Superconducting electronics

-from Josephson effects to quantum computing

-by Pascal Febvre and Paul Seidel
Superconducting electronics – Part 1
Josephson effects
Superconductors

Cooper pairs (CP)

CP density $n_s$

Common wave function $\psi$

Quantum mechanical phase

London penetration depth $\lambda$

Coherence length $\xi$

Abrikosov vortex
Tunneling effects

- 1961 Ivar Giaever
- Quasiparticle tunneling and energy gap

\[ N_N(\epsilon) \approx N_N(0) \]

\[ N_S(\epsilon) = N_N(0) \text{Re} \left[ \frac{|\epsilon|}{\left[\epsilon^2 - \Delta^2\right]^{1/2}} \right] \]
Tunneling experiments

[Daghero, Gonelli]

[Y. Noat]

[F. Massee]
Tunneling spectroscopy

\[ I_T \sim \int_{-\infty}^{\infty} dE \text{ } N_1(E) \text{ } N_2(E+eU) \text{ } [f_1(E)-f_2(E+eU)] \]

für \( T = 0 \), S-I-N, \( N_N(E) \equiv N(o) = \text{const.} \)

\[ I_T \sim \int_{0}^{eU} dE \text{ } N_S(E+eU) \]

\[ \rightarrow \frac{dI_T}{dU} \sim N_S(E+eU) \text{ “Tunneling spectrum”} \]
MgB₂: Simplest two-gap model

Partial DOS in the two bands

\[ N_S^L(E) = \Re \left[ \frac{E - i\Gamma}{\sqrt{(E - i\Gamma)^2 - \Delta_L^2}} \right] \]

\[ N_S^S(E) = \Re \left[ \frac{E - i\Gamma}{\sqrt{(E - i\Gamma)^2 - \Delta_S^2}} \right] \]

Measured tunneling DOS

\[ N(E) = \alpha N_S^S + (1 - \alpha) N_S^L \]

Tunneling selectivity

\[ \alpha = 0.13 \quad \Delta_L = 7.5 \text{ mV} \]
\[ 1 - \alpha = 0.87 \quad \Delta_S = 3.8 \text{ mV} \]
\[ \Gamma = 0.15 \text{ mV} \ll \Delta_S, \Delta_L \]

[ Y. Noat]
YBCO as d-wave superconductor

\[ d_{x^2-y^2} \]

\[ \Delta(k) = \Delta_0 \cos 2\theta \]

\[ \Delta/\Delta_0 \]

nodes

s-wave

anisotropic s-wave

d-wave
Density of states of a d-wave superconductor
Quasiparticle tunneling

N (E)  

T = 0

T > 0

N-I-N

\[ E_{F1} \]

\[ E_{F2} \]

\[ eU \]

\[ \Delta_1 \]

\[ \Delta_2 \]

\[ \frac{\Delta_1 + \Delta_1}{e} \]

\[ \frac{\Delta_2 + \Delta_1}{e} \]

\[ \frac{\Delta_2 - \Delta_1}{e} \]

N-I-S

S-I_S
Josephson effects

- 1962: Brian Josephson
- Tunneling of Cooper pairs!
- dc current without resistance
- ac currents (Josephson oscillations)
Different Josephson effects

- DC effect
- AC effect
- Inverse AC effect
- „Inverse“ AC effect (zero current steps)
- Intrinsic Josephson effect(s)
From SIS to different “weak links”
Different transport in “weak links”

- Tunneling of CP in SIS
- Multiple Andreev reflection (scattering) in SNS
- Direct ballistic transport in ScS
- Mixed transport in heterostructures

- Macroscopic Quantum effects
- Non-linear IV characteristics as basis of superconducting electronics
Josephson coupling after Feynman

\[ \psi_1 = \sqrt{n_{c1}} \exp(i \varphi_1) \]

bzw. \[ \psi_2 = \sqrt{n_{c2}} \exp(i \varphi_{21}) \]

\[ i \hbar \frac{\partial \psi_1}{\partial t} = E_1 \psi_1 + K \psi_2 \]

\[ i \hbar \frac{\partial \psi_2}{\partial t} = E_2 \psi_2 + K \psi_1 \]
\[
\frac{\dot{n}_{c_1}}{2 \sqrt{n_{c_1}}} \exp(i \varphi_1) + i \sqrt{n_{c_1}} \exp(i \varphi_1) \dot{\varphi}_1 = -\frac{i}{\hbar} \left\{ E_1 \sqrt{n_{c_1}} \exp(i \varphi_1) + K \sqrt{n_{c_2}} \exp(i \varphi_2) \right\}
\]

\[
\frac{\dot{n}_{c_2}}{2 \sqrt{n_{c_2}}} \exp(i \varphi_2) + i \sqrt{n_{c_2}} \exp(i \varphi_2) \dot{\varphi}_2 = -\frac{i}{\hbar} \left\{ E_2 \sqrt{n_{c_2}} \exp(i \varphi_2) + K \sqrt{n_{c_1}} \exp(i \varphi_1) \right\}
\]

\[
\frac{1}{2} \frac{\dot{n}_{c_1}}{\sqrt{n_{c_1}}} = \frac{K}{\hbar} \sqrt{n_{c_2}} \sin(\varphi_2 - \varphi_1)
\]

\[
\frac{1}{2} \frac{\dot{n}_{c_2}}{\sqrt{n_{c_2}}} = \frac{K}{\hbar} \sqrt{n_{c_1}} \sin(\varphi_1 - \varphi_2)
\]

\[
i \sqrt{n_{c_1}} \dot{\varphi}_1 = -\frac{i}{\hbar} \left\{ E_1 \sqrt{n_{c_1}} + K \sqrt{n_{c_2}} \cos(\varphi_2 - \varphi_1) \right\}
\]

\[
i \sqrt{n_{c_2}} \dot{\varphi}_2 = -\frac{i}{\hbar} \left\{ E_2 \sqrt{n_{c_2}} + K \sqrt{n_{c_1}} \cos(\varphi_1 - \varphi_2) \right\}
\]
Josephson coupling after Feynman

\[ \dot{n}_{c1} = -\dot{n}_{c2}, \quad n_{c1} = n_{c2} \]

\[ \dot{n}_{c1} = \frac{2K}{\hbar} n_{c1} \sin(\varphi_2 - \varphi_1) = -\dot{n}_{c2} \]

\[ I_s = I_{s,\text{max}} \sin(\varphi_2 - \varphi_1) \]

\[ I_{s,\text{max}} = \frac{2K \cdot 2e}{\hbar} \cdot V \cdot n_c \]
Josephson coupling after Feynman

\[
\frac{d}{dt}(\varphi_2 - \varphi_1) = \frac{1}{\hbar}(E_1 - E_2) = \frac{2eU}{\hbar}
\]

\[
(\varphi_2 - \varphi_1) = \frac{2eU}{\hbar} \cdot t + \varphi_0
\]

\[
\nu = \frac{2eU}{\hbar}
\]
DC Josephson current

- dc current periodically depends on phase difference of wave functions!
  
  \[ I_s = I_c \sin \phi, \quad \phi = \phi_2 - \phi_1 \]

- sometimes non-sinusoidal current phase relations
Critical Josephson current $I_c$

- For SIS and tunneling like junctions
\[ I_C = \frac{2}{eR_N} \frac{\Delta_L \Delta_R}{\Delta_L + \Delta_R}, \quad K \left( \frac{\left| \Delta_R - \Delta_L \right|}{\Delta_R + \Delta_L} \right) \]

K ellipt. Int. 1.Ord.

Only for \( \Delta_L \approx \Delta_R \) \( K(x) \approx \frac{\pi}{2} \) for \( x \to 0 \) and

\[ I_C = \frac{\pi}{eR_N} \frac{\Delta_L \Delta_R}{\Delta_L + \Delta_R} \]

and if \( \Delta_L = \Delta_R = \Delta \)

\[ V_C = I_C R_N = \left( \frac{\pi}{4} \right) V_G = 0.78 V_G \text{ mit } V_G = 2\Delta /e \]
Temperature dependence

for $T \neq 0$ Ambegaokar and Baratoff got

$$I_\text{c}(T) = \frac{2\Delta_1(T)\Delta_2(T)}{\pi k_B T e R_N} \sum_{l=0,1,2} \left\{ (2l+1)^2 + \left( \frac{\Delta_1(T)}{\pi k_B T} \right)^2 \right\} \left[ (2l+1)^2 + \left( \frac{\Delta_2(T)}{\pi k_B T} \right)^2 \right]^{-\frac{1}{2}}$$

for $\Delta_L = \Delta_R = \Delta$ one gets

$$I_\text{c}(T) = \frac{\pi}{2} \frac{\Delta(T)}{e R_N} \tanh \left( \frac{\Delta(T)}{2k_B T} \right)$$

Reminder: for $T \rightarrow 0 \tanh(\ )$ gives just 1
Measurement of $I_c(T)$

Experimentally verified 1964 by Fiske on Sn-SnO$_x$-Sn and Pb-PbO$_x$-Sn tunneling junctions

Near $T_C$ for SIS:

$$I_c \sim \Delta^2 \sim \left(1 - \frac{T}{T_C}\right)$$
Solid Curves: Calculation of Ambegaokar & Baratoff (1963 a, b)

\[ I_c(T)/I_c(0) \]

[ C. P. Poole et al. ]
DC effect at weak links

- Temperature dependence $I_c(T)$ is related to the contact type, e.g. near $T_c$:

$$T_c \sim (1 - T/T_c)^m$$

$m = 1$ (SIS), $2/3$ (SINS), $2$ (SNS)

→ reason: different coupling and transport mechanisms
Bridge-like junction
For bridge-like junctions length compared to the coherence length

[ B. H. Moeckly ]
Current-phase relations

[ E. Heinz ]
Dependence on barrier transmission coefficient
$I_C$ in an external magnetic field

Modulation of dc current in an external magnetic field (flux $\Phi = BA$) with the elementary Flux quantum $\Phi_0$

„Fraunhofer pattern“

$I_C = I_{cmax} \ abs(\sin x / x )$, $x = \pi \Phi / \Phi_0$
Current density modulation by external magnetic field for short junctions

[W. Buckel]
Homogeneity of $j_c$

Fluctuations

$I_c(H)$

YBCO

With and without illumination by light (Photodoping)

[J. Elly et al. PRB 56 (1997), R 8507.]
External magnetic field at long junctions

- For "long" or better "wide" junctions the spatial dependence of the phase has to be taken into account
- Results in a Sine-Gordon equation

\[
\frac{d^2 \varphi}{dx^2} = \frac{\sin \varphi (x)}{\lambda_J^2}
\]

- With \( \lambda_J = \left( \Phi_0 / 2\pi \mu_0 J_c d \right)^{1/2} \) as the Josephson penetration depth
Wide junction

\[ \frac{L}{\lambda_j} = 8.4 \quad [\text{N. W. Sawaritzki}] \]
AC Josephson effect

alternating current with a frequency $f$ connected to the voltage $V$ across the junction by

$$V = \Phi_0 \cdot f$$
rf currents!
AC Josephson effect

- Oscillating current if voltage applied

\[ \frac{d\phi}{dt} = \frac{2e}{\hbar} V(t) \]

\[ \omega = \frac{2e}{\hbar} I_C R_N \]

- Irradiation of electromagnetic waves

→ Josephson Oscillator
Experiments

1963: S. Shapiro indirect proof of the ac Josephson effect *(Shapiro steps)*

→ inverse AC Josephson effect
Inverse Josephson effect

- Response to external radiation with a frequency $f_{ex}$
- Voltage steps (Shapiro steps)

$$V_n = n\left(\frac{h}{2e}\right)f_{ex} = n\Phi_0 f_{ex}$$

$$f / V = 483.6MHz / \mu V$$

$$\Omega = \frac{\omega_{ex}}{\omega_0}$$
RCSJ model

\[ I = I_C \sin \varphi + \frac{V}{R} + C \frac{dV}{dt} \]
IVC in the RSJ model
RCSJ Model

Phase difference across the junction

\[ \beta_c \phi'' + \phi' + \sin \phi = i_o + i_1 \sin \Omega \tau + i_F (\tau) \]

\[ \beta_c = \frac{2e}{\hbar} I_C R_N C \quad \text{(McCumber-Parameter)} \]

\[ \tau = \omega_o t, \quad \omega_o = \frac{2e}{\hbar} I_C R_N, \quad \Omega = \frac{\omega_{ex}}{\omega_o} \]
Bessel function behaviour

Shapiro step height in the case of current biasing:
\[ \Delta I_n = 2 I_C \text{abs}(J_n(A)), \]
with \( J_n \) as Bessel function n-th order and

\[
A = \frac{i_1}{\Omega} \frac{1}{\sqrt{1 + \beta c^2 \Omega^2}}
\]

F. W. Bessel
(1784-1846)
Power dependence

[ P. Russer ]
Experiments on the ac effect

1965:
D. N. Langenberg, D. J. Scalapino, B.N. Taylor
and in parallel
L.K. Yanson, V. M. Svistunov, J. M. Dmitrenko
direct proof of the Josephson radiation

D. N. Langenberg et al. said that the detectable power was comparable to the light which a human eye gets from a 100 W lamp which is about 500 km away!
„Inverse“ AC Josephson effect

Zero current steps (voltage across junction without current biasing)

→ Voltage standard

Josephson constant

\[ K_J = 483\,597,898(19) \times 10^9 \text{ Hz/V} \]
Modelling

- Werthamer theory
- RSJ Model and variations
- Andreev reflexions, Weak-link models
Werthamer Model for SIS

[ M. Riedel ]
Riedel Peak

\[ V = 2\Delta/e \]

\[ \frac{I_c(f_j)}{I_c(0)} \]

[\text{C. P. Poole et al.}]
Intrinsic Josephson effects in anisotropic superconductors

Intrinsic Josephson Effects in Bi$_2$Sr$_2$CaCu$_2$O$_8$ Single Crystals

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(Received 21 August 1991; revised manuscript received 11 February 1992)

We have observed Josephson coupling between CuO double layers in Bi$_2$Sr$_2$CaCu$_2$O$_8$ single crystals by direct measurements of ac and dc Josephson effects with current flow along the c axis. The results show that a small Bi$_2$Sr$_2$CaCu$_2$O$_8$ single crystal behaves like a series array of Josephson junctions which can exhibit mutual phase locking.

PACS numbers: 74.50.+r, 74.60.Jg, 74.70.Jm
[ R. Kleiner, P. Müller ]
IVC with many branches

BSCCO

[ R. Kleiner ]
Mesa junction
Mesa junction

Josephson junctions with HTS
Applications of Josephson junctions in SC electronics

→ Voltage standard

→ Magnetic field sensors → SQUID, SQUIF

→ Radiation detectors

→ Josephson-Oscillator („Josephson-Laser“)

→ Digital electronics (logic, memory, ...)
Elementary flux quantum

\[ \Phi_0 = \frac{h}{2e} \]

\[ \Phi_0 = 2.07 \cdot 10^{-15} \text{Vs} \]
\[ = 2.07 \cdot 10^{-15} \text{T} \cdot \text{m}^2 \]
\[ = 2.07 \cdot 10^{-3} \text{V} \cdot 10^{-12} \text{s} = 2.07 \text{mV} \cdot \text{ps} \]