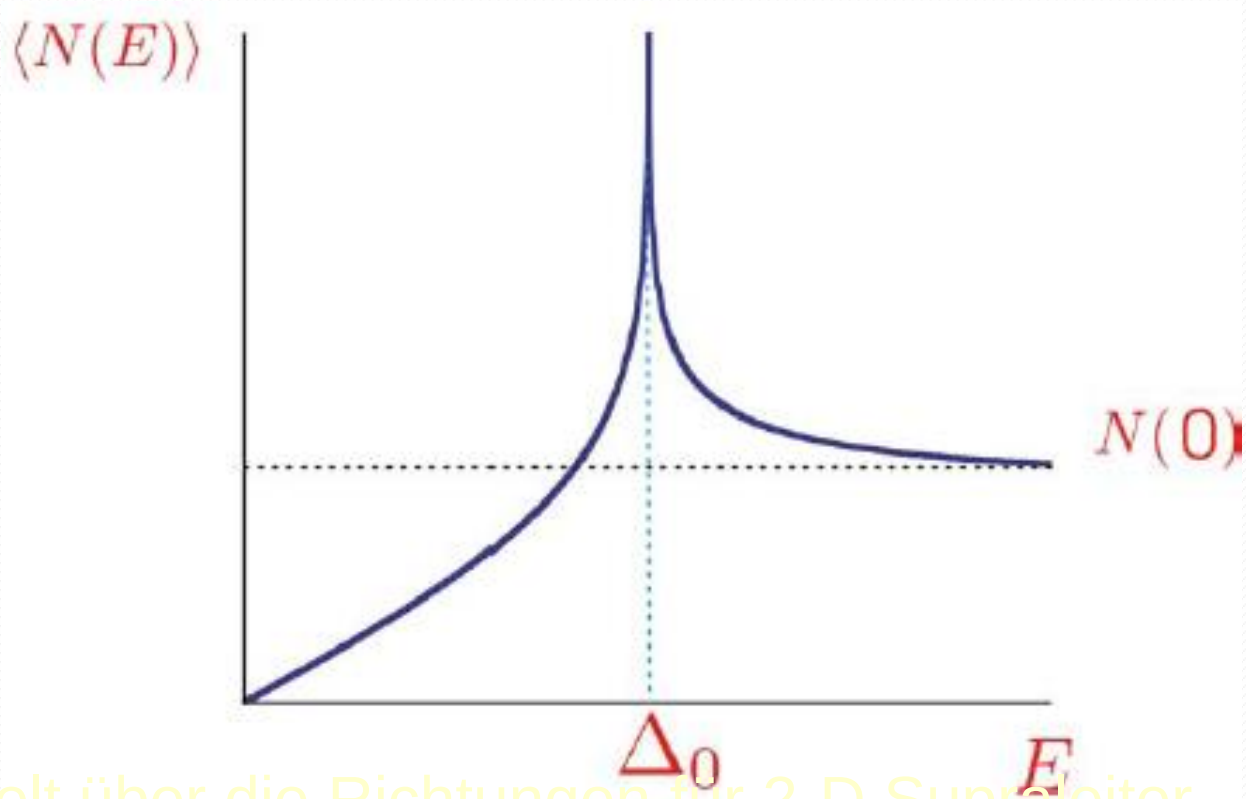






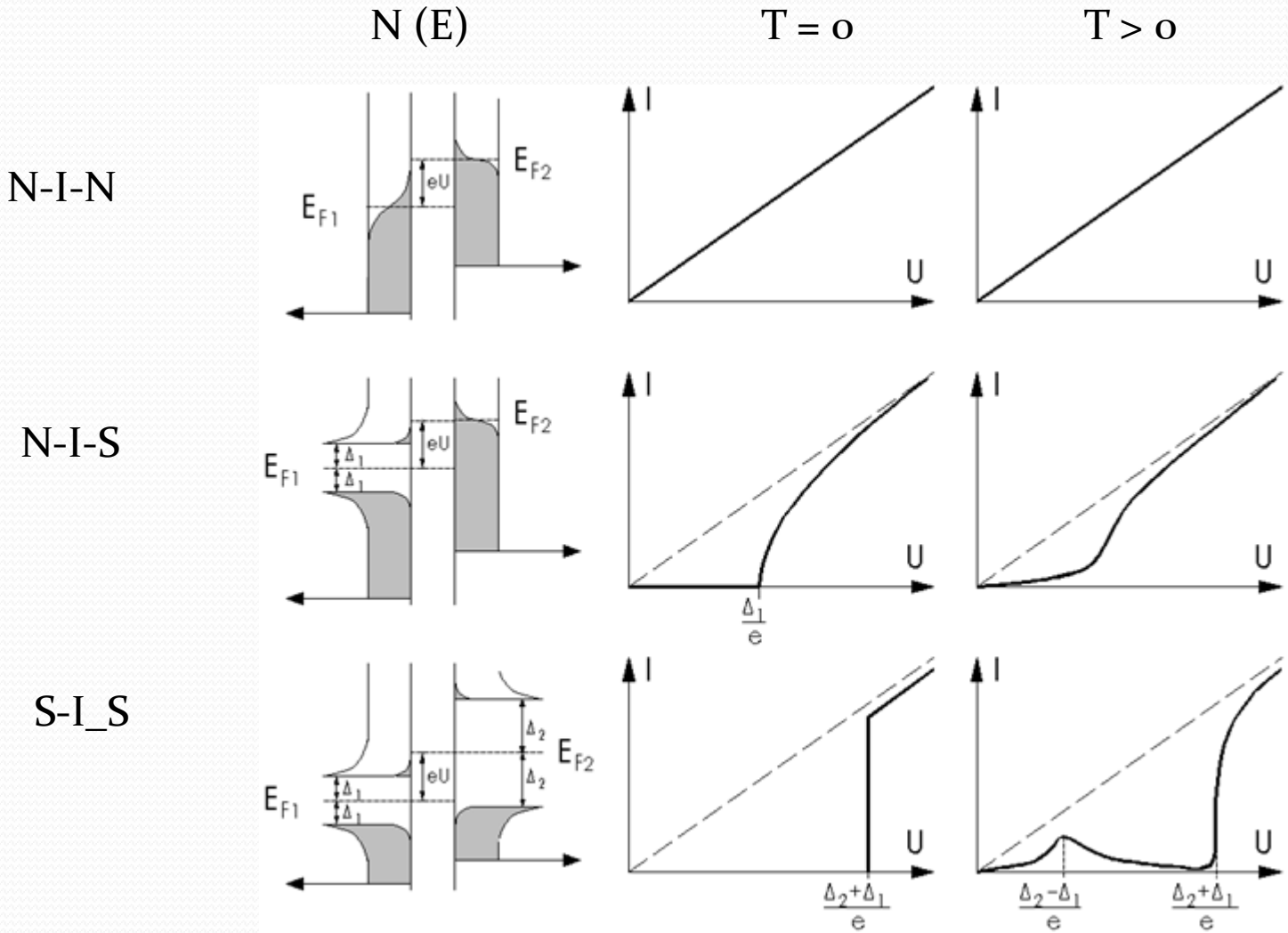
$\Delta(k) = \Delta_0 \cos 2\theta$

Density of states of a d-wave superconductor



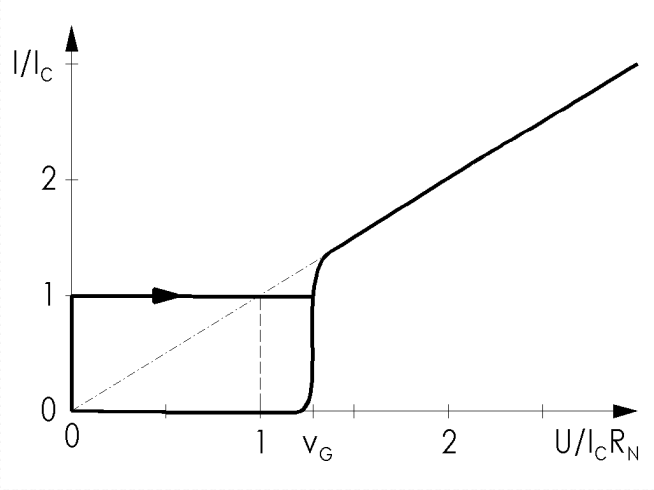
Gemittelt über die Richtungen für 2-D Supraleiter [A.Carrington]

Quasiparticle tunneling



Josephson effects

- 1962: Brian Josephson
- Tunneling of Cooper pairs!
- dc current without resistance
- ac currents (Josephson oscillations)

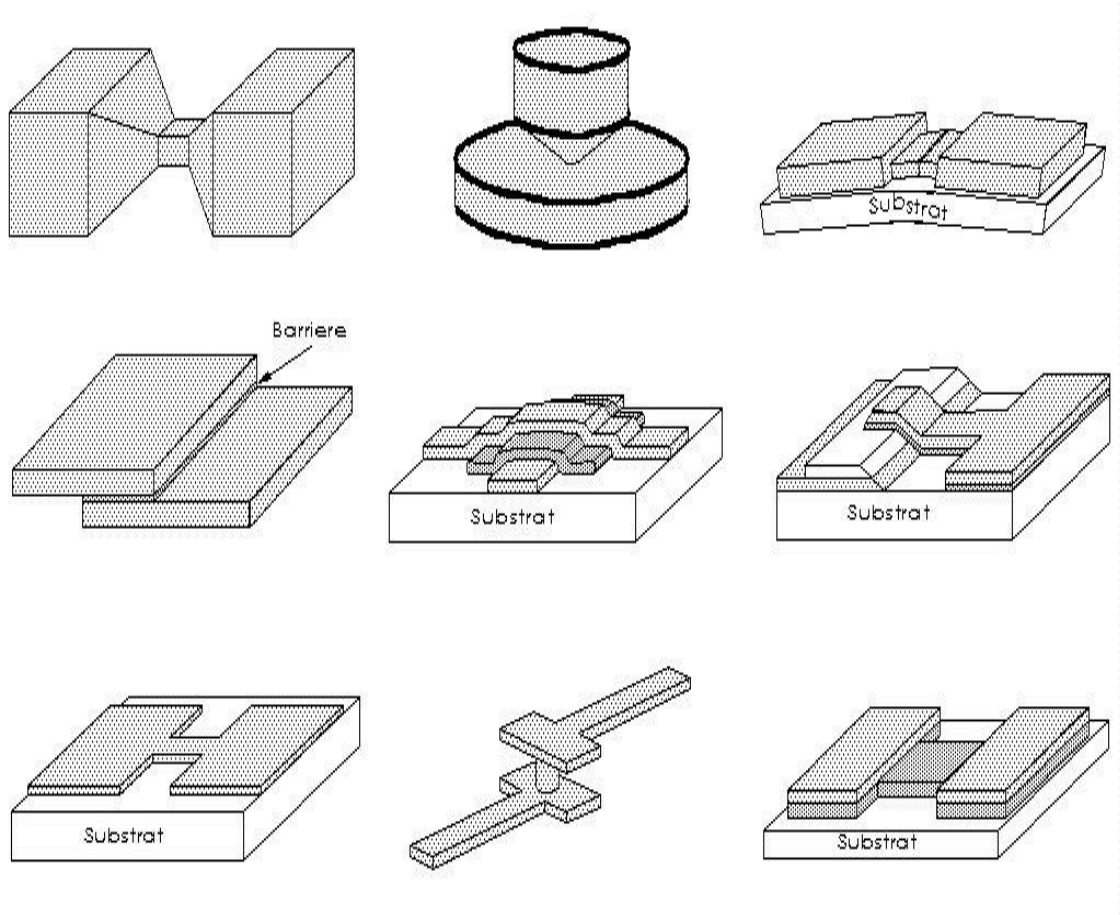


Different Josephson effects

- DC effect
- AC effect
- Inverse AC effect
- „Inverse“ AC effect (zero current steps)

- Intrinsic Josephson effect(s)

From SIS to different “weak links“



Different transport in “weak links“

- Tunneling of CP in SIS
- Multiple Andreev reflection (scattering) in SNS
- Direct ballistic transport in ScS
- Mixed transport in heterostructures

- Macroscopic Quantum effects
- Non-linear IV characteristics as basis of superconducting electronics

Josephson coupling after Feynman

$$\psi_1 = \sqrt{n_{c1}} \exp(i\varphi_1)$$

bzw. $\psi_2 = \sqrt{n_{c2}} \exp(i\varphi_{21})$

$$i\hbar \frac{\partial \psi_1}{\partial t} = E_1 \psi_1 + K \psi_2$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = E_2 \psi_2 + K \psi_1$$

$$\frac{\dot{n}_{c1}}{2\sqrt{n_{c1}}}\exp(i\varphi_1)+i\sqrt{n_{c1}}\exp(i\varphi_1)\dot{\varphi}_1=-\frac{i}{\hbar}\left\{E_1\sqrt{n_{c1}}\exp(i\varphi_1)+K\sqrt{n_{c2}}\exp(i\varphi_2)\right\}$$

$$\frac{\dot{n}_{c2}}{2\sqrt{n_{c2}}}\exp(i\varphi_2)+i\sqrt{n_{c2}}\exp(i\varphi_2)\dot{\varphi}_2=-\frac{i}{\hbar}\left\{E_2\sqrt{n_{c2}}\exp(i\varphi_2)+K\sqrt{n_{c1}}\exp(i\varphi_1)\right\}$$

$$\frac{1}{2}\frac{\dot{n}_{c1}}{\sqrt{n_{c1}}}=\frac{K}{\hbar}\sqrt{n_{c2}}\sin(\varphi_2-\varphi_1)$$

$$\frac{1}{2}\frac{\dot{n}_{c2}}{\sqrt{n_{c2}}}=\frac{K}{\hbar}\sqrt{n_{c1}}\sin(\varphi_1-\varphi_2)$$

$$i\sqrt{n_{c1}}\dot{\varphi}_1=-\frac{i}{\hbar}\left\{E_1\sqrt{n_{c1}}+K\sqrt{n_{c2}}\cos(\varphi_2-\varphi_1)\right\}$$

$$i\sqrt{n_{c2}}\dot{\varphi}_2=-\frac{i}{\hbar}\left\{E_2\sqrt{n_{c2}}+K\sqrt{n_{c1}}\cos(\varphi_1-\varphi_2)\right\}$$

Josephson coupling after Feynman

$$\dot{n}_{c1} = -\dot{n}_{c2}\ , n_{c1} = n_{c2}$$

$$\dot{n}_{c1} = \frac{2K}{\hbar} n_{c1} \sin(\varphi_2 - \varphi_1) = -\dot{n}_{c2}$$

$$I_S = I_{S,\max} \sin(\varphi_2 - \varphi_1)$$

$$I_{S,\max} = \frac{2K \cdot 2e}{\hbar} \cdot V \cdot n_c$$

Josephson coupling after Feynman

$$\frac{d}{dt}(\varphi_2 - \varphi_1) = \frac{1}{\hbar}(E_1 - E_2) = \frac{2eU}{\hbar}$$

$$(\varphi_2 - \varphi_1) = \frac{2eU}{\hbar} \cdot t + \varphi_0$$

$$\nu = \frac{2eU}{h}$$

DC Josephson current

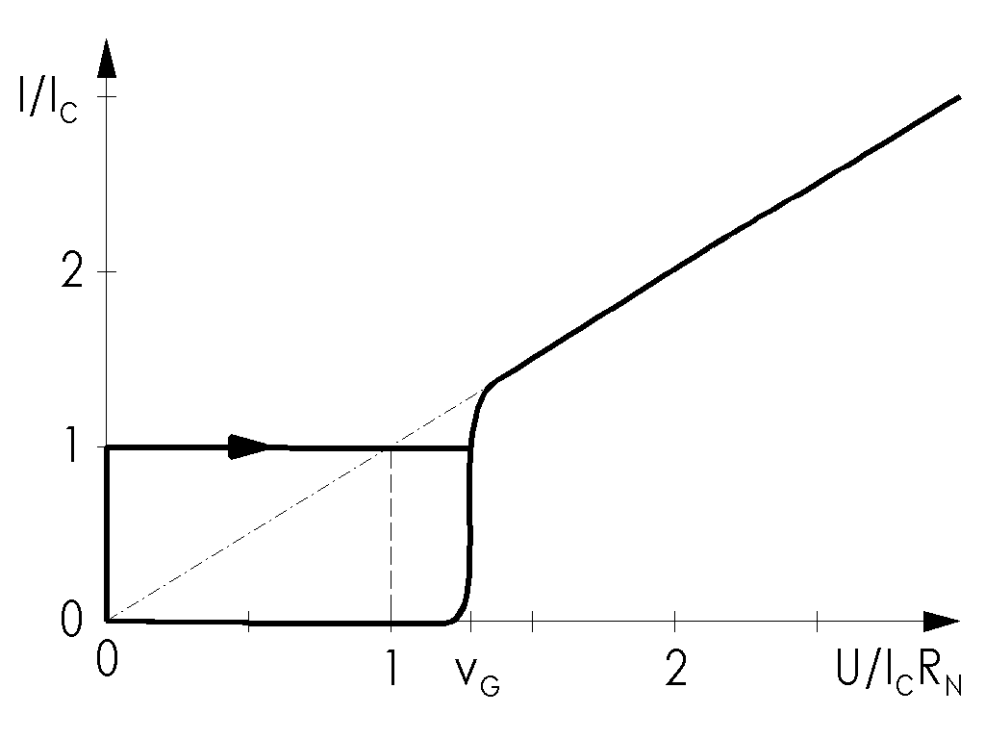
- dc current periodically depends on phase difference of wave functions!

- $I_s = I_c \sin \phi$, $\phi = \phi_2 - \phi_1$

- sometimes non-sinusoidal current phase relations

Critical Josephson current I_c

- For SIS and tunneling like junctions



$$I_C = \frac{2}{eR_N} \quad \frac{\Delta_L \Delta_R}{\Delta_L + \Delta_R} \quad K\left(\frac{|\Delta_R - \Delta_L|}{\Delta_R + \Delta_L}\right)$$

K ellipt. Int. 1.Ord.

Only for $\Delta_L \approx \Delta_R$ $\quad K(x) \approx \frac{\pi}{2}$ for $x \to 0$ and

$$I_C = \frac{\pi}{eR_N} \quad \frac{\Delta_L \Delta_R}{\Delta_L + \Delta_R}$$

and if $\Delta_L = \Delta_R = \Delta$

$$I_C = \frac{\pi}{2} \frac{\Delta}{eR_N}$$

$V_C = I_C R_N = (\pi/4)\, V_G = 0{,}78\, V_G$ mit $V_G = 2\Delta / e$

Temperature dependece

for T ≠0 Ambegaokar and Baratoff got

$$I_C(T) = \frac{2\Delta_1(T)\Delta_2(T)}{\pi k_B T\, eR_N} \sum_{l=0,1,2} \left\{ \left[(2l+1)^2 + \left(\frac{\Delta_1(T)}{\pi k_B T} \right)^2 \right] \left[(2l+1)^2 + \left(\frac{\Delta_2(T)}{\pi k_B T} \right)^2 \right] \right\}^{-\frac{1}{2}}$$

for $\Delta_L = \Delta_R = \Delta$ one gets

$$I_C(T) = \frac{\pi}{2} \frac{\Delta(T)}{eR_N} \tanh\left(\frac{\Delta(T)}{2k_B T} \right)$$

Reminder: for T → 0 tanh( ) gives just 1

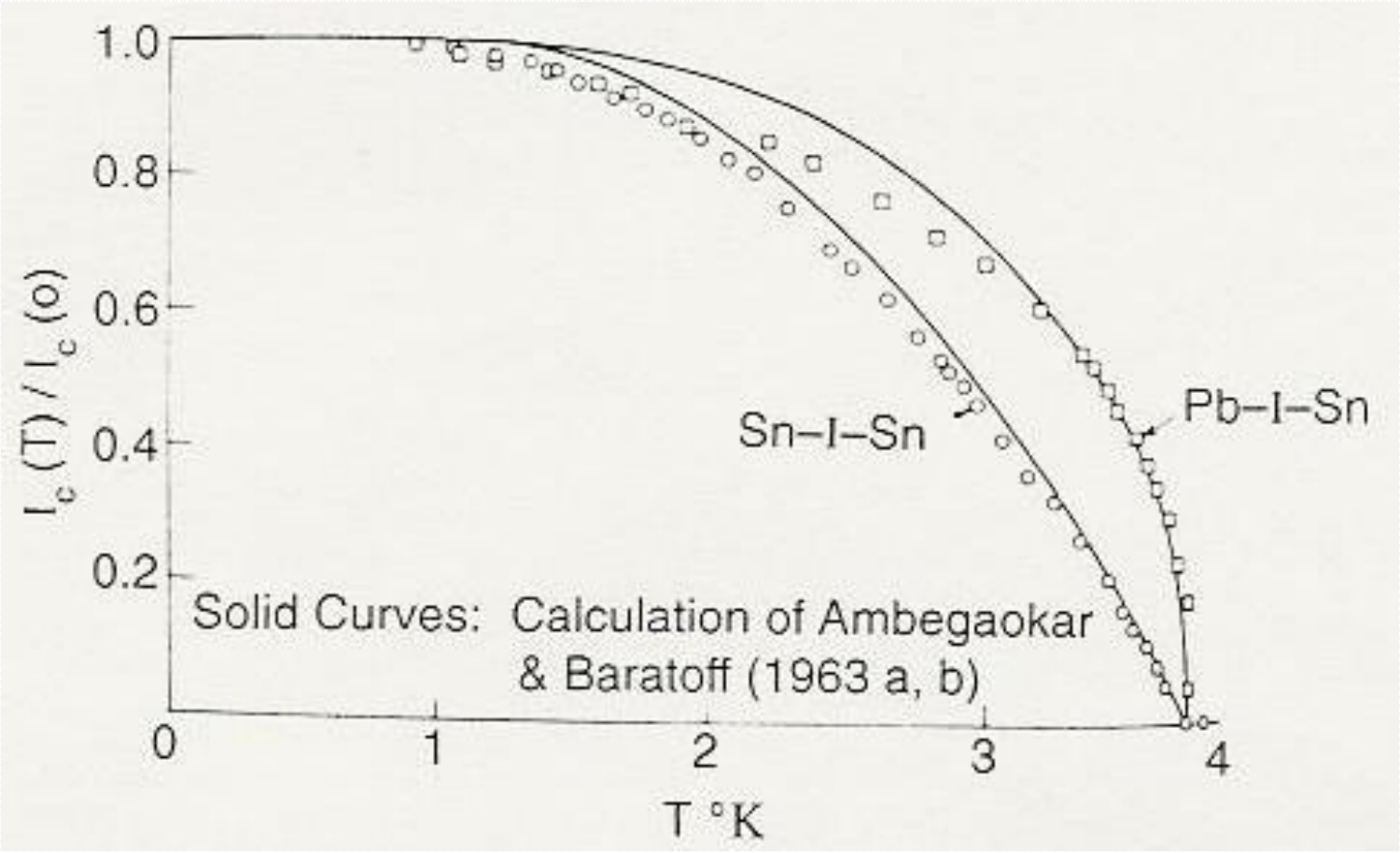
Measurement of I_c (T)

Experimentally verified 1964 by Fiske

on Sn-SnO_x-Sn and Pb-PbO_x-Sn tunneling junctions

Near T_C for SIS:

$$I_C \sim \Delta^2 \sim \left(1 - \frac{T}{T_C}\right)$$



[C. P. Poole et al.]

DC effect at weak links

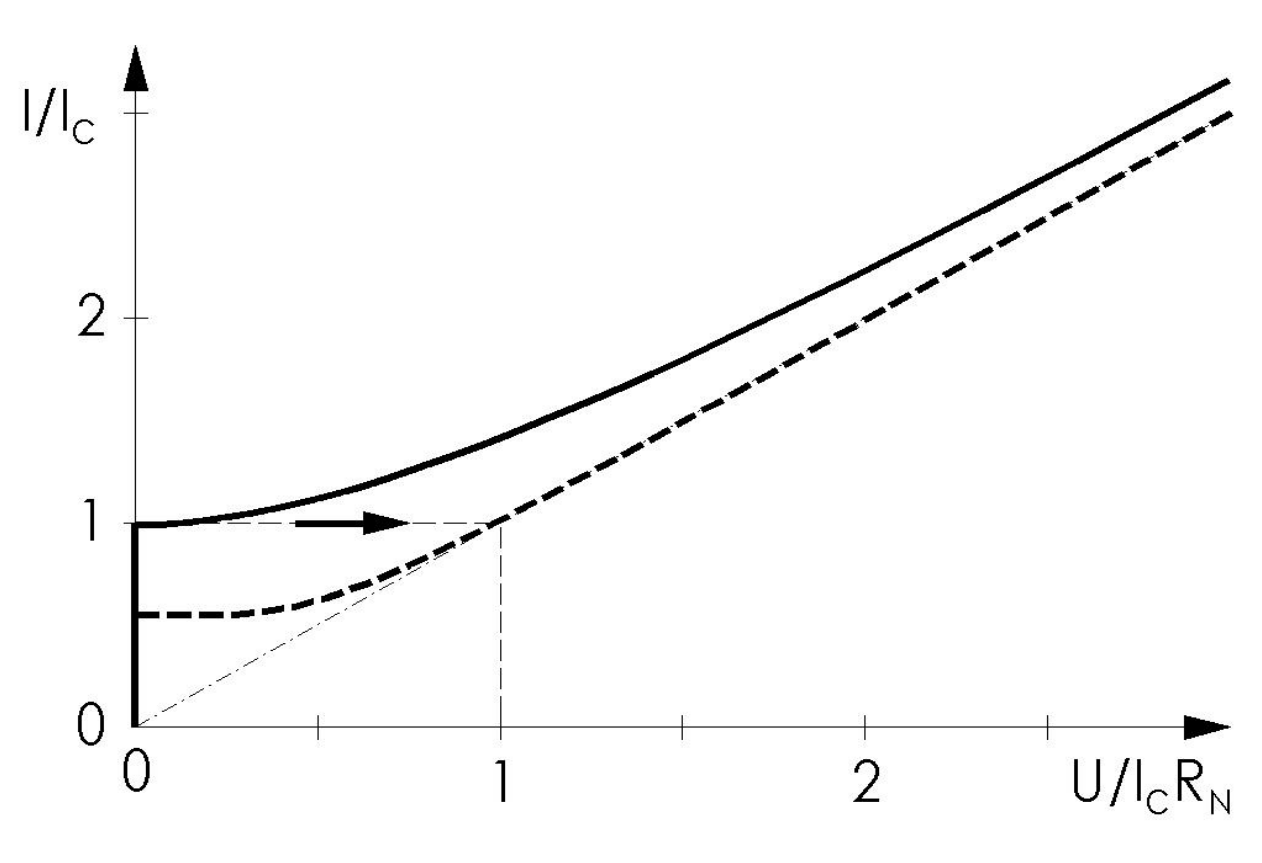
- Temperature dependence I_c (T)
  is related to the contact type, e.g. near T_c :

  $T_c \sim (1 - T/T_c)^m$

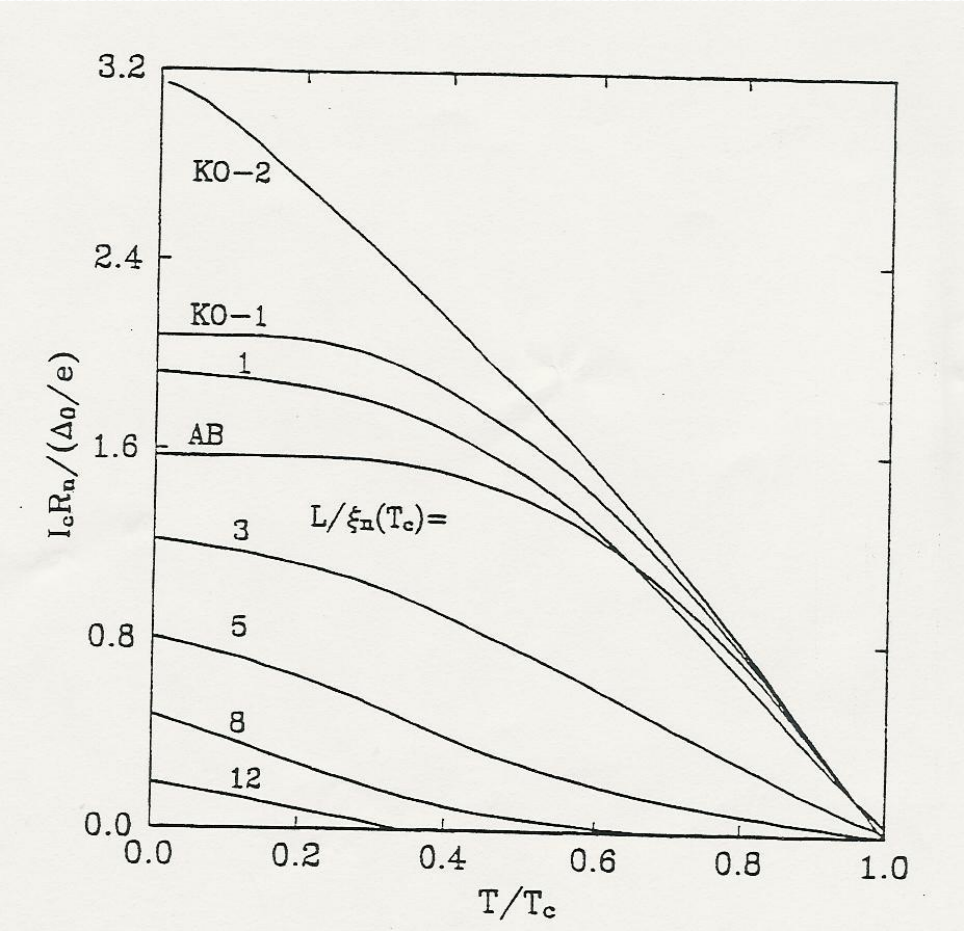
  m = 1 (SIS), 2/3 (SINS), 2 (SNS)

→ reason: different coupling and transport mechanisms

Bridge-like junction

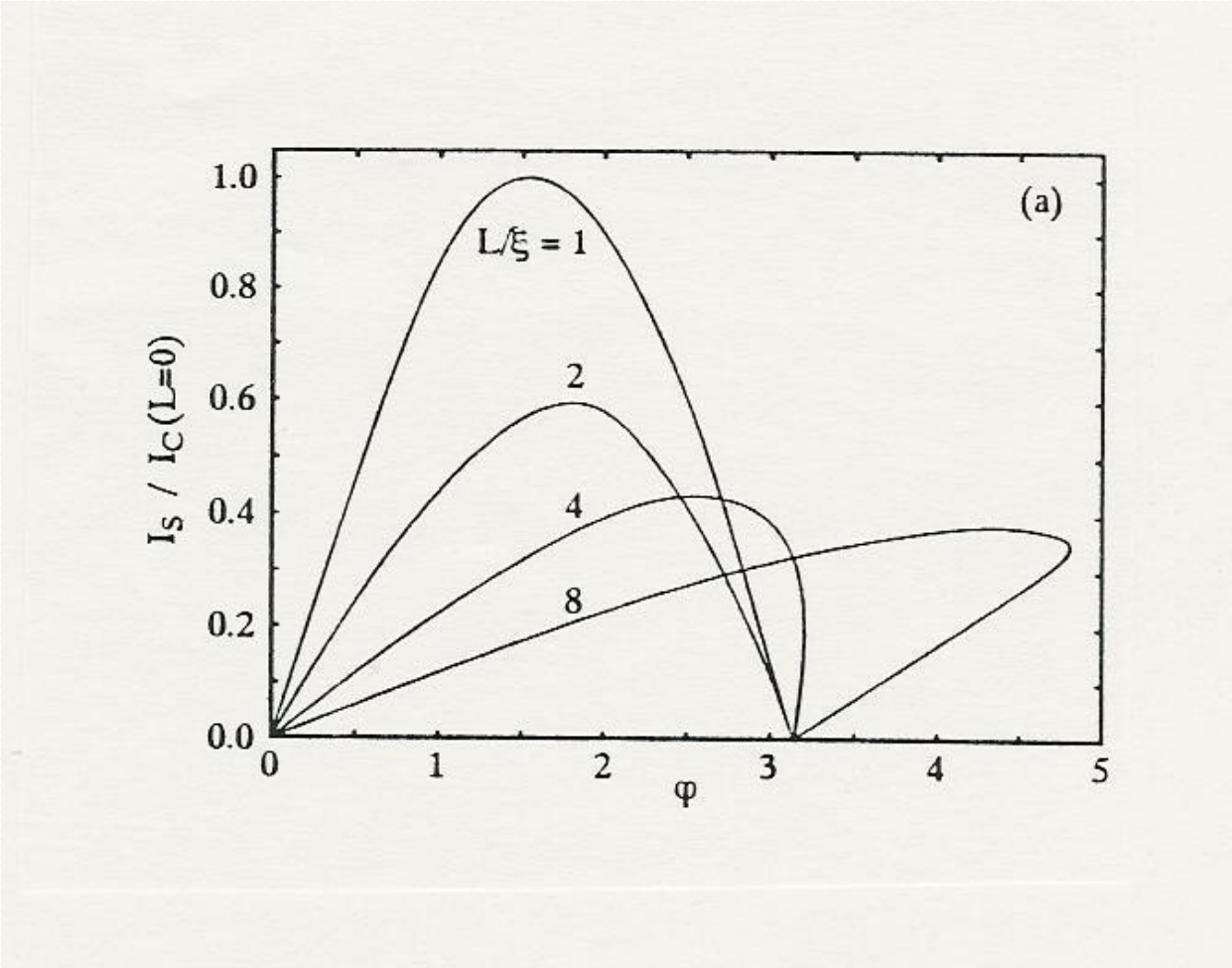


For bridge-like junctions length compared to the coherence length



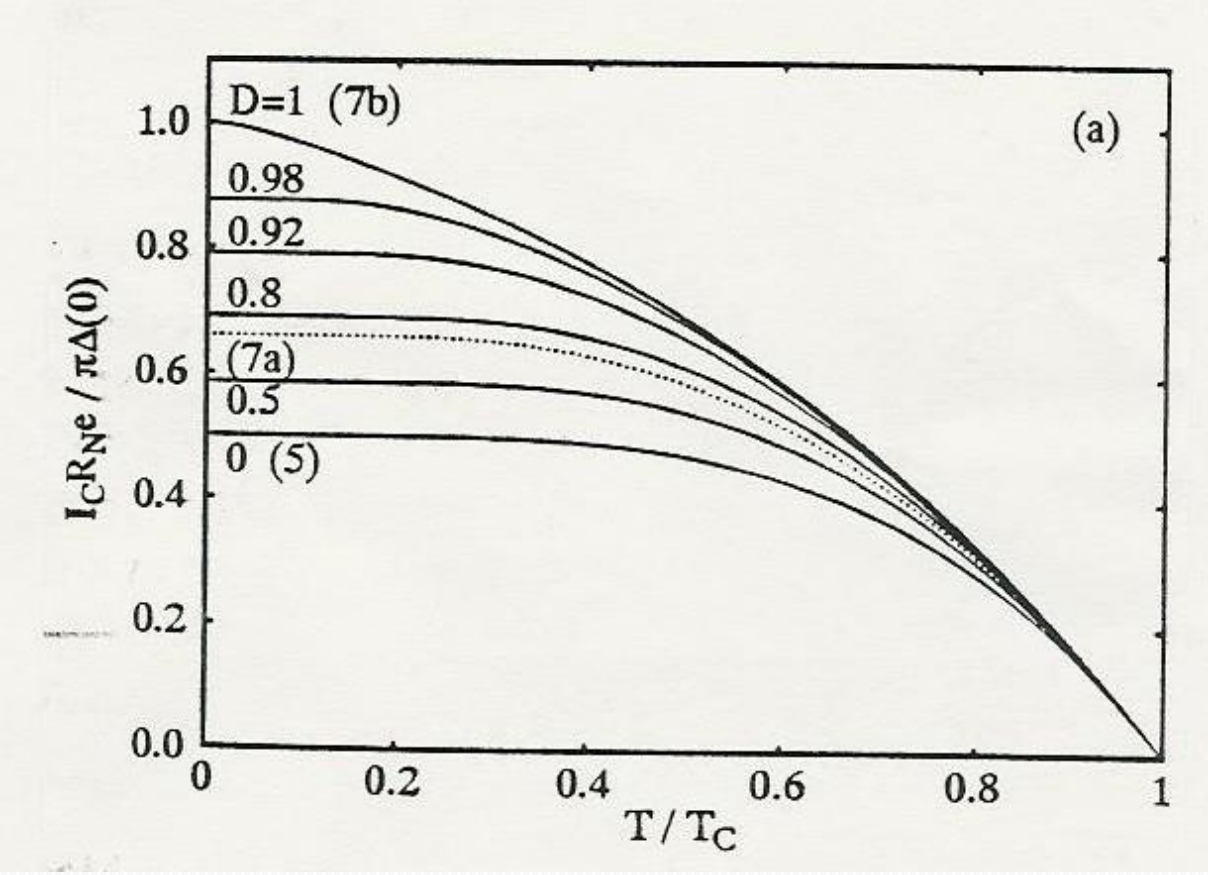
[B. H. Moeckly]

Current-phase relations



[E. Heinz]

Dependence on barrier transmission coefficient



[E. Heinz]

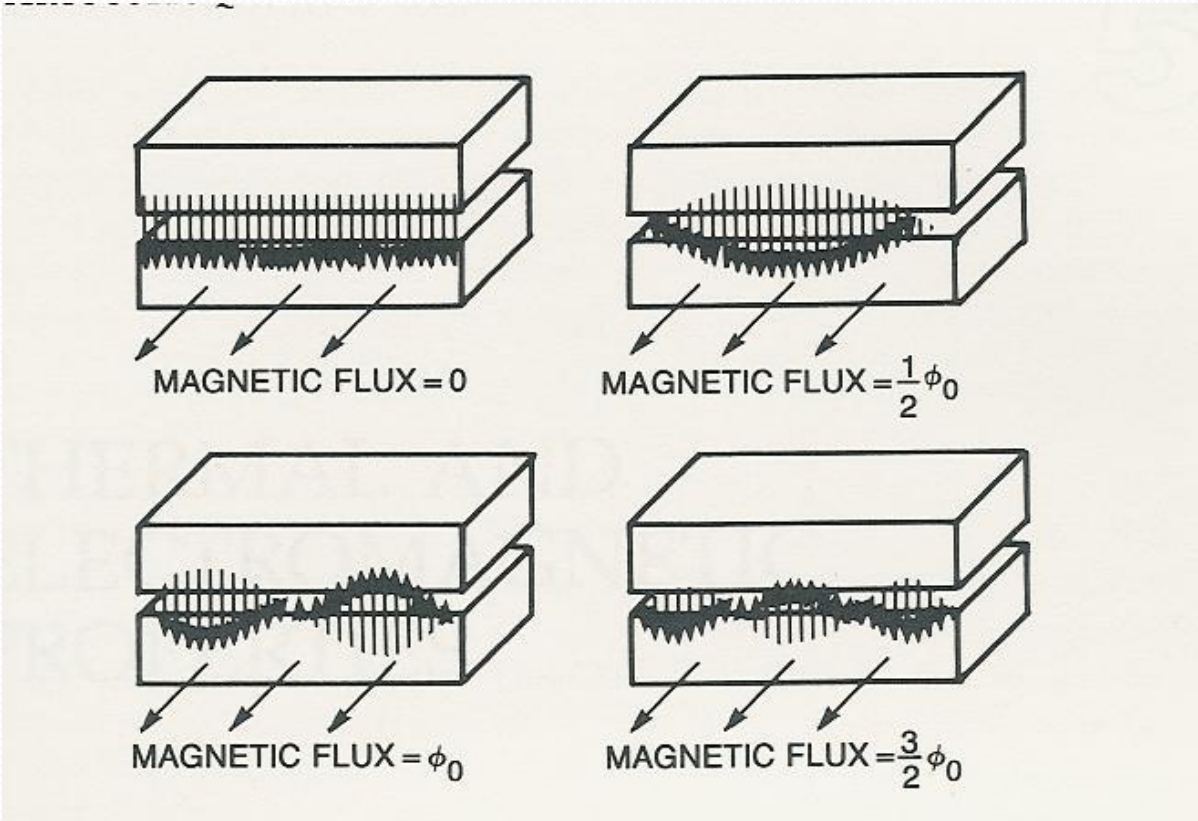
I_C in an external magnetic field

Modulation of dc current in an external magnetic field (flux Φ=BA)with the elementary Flux quantum Φ_0

„Fraunhofer pattern“

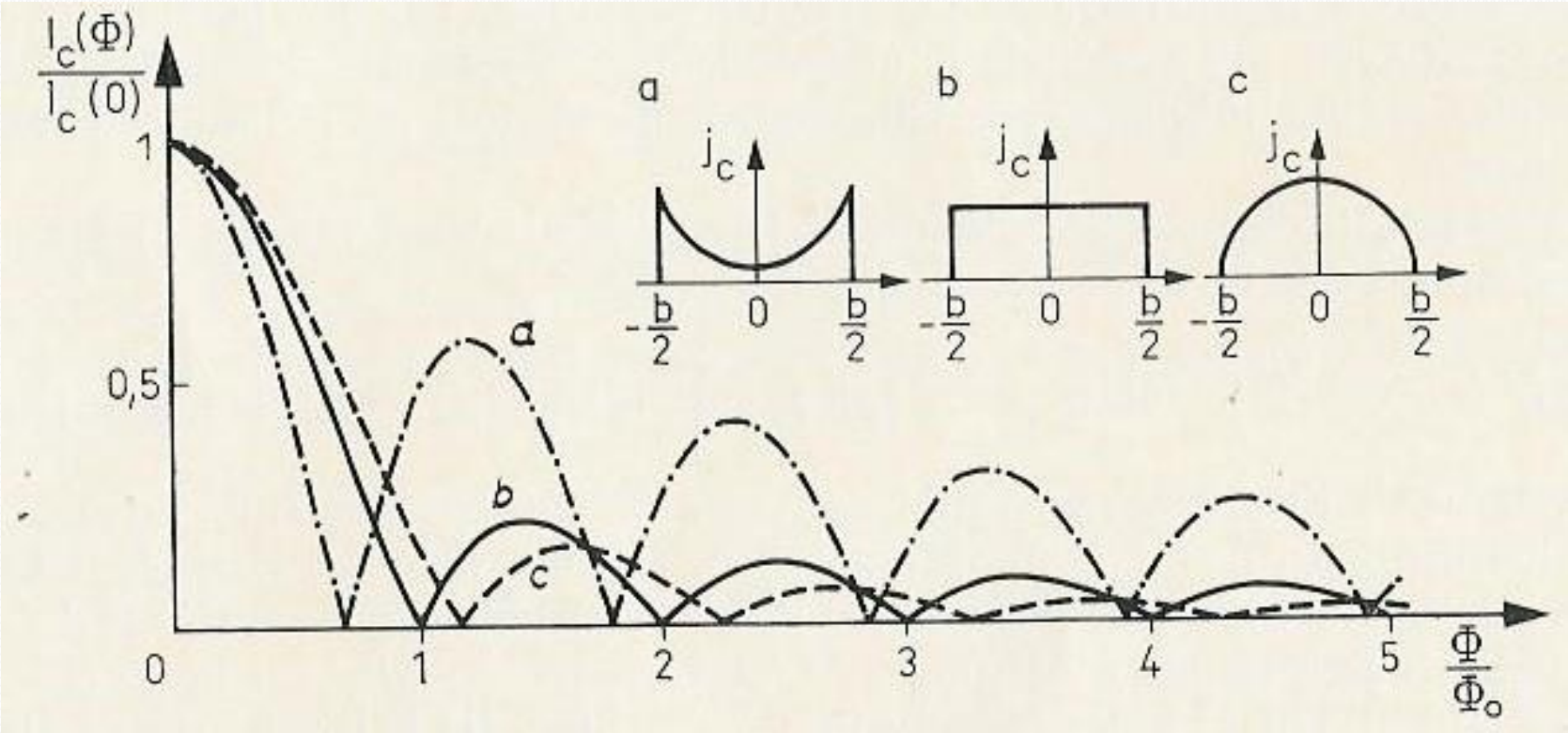
$$I_c = I_{cmax}\ \mathrm{abs}(\sin x / x), \qquad x = \pi\, \Phi/\Phi_0$$

Current density modulation by external magnetic field for short junctions



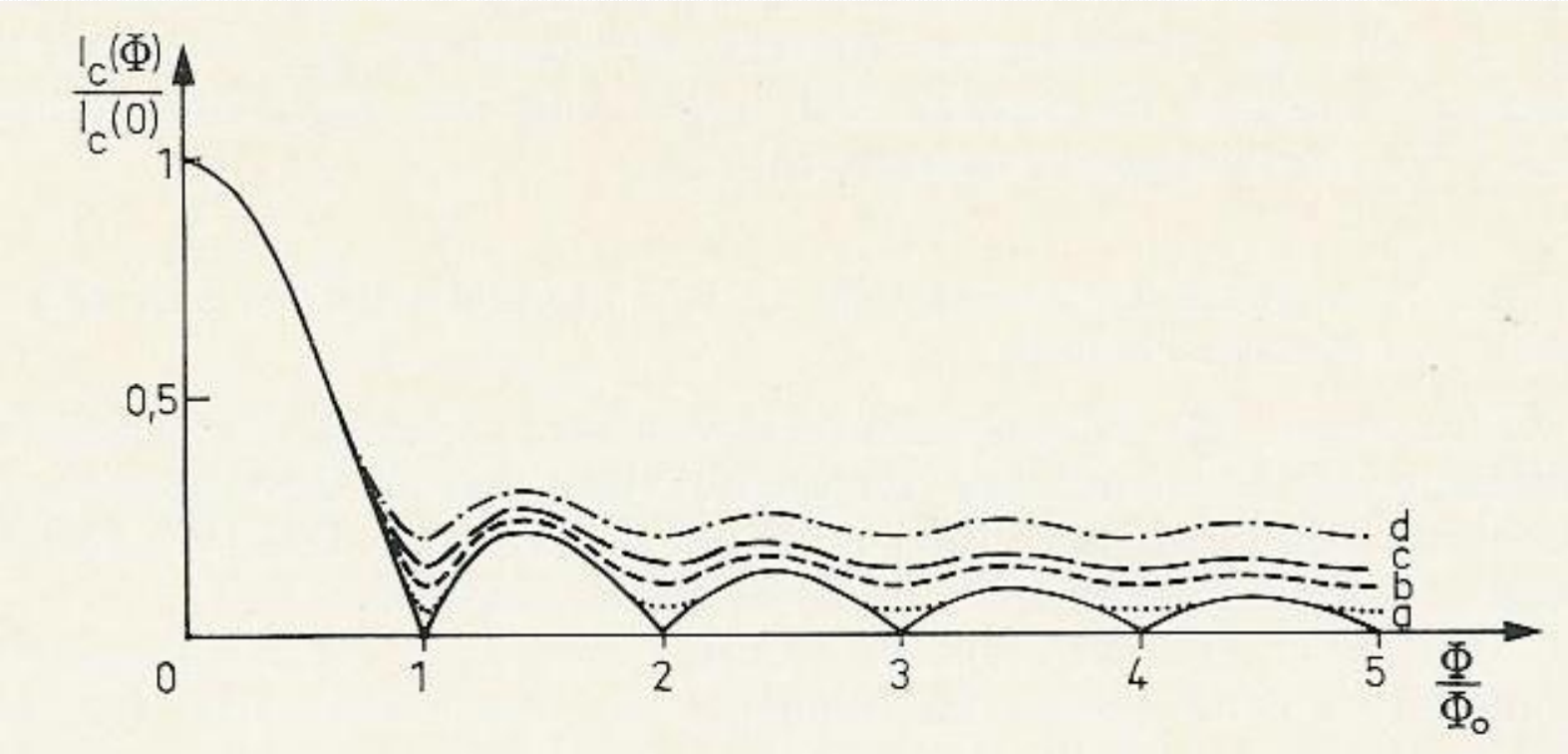
[W. Buckel]

Homogeneity of j_c



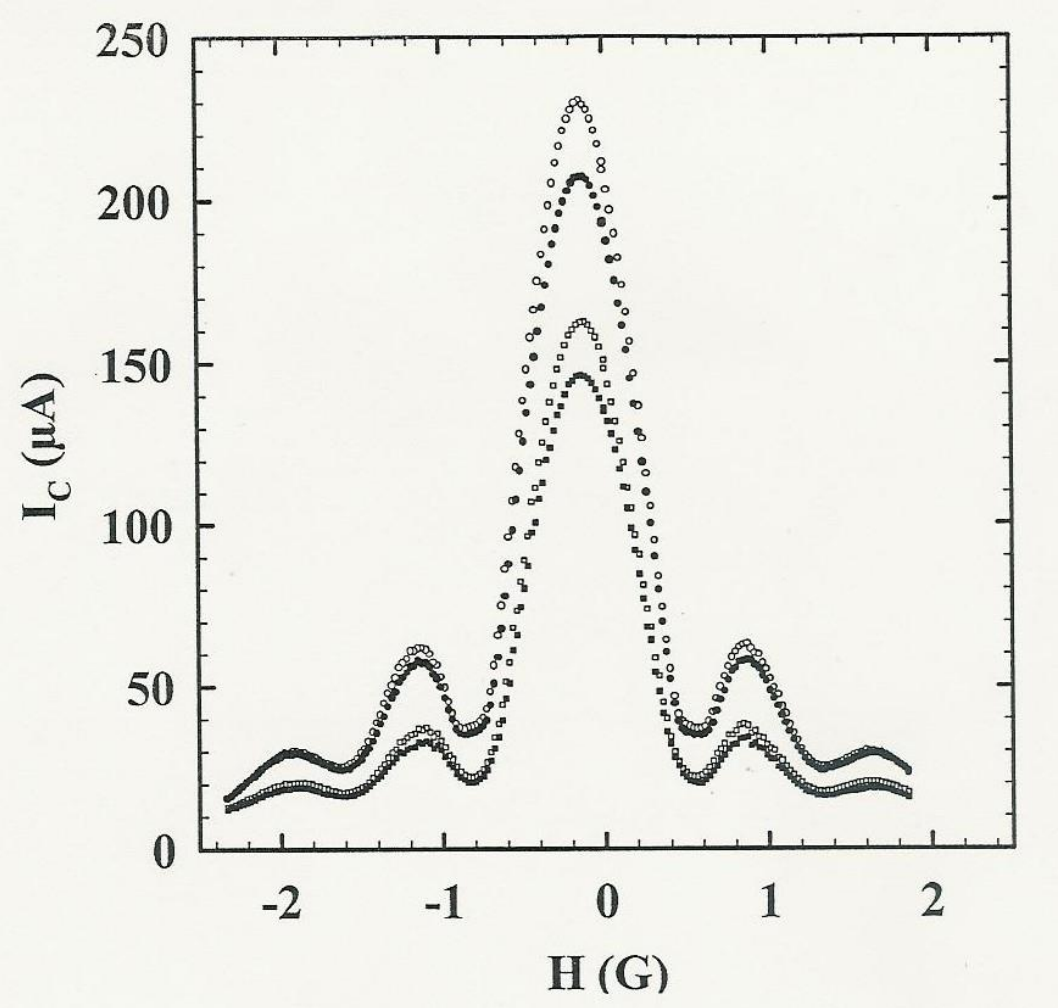
[J. Richter, P. Seidel, Exp. Tech. Phys. 26 (1978) 217]

Fluctuations



[J. Richter, P. Seidel, Exp. Tech. Phys. 26 (1978) 217]

$I_c(H)$



YBCO

With and without illumination by light (Photodoping)

[J. Elly et al. PRB 56 (1997), R 8507.]

External magnetic field at long junctions

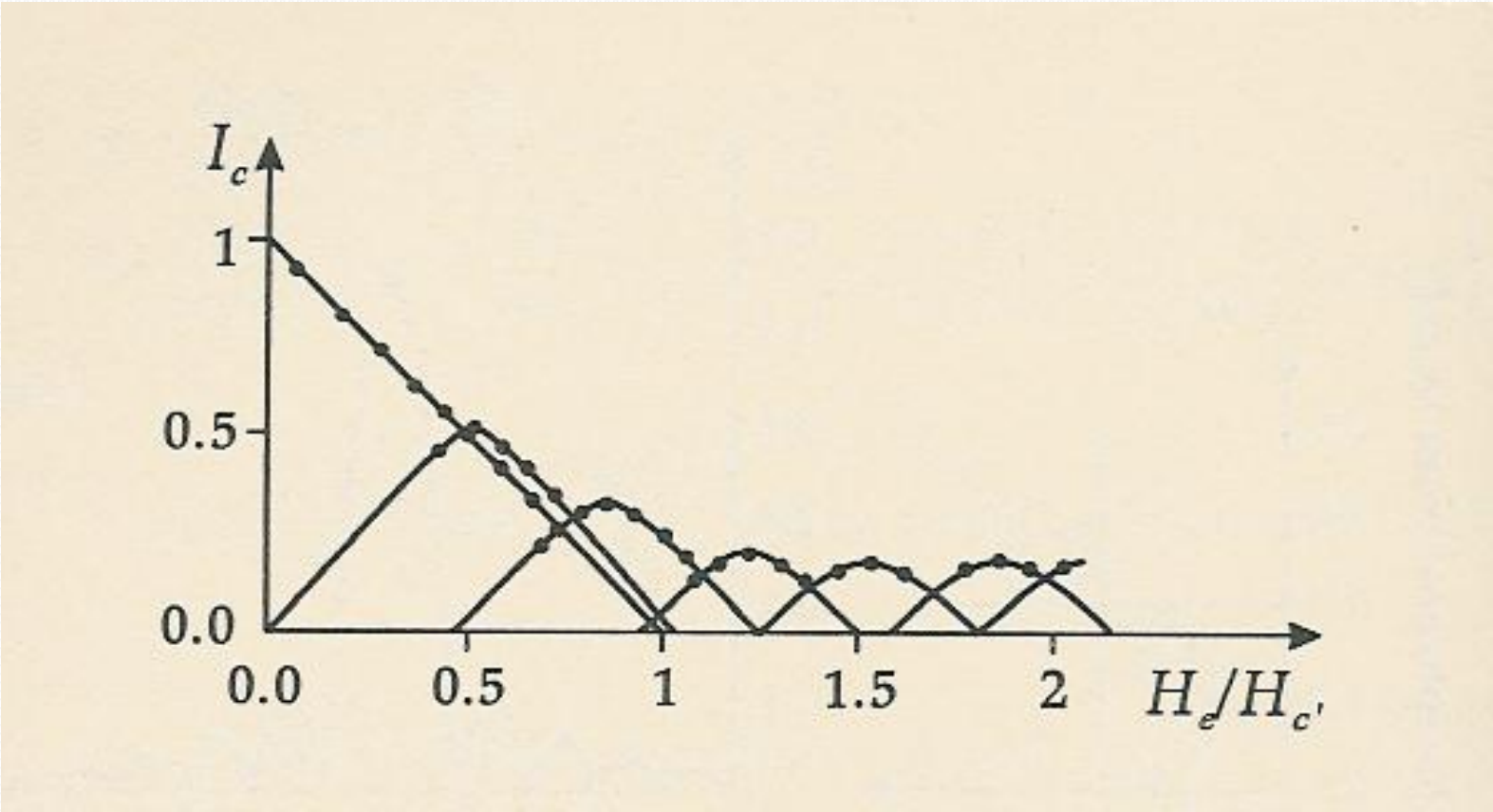
- For „long“ or better „wide“ junctions the spatial dependence of the phase has to be taken into account
- Results in a Sine-Gordon equation

$$\frac{d^2\varphi}{dx^2} = \frac{\sin\ \varphi(x)}{\lambda_J^2}$$

- With $\lambda_J = (\Phi_o / 2\pi\mu_o J_c d)^{1/2}$ as the

  Josephson penetration depth

Wide junction



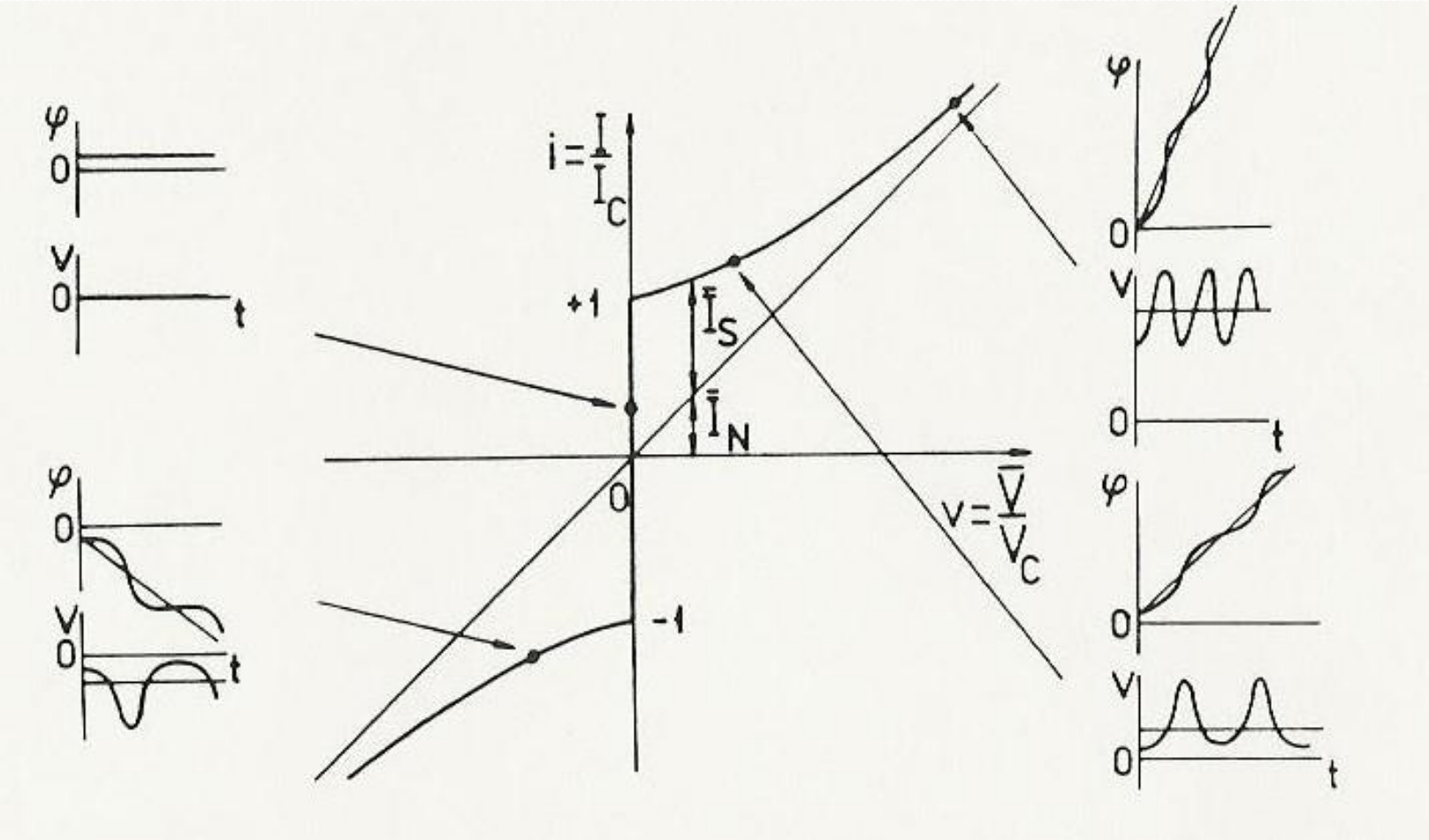
$L/\lambda_J = 8{,}4$ [N. W. Sawaritzki]

AC Josephson effect

alternating current with a frequency f connected to the voltage V across the junction by

$$V = \Phi_o \cdot f$$

rf currents !



[K. K. Likharev]

AC Josephson effect

- Oscillating current if voltage applied

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V(t)$$

$$\omega = \frac{2e}{\hbar} I_C R_N$$

- Irradiation of electromagnetic waves

→ Josephson Oscillator

Experiments

1963: S. Shapiro indirect proof of the ac Josephson effect (*Shapiro steps*)

→ inverse AC Josephson effect

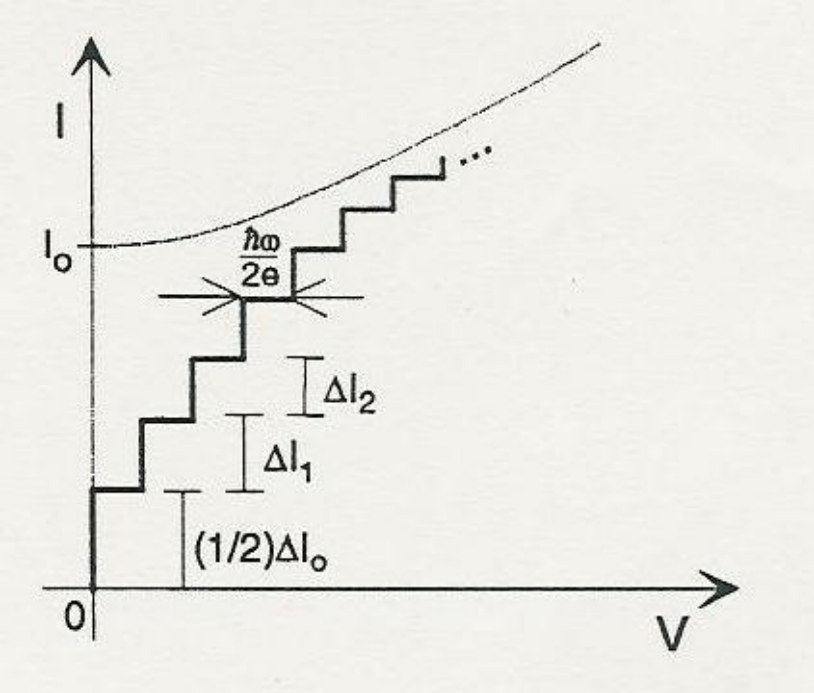
Inverse Josephson effect

- Response to external radiation with a frequency f_{ex}
- Voltage steps (Shapiro steps)

$$V_n = n\left(\frac{h}{2e}\right) f_{ex} = n\Phi_0 f_{ex}$$

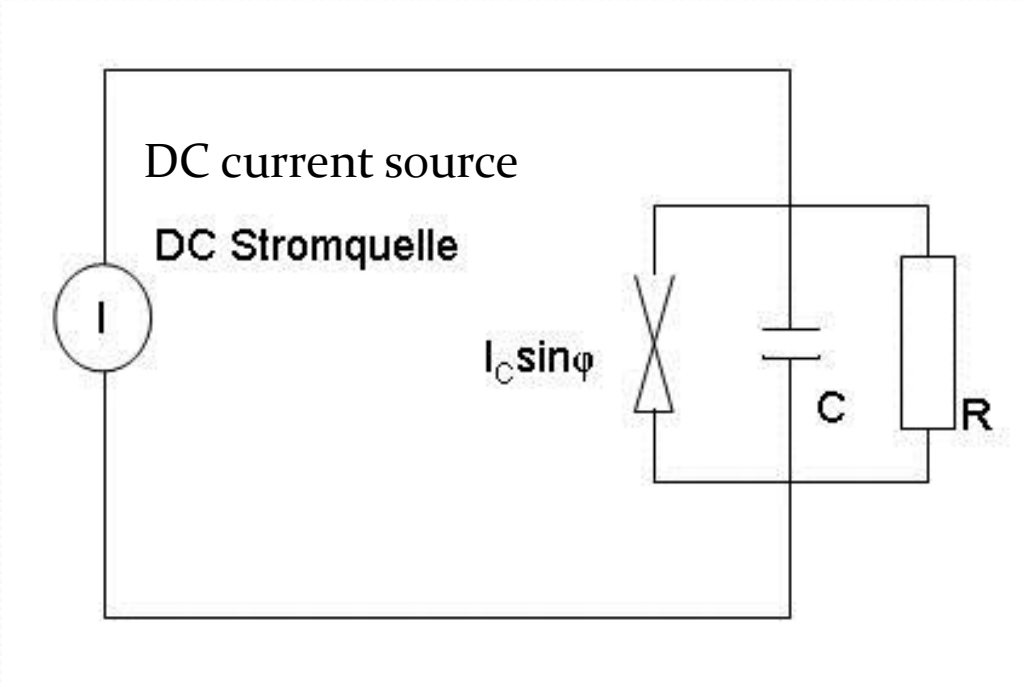
$$f / V = 483{,}6 MHz / \mu V$$

$$\Omega = \frac{\omega_{ex}}{\omega_0}$$

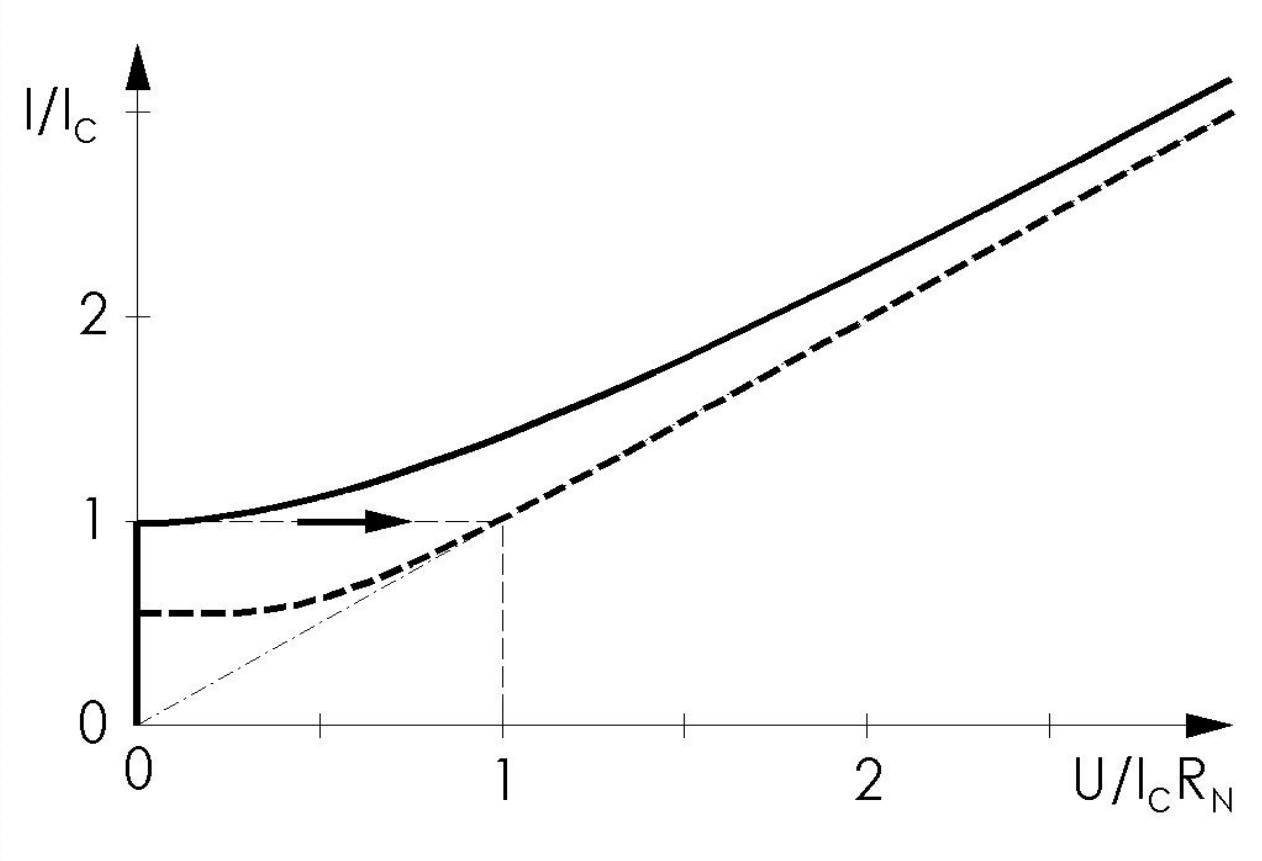


RCSJ model

$$I = I_C \sin\varphi + \frac{V}{R} + C\frac{dV}{dt}$$



IVC in the RSJ model



RCSJ Model

Phase difference across the junction

$$\beta_c \phi'' + \phi' + \sin \phi = i_o + i_1 \sin \Omega\tau + i_F(\tau)$$

$$\beta_c = (2e/\hbar)\, I_C R_N C$$ (McCumber-Parameter)

$$\tau = \omega_o t\,,\ \omega_o = (2e/\hbar)\, I_C R_N\,,\ \Omega = \omega_{ex} / \omega_o$$

Bessel function behaviour

Shapiro step heigth in the case of current biasing:

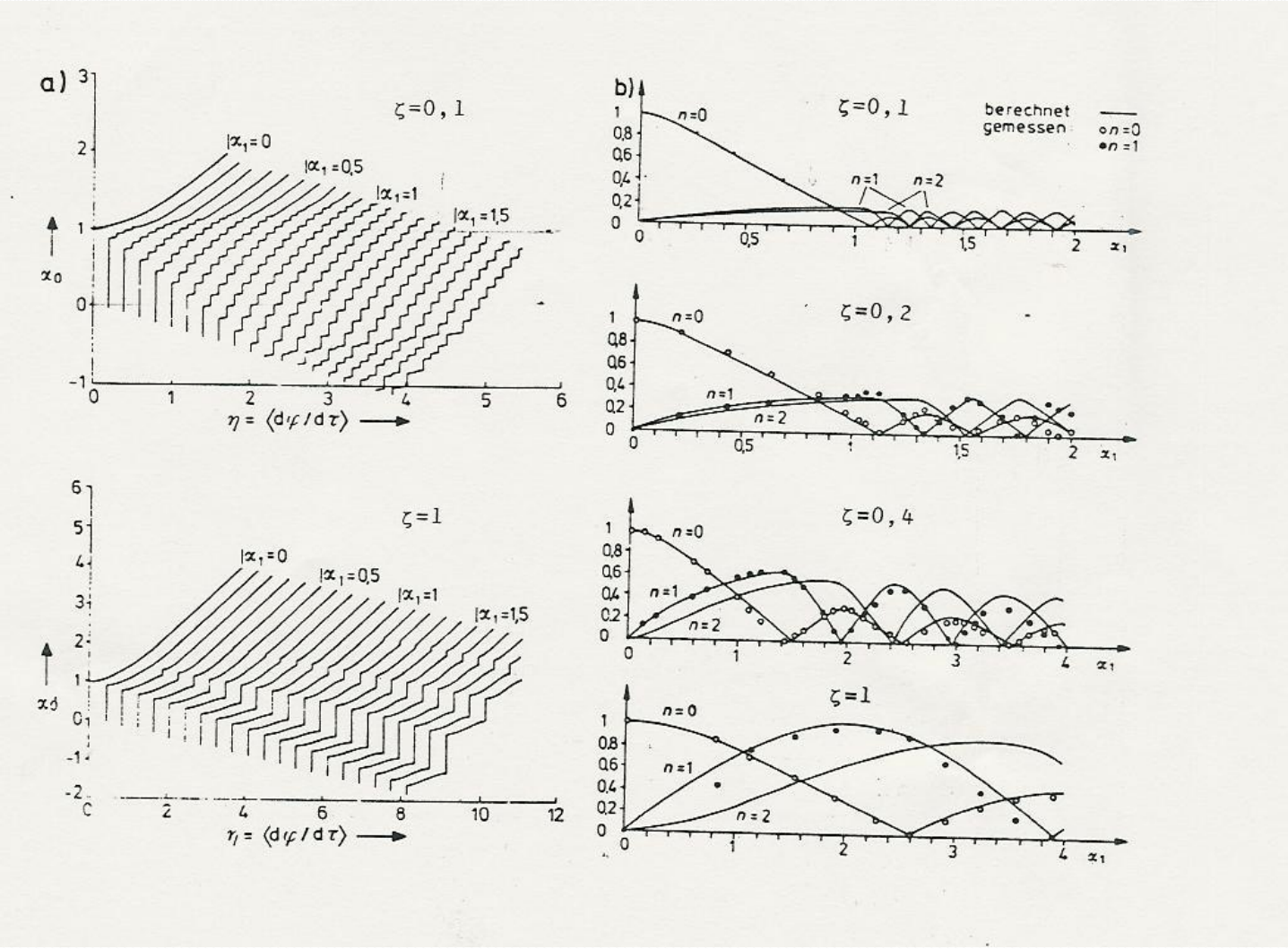
$\Delta I_n = 2 I_C \, abs(J_n (A))$,

with J_n as Bessel function n-th order and

$$A = \frac{i_1}{\Omega} \frac{1}{\sqrt{1 + \beta_C^2 \Omega^2}}$$

F. W. Bessel
(1784-1846)

Power dependence



[P. Russer]

Experiments on the ac effect

1965:

D. N. Langenberg, D. J. Scalapino, B.N. Taylor

and in parallel

L.K. Yanson, V. M. Svistunov, J. M. Dmitrenko

direct proof of the Josephson radiation

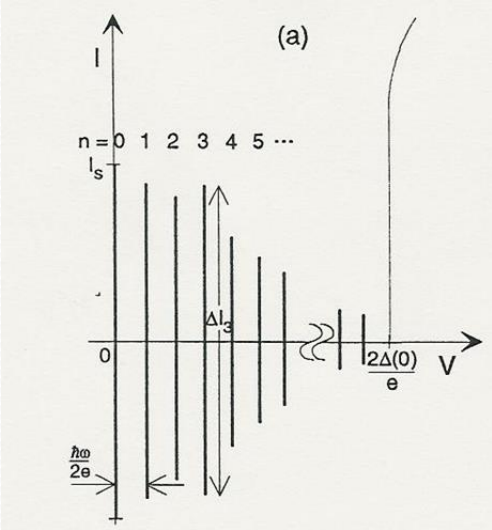
D. N. Langenberg et al. said that the detectable power was compareable to the light which a human eye gets from a 100 W lamp which is about 500 km away!

„Inverse“ AC Josephson effect

Zero current steps (voltage across junction without current biasing)

→ *Voltage standard*

Josephson constant

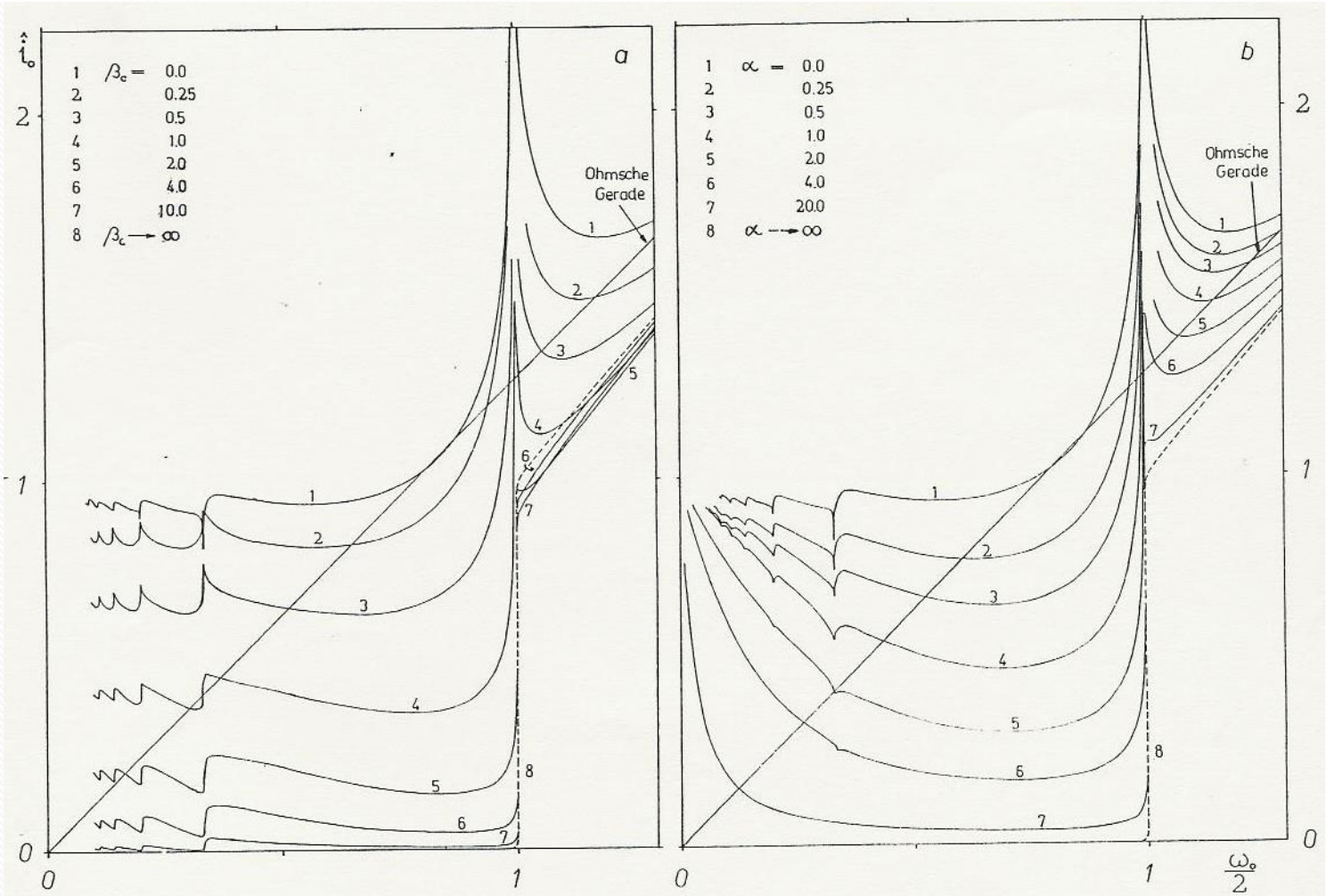


$K_J = 483\,597{,}898(19)\ 10^9\ \mathrm{Hz/V}$

Modelling

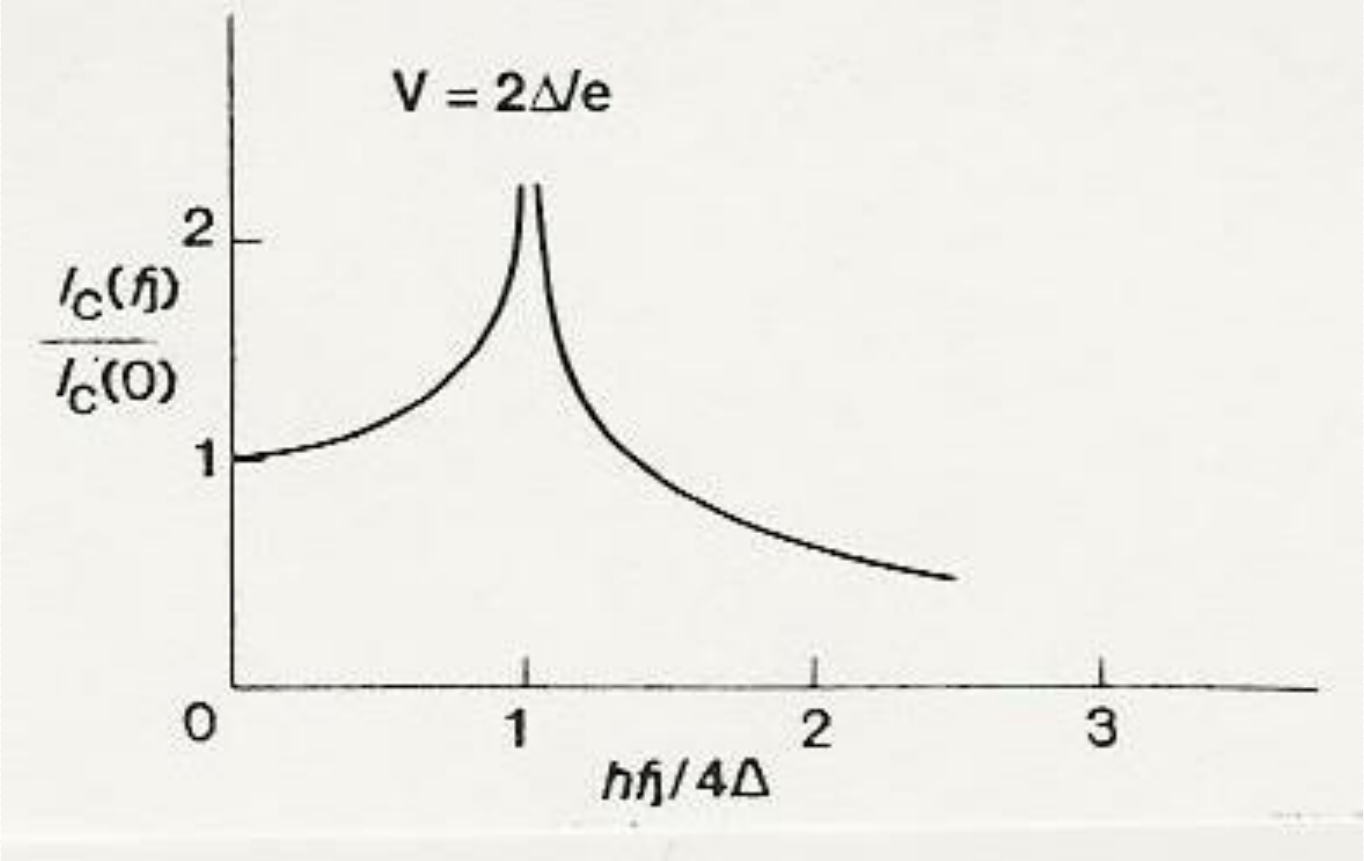
- Werthamer theory
- RSJ Model and variations
- Andreev reflexions, Weak-link models

Werthamer Model for SIS



[M. Riedel]

Riedel Peak



[C. P. Poole et al.]

Intrinsic Josephson effects in anisotropic superconductors

VOLUME 68, NUMBER 15 PHYSICAL REVIEW LETTERS 13 APRIL 1992

Intrinsic Josephson Effects in $Bi_2Sr_2CaCu_2O_8$ Single Crystals

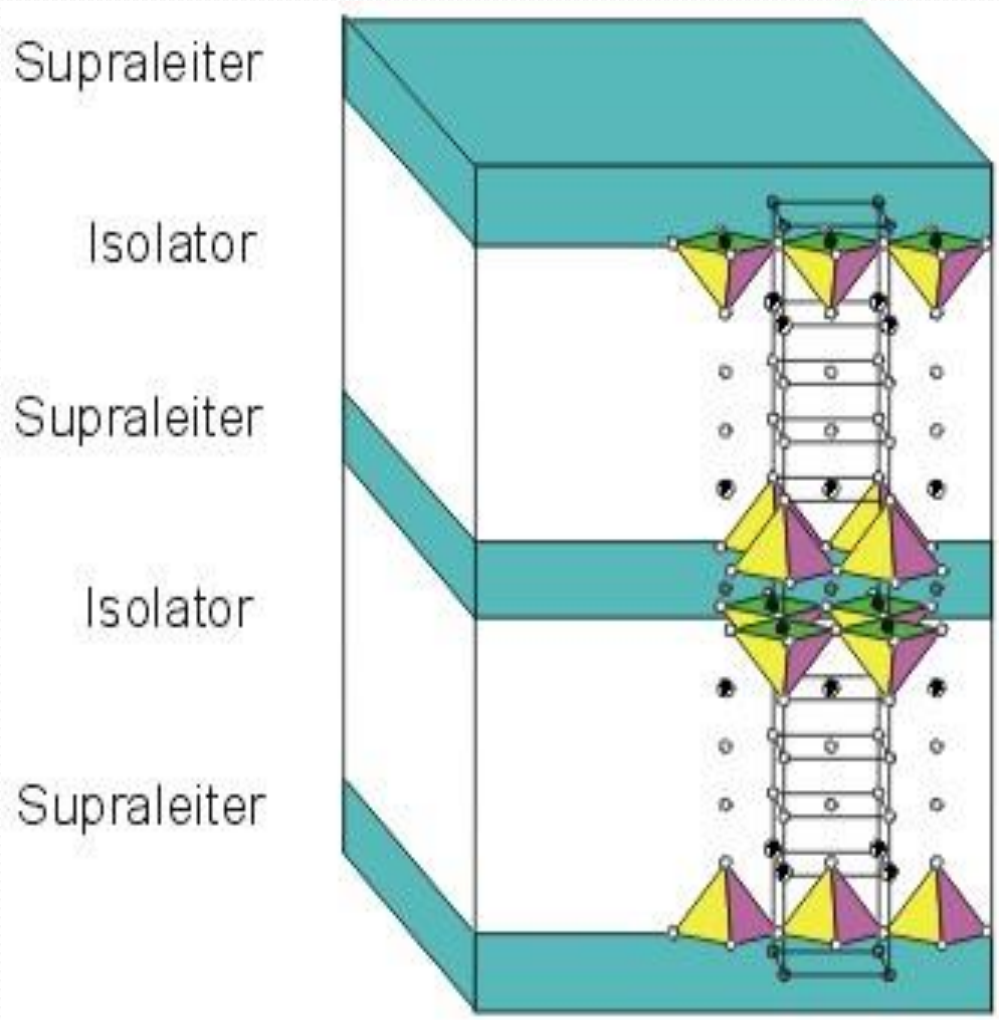
R. Kleiner, F. Steinmeyer, G. Kunkel, and P. Müller

Walther-Meissner-Institut, Walther-Meissner-Strasse 8, W-8046 Garching, Germany

(Received 21 August 1991; revised manuscript received 11 February 1992)

We have observed Josephson coupling between CuO double layers in $Bi_2Sr_2CaCu_2O_8$ single crystals by direct measurements of ac and dc Josephson effects with current flow along the *c* axis. The results show that a small $Bi_2Sr_2CaCu_2O_8$ single crystal behaves like a series array of Josephson junctions which can exhibit mutual phase locking.

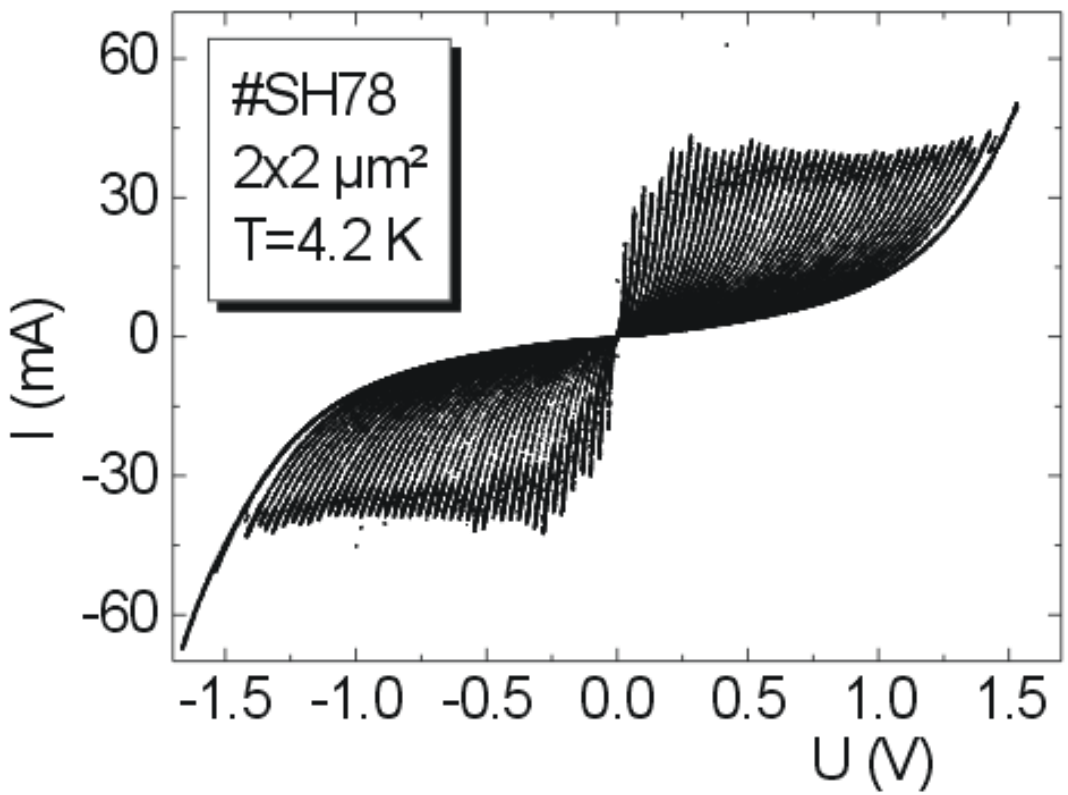
PACS numbers: 74.50.+r, 74.60.Jg, 74.70.Jm



$Bi_2Sr_2CaCu_2O_8$

[R. Kleiner, P. Müller]

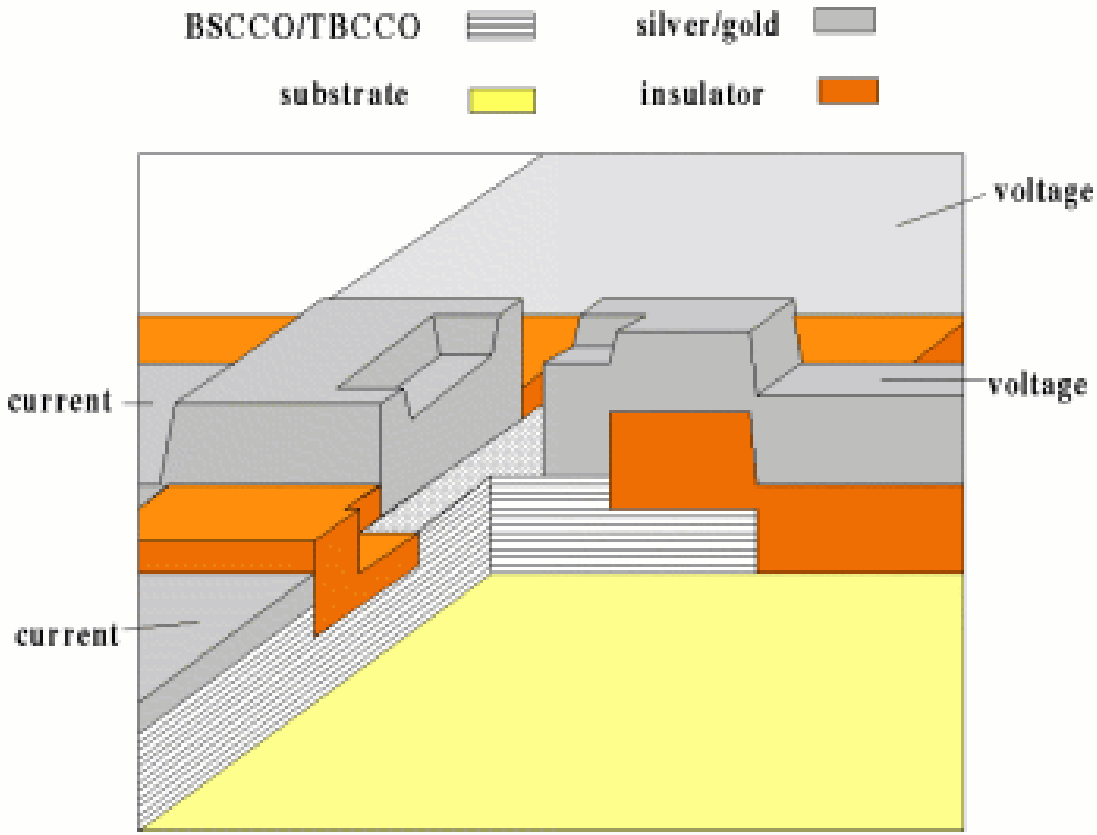
IVC with many branches



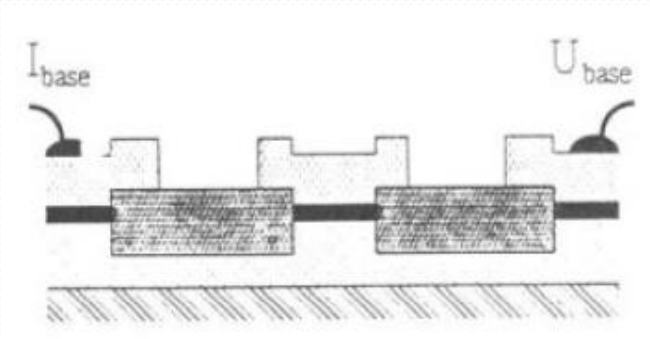
BSCCO

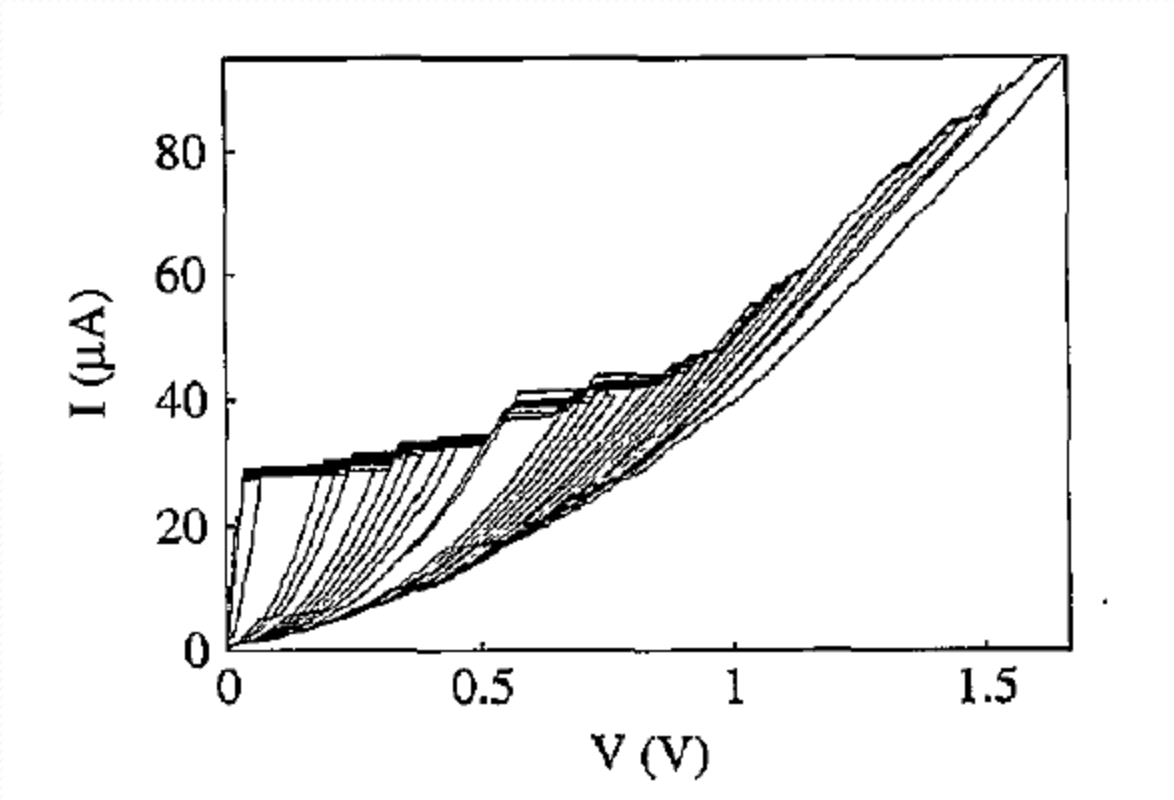
[R. Kleiner]

Mesa junction



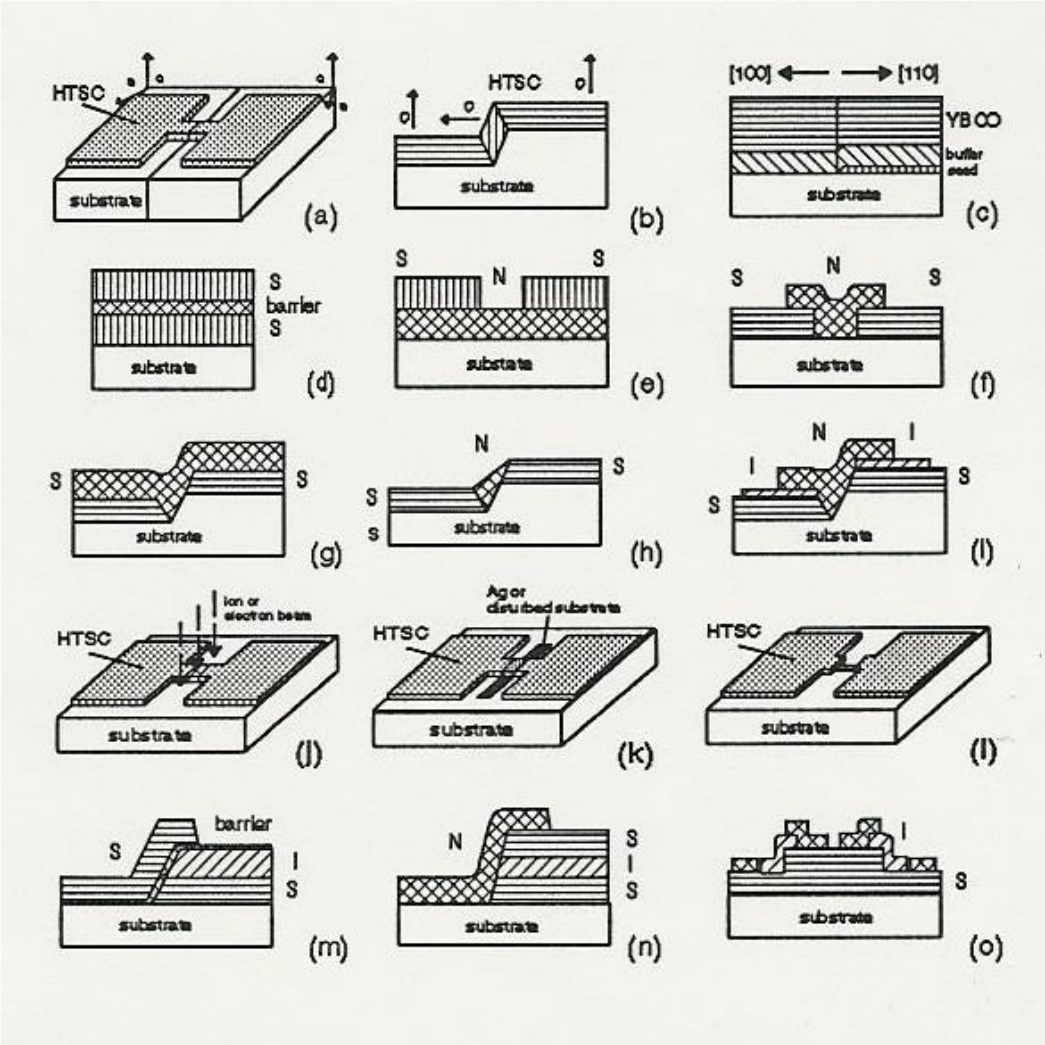
Mesa junction





Supercond. Sci. Technol. 8 (1995) 740–743.

Josephson junctions with HTS



Applications of Josephson junctions in sc electronics

→ Voltage standard

→ Magnetic field sensors → SQUID, SQUIF

→ Radiation detectors

→ Josephson-Oscillator („Josephson-Laser“)

→ Digital electronics (logic, memory, …)

Elementary flux quantum

$$\Phi_o = \frac{h}{2e}$$

$$\Phi_o = 2{,}07 \cdot 10^{-15}\ \mathrm{Vs}$$

$$= 2{,}07 \cdot 10^{-15}\ \mathrm{T} \cdot \mathrm{m}^2$$

$$= 2{,}07 \cdot 10^{-3}\ \mathrm{V} \cdot 10^{-12}\ \mathrm{s} = 2{,}07\ \mathrm{mV} \cdot \mathrm{ps}$$